Robust Stabilization of Nonlinear Systems Based on a Switched Fuzzy Control Law

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Abstract: This paper deals with robust stabilization of nonlinear systems represented by uncertain and disturbed switched Takagi-Sugeno fuzzy systems. First, stabilization of uncertain switched fuzzy systems is considered without external disturbances. A stabilization criterion is proposed as sufficient Linear Matrix Inequality (LMI) conditions. These ones allow designing a switched parallel distributed compensation fuzzy control law based on a candidate switched Lyapunov function. Then, an extension to systems subject to external disturbances is provided based on a H-infinity criterion. To illustrate the effectiveness of the proposed stabilization criterion and controller design approaches, a designed numerical example is studied and some simulations are provided.

Keywords: Switched fuzzy models, Takagi-Sugeno, uncertainties, external disturbances, switched PDC control law, Linear Matrix Inequalities (LMI), switched Lyapunov function..

1. INTRODUCTION

With the growing complexity of some control engineering problems, control techniques drawn from linear theory have shown their limits. Among nonlinear theory, new control approaches have appeared in the last decades such as hybrid or fuzzy techniques.

A hybrid dynamical system (HDS) consists of continuous (or discrete) time dynamics associated with discrete events following some logical or decision-making rules. For instance, power transmission and distribution, constrained robotic systems and intelligent vehicle highway systems may be considered as HDS. Among HDS, switched linear systems have attracted extensive research, see e.g. (Chiou (2006), Daafouz et al. (2002), Fang et al. (2004), Hetel et al. (2006), Mansouri et al. (2008), Liberzon et al. (2009), Ni et al. (2008)). To obtain stability conditions, two techniques are usually employed. Some authors consider the dwell time concept (Liberzon et al. (2009), Chiou (2006), Chiou (2006)). In these works, authors proved that, when the linear subsystems are Hurwitz, the overall switched system is stable if the time between consecutive switching is sufficiently large. The second technique, generally based on Lyapunov theory, aims at designing a control law able to stabilize the overall linear switched system without considering a particular switching law (Hetel et al. (2006), Mansouri et al. (2008)). Note that, all these studies consider HDS described as a collection of linear systems switching together. However, stability and stabilisation issue for nonlinear switched systems has been seldom treated in the literature (Hespanha and Morse (1999), Palm and Driankov (1998)).

Independently to the works on HDS, other studies have focussed on fuzzy modelling and control approaches. Starting from basic fuzzy control techniques (Mamdani et al. (1974)), the last three decades have shown Takagi-Sugeno (T-S) fuzzy modelling and control techniques arising (Takagi and Sugeno (1985), Tanaka and Wang (2001)). Indeed, T-S fuzzy models have then attracted interest when dealing with nonlinear systems. These are constituted by a set of linear models interconnected by fuzzy membership functions. Thus, using a convenient convex polytopic transformation, a T-S model can match exactly an affine (bounded) nonlinear system in a compact set of the state space (Tanaka and Wang (2001)). Based on the polytopic structure of T-S models, the merit of T-S fuzzy control approaches is that they make possible the extension of some linear concept to the case of nonlinear systems (Tanaka and Wang (2001), Sala et al. (2005), Bouarar et al. (2007), (2010), Zerar et al. (2008), Mansouri et al. (2009), Lendek et al. (2010)). Nevertheless, an inherent drawback remains since the number of fuzzy rules of a TS model increase exponentially with the number of nonlinearities constituting the matched nonlinear system (Delmotte et al. (2008)). This makes fuzzy controller design and implementation difficult as the complexity of the nonlinear system to be controlled increases.

To outline the problem of rules explosion in T-S modelling, some authors have proposed to combine the merit of switched systems with T-S ones to deal with nonlinear control problems (Othake et al. (2002), Othake et al. (2006), Lam (2009)). To do so, partitioning the state space of a nonlinear system allows defining a switched nonlinear system. Then, inside each partition, a T-S model can be obtained. So, as stated in (Yang and Zhao (2007)), the resulting switched T-S system inherits some essential features of hybrid systems and maintains all the information and knowledge representation capacity of fuzzy systems. Few papers have studied stabilization issues of switched fuzzy systems based on quadratic approaches (Palm and Driankov (1998), Lam, (2009), Yand et al. (2008), Ojleska and Stojanovski (2008), Yang and Zhao (2007)) or switching Lyapunov function (Othake et al. (2002), Othake et al. (2006)). Note that these studies only consider nominal systems and so, they are irrelevant when dealing with robustness of the designed controller. Therefore, a robust controller design has been proposed in (Yang and Zhao (2007)) for uncertain switched T-S systems. Nevertheless, in the latter study, a classical quadratic Lyapunov approach has been employed leading to conservative results since it needs to check the existence of a common Lyapunov matrix for a set of linear matrix inequalities (LMI) constraints. Following the work on switched linear systems (Dafouz et al. (2002), Fang et al. (2004)), less conservative LMI conditions for T-S switched systems have been provided by employing switched Lyapunov function (Othake et al. (2006)). The aim of this paper is then to extend these works to the case of robust switched fuzzy Parallel Disturbed Compensation (PDC) controller design for the class of uncertain and disturbed switched T-S fuzzy systems.

The paper is organized as follows. In the first section, the class of uncertain and disturbed switched T-S fuzzy systems is depicted as well as the considered switched PDC control law and switched Lyapunov candidate function are presented. After some useful lemmas and notations, the second section presents the main result: a stabilization criterion is proposed as LMI conditions for uncertain switched T-S systems. Then, this result is extended to the class of uncertain switched fuzzy system subject to external disturbances using a H_∞ criterion. Finally, a simulation example, followed by a conclusion, is provided to illustrate the efficiency of the proposed approaches.

2. PROBLEM STATEMENT

2.1 From a nonlinear system to its switched fuzzy representation:

Consider the following nonlinear system:

$$\begin{aligned}\dot{x}(t) = &\left(f\left(x(t)\right) + \Delta f\left(x(t)\right)\right)x(t) \\ &+ \left(g\left(x(t)\right) + \Delta g\left(x(t)\right)\right)u(t) + d\left(x(t)\right)\varphi(t)\end{aligned} \tag{1}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_n(t)\end{bmatrix} \in \Re^n$, $u(t) = \begin{bmatrix} u_1(t) & u_2(t) & \cdots & u_m(t)\end{bmatrix} \in \Re^m$ and $\varphi(t) \in \Re^p$ are respectively the state, the input and the external disturbances vectors. $f\left(x(t)\right) \in \Re^{n\times n}$, $g\left(x(t)\right) \in \Re^{n\times m}$ are nonlinear matrices defining the nominal part of (1). $\Delta f\left(x(t)\right) \in \Re^{n\times n}$ and $\Delta g\left(x(t)\right) \in \Re^{n\times m}$ represent Lebesgue measurable structural uncertainties due, for instance, to modelling approximations. The quantity $d\left(x(t)\right)\varphi(t)$, with $d\left(x(t)\right) \in \Re^{n\times p}$, represent external disturbances to the state dynamics.

Following the way proposed in (Othake et al. (2002)) for nominal systems, using the sector nonlinearity (SNL) approach (Tanaka and Wang (2001)) and a convenient state space partitioning, the uncertain and disturbed nonlinear system (1) can be rewritten as an uncertain and disturbed switched Takagi-Sugeno model described as follows:

$$\begin{aligned}\dot{x}(t) = &\sum_{q=1}^{Q}\sum_{i=1}^{r_q} v_q\left(x(t)\right)h_{qi}\left(x(t)\right)\Big(\left(A_{qi} + \Delta A_{qi}(t)\right)x(t) \\ &+ \left(B_{qi} + \Delta B_{qi}(t)\right)u(t) + G_{qi}\varphi(t)\Big)\end{aligned} \tag{2}$$

where Q denotes the number of partitioned regions of the state space and r_q is the number of rules in each region. $A_{qi} \in \Re^{n\times n}$, $B_{qi} \in \Re^{n\times m}$ and $G_{qi} \in \Re^{n\times p}$ are constant matrices with appropriate dimensions for all $i = 1,...,r$ and $q = 1,...,Q$. The matrices $\Delta A_{qi}(t) \in \Re^{n\times n}$, $\Delta B_{qi}(t) \in \Re^{n\times m}$ represent the uncertain norm-bounded (lebesgue measurable) matrices which can be rewritten such that:

$$\begin{cases}\Delta A_{qi}(t) = H_{aqi}F_a(t)N_{aqi} \\ \Delta B_{qi}(t) = H_{bqi}F_b(t)N_{bqi}\end{cases} \tag{3}$$

where H_{aqi}, H_{bqi}, N_{aqi} and N_{bqi} are known real matrices of appropriate dimension, $F_a(t)$ and $F_b(t)$ are unknown normalized functions satisfying respectively $F_a^T(t)F_a(t) \le I$ and $F_b^T(t)F_b(t) \le I$.

Moreover, in (2), $h_{qi}\left(x(t)\right) \ge 0$ are the fuzzy membership functions verifying the convexes sum propriety $\sum_{i=1}^{r_q} h_{qi}\left(x(t)\right) = 1$ and $v_q\left(x(t)\right)$ are the switched laws defined by:

$$v_q\left(x(t)\right) = \begin{cases}1 & if\ x(t) \in \text{region } R_q\left(s_{1q}, s_{2q}, \ldots, s_{nq}\right) \\ 0 & if\ x(t) \notin \text{region } R_q\left(s_{1q}, s_{2q}, \ldots, s_{nq}\right)\end{cases} \tag{4}$$

Consider the state vector $x(t) = \begin{bmatrix} x_1(t) & \ldots & x_n(t)\end{bmatrix}$, the q^{th} region $R_q\left(s_{1q}, s_{2q}, \ldots, s_{nq}\right)$ follows:

$$s_{kq} = \begin{cases}1 & if\ x_k(t) \ge 0 \\ 0 & if\ x_k(t) < 0\end{cases}, \qquad k = 1,...,n \tag{5}$$

To illustrate this modelling approach, based on the switched laws (4), the state space partitions of a second order switched T-S system leads to four regions $R_q\left(s_{1q}, s_{2q}\right)$ depicted in Fig.1.

Note that, using the modelling methodology proposed by (Othake et al. (2002)), the switched T-S system (2) represents exactly the nonlinear system (1) on a compact set of the state space. That is to say that the robust controller design proposed in the sequel is valid on the whole state space if the nonlinearities of the uncertain system (1) are bounded (global SNL) or, in the contrary, on a restricted region (local SNL), see (Ohtake et al. (2002)) for more details.

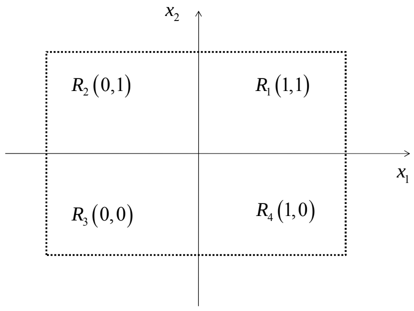

Fig. 1. Example of a second order state space partition.

To lead to the LMI conditions proposed in the next section, an extended state space system can be employed (Othake et al. (2006)). Following this way, let us consider a stable autonomous linear system such that:

$$\dot{\hat{x}}(t) = C\hat{x}(t) \tag{6}$$

where $\hat{x}(t) = \begin{bmatrix} \hat{x}_1(t) & \hat{x}_2(t) & \cdots & \hat{x}_n(t) \end{bmatrix} \in \Re^n$ is a state vector and $C \in \Re^{n\times n}$ is a Hurwitz matrix.

Let $\tilde{x}(t) = \begin{bmatrix} x(t) & \hat{x}(t) \end{bmatrix}$ be an extended state vector, (2) can be extended with (6) leading to:

$$\begin{aligned}\dot{\tilde{x}}(t) = \sum_{q=1}^{Q}\sum_{i=1}^{r_q} v_q\left(x(t)\right)h_{qi}\left(x(t)\right)\Big(\left(\tilde{A}_{qi} + \Delta\tilde{A}_{qi}(t)\right)\tilde{x}(t) \\ + \left(\tilde{B}_{qi} + \Delta\tilde{B}_{qi}(t)\right)u(t) + \tilde{G}_{qi}\varphi(t)\Big)\end{aligned} \tag{7}$$

with $\tilde{A}_{qi} = \begin{bmatrix} A_{qi} & 0 \\ 0 & C \end{bmatrix}$, $\Delta\tilde{A}_{qi} = \begin{bmatrix} \Delta A_{qi} & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{B}_{qi} = \begin{bmatrix} B_{qi} \\ 0 \end{bmatrix}$, $\Delta\tilde{B}_{qi} = \begin{bmatrix} \Delta B_{qi} \\ 0 \end{bmatrix}$ and $\tilde{G}_{qi} = \begin{bmatrix} G_{qi} \\ 0 \end{bmatrix}$.

Note that, $\Delta\tilde{A}_{qi}$ and $\Delta\tilde{B}_{qi}$ can be rewritten as follow:

$$\begin{cases} \Delta\tilde{A}_{qi}(t) = \tilde{H}_{aqi}F_a(t)\tilde{N}_{aqi} \\ \Delta\tilde{B}_{qi}(t) = \tilde{H}_{bqi}F_b(t)\tilde{N}_{bqi} \end{cases}$$

with $\tilde{H}_{aqi} = \begin{bmatrix} \tilde{H}_{aqi} & 0 \end{bmatrix}$, $\tilde{H}_{bqi} = \begin{bmatrix} \tilde{H}_{bqi} & 0 \end{bmatrix}$, $\tilde{N}_{aqi} = \begin{bmatrix} N_{aqi} \\ 0 \end{bmatrix}$ and $\tilde{N}_{bqi} = \begin{bmatrix} N_{bqi} \\ 0 \end{bmatrix}$.

Note that, as shown in (Ohtake et al. (2006)), to lead to LMI conditions, it is convenient to choose $C = -\alpha I_{n\times n}$ where α is an arbitrary positive scalar and $I_{n\times n} \in \Re^{n\times n}$ is a unit matrix. Moreover, the trajectories of (2) with the initial state $x(0) = x_0$ are equal to the first n trajectories of (7) with the initial state $\tilde{x}(0) = \begin{bmatrix} x_0^T & 0_{n\times 1}^T \end{bmatrix}^T$.

Remark: The proposed control approach is dedicated to nonlinear systems (1) (instead of switched nonlinear ones) by rewriting them as switched TS systems (2). Other studies were focused on switched nonlinear systems regarded as sets of TS systems switching together, see e.g. (Guelton et al. (2010), Jabri et al. (2011)).

2.2 Switched PDC controller and switched Lyapunov candidate:

Now, in order to stabilize the switched T-S systems (7), consider the following switched Parallel Distributed Compensation (PDC) control law Ohtake et al. (2006)):

$$u(t) = \sum_{q=1}^{Q}\sum_{i=1}^{r_q} v_q\left(x(t)\right)h_{qi}\left(x(t)\right)K_{qi}E_q\tilde{x}(t) \tag{8}$$

where $K_{qi} \in R^{m\times 2n}$ are the gain matrices and $E_q \in R^{2n\times 2n}$ are non singular matrices defined such that:

$$E_q = \begin{bmatrix} s_{1q} & 0 & \dots & 0 & 0 & 1-s_{1q} & 0 & \dots & 0 & 0 \\ 0 & s_{2q} & \dots & 0 & 0 & 0 & 1-s_{2q} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & s_{(r-1)q} & 0 & 0 & 0 & \dots & 1-s_{(r-1)q} & 0 \\ 0 & 0 & \dots & 0 & s_{rq} & 0 & 0 & \dots & 0 & 1-s_{rq} \\ 1-s_{1q} & 0 & \dots & 0 & 0 & s_{1q} & 0 & \dots & 0 & 0 \\ 0 & 1-s_{2q} & \dots & 0 & 0 & 0 & s_{2q} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1-s_{(r-1)q} & 0 & 0 & 0 & \dots & s_{(r-1)q} & 0 \\ 0 & 0 & \dots & 0 & 1-s_{rq} & 0 & 0 & \dots & 0 & s_{rq} \end{bmatrix} \tag{9}$$

with s_{kq}, for $k = 1,...,n$, defined above in (5).

Substituting (8) into (7), one obtains the following closed-loop uncertain and disturbed system :

$$\begin{aligned}\dot{\tilde{x}}(t) = \sum_{q=1}^{Q}\sum_{i=1}^{r_q} v_q\left(x(t)\right)h_{qi}\left(x(t)\right)\Big(\Big(\left(\tilde{A}_{qi} + \Delta\tilde{A}_{qi}(t)\right) \\ + \left(\tilde{B}_{qi} + \Delta\tilde{B}_{qi}(t)\right)K_{qj}E_q\Big)\tilde{x}(t) + \tilde{G}_{qi}\varphi(t)\Big)\end{aligned} \tag{10}$$

The goal is now to design the matrices K_{qi}, for $i = 1,...,r$ and $q = 1,...,Q$, ensuring the stability of the closed loop system (10). The following results will be obtained through the use of a candidate switched Lyapunov function given by:

$$V\left(\tilde{x}(t)\right)=\begin{cases}\tilde{x}^T(t)P_1\tilde{x}(t), & \tilde{x}(t)\in R_1\left(s_{11},s_{21},\ldots,s_{n1}\right)\\ \tilde{x}^T(t)P_2\tilde{x}(t), & \tilde{x}(t)\in R_2\left(s_{12},s_{22},\ldots,s_{n2}\right)\\ \vdots & \\ \tilde{x}^T(t)P_Q\tilde{x}(t), & \tilde{x}(t)\in R_Q\left(s_{1Q},s_{2Q},\ldots,s_{nQ}\right)\end{cases} \tag{11}$$

where $P_q \in \Re^{2n\times 2n}$, for $q=1,...,Q$, are positive definite real matrices.

Note that, to guarantee the continuity of the candidate Lyapunov function (11) on region boundaries (Liberzon and Morse (2009)), P_q can be rewritten such that:

$$P_q = E_q^T P E_q \tag{12}$$

where P is a definite positive matrix with E_q defined above.

Let us assume that for each instant t, only one region can be activated, thus (11) can be rewritten as:

$$V\left(\tilde{x}(t)\right)=\tilde{x}^T(t)\sum_{q=1}^{Q}E_q^T P E_q\tilde{x}(t) \tag{13}$$

Therefore, if there exist $P>0$ such that (13) is strictly decreasing, the closed loop system (10) is stable. The main result will be provided in the next section in terms of LMIs.

2.3 Notations and Lemmas:

The following notations will be used in the sequel to clarify the mathematical expression:

$$A_{vh}=\sum_{q=1}^{Q}\sum_{i=1}^{r_q}v_q\left(x(t)\right)h_{qi}\left(x(t)\right)A_{qi},\ E_v=\sum_{q=1}^{Q}v_q\left(x(t)\right)E_q,$$

$$X_hY_h=\left(\sum_{i=1}^{r_q}h_i\left(x(t)\right)X_i\right)\left(\sum_{i=1}^{r_q}h_i\left(x(t)\right)Y_i\right).$$

Note that, assuming only one region is allowable at each instant. Thus one has:

$$X_vY_v=\left(\sum_{q=1}^{Q}v_q\left(x(t)\right)X_q\right)\left(\sum_{q=1}^{Q}v_q\left(x(t)\right)Y_q\right)=\sum_{q=1}^{Q}v_q\left(x(t)\right)X_qY_q$$

As usual, a star (*) indicates a transpose quantity in a matrix. The time t will be omitted when there is no ambiguity. I denote identity matrices with appropriate dimensions.

Moreover, the following lemmas will be used in the sequel to derive relaxed LMI stability conditions.

Lemma 1 (Tuan et al. (2001)). The inequality :

$$\sum_{i=1}^{r_q}\sum_{j=1}^{r_q}h_i\left(z(t)\right)h_j\left(z(t)\right)\Gamma_{ij}<0 \tag{14}$$

is verified if, for all $i=1,...,r$ and $j=1,...,r$, $j\neq i$:

$$\Gamma_{ii}<0 \tag{15}$$

and

$$\frac{1}{r_q-1}\Gamma_{ii}+\frac{1}{2}\left(\Gamma_{ij}+\Gamma_{ji}\right)<0 \tag{16}$$

Lemma 2 (Zhou and Khargonedkar (1988)). Let us consider X and Y two matrices of appropriate dimensions, there exists a positive scalar $\beta>0$ such that :

$$X^TY+Y^TX\leq\beta X^TX+\beta^{-1}Y^TY \tag{17}$$

2. ROBUST LMI BASED SWITCHED CONTROLLER DESIGN

In this section, one firstly proposes to study the stabilisation of the system (2) without considering external disturbances (i.e. $\varphi(t)=0$). Using the above defined notations, the closed loop uncertain switched T-S system is given as follows:

$$\dot{\tilde{x}}(t)=\left(\left(\tilde{A}_{vh}+\Delta\tilde{A}_{vh}\right)+\left(\tilde{B}_{vh}+\Delta\tilde{B}_{vh}\right)K_{vh}E_v\right)\tilde{x}(t) \tag{18}$$

Sufficient LMI conditions for the design of a switched controller (8) guaranteeing the stability of (18) are proposed in the following theorem.

Theorem 1. The uncertain switched fuzzy system (2) without external disturbances ($\varphi(t)=0$) is GAS (globally asymptotically stabilized) using the PDC switched fuzzy control law (8) if there exist the matrices $X=X^T>0$, M_{qi}, the scalars $\gamma_{qi}>0$, $\lambda_{qi}>0$ satisfying the following LMIs for all $q=1,...,Q$, $i,j=1,...,r_q$ and $i<j$:

$$\Gamma_{ii}<0 \tag{19}$$

and

$$\frac{1}{r_q-1}\Gamma_{ii}+\frac{1}{2}\left(\Gamma_{ij}+\Gamma_{ji}\right)<0 \tag{20}$$

with $\Gamma_{qij}=\begin{bmatrix}\Gamma_{qij}^{(1,1)} & (*) & (*)\\ N_{aqi}E_q^{-1}X & -\gamma_{qi}I & 0\\ N_{bqj}M_{qj} & 0 & -\lambda_{qj}I\end{bmatrix}$ and

$$\begin{aligned}\Gamma_{qij}^{(1,1)}=&XE_q^{-T}\tilde{A}_{qi}^TE_q^T+E_q\tilde{A}_{qi}E_q^{-1}X+M_{qi}^T\tilde{B}_{qj}^TE_q^T+E_q\tilde{B}_{qj}M_{qi}\\&+\gamma_{qi}E_qH_{qi}H_{qi}^TE_q^T+\lambda_{qj}E_qH_{qj}H_{qj}^TE_q^T\end{aligned}$$

Then, the switched PDC controller gain matrices are obtained through the bijective change of variables $K_{qi}=M_{qi}X^{-1}$.

Proof. Consider the Lyapunov candidate (13), the closed loop system is asymptotically stable if

$$\dot{\tilde{x}}^T(t)E_v^TPE_v\tilde{x}(t)+\tilde{x}^T(t)E_v^TPE_v\dot{\tilde{x}}(t)<0 \tag{21}$$

Considering (18), inequality (21) is verified for all $\tilde{x}(t)$ if:

$$\begin{aligned}&\left(\left(\tilde{A}_{vh}+\Delta\tilde{A}_{vh}\right)+\left(\tilde{B}_{vh}+\Delta\tilde{B}_{vh}\right)K_{vh}E_v\right)^TE_v^TPE_v\\&+E_v^TPE_v\left(\left(\tilde{A}_{vh}+\Delta\tilde{A}_{vh}\right)+\left(\tilde{B}_{vh}+\Delta\tilde{B}_{vh}\right)K_{vh}E_v\right)<0\end{aligned} \tag{22}$$

Multiplying left by E_v^{-T} and right by E_v^{-1}, (22) becomes:

$$E_v^{-T}\left(\tilde{A}_{vh}+\Delta\tilde{A}_{vh}\right)^T E_v^T P + K_{vh}^T\left(\tilde{B}_{vh}+\Delta\tilde{B}_{vh}\right)^T E_v^T P \\ +PE_v\left(\tilde{A}_{vh}+\Delta\tilde{A}_{vh}\right)E_v^{-1}+PE_v\left(\tilde{B}_{vh}+\Delta\tilde{B}_{vh}\right)K_{vh}<0 \tag{23}$$

which can be rewritten in its extended form considering (3) as:

$$E_v^{-T}\tilde{A}_{vh}^T E_v^T P + PE_v\tilde{A}_{vh}E_v^{-1}+K_{vh}^T\tilde{B}_{vh}^T E_v^T P + PE_v\tilde{B}_{vh}K_{vh} \\ +E_v^{-T}\tilde{N}_{avh}^T F_a^T(t)\tilde{H}_{avh}^T E_v^T P + PE_v\tilde{H}_{avh}F_a(t)\tilde{N}_{avh}E_v^{-1} \\ +K_{vh}^T\tilde{N}_{bvh}^T F_b^T(t)\tilde{H}_{bvh}^T E_v^T P + PE_v\tilde{H}_{bvh}F_b(t)\tilde{N}_{bvh}K_{vh}<0 \tag{24}$$

Let us consider $X = P^{-1}$, left and right multiplying (24) by X, one obtains :

$$XE_v^{-T}\tilde{A}_{vh}^T E_v^T + E_v\tilde{A}_{vh}E_v^{-1}X + M_{vh}^T\tilde{B}_{vh}^T E_v^T + E_v\tilde{B}_{vh}M_{vh} \\ +XE_v^{-T}\tilde{N}_{avh}^T F_a^T(t)\tilde{H}_{bvh}^T E_v^T + E_v\tilde{H}_{avh}F_b(t)\tilde{N}_{avh}E_v^{-1}X \\ +M_{vh}^T\tilde{N}_{bvh}^T F_b^T(t)\tilde{H}_{bvh}^T E_v^T + E_v\tilde{H}_{bvh}F_b(t)\tilde{N}_{bvh}M_{vh}<0 \tag{25}$$

Recall that $F_a^T(t)F_a(t)\le I$ and $F_b^T(t)F_b(t)\le I$, applying lemma 2 and the Schur complement, (25) is verified if:

$$\Gamma_{vhh}=\begin{bmatrix}\Gamma_{vhh}^{(1,1)} & (*) & (*)\\ \tilde{N}_{avh}E_q^{-1}X & -\gamma_{vh}I & 0\\ \tilde{N}_{bvh}M_{vh} & 0 & -\lambda_{vh}I\end{bmatrix}<0 \tag{26}$$

with

$$\Gamma_{vhh}^{(1,1)} = XE_v^{-T}\tilde{A}_{vh}^T E_v^T + E_v\tilde{A}_{vh}E_v^{-1}X + \tilde{M}_{vh}^T B_{vh}^T E_v^T + E_v B_{vh}\tilde{M}_{vh} \\ +\gamma_{vh}E_v\tilde{H}_{vh}\tilde{H}_{vh}^T E_v^T + \lambda_{vh}E_v\tilde{H}_{vh}\tilde{H}_{vh}^T E_v^T$$

Finally, applying Lemma 1 on (26), the proof is completed. ■

Now, the purpose is to extend theorem 1 to the design of robust controllers (8) stabilizing uncertain switched T-S systems (2) subject to external disturbances ($\varphi(t)\neq 0$). To do so, the following H_∞ criterion is employed to minimize the effect of the external disturbances on the state dynamics:

$$\int_0^{tf}\tilde{x}(t)^T W\tilde{x}(t)dt \le \eta^2\int_0^{tf}\varphi(t)^T\varphi(t)dt \tag{27}$$

where $W\in\Re^{2n\times 2n}$ is a weighting positive definite real matrix and η is the disturbances attenuation level.

The result is summarized in the following theorem.

Theorem 2. The uncertain switched fuzzy system (2) subject to external disturbances is GAS using the PDC switched fuzzy control law (8) if there exist the matrices $X = X^T > 0$, M_{qi}, the scalars $\gamma_{qi}>0$, $\lambda_{qi}>0$ and $\rho>0$ satisfying the following LMIs for all $q=1,...,Q$, $i,j=1,...,r_q$ and $i<j$:

Minimize ρ such that :

$$\Psi_{ii}<0 \tag{28}$$

and

$$\frac{1}{r_q-1}\Psi_{ii}+\frac{1}{2}\left(\Psi_{ij}+\Psi_{ji}\right)<0 \tag{29}$$

with $\Psi_{qij}=\left[\begin{array}{ccc|cc}\Gamma_{qij} & & & (*) & (*)\\ \hline E_q^{-1}X & 0 & 0 & -W^{-1} & 0\\ \tilde{G}_{qi}^{\ T}E_q^T PE_q & 0 & 0 & 0 & -\eta^2 I\end{array}\right]<0$ and Γ_{qij} defined in theorem 1.

Therefore, the robust switched PDC controller gain matrices are obtained using the bijective change of variables $K_{qi}=M_{qi}X^{-1}$ and the designed control law ensures a H_∞ performance $\eta=\sqrt{\rho}$.

Proof. Consider the Lyapunov candidate function (13) and the H_∞ criterion given in (27). The close loop system (10) is stable and the H_∞ performance η is guaranteed if:

$$\dot{\tilde{x}}^T(t)E_v^T PE_v\tilde{x}(t)+\tilde{x}^T(t)E_v^T PE_v\dot{\tilde{x}}(t) \\ +\tilde{x}^T(t)R\tilde{x}(t)-\eta\varphi^T(t)\varphi(t)<0 \tag{30}$$

Considering the same steps as for the proof of theorem 1, (30) is verified for all $x(t)$ and $\varphi(t)$ if:

$$\left[\begin{array}{ccc|c}\Gamma_{vhh}+\bar{W} & & & (*)\\ \hline \tilde{G}_{vh}^{\ T}E_v^T PE_v & 0 & 0 & -\eta^2 I\end{array}\right]<0 \tag{31}$$

with Γ_{vhh} defined in (26) (proof of theorem 1) and $\bar{W}=\begin{bmatrix}XE_v^{-T}RE_v^{-1}X & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$.

Then, applying the Schur complement, (31) is verified if:

$$\Psi_{vhh}=\left[\begin{array}{ccc|cc}\Gamma_{vhh} & & & (*) & (*)\\ \hline E_v^{-1}X & 0 & 0 & -W^{-1} & 0\\ \tilde{G}_{vh}^{\ T}E_v^T PE_v & 0 & 0 & 0 & -\eta^2 I\end{array}\right]<0 \tag{32}$$

Finally, applying lemma 2, one obtains the LMI stability conditions proposed in theorem 2. This ends the proof. ■

3. NUMERICAL EXAMPLE

Let us consider the following uncertain nonlinear system inspired from the nominal one given in Ohtake et al. (2006)):

$$\begin{cases}\dot{x}_1(t)=x_2(t)+0.05F_a(t)x_1(t)\\ \qquad +0.02F_a(t)x_2(t)+0.05F_b(t)u(t)+\varphi(t)\\ \dot{x}_2(t)=f(x(t))+g(x(t))u(t)+0.0375F_a(t)x_1(t)\\ \qquad +0.015F_a(t)x_2(t)+0.0375F_b(t)u(t)+\varphi(t)\end{cases} \tag{33}$$

with $x_1\in[-d,d]$ and $x_2\in[-d,d]$, $d=0.5$, $f(x)=-x_1^3-x_2^3+5x_1^2x_2+5x_1x_2^2-3x_1x_2-x_1-x_2$ and $g(x)=-0.7+x_1(t)x_2(t)$.

The state space partition is chosen as in Fig.1 with four regions $R_1(1,1)$, $R_2(0,1)$, $R_3(1,0)$ and $R_4(0,0)$. Thus, the considered switched law is defined by:

$$v_1(x(t)) = \begin{cases} 1 & \text{if } 0 \le x_1 \le \mathrm{d},\ 0 \le x_2 \le \mathrm{d} \\ 0 & \text{otherwise} \end{cases}$$

$$v_2(x(t)) = \begin{cases} 1 & \text{if } -\mathrm{d} \le x_1 < 0,\ 0 \le x_2 \le \mathrm{d} \\ 0 & \text{otherwise} \end{cases}$$

$$v_3(x(t)) = \begin{cases} 1 & \text{if } 0 \le x_1 \le \mathrm{d},\ -\mathrm{d} \le x_2 < 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$v_4(x(t)) = \begin{cases} 1 & \text{if } -\mathrm{d} \le x_1 < 0,\ -\mathrm{d} \le x_2 < 0 \\ 0 & \text{otherwise} \end{cases}.$$

Using the sector nonlinearity approach (Tanaka and Wang (2001)) and the previous state partition, the nonlinear system (33) can be constructed as an uncertain switched TS disturbed system given by:

$$\begin{aligned}\dot{x}(t) = \sum_{q=1}^{Q}\sum_{i=1}^{r} v_q(x(t)) h_{qi}(x(t)) \big((A_{qi} + \Delta A_{qi}(t)) x(t) \\ + (B_{qi} + \Delta B_{qi}(t)) u(t) + G_{qi}\varphi(t) \big)\end{aligned} \tag{34}$$

with $A_{11} = A_{13} = \begin{bmatrix} 0 & 1 \\ -0.246 & -0.246 \end{bmatrix}$,

$A_{12} = A_{14} = \begin{bmatrix} 0 & 1 \\ -1.25 & -1.25 \end{bmatrix}$, $A_{21} = A_{23} = \begin{bmatrix} 0 & 1 \\ -1.952 & -0.246 \end{bmatrix}$,

$A_{22} = A_{24} = \begin{bmatrix} 0 & 1 \\ 0.75 & -1.25 \end{bmatrix}$, $A_{31} = A_{33} = \begin{bmatrix} 0 & 1 \\ -0.246 & -1.952 \end{bmatrix}$,

$A_{32} = A_{34} = \begin{bmatrix} 0 & 1 \\ -1.25 & 0.75 \end{bmatrix}$ $A_{41} = A_{43} = \begin{bmatrix} 0 & 1 \\ -1.952 & -1.952 \end{bmatrix}$,

$A_{42} = A_{44} = \begin{bmatrix} 0 & 1 \\ 0.75 & 0.75 \end{bmatrix}$,

$B_{11} = B_{12} = B_{41} = B_{42} = [0 \quad -0.45]$,

$B_{13} = B_{14} = B_{43} = B_{44} = [0 \quad -0.7]$,

$B_{21} = B_{23} = B_{31} = B_{32} = [0 \quad -0.7]$,

$B_{22} = B_{24} = B_{33} = B_{34} = [0 \quad -0.95]$,

$G_{11} = G_{12} = G_{21} = G_{22} = G_{31} = G_{32} = G_{41} = G_{42} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the external disturbances defined by $\Delta A_{qi}(t) = H_{aqi} F(t) N_{aqi}$ and $\Delta B_{qi}(t) = H_{bqi} F(t) N_{bqi}$ with, for $q = 1,\dots,4$ and $i = 1,\dots,4$, $H_{aqi} = H_{bqi} = [0.2 \quad 0.15]^T$ and $N_{aqi} = N_{bqi} = [0.25 \quad 0.1]$.

Recall that E_q, $q = 1,\dots,4$, are defined in (9) and lead, for the considered partition, to:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ E_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The membership functions are given by $h_{qi}(x(t)) = \Gamma_{qi} w_{qi}$ with:

$$\Gamma_{11} = \Gamma_{13} = \begin{cases} \dfrac{f(x) + 1.25x_1 + 1.25x_2}{1.004x_1 + 1.004x_2}, & (x_1, x_2) \ne (0,0) \\ 1 & \text{otherwise} \end{cases}$$

$$\Gamma_{12} = \Gamma_{14} = \begin{cases} \dfrac{-0.246x_1 - 0.246x_2 - f(x)}{1.004x_1 + 1.004x_2}, & (x_1, x_2) \ne (0,0) \\ 0 & \text{otherwise} \end{cases}$$

$$\Gamma_{21} = \Gamma_{23} = \begin{cases} \dfrac{f(x) - 0.75x_1 + 1.25x_2}{-2.702x_1 + 1.004x_2}, & (x_1, x_2) \ne (0,0) \\ 1 & \text{otherwise} \end{cases}$$

$$\Gamma_{22} = \Gamma_{24} = \begin{cases} \dfrac{-1.952x_1 - 0.246x_2 - f(x)}{-2.702x_1 + 1.004x_2}, & (x_1, x_2) \ne (0,0) \\ 0 & \text{otherwise} \end{cases}$$

$$\Gamma_{31} = \Gamma_{33} = \begin{cases} \dfrac{f(x) + 1.25x_1 - 0.75x_2}{1.004x_1 - 2.702x_2}, & (x_1, x_2) \ne (0,0) \\ 1 & \text{otherwise} \end{cases}$$

$$\Gamma_{32} = \Gamma_{34} = \begin{cases} \dfrac{-0.246x_1 - 1.952x_2 - f(x)}{1.004x_1 - 2.702x_2}, & (x_1, x_2) \ne (0,0) \\ 0 & \text{otherwise} \end{cases}$$

$$\Gamma_{41} = \Gamma_{43} = \begin{cases} \dfrac{f(x) - 0.75x_1 - 0.75x_2}{-2.702x_1 - 2.702x_2}, & (x_1, x_2) \ne (0,0) \\ 1 & \text{otherwise} \end{cases}$$

$$\Gamma_{42} = \Gamma_{44} = \begin{cases} \dfrac{-1.952x_1 - 1.952x_2 - f(x)}{-2.702x_1 - 2.702x_2}, & (x_1, x_2) \ne (0,0) \\ 0 & \text{otherwise} \end{cases}$$

and $w_{13} = w_{14} = w_{43} = w_{44} = 1 - 4x_1x_2$,

$w_{11} = w_{12} = w_{41} = w_{42} = 4x_1x_2$,

$w_{21} = w_{22} = w_{31} = w_{32} = 4x_1x_2 + 1$,

$w_{23} = w_{24} = w_{33} = w_{34} = \dfrac{-1.4 - x_1x_2}{0.25}$.

In order to be able to apply the above proposed theorems, the extended form of (34) is built by adding a stable linear system (6) with $C=-I$. Hence, the augmented switched fuzzy system yields:

$$\begin{aligned}\dot{\tilde{x}}(t)=\sum_{q=1}^{Q}\sum_{i=1}^{r}v_q\left(x(t)\right)h_{qi}\left(x(t)\right)\Big(\left(\tilde{A}_{qi}+\Delta\tilde{A}_{qi}(t)\right)\tilde{x}(t)\\ +\left(\tilde{B}_{qi}+\Delta\tilde{B}_{qi}(t)\right)u(t)+\tilde{G}_q\varphi(t)\Big)\end{aligned} \tag{35}$$

with $\tilde{A}_{11}=\tilde{A}_{13}=\begin{bmatrix}0&1&0&0\\ -0.246&-0.246&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{12}=\tilde{A}_{14}=\begin{bmatrix}0&1&0&0\\ -1.25&-1.25&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{21}=\tilde{A}_{23}=\begin{bmatrix}0&1&0&0\\ -1.952&-0.246&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{22}=\tilde{A}_{24}=\begin{bmatrix}0&1&0&0\\ 0.75&-1.25&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{31}=\tilde{A}_{33}=\begin{bmatrix}0&1&0&0\\ -0.246&-1.952&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{32}=\tilde{A}_{34}=\begin{bmatrix}0&1&0&0\\ -1.25&0.75&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{41}=\tilde{A}_{43}=\begin{bmatrix}0&1&0&0\\ -1.952&-1.952&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{A}_{42}=\tilde{A}_{44}=\begin{bmatrix}0&1&0&0\\ 0.75&0.75&0&0\\ 0&0&-1&0\\ 0&0&0&-1\end{bmatrix}$,

$\tilde{B}_{11}=\tilde{B}_{12}=\tilde{B}_{41}=\tilde{B}_{42}=\begin{bmatrix}0\\ -0.45\\ 0\\ 0\end{bmatrix}$,

$\tilde{B}_{13}=\tilde{B}_{14}=\tilde{B}_{43}=\tilde{B}_{44}=\begin{bmatrix}0\\ -0.7\\ 0\\ 0\end{bmatrix}$,

$\tilde{B}_{21}=\tilde{B}_{23}=\tilde{B}_{31}=\tilde{B}_{32}=\begin{bmatrix}0\\ -0.7\\ 0\\ 0\end{bmatrix}$,

$\tilde{B}_{22}=\tilde{B}_{24}=\tilde{B}_{33}=\tilde{B}_{34}=\begin{bmatrix}0\\ -0.95\\ 0\\ 0\end{bmatrix}$ and for $q=1,2$ and

$i=1,..,4$, $\tilde{G}_{iq}=\begin{bmatrix}1\\ 1\\ 0\\ 0\end{bmatrix}$, $\tilde{H}_{aqi}=\tilde{H}_{bqi}=\begin{bmatrix}0.2\\ 0.15\\ 0\\ 0\end{bmatrix}$

and $\tilde{N}_{aqi}=\tilde{N}_{bqi}=\begin{bmatrix}0.25 & 0.1 & 0 & 0\end{bmatrix}$.

In order to show the efficiency of theorem 1, let us consider the system (2) without external disturbances (i.e. $\varphi(t)=0$). The Matlab LMI toolbox is used to solve the LMI conditions leading to the synthesis of a switched fuzzy controller given by:

$$\mathrm{P}=\begin{bmatrix}5.358&3.198&3.268&3.360\\ 3.198&5.308&3.340&3.149\\ 3.268&3.340&5.899&3.704\\ 3.360&3.149&3.704&5.884\end{bmatrix},$$

$\tilde{K}_{11}=\begin{bmatrix}6.334 & 6.204 & 6.206 & 5.992\end{bmatrix}$,

$\tilde{K}_{12}=\begin{bmatrix}4.514 & 4.360 & 5.721 & 5.524\end{bmatrix}$,

$\tilde{K}_{13}=\begin{bmatrix}4.432 & 4.532 & 4.167 & 4.016\end{bmatrix}$,

$\tilde{K}_{14}=\begin{bmatrix}2.748 & 2.759 & 3.714 & 3.581\end{bmatrix}$,

$\tilde{K}_{21}=\begin{bmatrix}4.274 & 4.521 & 3.115 & 4.389\end{bmatrix}$,

$\tilde{K}_{22}=\begin{bmatrix}4.008 & 3.152 & 5.692 & 4.116\end{bmatrix}$,

$\tilde{K}_{23}=\begin{bmatrix}3.225 & 3.653 & 2.026 & 3.301\end{bmatrix}$,

$\tilde{K}_{24}=\begin{bmatrix}2.904 & 2.196 & 4.492 & 2.981\end{bmatrix}$,

$\tilde{K}_{31}=\begin{bmatrix}4.356 & 3.905 & 4.427 & 2.845\end{bmatrix}$,

$\tilde{K}_{32}=\begin{bmatrix}3.358 & 3.959 & 4.491 & 5.888\end{bmatrix}$,

$\tilde{K}_{33}=\begin{bmatrix}3.276 & 2.806 & 3.181 & 1.698\end{bmatrix}$,

$\tilde{K}_{34}=\begin{bmatrix}2.479 & 3.034 & 3.454 & 4.964\end{bmatrix}$,

$\tilde{K}_{41}=\begin{bmatrix}5.802 & 5.563 & 4.421 & 4.328\end{bmatrix}$,

$\tilde{K}_{42}=\begin{bmatrix}5.707 & 5.467 & 8.029 & 8.100\end{bmatrix}$,

$\tilde{K}_{43}=\begin{bmatrix}3.688 & 3.533 & 2.270 & 2.281\end{bmatrix}$

and $\tilde{K}_{44}=\begin{bmatrix}3.924 & 3.750 & 5.941 & 6.216\end{bmatrix}$.

A simulation has been performed with the initial states $x(0)=\begin{bmatrix}-0.3 & -0.3\end{bmatrix}^T$. The close-loop dynamics, the control signal and the switched Lyapunov function evolutions are depicted respectively in Fig.2, Fig.3 and Fig.4. These show a stable behaviour.

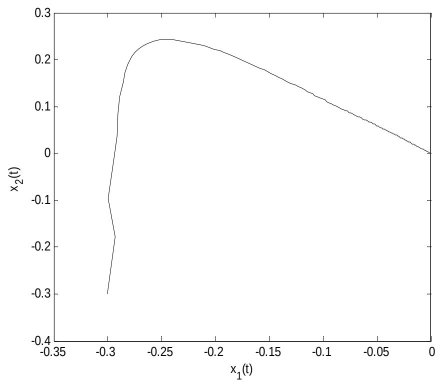

Fig. 2. Phase plan of the closed-loop uncertain switched fuzzy system.

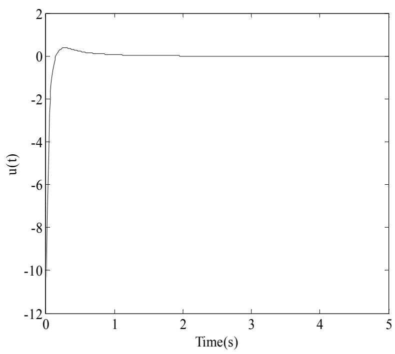

Fig. 3. Control law evolution of the closed-loop uncertain switched fuzzy system.

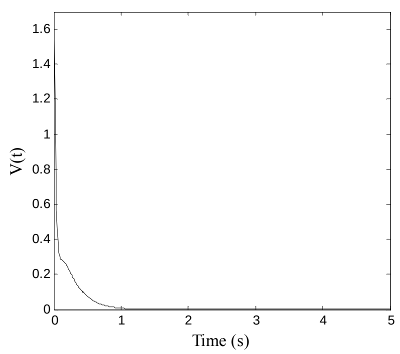

Fig. 4. Lyapunov function evolution of the closed-loop uncertain switched fuzzy system.

Let us now consider (33) subject to external disturbances $\varphi(t)$. Then, using the Matlab LMI toolbox to solve the LMI conditions proposed in the theorem 2, one obtains:

$$P=\begin{bmatrix} 9.071 & 6.780 & 6.996 & 7.123 \\ 6.780 & 8.947 & 6.943 & 7.032 \\ 6.996 & 6.943 & 9.560 & 7.419 \\ 7.123 & 7.032 & 7.419 & 9.844 \end{bmatrix} > 0 ,$$

$K_{11}=[10.924 \quad 10.707 \quad 10.990 \quad 11.147]$,

$K_{12}=[9.749 \quad 9.531 \quad 10.990 \quad 11.147]$,

$K_{13}=[7.022 \quad 7.048 \quad 6.692 \quad 6.787]$,

$K_{14}=[5.601 \quad 5.628 \quad 6.692 \quad 6.787]$,

$K_{21}=[10.916 \quad 10.890 \quad 9.570 \quad 11.394]$,

$K_{22}=[10.916 \quad 9.715 \quad 12.732 \quad 11.394]$,

$K_{23}=[6.639 \quad 7.152 \quad 4.872 \quad 6.927]$,

$K_{24}=[6.639 \quad 7.152 \quad 4.872 \quad 6.927]$,

$K_{31}=[10.916 \quad 10.641 \quad 11.163 \quad 9.522]$,

$K_{32}=[9.741 \quad 10.641 \quad 11.163 \quad 12.685]$,

$K_{33}=[7.041 \quad 6.501 \quad 6.823 \quad 5.045]$,

$K_{34}=[5.620 \quad 6.501 \quad 6.823 \quad 8.868]$,

$K_{41}=[10.674 \quad 10.552 \quad 9.326 \quad 9.474]$,

$K_{42}=[10.674 \quad 10.552 \quad 12.487 \quad 12.637]$,

$K_{43}=[6.526 \quad 6.451 \quad 4.766 \quad 5.018]$,

$K_{44}=[6.526 \quad 6.451 \quad 8.589 \quad 8.841]$

and with the disturbance attenuation level $\eta = 1.932$.

A simulation has been performed with the initial states $x(0)=[-0.4 \ -0.4]^T$ and a disturbance $\varphi(t)$ chosen as an uncorrelated Gaussien white noise with a variance equal to 0.1. The close-loop dynamics, the switched law and the control signal are depicted respectively in Fig.6, Fig.7 and Fig.8. One more time, they show, as expected, a stable behaviour.

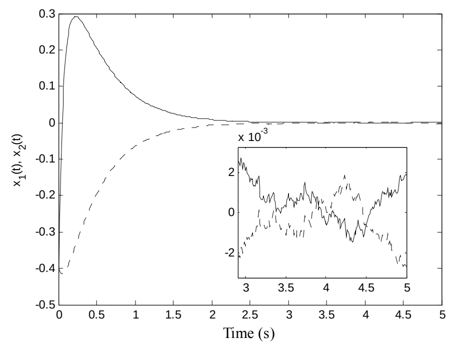

Fig. 5. State's dynamics of the closed-loop uncertain and disturbed system, (x_1) dotted line, (x_2) solid line.

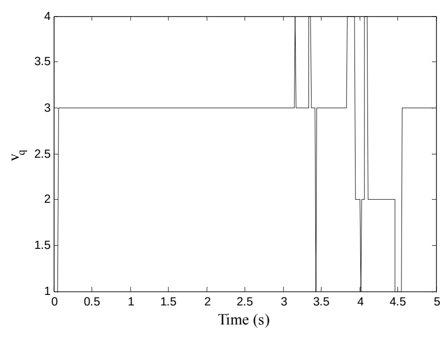

Fig. 6. Switched law evolution of the closed-loop uncertain switched fuzzy disturbed system.

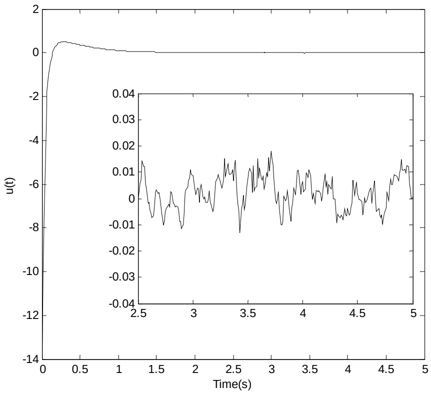

Fig. 7. Control law evolution of the closed-loop uncertain switched fuzzy disturbed system.

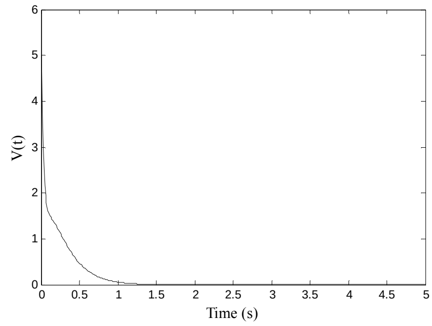

Fig. 8. Lyapunov function evolution of the closed-loop uncertain switched fuzzy disturbed system.

4. CONCLUSION

In this paper, the stabilization of a class of nonlinear systems represented by uncertain and disturbed switched Takagi-Sugeno fuzzy systems has been studied. The interest of this approach is to benefit from the well-known information of T-S systems with association to the characteristic of the switched systems. Moreover, to cope with uncertainties and external disturbances, LMI conditions for robust switched fuzzy PDC controller design have been obtained based on switched quadratic Lyapunov function and a H_∞ criterion. Finally, a numerical example has been provided to illustrate the efficiency of the proposed switched fuzzy PDC controller design methodology.

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