STATE FEEDBACK Ω-STABILIZATION WITH ROBUST H^∞ PERFORMANCE FOR SYSTEMS WITH LFT BASED PARAMETRIC UNCERTAINTY

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Abstract: *The paper presents a state feedback stabilization method for uncertain systems subjected to parametric time-varying structured uncertainty. The proposed design approach also allows to obtain an imposed robust H^∞ performance with respect to the class of uncertainty considered. Necessary and sufficient conditions for the existence of such a state feedback control are derived in terms of the feasibility of a system of linear matrix inequalities. An application of the theoretical developments concerning the prevention of pilot induced oscillations occurrence is also presented.*

Keywords: *Uncertain systems, parametric time-varying uncertainty, LFT description, Ω-stability, H^∞ robust performance, LMI.*

1. INTRODUCTION

The purpose of this paper is to present a state feedback design method for systems subjected to *linear fractional transformations* (LFT) *based parametric uncertainty* in order to achieve some specific robust stabilizing properties and in the same time, to accomplish an H^∞- type performance for any uncertainty in the admissible set. This admissible set includes *norm bounded time-varying structured uncertainty*. There are some important reasons which motivate the interest for this class of uncertainty. Firstly, in many engineering applications the uncertain parameters are not time-invariant. This means that the condition $S_T\delta = \delta S_T$ is not fulfilled $\forall T > 0$ as in the time-invariant case, where δ denotes the uncertain parameter and S_T is the shift operator defined as $S_T f(t) = 0,\ \forall t < T$ and $S_T f(t) = f(t-T),\ \forall t \geq T$. Secondly, the stability domain with respect to time-varying uncertainty is usually smaller than the one corresponding to the time-invariant case. Useful aspects emphasizing the sharp difference between the time-invariant and the time-varying uncertainty can be found for instance in (Megretski, 1993), (Rotea *et al*., 1993) and in (Shamma, 1994). The *quadratic stability* (Petersen and Hollot, 1986) is frequently used to handle time-varying uncertainty. This kind of stability is stronger than the robust stability (see *e.g*. (Rotea *et al*., 1993)) and it can be successfully applied if the elements of the state matrix of the uncertain system are affine functions with respect to the uncertain parameters. Unfortunately the affine parameter dependent representation may be very conservative because possible joint parameter dependencies are ignored. A method allowing to overcome this disadvantage is to use the LFT based uncertainty description ((Doyle *et al.*, 1991), (Varga *et al.*, 1995)). Its main advantage is the more accurate representation of the parametric uncertainty including polynomial and rational approximations. For a plant with LFT based uncertainty description, the Small Gain Theorem ((Zames, 1966)) gives sufficient robust stability conditions. If the uncertainty is time-varying these conditions are also necessary ((Megretski, 1993), (Shamma, 1994), (Rotea *et al.*, 1993)). In the case when the structure of the uncertainty is known, a modified small gain condition is used in order to reduce the conservativeness of the results. According with the terminology adopted in (Doyle *et al.*, 1991) this small gain type condition defines the so-called Q-*stability* of a stable system. Its definition and analysis in terms of the feasibility of a specific linear matrix inequality (LMI) are given in the next section.

On the other hand, in many applications it is important to guarantee a certain robust performance for all admissible uncertainty. In the present paper this performance is the upper bound of an H^∞-type norm. The state feedback Q-stabilization problem with imposed H^∞ robust performance is formulated and solved in Section 3. Necessary and sufficient conditions for the existence of such feedback gain are expressed in terms of the solutions of an appropriate LMIs system. Some of the theoretical results are illustrated in Section 4 by an application concerning the detection of the so-called *Pilot-Induced- Oscillations* (PIO). This problem received much interest in the aeronautical engineering over the last years and it is determined by the interaction between the human pilot and the aircraft. Some of the methods developed to analyse the PIO occurrence imply in fact to determine the stability domain with respect to time-varying uncertain parameters (see for details (Amato *et al.*, 1999) and (Stoica, 2004)).

Some concluding remarks are given in the last section of the paper.

2. NOTATIONS, DEFINITIONS AND PRELIMINARY RESULTS

Consider the following class of uncertain systems with LFT based parametric uncertainty:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Kw(t) + Bu(t)\\ z(t) &= Lx(t) + Mw(t) + Nu(t)\\ y(t) &= Cx(t) + Hw(t) + Du(t),\ \ w(t) = \Delta(t)z(t)\end{aligned} \tag{1}$$

where x denotes the state vector, w and u are the exogenous and the control input, respectively, z is the controlled output and y denotes the measurement vector. It is assumed that the norm-bounded time-varying structured uncertainty satisfies the condition $\Delta \in \mathsf{B}_\mathsf{D}$ where

$$\mathsf{B}_\mathsf{D} := \left\{\Delta : \mathbf{R} \to \mathbf{R}^{m\times m},\ \|\Delta\| \le 1,\ \Delta(t) \in \mathsf{D}\right\}$$

and

$$\begin{aligned}\mathsf{D} = \Big\{&\text{block diag}\left(\delta_1 \boldsymbol{I}_{k_1}, \dots, \delta_r \boldsymbol{I}_{k_r}, \Delta_1, \dots, \Delta_\ell\right),\\ &\delta_i \in \mathbf{R},\ \Delta_i \in \mathbf{R}^{k_{r+i}\times k_{r+i}}\Big\}\end{aligned} \tag{2}$$

with $\sum_{i=1}^{r+\ell} k_i = m$. Since the uncertainty Δ is square, it is implicitly assumed that w and z have the same dimension $(m\times 1)$. If $I - M\Delta$ is invertible, the uncertain system (1) can be rewritten as:

$$\begin{aligned}\dot{x} &= \left(A + K\Delta(I - M\Delta)^{-1}L\right)x\\ &\quad + \left(B + K\Delta(I - M\Delta)^{-1}N\right)u\\ y &= \left(C + H\Delta(I - M\Delta)^{-1}L\right)x\\ &\quad + \left(D + H\Delta(I - M\Delta)^{-1}N\right)u\end{aligned} \tag{3}$$

According with the Small Gain Theorem, if A is Hurwitz and

$$\left\|L(sI - A)^{-1}K + M\right\|_\infty < 1 \tag{4}$$

where $\|\cdot\|_\infty$ denotes the H^∞ norm, then (3) is stable for all Δ with $\|\Delta\| < 1$. It is important to note that the above condition guarantees *quadratic stability* and therefore robust stability conditions with respect to time-varying uncertainty. The condition (4) is not only sufficient but also necessary in the case of complex unstructured uncertainty Δ (see *e.g.* (Khargonekar *et al.*, 1990)). Since in the present paper it was assumed that Δ is structured, condition (4) may provide conservative results. An effective approach (see *e.g.* (Doyle *et al.*, 1991)) to reduce the conservativeness generated by the small gain condition (4) in the case of structured uncertainty of form (2) is the scale it as $S^{\frac{1}{2}}\Delta S^{-\frac{1}{2}}$ where S is a positive matrix with the following structure

$$S=\text{block diag}\left(S_1,,,S_r,s_1I_{k_{r+1}},\ldots,s_\ell I_{k_{r+\ell}}\right),\quad s_i\in\mathbf{R},S_i\in\mathbf{R}^{k_i\times k_i}. \tag{5}$$

Definition 1. *A stable system having the transfer function* $T(s)=C(sI-A)^{-1}B+D$ *with* D *square is called* Q*-stable if it exists a symmetric matrix* $S>0$ *such that* $\left\|S^{\frac{1}{2}}TS^{-\frac{1}{2}}\right\|_\infty<1$.

For the imposed structure (2) of the uncertainty Δ the matrix S in the above definition has the form (5). A direct consequence of the LMI version of the Bounded Real Lemma is the following result ((Skelton *et al.*, 1998), (Stoica, 2004)):

Proposition 1. *A stable system with the transfer function* $T(s)=C(sI-A)^{-1}B+D$ *is* Q*-stable if and only if there exist the symmetric matrices* $S>0$ *of form (5) and* $P>0$ *such that:*

$$\begin{bmatrix} A^TP+PA+C^TSC & PB+C^TSD \\ B^TP+D^TSC & -S+D^TSD\end{bmatrix}<0. \quad \square$$

Consider now the uncertain system (1) and assume that it is Q-stable for all $\Delta\in\mathsf{B}_\mathsf{D}$. Then define the cost function

$$J_{H^\infty}:=\sup_{u,\Delta}\left\{\int_0^\infty y^T(t)y(t)dt,\ \|u\|_2\le 1,\ \Delta\in\mathsf{B}_\mathsf{D}\right\}$$

where $\|(\cdot)\|_2$ denotes the norm on Lebesgue space of square integrable valued functions on $\mathbf{R}$ and $y(t)$ is the output determined by the control u in presence of Δ with the initial state $x(0)=0$. For the nominal system $(\Delta\equiv 0)$, the performance index J_{H^∞} coincides with the H^∞ norm of $T(s)$. For the perturbed system $(\Delta\ne 0)$ J_{H^∞} represents a measure of the robust H^∞-type (energy-to-energy gain) performance bound.

The next result which proof may be found in (Doyle *et al.*, 1991) and (Skelton *et al.*, 1998) gives a characterization of the robust H^∞ performance bound.

Proposition 2. *The following assertions are equivalent:*

(i) The uncertain system (1) is Q*-stable for all* $\Delta\in\mathsf{B}_\mathsf{D}$*;*

(ii) There exist a scalar $\gamma>0$ *and the symmetric matrices* $P>0$ *and* $S>0$ *with* S *having the form (5), such that:*

$$\begin{bmatrix} A^TP+PA & PK & PB \\ K^TP & -S & 0 \\ B^TP & 0 & -\gamma^2I\end{bmatrix} + \begin{bmatrix} L^T & C^T \\ M^T & H^T \\ N^T & D^T\end{bmatrix}\begin{bmatrix} S & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix} L & M & N \\ C & H & D\end{bmatrix}<0 \tag{6}$$

Moreover, if the above statements hold then $J_{H^\infty}<\gamma$. $\square$

3. STATE FEEDBACK Q-STABILIZATION WITH ROBUST H^∞ PERFORMANCE

In this section the following system with LFT based parametric uncertainty is considered:

$$\begin{aligned} \dot{x}(t)&=Ax(t)+K_1w_1(t)+K_2w_2(t)+Bu(t)\\ z_1(t)&=L_1x(t)+M_{11}w_1(t)+M_{12}w_2(t)+N_1u(t)\\ z_2(t)&=L_2x(t)+M_{21}w_1(t)+M_{22}w_2(t)+N_2u(t)\\ y(t)&=x(t),\quad w_1(t)=\Delta(t)z_1(t)\end{aligned} \tag{7}$$

with $\Delta\in\mathsf{B}_\mathsf{D}$. Then the problem has the following statement: given $\gamma>0$, determine a state feedback control $u(t)=Fx(t)$ such that the resulting system:

$$\begin{aligned} \dot{x}&=(A+BF)x+K_1w_1+K_2w_2\\ z_1&=(L_1+N_1F)x+M_{11}w_1+M_{12}w_2\\ z_2&=(L_2+N_2F)x+M_{21}w_1+M_{22}w_2,\quad w_1=\Delta z_1\end{aligned} \tag{8}$$

is Q-stable for all $\Delta\in\mathsf{B}_\mathsf{D}$ and $J_{H^\infty}<\gamma$.

The next result provides necessary and sufficient conditions for the solvability of this problem.

Theorem 1. *There exists a state feedback gain* F *such that the system (8) with LFT based parametric uncertainty is* Q*-stable for all* $\Delta\in\mathsf{B}_\mathsf{D}$ *and* $J_{H^\infty}<\gamma$ *if and only if:*

(i) $\gamma>\bar{\sigma}(M_{22})$*, where* $\bar{\sigma}(\cdot)$ *denotes the maximal singular value of* $(\cdot)$*;*

(ii) There exist the symmetric matrices $X>0$ *and* $Y>0$ *with* Y *having the structure (5) such that:*

$$\tilde{A}X\mathsf{N}_{11} + \mathsf{N}_{11}{}^T X\tilde{A}^T + \tilde{K}_1 Y\tilde{K}_1{}^T - \mathsf{N}_{12}{}^T Y\mathsf{N}_{12} + \gamma^{-2}\tilde{K}_2\tilde{K}_2{}^T - \mathsf{N}_{13}{}^T\mathsf{N}_{13} < 0 \quad (9)$$

and

$$\begin{bmatrix} \mathsf{M}_{11}(Y) & \mathsf{M}_{12}(Y) \\ \mathsf{M}_{12}{}^T(Y) & \mathsf{M}_{22}(Y) \end{bmatrix} < 0 \quad (10)$$

where

$$\tilde{A} := \mathsf{N}_{11}{}^T A + \mathsf{N}_{12}{}^T L_1 + \mathsf{N}_{13}{}^T L_2$$
$$\tilde{K}_1 := \mathsf{N}_{11}{}^T K_1 + \mathsf{N}_{12}{}^T M_{11} + \mathsf{N}_{13}{}^T M_{21}$$
$$\tilde{K}_2 := \mathsf{N}_{11}{}^T K_2 + \mathsf{N}_{12}{}^T M_{12} + \mathsf{N}_{13}{}^T M_{22}$$
$$\mathsf{M}_{11}(Y) := M_{11}YM_{11}{}^T - Y + \gamma^{-2}M_{12}M_{12}{}^T$$
$$\mathsf{M}_{12}(Y) := M_{11}YM_{21}{}^T + \gamma^{-2}M_{12}M_{22}{}^T$$
$$\mathsf{M}_{22}(Y) := -I + M_{21}YM_{21}{}^T + \gamma^{-2}M_{22}M_{22}{}^T$$

and $\begin{bmatrix} \mathsf{N}_{11}{}^T & \mathsf{N}_{12}{}^T & \mathsf{N}_{13}{}^T \end{bmatrix}^T$ *is any basis of the null space of* $\begin{bmatrix} B^T & N_1{}^T & N_2{}^T \end{bmatrix}$.

Proof. Applying the last part of Proposition 2 for the closed loop system (8) it follows that there exist the symmetric matrices $P > 0$ and $S > 0$ with S having the structure (5) such that:

$$\begin{bmatrix} (A+BF)^T P + P(A+BF) & PK_1 & PK_2 \\ K_1{}^T P & -S & 0 \\ K_2{}^T P & 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} (L_1+N_1F)^T & (L_2+N_2F)^T \\ M_{11}{}^T & M_{21}{}^T \\ M_{12}{}^T & M_{22}{}^T \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & I \end{bmatrix} \times \begin{bmatrix} L_1+N_1F & M_{11} & M_{12} \\ L_2+N_2F & M_{21} & M_{22} \end{bmatrix} < 0 .$$

Based on a Schur complement argument, the above condition is equivalent with:

$$\begin{bmatrix} \mathsf{L}_0(P,F) & PK_1 & PK_2 & \mathsf{L}_1{}^T(F) & \mathsf{L}_2{}^T(F) \\ K_1{}^T P & -S & 0 & M_{11}{}^T & M_{21}{}^T \\ K_2 P & 0 & -\gamma^2 I & M_{12}{}^T & M_{22}{}^T \\ \mathsf{L}_1(F) & M_{11} & M_{12} & -S^{-1} & 0 \\ \mathsf{L}_2(F) & M_{21} & M_{22} & 0 & -I \end{bmatrix} < 0$$

where the following notions have been introduced:

$$\mathsf{L}_0(P,F) := (A+BF)^T P + P(A+BF)$$
$$\mathsf{L}_1(F) := L_1 + N_1F;$$
$$\mathsf{L}_2(F) := L_2 + N_2F.$$

The above inequality can be rewritten in the equivalent form:

$$Z + \mathsf{P}^T F\mathsf{R} + \mathsf{R}^T F^T \mathsf{P} < 0 \quad (11)$$

where:

$$Z := \begin{bmatrix} A^T P + PA & PK_1 & PK_2 & L_1^T & L_2^T \\ K_1{}^T P & -S & 0 & M_{11}{}^T & M_{21}{}^T \\ K_2{}^T P & 0 & -\gamma^2 I & M_{12}{}^T & M_{22}{}^T \\ L_1 & M_{11} & M_{12} & -S^{-1} & 0 \\ L_2 & M_{21} & M_{22} & 0 & -I \end{bmatrix},$$

$$\mathsf{P} := \begin{bmatrix} B^T P & 0 & 0 & N_1 & N_2 \end{bmatrix}, \quad (12)$$
$$\mathsf{R} := \begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}.$$

According with the Projection Lemma (see *e.g.* (Boyd *et al.*, 1994), (Skelton *et al.,* 1998)) there exists F satisfying (11) if and only if:

$$W_{\mathsf{P}}{}^T Z W_{\mathsf{P}} < 0 \quad (13)$$

and

$$W_{\mathsf{R}}{}^T Z W_{\mathsf{R}} < 0 , \quad (14)$$

where W_{P} and W_{R} denote any bases of the null spaces of P and R, respectively.
Since

$$W_{\mathsf{P}} = \begin{bmatrix} P^{-1}\mathsf{N}_{11} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ \mathsf{N}_{12} & 0 & 0 \\ \mathsf{N}_{13} & 0 & 0 \end{bmatrix}$$

with $\mathsf{N}_{11}, \mathsf{N}_{12}$ and N_{13} defined in the statement, direct algebraic computations together with Schur complement arguments show that condition (13) is equivalent with (9), where $X := P^{-1}$ and $Y := S^{-1}$. Then based on the fact that

$$W_{\mathsf{R}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

it follows that (14) is equivalent with:

$$\begin{bmatrix} -S & 0 & \vdots & M_{11}{}^T & M_{21}{}^T \\ 0 & -\gamma^2 I & \vdots & M_{12}{}^T & M_{22}{}^T \\ \cdots & \cdots & \vdots & \cdots & \cdots \\ M_{11} & M_{12} & \vdots & -S^{-1} & 0 \\ M_{21} & M_{22} & \vdots & 0 & -I \end{bmatrix} < 0.$$

The Schur complement of the bloc (1,1) in the above inequality directly gives (10).

Part (i) of the statement directly follows from the block (2,2) of the LMI in (10). □

Remark 1. *If the conditions in the statement of Theorem 1 are fulfilled then a state feedback gain F can be easily determined by solving the basic LMI (11) in which P and S are replaced by X^{-1} and by Y^{-1}, respectively .*

4. A CASE STUDY

In the present section a case study concerning the prevention of pilot induced oscillations (PIO) is described. Although these phenomena are not new, several recent incidents in which advanced fighters prototypes as YF-22, Saab Gripen and transport aircrafts as the B777 were involved. PIO have many sources and a detailed model of the pilot-aircraft dynamics is difficult to be derived. The case study described in this section considers the so-called Category II of PIO (see *e.g.* (Klyde *et al.*, 1996)). These oscillations are mainly induced by nonlinearities determined by rate or position limits of surface actuators. The resulting pilot-aircraft system is linear, except rate or position saturation. Considering for example a position saturation of the actuator, the behaviour of this nonlinear element is equivalent with a linear unknown gain $L \in [L_{\min}, 1]$, as illustrated in Figure 1 where $\psi_{sat} = u_{sat}$ (for details, see (Amato *et al.*, 1994)). A large value of the predicted maximal command $u_{\max}$ corresponds to a small value of $L_{\min}$. Therefore for a wide range of the control variable u, a low limit $L_{\min}$ of the gain L is required. One can directly check that the gain L is not time-invariant.

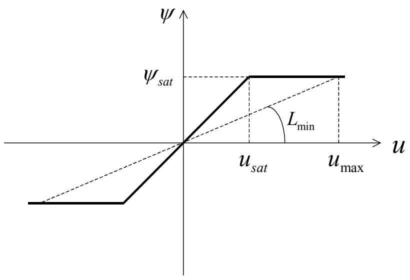

Fig. 1. The saturation nonlinear element

Taking into account that L is time-varying one expects to obtain a larger value of $L_{\min}$ than in the case when L is assumed time-constant.

A typical control configuration for the linearized longitudinal dynamics of an aircraft with rate limited actuator is shown in Figure 2.

In this figure K_p denotes the human pilot gain, $\tau = 0.04\sec$ is the time constant of the first order actuator dynamics and

$$\frac{\theta(s)}{\delta_e(s)} = \frac{3.457(s+0.0292)(s+0.883)}{(s^2+0.019s+0.01)(s^2+0.8418s+5.29)}$$

is the transfer function from the elevator position δ_e to the longitudinal attitude angle θ of the aircraft.

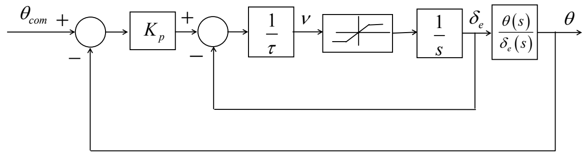

Fig. 2. The closed-loop system with rate saturation actuator

As shown above the saturation element in the inner loop of Figure 2 can be replaced by a linear unknown gain L. Since the pilot gain is variable, two uncertain parameters will be considered in this problem, namely K_p and L. The PIO detection implies to determine the pairs (L, K_p) separating the stability and the instability regions in the parameters plane $L - K_p$.

In Figure 3 the stability regions in the case when L is assumed constant (solid frontier) and when it is assumed time-varying are illustrated (dashed frontier), respectively (see (Stoica, 2004)).

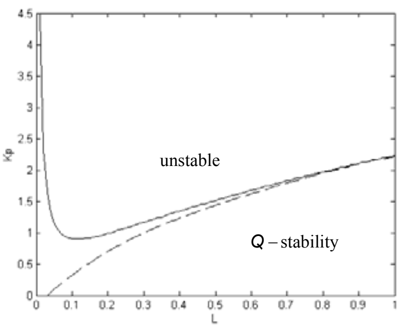

Fig. 3. Stability regions in constant and time-varying cases

For the configuration shown in Figure 2, the H^{∞} norm of $T_{v\theta_{com}}$ denoting the mapping from θ_{com} to the input v of the saturation element replaced by the uncertain gain L has been determined. Denoting by (A,B,C) a realization of the transfer function $\theta(s)/\delta_e(s)$ of the aircraft dynamics, the following state space representation of $T_{v\theta_{com}}$ is obtained:

$$\begin{bmatrix}\dot{x}\\ \dot{\delta}_e\end{bmatrix}=\begin{bmatrix}A & B\\ -\frac{K_pL}{\tau}C & -\frac{L}{\tau}\end{bmatrix}\begin{bmatrix}x\\ \delta_e\end{bmatrix}+\begin{bmatrix}0\\ \frac{K_pL}{\tau}\end{bmatrix}\theta_{com} \tag{15}$$

$$v=\begin{bmatrix}-K_pC & -1\end{bmatrix}\begin{bmatrix}x\\ \delta_e\end{bmatrix}+K_p\theta_{com}$$

Denoting $\delta_1 = K_p$ and $\delta_2 = L$, (15) may be written as un uncertain system with LFT based parametric uncertainty of form (3) where:

$$\Delta=\begin{bmatrix}\delta_1 & 0\\ 0 & \delta_2\end{bmatrix},\ K=\begin{bmatrix}0 & 0\\ 0 & -\frac{1}{\tau}C\end{bmatrix},\ L=\begin{bmatrix}1 & 0\\ 0 & C^+\end{bmatrix},$$

$$M=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix},N=\begin{bmatrix}-C^+\\ 0\end{bmatrix},H=\begin{bmatrix}-C & 0\end{bmatrix},$$

with C^+ denoting the pseudo-inverse of C.
Numerical results for several values of the uncertain parameters K_p and L are given in Table 1.

Table 1. Numerical results

K_p	θ^*_{com} (deg)	L	$\lVert T_{v\theta_{com}}\rVert_\infty$
$K_p = 1.2$	-	1	30
		0.7	35.9794
$K_p = 1.3$	17.8	1	32.5000
		0.7	49.6522
$K_p = 1.5$	10.4	1	43.5498
		0.7	107.1137
$K_p = 1.75$	6.7	1	91.4996
		0.7	615.9411

In the above table, θ^*_{com} denotes the minimal amplitude of the step command θ_{com} at which PIO occur. It is desirable to obtain large values of θ^*_{com} since it means that only such large amplitude commands can generate PIO. The time response of the pitch angle θ corresponding to $\theta^*_{com} = 6.7\,\text{deg}$ and $K_p = 1.3$ determined by the configuration in Figure 2 with a rate limit of $\pm 15\,\text{deg/sec}$ is plotted in Figure 4.

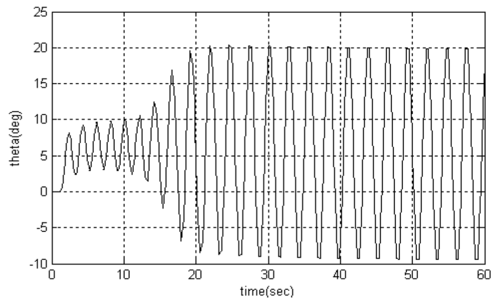

Fig. 4. PIO occurrence

Analysing these numerical results, it clearly results the relationship between θ^*_{com} and the H^∞ norm of $T_{v\theta_{com}}$. One can also see that a small value of $\lVert T_{v\theta_{com}}\rVert_\infty$ over the interval of variation of L implies large values of the command θ_{com} that induces oscillations.

5. CONCLUSIONS

The design method proposed in this paper is a state feedback robust stabilization for uncertain systems with structured time-varying LFT based parametric uncertainty. This class of uncertainty includes the affine parameter dependent representation. In order to reduce the conservativeness, a Q-stability condition is imposed which gives necessary and sufficient robustness conditions with respect to time-varying parametric uncertainty. Additionally, an H^∞ type robust performance has been also included in order to provide an upper bound of the energy-to-energy gain for all uncertainty in the admissible set. The solvability conditions are expressed in terms of the feasibility of a system of linear matrix inequalities.

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