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EXPLORING THE COMPONENTWISE ABSOLUTE STABILITY OF ENDEMIC EPIDEMIC SYSTEMS VIA SIR MODELS

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Abstract: *Results of previous work on componentwise asymptotic stability (characterizations based on the flow-invariance method) are applied for the study of a class of bilinear differential equations, describing the dynamics of (compartmental) endemic epidemic systems. Using the standard form of a SIR model, a necessary and sufficient condition is formulated for the componentwise absolute stability on a given closed and bounded set, which includes the equilibrium point of the system. It is shown that the class of approachable sets can be considerably enlarged, by applying adequate bijective transformations of the original coordinates, used for the initial statement of the problem. A detailed example illustrates the whole procedure for the exploration of the componentwise absolute stability on a set resulting from a nonsingular linear transformation applied to the original state variables.*

Keywords: *Biomedical systems, Environmental systems, Bilinear systems, Componentwise absolute stability*

1. INTRODUCTION

In the last decades a lot of attention has been attracted to the mathematical modelling of biological systems and to the study of their properties. Beginning with Vito Volterra's pioneering works on predator-prey models the stability of biological systems remains a perennial research problem that has always to deal with new emerging mathematical models and/or by using new analysis methods.

This paper is concerned with epidemic systems. It is already accepted that the most suitable models for the microparasitic, usually short infections are the compartmental models. General compartmental models for epidemic systems have been formulated and, by using the Lyapunov direct method, their stability properties have been studied, [1]. In this respect the purpose of this paper is to refine in a certain sense the mentioned results by applying the flow-invariance method, [2], and more specific, the subsequent concept of *componentwise absolute stability*, [2] ÷ [6], to the compartmental epidemic systems which are modelled by a class of bilinear differential equations. The text is organized according to the following plan. Section 2 creates an overview of the componentwise absolute stability. Section 3 formulates the problem for endemic epidemic systems, presents the main results and discusses possibilities for enlarging the class of sets on which the componentwise absolute stability can be explored. Section 4 illustrates, by the help of an example, the whole procedure for the analysis of the componentwise absolute stability. Section 5 comments the novelty of the work, the applicability of the results and possible future developments.

2. BRIEF OVERVIEW OF COMPONENTWISE ABSOLUTE STABILITY

Notations

Let $P,Q\in\mathbf{R}^{n\times r}$, with $r\in\{1,n\}$ and $P:=(p_{ij}),\ Q:=(q_{ij})$. One denotes by $P\le Q$ $(P<Q)$ or by $P\ge Q$ $(P>Q)$ the inequalities $p_{ij}\le q_{ij}$ $(p_{ij}<q_{ij})$ or $p_{ij}\ge q_{ij}$ $(p_{ij}>q_{ij})$ respectively, by $|P|$ the matrix with the elements $|p_{ij}|$, by $\overline{P}:=(\overline{p}_{ij})$ the matrix with $\overline{p}_{ii}=p_{ii}$ and $\overline{p}_{ij}=|p_{ij}|$, $i\ne j$, and by $\overline{P}_k$ the leading principal minors of $\overline{P}$.

Let $V\subset\mathbf{R}^n$ and $\Phi:V\to\mathbf{R}^{n\times n}$ be a continuous function with $\Phi(x):=(\varphi_{ij}(x))$ and let $v:=(v_i)\in V$, $z:=(z_i)\in V$. One denotes by $\complement_v^z\{\Phi(v)\}$ the operation which "catches" each column $\varphi_j(v)$ of $\Phi(v)$ in a diagonal manner in z, i.e.

$$\complement_v^z\{\varphi_j(v)\}=$$
$$=\left[\varphi_{1j}(z_1,v_2,..,v_n),\varphi_{ij}(v_1,..,z_i,..,v_n),..,\varphi_{nj}(v_1,..,v_{n-1},z_n)\right]^T,$$
$$j=\overline{1,n}$$

where $[\cdot]^T$ signifies the transposition.

Definition and characterization

Consider the nonlinear dynamical system:

$$\dot{x}=F(t,x)x,\ t\in\mathbf{R}_+,\ x\in V\subset\mathbf{R}^n, \quad (1)$$

$$x(t_0)=x_0,\ t_0\in\mathbf{R}_+,\ x_0\in V\subset\mathbf{R}^n, \quad (2)$$

where V is a closed and bounded domain with $0\in V$ and $F(t,x)$ belongs to a class $\mathrm{F}_{\overline{M}}$ of real $(n\times n)$ matrices which are continuous and adequately bounded. Pertaining the stability of equilibrium point

$$x=0 \quad (3)$$

of system (1) this boundedness must be understood in the following sense: for a given real constant matrix M there exist $\alpha>0$ $(\alpha\in V)$, a maximal $\rho_m\ \ge\ 1$ with $\rho\alpha\in V$ for each $\rho\in[1,\rho_m]$, and $\beta>0$ (scalar) such that:

$$\complement_v^{\pm\alpha}\left\{\overline{F}\left(t,\rho v e^{-\beta t}\right)\right\}\le\overline{M},\ t\in\mathbf{R}_+,\ |v|<\alpha,\ \rho\in[1,\rho_m]. \quad (4)$$

Clearly there exists a nonempty class $\mathrm{F}_{\overline{M}}$ of continuous matrices $F(t,x)$, because at least $M\in\mathrm{F}_{\overline{M}}$. M is called an *elementwise C - majorant* of *F(t,x)* on *V*.

Definition ([6], cf. Definition 6)

System (1) is called *componentwise absolutely stable in V* (CWABSV) if it is *componentwise exponential asymptotically stable in V* (CWEASV) for all $F\in\mathrm{F}_{\overline{M}}$, i.e. there exist $\alpha>0$, a maximal $\rho_m\ge 1$ with $\rho\alpha\in V$ for each $\rho\in[1,\rho_m]$, and $\beta>0$ such that for each $t_0\in\mathbf{R}_+$, for each $|x_0|\le\rho\alpha$, for each $\rho\in[1,\rho_m]$ with $\rho\alpha\in V$ and for each $F\in\mathrm{F}_{\overline{M}}$ the solution of (1), (2) satisfies:

$$|x(t)|\le\rho\alpha e^{-\beta(t-t_0)},\ t\ge t_0;\ \rho\in[1,\rho_m]. \quad (5)$$

Theorem 1 ([6], cf. Theorems 9 and 8)

System (1) is CWABSV if and only if $M:=(m_{ij}),\alpha:=(\alpha_i)>0$, with $\rho\alpha\in V$, $\rho\in[1,\rho_m]$, and $\beta>0$, satisfy one of the five following equivalent conditions:

1° $\overline{M}\alpha\le-\beta\alpha$;

2° $\beta\le\min\limits_i\left(-m_{ii}-\dfrac{1}{\alpha_i}\sum\limits_{\substack{j=1\\j\ne i}}^{n}|m_{ij}|\alpha_j\right)$;

3° $\overline{M}\alpha<0$;

4° $(-1)^k\,\overline{M}_k>0,\ k=\overline{1,n}$;

5° $\overline{M}$ is Hurwitzian.

It is worth to be noticed here that each of the equivalent conditions 1° - 5° from ***Theorem 1*** is a necessary and sufficient condition such that the *linear C - majorant* system:

$$\dot{x} = Mx,\ t \in \mathrm{R}_+,\ x \in \mathrm{R}^n, \quad (6)$$

be componentwise exponential asymptotically stable (CWEAS) on $\mathbf{R}^n$, [3], [4], [6]. CWEAS of (6) corresponds to a certain *diagonal dominance along the rows* of M, necessarily implying that the diagonal elements are negative.

3. EPIDEMIC SYSTEMS

3.1. *SIR models*

The epidemic systems are described by compartmental models, [1]. The usual starting point case is the SIR model, which is based on the division of the considered population into the following three classes (compartments):

- S - susceptibles; they are capable of contracting the disease and becoming infective;
- I - infectives; they are capable of transmitting the disease to susceptibles;
- R - removed individuals; having contracted the disease, they died or are permanently immune, or have been isolated.

The transfer of the individuals from a compartment to another is illustrated by the scheme in fig. 1, where r (with specified subscripts) denotes the appropriate transfer rates.

The classes (compartments) S, I and R can be divided into more subclasses (subcompartments) according to some specific criteria issued from the epidemiological casuistic.

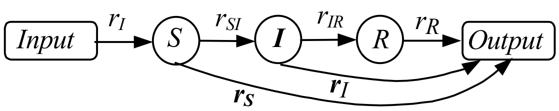

Fig.1. Schematic representation of the transfer of individuals between the compartments of the SIR model.

3.2. *General epidemic bilinear models*

The general bilinear model of an epidemic system divided into the compartments K_i, $i = \overline{1,m}$, [1], is the following:

$$\dot{z} = \mathrm{diag}(z)(e + Az) + Bz + c,\ t \in \mathbf{R}_+,\ z \in \mathbf{R}^m, \quad (7)$$

with $e \in \mathbf{R}^m, c \in \mathbf{R}^m, A \in \mathbf{R}^{m \times m}, B := (b_{ij}) \in \mathbf{R}^{m \times m}$, $b_{ii} = 0$ and $b_{ij} \geq 0$, $i \neq j$, and $z := (z_i)$, $\mathrm{diag}(z) := \mathrm{diag}(z_1, \ldots, z_m)$.

The components z_i, $i = \overline{1,m}$, of state z are the number of individuals in K_i, $i = \overline{1,m}$, respectively, and

$$\textstyle\sum_1^m z_i = z_{m+1} \quad (8)$$

is the total population of the system.

In the case z_{m+1} = constant one considers that the total population is maintained by compensation of the output by an equal input in the class S (e.g the birth rate is equal to the death rate as the level of the overall system). Usually $z_{m+1} = 1$ and z_i, $i = \overline{1,m}$, represents fractions of the total population. Certainly one can write:

$$\dot{z}_{m+1} = h^T \dot{z} = 0 \quad (9)$$

with $h := [1 \ldots 1]^T \in \mathbf{R}^m$. Equation (7) must satisfy (9).

Further one can ascertain that any trajectory $\{z(t),\ t \in \mathbf{R}_+\}$ of system (7) is contained in a bounded domain $\Omega \in \mathbf{R}^m$ which is positively invariant. According to this invariance of Ω and to the fact that the right side term of (7) belongs to $C^1(\Omega)$, it follows that standard fixed point theorems assure the existence of at least one equilibrium point $z_e \in \Omega$, [1], satisfying:

$$\mathrm{diag}(z_e)(e + Az_e) + Bz_e + c = 0. \quad (10)$$

Equilibrium point satisfying $z_e > 0$ is called *endemic point*, because in this case the epidemic disease is endemic. It can be shown that $c > 0$ implies $z_e > 0$, [1].

For the endemic point z_e several sufficient conditions for global asymptotic stability, based on Lyapunov direct method, are available, [1].

3.3. *Problem statement for endemic points*

For system (7) one can use the transformation:

$$\begin{bmatrix} z_1 \\ \vdots \\ z_{m-1} \\ z_{m+1} \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \\ 1 & \cdots & 1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_{m-1} \\ z_m \end{bmatrix}.$$

Under these circumstances the last scalar equation of (7) will be replaced by (9). The first m - 1 equations of (7) does not depend on z_{m+1} and consequently one has to study only the system of the first m - 1 scalar equations of (7).

For the sake of simplicity of writing let us observe that by denoting $n:=m-1$ the system under consideration has the same form as (7), i.e.

$$\dot{z}=\mathrm{diag}(z)(e+Az)+Bz+c,\ t\in\mathbf{R}_+,\ z\in\Omega\in\mathbf{R}^n, \quad (11)$$

where e, c, A, B have the properties specified in connection with (7) (mutatis mutandis), and

$$\Omega=\left\{z\in\mathbf{R}_+^n \mid 0\le h^T z\le 1\right\}. \quad (12)$$

Let $z_e\in\Omega$ be an equilibrium point of (11) according to:

$$\mathrm{diag}(z_e)(e+Az_e)+Bz_e+c=0. \quad (13)$$

Using the transformation:

$$x=z-z_e, \quad (14)$$

from (1) (with $F(t,x)\equiv F(x)$) and (11), with (13), one obtains:

$$\dot{x}=F(x)x,\ t\in\mathbf{R}_+,\ x\in V\subset\mathbf{R}^n, \quad (15)$$

where:

$$F(x)=\mathrm{diag}(x+z_e)A+\mathrm{diag}(e+Az_e)+B \quad (16)$$

and

$$V=\left\{x\in\mathbf{R}^n \mid 0\le h^T(x+z_e)\le 1\right\}. \quad (17)$$

Notice that the endemic point

$$x=0, \quad (18)$$

corresponding to $z_e>0$, satisfies $0\in\mathrm{Int}V$. Consequently and in connection with CWABSV, for system (13) a symmetric exponentially time-dependent hyperinterval belonging to V can be considered.

3.4. *Main results for endemic points*

In order to apply the operator C to matrix (16) let us denote:

$\alpha_m:=\rho_m\alpha$, with $\pm\alpha_m\in V$,

$A_d:=\mathrm{diag}(a_{11},\ldots,a_{nn})$,

$\mathrm{sgn}A_d=\mathrm{diag}(\mathrm{sgn}a_{11},\ldots,\mathrm{sgn}a_{nn})$ with $\mathrm{sgn}0=0$,

$A_{0d}:=A-A_d$.

Lemma: There exists an elementwise C - majorant of $F(x)$ on V.

$$\begin{aligned}\overline{M}\ge\overline{M}^0=M^0:=&\mathrm{diag}[(\mathrm{sgn}A_d)\alpha_m+z_e]A_d+\\&+\mathrm{diag}(\alpha_m+z_e)|A_{0d}|+\mathrm{diag}(e+Az_e)+B\end{aligned} \quad (19)$$

Proof: It can be shown that any matrix M satisfying (19) is an elementwise C - majorant of $F(x)$ on V.

Clearly, by applying C to matrix (16) one can successively write:

$$\begin{aligned}
&\mathrm{C}_v^{\pm\alpha}\left\{F\left(\rho v e^{-\beta t}\right)\right\}=\\
&=\mathrm{C}_v^{\pm\alpha}\left\{\mathrm{diag}\left(\rho v e^{-\beta t}+z_e\right)A+\mathrm{diag}(e+Az_e)+B\right\}\le\\
&\le\mathrm{C}_v^{\pm\alpha}\left\{\mathrm{diag}\left(\rho v e^{-\beta t}\right)A\right\}+\mathrm{diag}(z_e)\overline{A}+\\
&+\mathrm{diag}(e+Az_e)+B=\\
&=\mathrm{C}_v^{\pm\alpha}\left\{\mathrm{diag}\left(\rho v e^{-\beta t}\right)A_d+\mathrm{diag}\left(\rho v e^{-\beta t}\right)A_{0d}\right\}+\\
&+\mathrm{diag}(z_e)\overline{A}+\mathrm{diag}(e+Az_e)+B=\\
&=\mathrm{C}_v^{\pm\alpha}\left\{\mathrm{diag}\left(\rho v e^{-\beta t}\right)A_d\right\}+\mathrm{C}_v^{\pm\alpha}\left\{\mathrm{diag}\left(\rho v e^{-\beta t}\right)A_{0d}\right\}+\\
&+\mathrm{diag}(z_e)\overline{A}+\mathrm{diag}(e+Az_e)+B\le\\
&\le\mathrm{diag}(\alpha_m)|A_d|+\mathrm{diag}(\alpha_m)|A_{0d}|+\mathrm{diag}(z_e)\overline{A}+\\
&+\mathrm{diag}(e+Az_e)+B=\mathrm{diag}(\alpha_m)(\mathrm{sgn}A_d)A_d+\\
&+\mathrm{diag}(\alpha_m)|A_{0d}|+\mathrm{diag}(\alpha_m)|A_d|+\\
&+\mathrm{diag}(z_e)|A_{0d}|+\mathrm{diag}(e+Az_e)+B=\\
&=\mathrm{diag}[(\mathrm{sgn}A_d)\alpha_m+z_e]A_d+\\
&+\mathrm{diag}(\alpha_m+z_e)|A_{0d}|+\mathrm{diag}(e+Az_e)+B=\\
&=\overline{M}^0=M^0.
\end{aligned}$$

Theorem 2: System (15) with (16) and an elementwise C - majorant M according to (19) is CWABSV if and only if in terms of ***Theorem 1*** one of the equivalent conditions 1^o - 5^o is satisfied.

Theorem 3: A necessary condition such that system (15) with (16) and an elementwise C - majorant according to (19) be CWABSV is that:

$$(A_d+A)z_e+|A_d|\alpha_m+e<0. \quad (20)$$

Proof: It relies on the condition that, according to ***Theorem 1*** (5^o) and (19), M and M^0 must satisfy:

$$M_d^0\le M_d<0, \quad (21)$$

where subscript d signifies the diagonal matrices extracted from M^0 and M.

Extracting $M_d=\overline{M}_d$ and $M_d^0=\overline{M}_d^0$ from (19), with (21) it results (20).

3.5. *Enlarging the class of approachable sets*

In some situations, it might be of great interest to operate with sets resulting from bijective transformations of the original coordinates. For example, in the most usual case when a nonsingular linear transformation:

$$x = T\tilde{x},\ \det T \neq 0 \quad (22)$$

is used, equation (15) becomes:

$$\dot{\tilde{x}} = \tilde{F}(\tilde{x})\tilde{x},\ t \in \mathbf{R}_+, \quad (23)$$

where:

$$\tilde{F}(\tilde{x}) = T^{-1}F(T\tilde{x})T\,. \quad (24)$$

The factorization occurring in the right hand side of (23) means that one first performs all the calculations requested by the usage of transformation (22) in equation (15), and, afterwards, extracts vector $\tilde{x}$ to the right side of operator $\tilde{F}$ (24).

Obviously, the results obtained in the previous paragraph (3.4) can be applied for the transformed system (23).

Considering nonsymmetrical exponentially time-dependent hyper-intervals represents another possibility to enlarge the applicability of componentwise absolute stability to the study of endemic, epidemic systems. This opens new research opportunities, for which there already exist a general result (on flow-invariance of nonsymmetrical time-dependent hyper-intervals, [6]) and some special results (for linear constant systems, extending CWEAS for nonsymmetrical intervals, [7], [8], and for interval matrix systems, [9]).

4. ILLUSTRATIVE EXAMPLE

Consider a second order system of form (11), described (according to [1]) by the matrices A and B and the vectors e and c as follows:

$$A = \begin{bmatrix} 0 & -k \\ k & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ e = \begin{bmatrix} -\delta \\ -(\gamma+\delta) \end{bmatrix},\ c = \begin{bmatrix} \delta \\ 0 \end{bmatrix},$$

where the meaning of parameters is given below (in accordance with fig. 1):

- k – rate of transfer from class S to class I
- γ - rate of transfer from class I to class R
- δ - rate of birth/death

The state variables are defined as follows:

- x_1 – number of individuals in class S;
- x_2 – number of individuals in class I;
- x_3 - number of individuals in class R (already eliminated as shown in equation (11));

The equilibrium point of the system is:

$$z_e = \begin{bmatrix} \dfrac{\gamma+\delta}{k} \\ \delta\left(\dfrac{1}{\gamma+\delta} - \dfrac{1}{k}\right) \end{bmatrix},$$

where the parameters fulfil the condition $k > \gamma + \delta$.

The operator $F(x)$ used in (15), with the detailed expression (16), has (in our concrete case) the form:

$$F(x) = \begin{bmatrix} -\dfrac{k\delta}{\gamma+\delta} & -kx_1 - \gamma - \delta \\ kx_2 + \dfrac{k\delta}{\gamma+\delta} - \delta & 0 \end{bmatrix}.$$

After applying linear transformation (22) with:

$$T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix},\ T^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

one obtains the following expression for operator $\tilde{F}(\tilde{x})$ used in the right hand side of equation (23):

$$\tilde{F}(\tilde{x}) = \begin{bmatrix} -\delta & -\gamma \\ k\tilde{x}_2 + \dfrac{\delta k}{\gamma+\delta} - \delta & -k\tilde{x}_2 - \dfrac{\delta k}{\gamma+\delta} + \delta \end{bmatrix}.$$

Now, according to Theorem 3, the following necessary condition should be met:

$$\mp k\tilde{\rho}\tilde{\alpha}_2 e^{-\tilde{\beta}t} - \frac{\delta k}{\gamma+\delta} + \delta < 0,$$

which yields the situations detailed below:

(i) for $t \to \infty$, $-\dfrac{\delta k}{\gamma+\delta} + \delta < 0$

meaning $k > \gamma + \delta$ (already requested by the existence of the equilibrium point)

(ii) for $t = 0$, $k\tilde{\rho}\tilde{\alpha}_2 - \dfrac{\delta k}{\gamma+\delta} + \delta < 0$,

meaning $\tilde{\rho}\tilde{\alpha}_2 < \dfrac{\delta k}{\gamma+\delta} - \dfrac{\delta}{k} = z_{e_2}$.

On the other hand, according to Theorem 2, after evaluating (4):

$$\mathrm{C}_v^{\pm\alpha}\left\{\bar{\tilde{F}}\left(\tilde{\rho}\tilde{v}e^{-\tilde{\beta}t}\right)\right\} \le \bar{M} =$$

$$= \begin{bmatrix} -\delta & \gamma \\ -k\tilde{\rho}\tilde{\alpha}_2 + k\delta\left(\dfrac{1}{\gamma+\delta} - \dfrac{1}{k}\right) & \underbrace{k\tilde{\rho}\tilde{\alpha}_2 - k\delta\left(\dfrac{1}{\gamma+\delta} - \dfrac{1}{k}\right)}_{<0} \end{bmatrix}$$

we also need the fulfillment of one of the five conditions presented in Theorem 1. For instance, using condition 4° in Theorem 1, together with the negativity of -δ, we need only $\det M > 0$, i.e.

$$\det M = -k\left[\tilde{\rho}\tilde{\alpha}_2 - \delta\left(\frac{1}{\gamma+\delta} - \frac{1}{k}\right)\right]\det\begin{bmatrix} -\delta & \gamma \\ 1 & -1 \end{bmatrix} = = -k\left[\tilde{\rho}\tilde{\alpha}_2 - \delta\left(\frac{1}{\gamma+\delta} - \frac{1}{k}\right)\right](\delta - \gamma) > 0.$$

As condition (*ii*) is already satisfied from the above discussion, the positivity of det*M* brings a single supplementary condition, namely:

(iii) $\delta > \gamma$.

The final results should be formulated in terms of $\tilde{\alpha}_1, \tilde{\alpha}_2$, and $\tilde{\rho}$. Thus, because $\tilde{\alpha}_1$ was not involved in any condition, the only request for $\tilde{\rho}\tilde{\alpha}_1$ refers to the location of the equilibrium point, and $\tilde{\rho}\tilde{\alpha}_2$ refers to (*ii*), (*iii*). The complete set of conditions for the componentwise absolute stability of the transformed system is given below:

$$\begin{cases} 0 < \tilde{\rho}\tilde{\alpha}_1 < \dfrac{\gamma+\delta}{k} \quad (= z_{e_1}) \\ 0 < \tilde{\rho}\tilde{\alpha}_2 < \delta\left(\dfrac{1}{\gamma+\delta} - \dfrac{1}{k}\right) \quad (= z_{e_2}) \\ \delta > \gamma \end{cases}.$$

5. CONCLUDING REMARKS

The main result (***Theorem 2***) shows that by satisfying one of the equivalent conditions 1° ÷ 5° from ***Theorem 1*** there exist classes of endemic epidemic systems which may be CWABSV, i.e. with a symmetric exponentially time-dependent hyperinterval (included in *V*) as flow-invariant set for an entire class of endemic epidemic systems. The premise of the symmetry of this hyperinterval with respect to the endemic point $x = 0$ is relatively strong, taking into account the polyhedron represented by *V* and the position of the endemic point $x = 0$ in *V*. In order to enlarge the approachable classes of endemic epidemic systems one may consider sets resulting from bijective transformations of the original coordinates. Such a case is analyzed for an illustrative example, where a nonsingular linear transformation of the state variables was used to define the new set, with respect to which the componentwise absolute stability is explored. Considering nonsymmetrical exponentially time-dependent hyperintervals represents another possibility to enlarge the applicability of componentwise absolute stability to the study of endemic, epidemic systems, which creates new opportunities for future research.

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