CONTROLLING A MOBILE ROBOT ALONG PLANNED TRAJECTORIES

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Abstract: *The previous work presents and analyses some results in path tracking control on complex plane trajectories for a differential wheeled mobile robot. For the same model of the robot, the control uses alternatively two different algorithms. The first one is a bang-bang algorithm and the second is a classical solution in path tracking control. If the path given by the global path planner is a complex trajectory concerning straight lines, circular arcs, quick turning motion or lane change motion, finally the bang-bang algorithm offers better results in accurate and time execution. The results confirm the capability of this tracking method to be practically efficient for general path tracking of a wheeled differential mobile robot.*

Keywords: *wheeled mobile robot, control on the trajectory*

1. INTRODUCTION

For isostatic equilibrium, a differential mobile robot has three wheels (Fig.1) or many wheels in the case of hyper static equilibrium. Two driving wheels are obligatory fixed at each side of the robot. The robot can move back and forth and change its heading angle through velocity control of the driving wheels. Depending of mechanical construction (i.e. the type of implementing equilibrium), one, two or many supplementary castor wheels support the robot, at the front and at rear sides respectively. In consequence, this type of wheeled mobile robot has two degrees of freedom, of linear and rotational motions.

Because wheeled mobile robots have non-linear cinematic characteristics as well as the non-holonomic constraint problem, path planning and path control become very difficult to design from the viewpoints of optimality and stability.

In particular, stable and smooth motions are very important for wheeled mobile robots to avoid possible slippage between wheels and floor or mechanical shocks arising from abrupt changes in

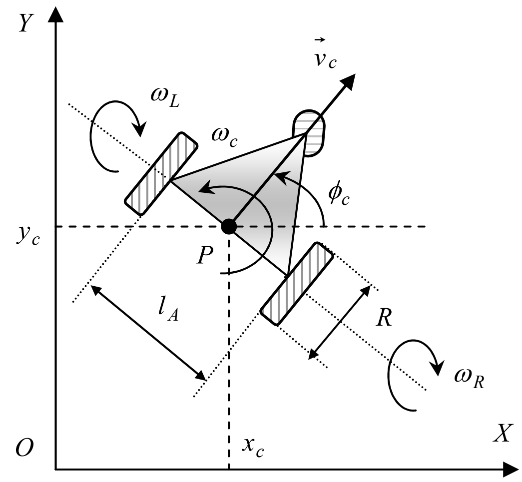

Fig. 1. Parameters for the mobile robot

motion (Andrea and Bastin, 1991; Bicchi, *et al.*, 1995; De Santis, 1995). There exists inevitable path error between the current position of the robot and the desired path due to the imperfect tracking control of driving wheel velocities or environmental disturbances. This path deviation should be corrected by on-line adjustment of the desired linear and rotational velocities of the robot in the execution level called path-tracking control.

The hardware environment implementing the control system was designed based on an IBM-PC. To investigate the performance of the control system, in the same priory conditions, a series of path tracking experiments was conducted using a time optimal bang-bang algorithm and a classical algorithm in path tracking control.

2. GENERAL ROBOT CONTROL SYSTEM

As shown in Fig. 2, the control system contains four essential elements: path planner, path-tracking controller, wheel servo controller and position estimation.

The path planner makes a plan for a reference path trajectory in two modes: off-line and on-line. In the off-line mode, the global path is a sequence of prescribed status starting from an initial point and ending at a goal point designed, while the local path (based on the global path) is specifically planned in the on-line mode.

During the navigation, the robot usually makes typical motions as straight lines, circular arcs, quick turning motions or lane change motions. A complex plane trajectory can include different combinations between these elementary situations. Therefore, multiple path segments corresponding to such motions in local path planning mode approximate the global path.

The motion of the robot can be described (Fig. 1) by linear and rotational velocity which are calculated from the direct cinematic transformations:

$$v_C = R \cdot (\omega_R + \omega_L)/2$$
$$\omega_C = R \cdot (\omega_R - \omega_L)/2 \qquad (1)$$

The posture of the mobile robot is defined in operational space by two positioned variables (x_C, y_C) and a direction variable (ϕ_C):

$$\dot{x}_C = v_C \cdot \cos\phi_C \quad \dot{y}_C = v_C \cdot \sin\phi_C \quad \dot{\phi}_C = \omega_C \qquad (2)$$

and the non-holonomic constraint can be derived:

$$\dot{y}_c = \dot{x}_c \cdot \tan\phi_c \qquad (3)$$

This means that the robot cannot change position and direction independently and therefore makes it difficult to design the path-tracking controller.

The path-tracking controller consists of two independent control algorithms, the first determining the desired linear and the second determining the rotational velocities in the execution level.

The inverse cinematic transformations give the desired velocities (v_c^d, ω_c^d) and finally the reference wheel velocities through the following transformation:

$$\omega_R^d = R^{-1} \cdot \left(v_c^d + l_A \cdot \omega_c^d / 2\right)$$
$$\omega_L^d = R^{-1} \cdot \left(v_c^d - l_A \cdot \omega_c^d / 2\right) \qquad (4)$$

Given the reference wheel velocities (ω_R^d, ω_L^d), the wheel servo controller, independently designed for each wheel, controls the driving wheel to follow the reference velocities.

To navigate accurately along the desired path, the robot has to know its actual position and orientation. Odometric method (Nițulescu, 1995) using wheel encoders estimate them.

For rotational motion ($\omega_c \neq 0$), finally we obtain:

$$x_c = x_c^p + v_c \cdot \left(\sin\phi_c - \sin\phi_c^p\right)/\omega_c$$
$$y_c = y_c^p - v_c \cdot \left(\cos\phi_c - \cos\phi_c^p\right)/\omega_c$$
$$\phi_c = \phi_c^p + \omega_c \cdot T_s \quad (5)$$

and for linear motion ($\omega_c \approx 0$) equations are:

$$x_c = x_c^p + v_c \cdot T_s \cdot \cos\phi_c^p$$
$$y_c = y_c^p + v_c \cdot T_s \cdot \sin\phi_c^p$$
$$\phi_c = \phi_c^p \quad (6)$$

where (T_s) is the sampling period and the super-script (p) denotes the value at the last sampling time.

The path can be represented by a sequence of target positions. Similar, the current motion of the mobile robot is described by its position and its linear and rotational velocities. As shows in Fig. 3, the path errors are defined in the local co-ordinates fixed on the target vehicle and can be calculated using the equations:

$$e_x = (x_t - x_c)\cdot\cos\phi_t + (y_t - y_c)\cdot\sin\phi_t$$
$$e_y = (x_c - x_t)\cdot\sin\phi_t + (y_t - y_c)\cdot\cos\phi_t$$
$$e_\phi = \phi_t - \phi_c \quad (7)$$

where (e_x), (e_y) and (e_ϕ) are respectively the tangential, lateral and orientation errors.

In long-range navigation, it is necessary to periodically correct the accumulated error caused by the estimation. Therefore, in the real case, the mobile differential robot introduces some dynamic restrictions:

$$|v_c| \le v_{\max} \qquad |a_c| \le a_{\max}$$
$$|\omega_c| \le \omega_{\max} \qquad |\alpha_c| \le \alpha_{\max} \quad (8)$$

3. TIME OPTIMAL BANG-BANG CONTROL ALGORITHM AND EXPERIMENTS

Then, the problem of finding an acceleration control law that enables the wheeled differential mobile robot to reach the target position and velocity in minimum time, while satisfying the limit condition, can by solved by a typical time optimal bang-bang control method which was detailed described in Koh and Cho (1994).

Summary, it consists in two different algorithms, for linear velocity control and for rotational velocity control. The desired linear velocity is determined to reduce the tangential error, while the desired rotational velocity is determined to reduce simultaneously the lateral and the orientation errors.

Due to the non-holonomic restriction (3), the rotational velocity control algorithm is designed by combining the bang-bang control with an on-line curve design scheme. This landing curve, passing the center of the real mobile robot and contacting an asymptotic tangent line of the target vehicle imposed by the global path planner, is a minimal order polynomial curve satisfying the continuity conditions for position, orientation and curvature at the contact point on the tangential line:

$$y_L(x_L) = C_x \cdot (x_L)^3 \quad (9)$$

The stability of the tracking motion of the mobile robot depends on the coefficient (C_x) of the landing curve. To avoid this, a method of determining the coefficient is developed in (Koh and Cho, 1994) through curvature analysis of the landing curve. The dynamic constraints of the robot, such as the acceleration limits or the curvature constraints have been considered for smooth motions. The final conditions for (C_x) are:

$$|C_x| \le \beta \cdot (\omega_{\max}/v_t)^2 \text{, where } \beta = 18 \cdot 5^{-5/2}$$
$$|C_x| \le \alpha_{\max}/6 \cdot v_t^2 \quad (10)$$

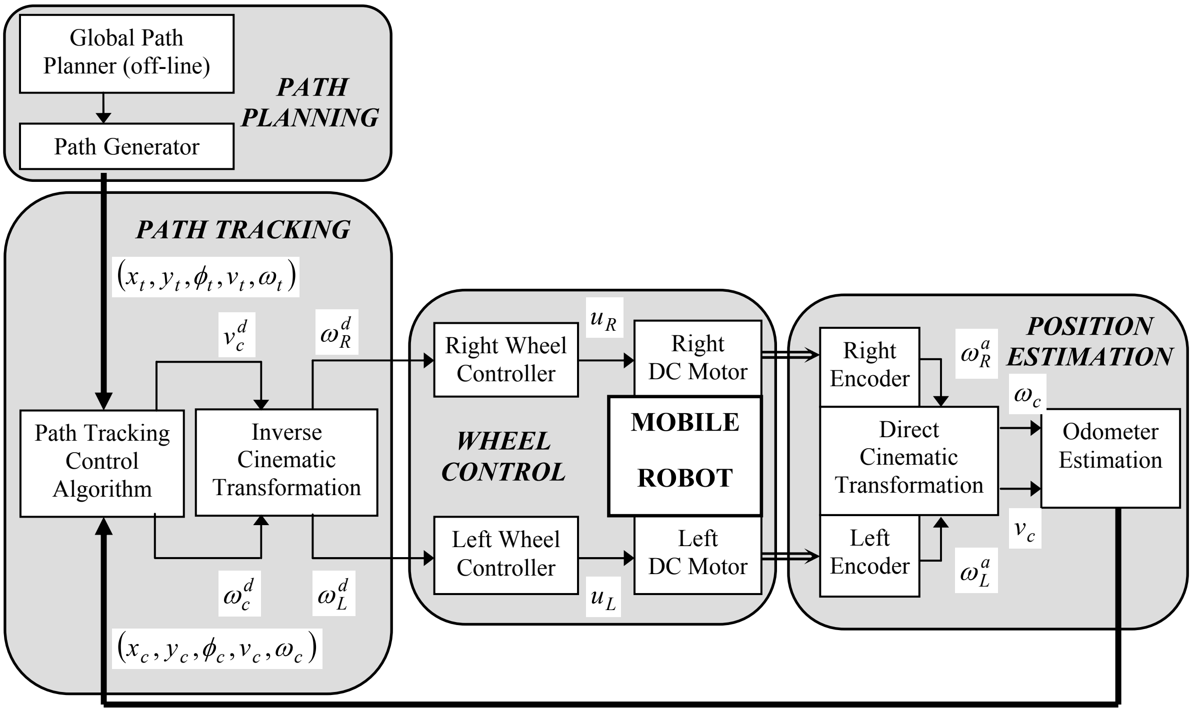

Fig. 2. General block diagram of the path-tracking control system

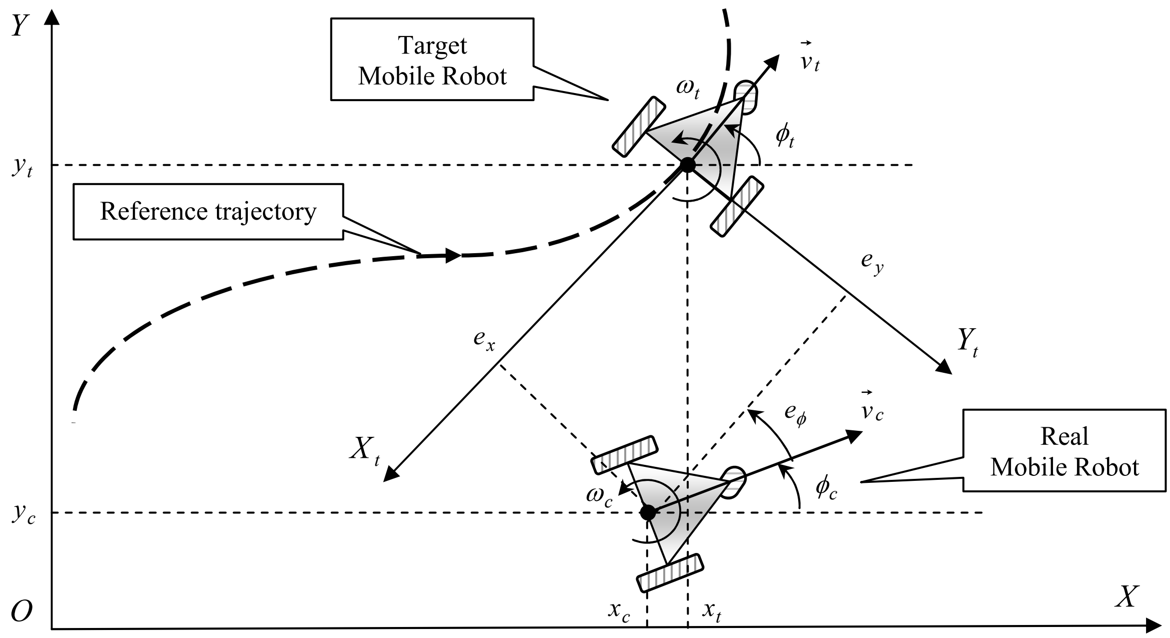

Fig. 3. Path errors expressed in a local co-ordinate

For the experimental simulations discussed below, this method is namely here "Method A".

The classical method (Kriegman and Triendl, 1987) is a linear path tracking control algorithm given by:

$$v_c^d = v_t + K_x \cdot e_x$$

$$\omega_c^d = \omega_t + K_y \cdot e_y + K_\phi \cdot e_\phi \quad (11)$$

To compare the performances of the two tracking algorithms, various combinations of the gain set (K_x, K_y, K_ϕ) were selected through trial in the same constraints on the velocities and accelerations. This second method is namely here "Method B".

A program was developed to compare the two algorithms, where the mobile robot is described by its parameters: $R = 5cm$, $l_A = 0.4m$, $T_s = 20ms$, $v_{\max} = 0.2m/s$, $a_{\max} = 0.2m/s^2$, $v_t = 0.1m/s$, $\alpha_{\max} = \pi/4rad/s^2$ and $\omega_{\max} = \pi/4rad/s$.

In (Nițulescu, 1999a,b) was reported different experiments with these two algorithms. The experiments were performed for the four typical types of path: straight lines, quick turning motions, circular arcs, and lane change motions and for various combinations of the gain set implied by the "Method B". Now, the path tracking experiments were performed for different complex global trajectories.

Fig. 4 resumes the results in the case of the real complex plane trajectory that involve all the basic situations presented above. For this case, Figures 5-16 offer many possibilities to compare the two method's algorithm in different aspects concerning:

- Cartesian evolution $y(x)$, in Fig.5-6;
- X-time evolution $x(t)$, in Fig.7-8;
- Y-time evolution $y(t)$, in Fig.9-10;
- Linear velocity $v(t)$, in Fig.11-12;
- Angular velocity $\omega(t)$, in Fig.13-14;
- Direction variable $\phi(t)$, in Fig.15-16.

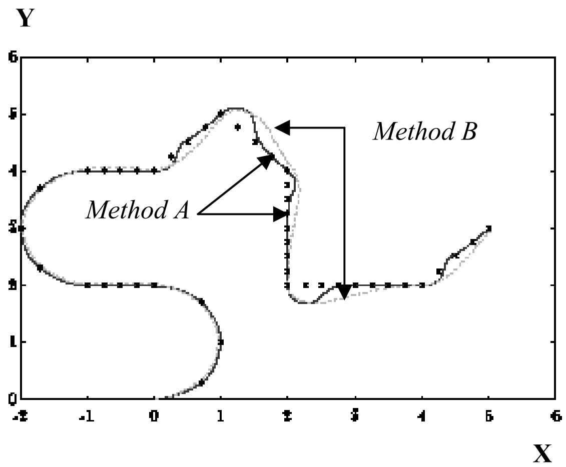

Fig. 4. Experiment on a complex trajectory

4. CONCLUSIONS

This paper presents some results obtained in the control of a mobile robot along planned trajectories. A general path tracking control system for wheeled differential mobile robot has been tested and evaluated using two different algorithms.

As a model, a three wheeled differential mobile robot it has used. Each complex and plane trajectory can be constructed using only four typical types of path: straight lines, quick turning motions, circular arcs, and lane change motions. Therefore, on a desired planned trajectory each current path segment can be followed be any of the four-type path segments.

The entire control strategy is adopted in such a way that the local paths are as simply designed as possible and a path tracking control algorithm guarantees smooth and stable motions along different plane and complex trajectories.

For the same model of the robot and the same planned trajectory, the control uses alternatively two different algorithms, which are in short presented in the paper. The first one, namely here "Method A", is a bang-bang algorithm and the second, namely "Method B", is a classical solution in path tracking control. Globally speaking, controlling the differential wheeled mobile robot along planned trajectories with the "Method A" the entire obtained performances are certainly better.

Control performances using “Method A”:

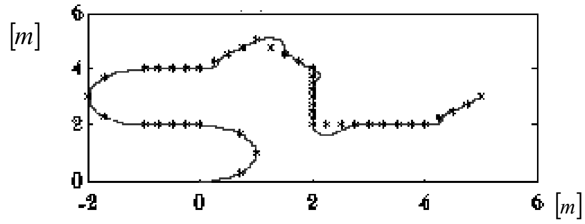

Fig. 5. Cartesian evolution $y(x)$

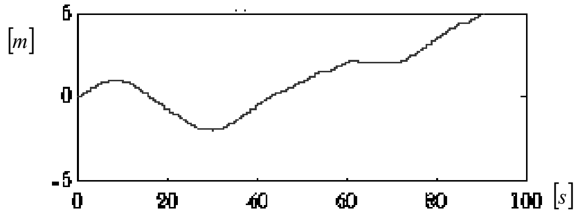

Fig. 7. X-time evolution $x(t)$

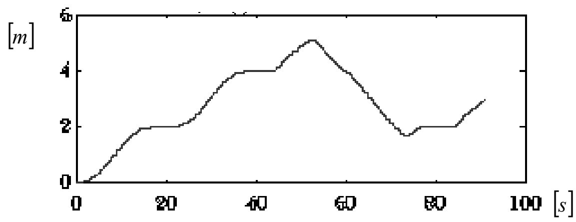

Fig. 9. Y-time evolution $y(t)$

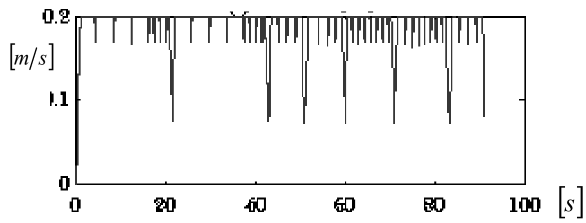

Fig. 11. Linear velocity $v(t)$

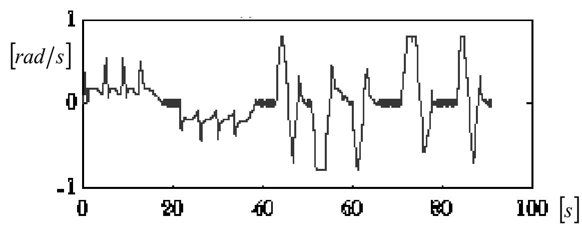

Fig. 13. Angular velocity $\omega(t)$

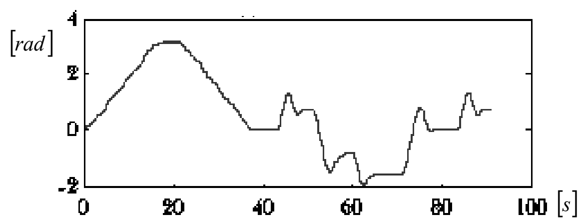

Fig. 15. Direction variable $\phi(t)$

Control performances using “Method B”:

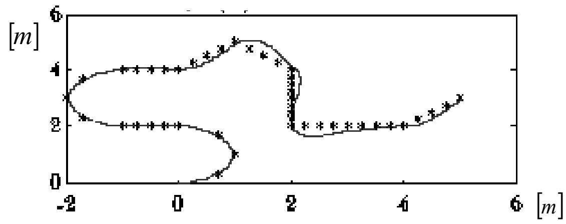

Fig. 6. Cartesian evolution $y(x)$

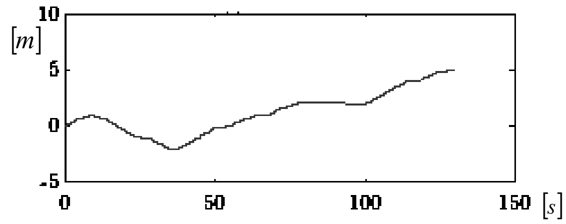

Fig. 8. X-time evolution $x(t)$

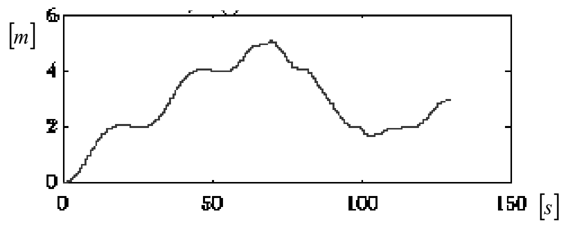

Fig. 10. Y-time evolution $y(t)$

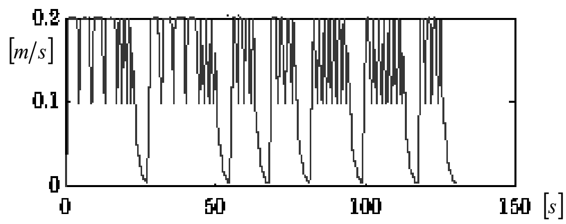

Fig. 12. Linear velocity $v(t)$

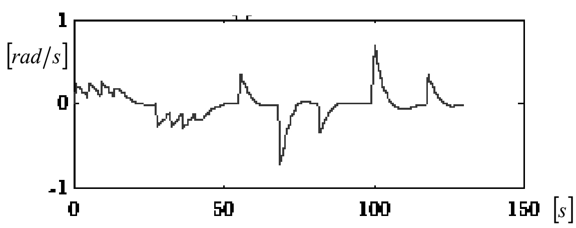

Fig. 14. Angular velocity $\omega(t)$

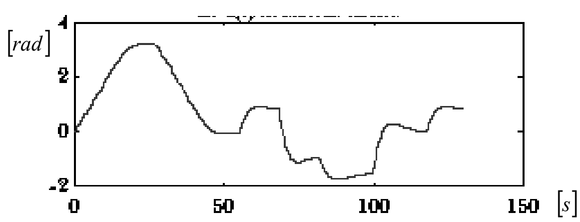

Fig. 16. Direction variable $\phi(t)$

If it analyzes in details the experimental results that was obtained, it can be appreciate that the “Method A” described so far have the following features:

First, it considers the dynamic constraints for robot motion $(v_{\max}, a_{\max}, \omega_{\max}, \alpha_{\max})$ and employs the bang-bang control scheme to make the tracking motion as smooth as possible in minimum time under the constraints.

Secondly, by introducing the concept of the landing curve, the non-holonomic problem associated with steering of wheeled mobile robot can be solved.

From the figures, it can be observed that “Method A” possesses robust tracking performance and a little faster convergence for various turning angles and different complex trajectories. Therefore, the path tracking error occurs at the beginning and sometime at the end of the current path due to the curvature discontinuity.

The results confirm the capability of this tracking method that guarantee smoothly and fast converging tracking motion for such roughly designed path trajectories, to be practically efficient for controlling a mobile robot along planned trajectories.

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