PERFORMANCE GUIDED HYBRID LQ CONTROLLER FOR TIME-DELAY SYSTEMS

Vojislav Z. Filipovic

RCT, POBox 126, 15300 Loznica, Serbia and Montenegro
Fax: ++381 15 881 945, E-mail: vfilip@eunet.yu

Abstract: *In this paper the concept of multiple models and concept of switching controllers is used. The analog part of the system is described by finite set of discrete-time models. It is considered general case when time-delay of real process is not equal to integer multiple of sampling time. As a set of controllers is used a finite set of LQ controllers with the prescribed degree of stability. The switching rule is based on the selection of the best performance of the closed-loop system. In the form of theorem is proved that hybrid system is asymptotically stable in the Lypunov sense and performance of system is no worse then the best non-switching strategy.*

Keywords: *Multiple models, discrete-time LQ controller, hybrid control, asymptotic stability, optimality*

1. INTRODUCTION

Hybrid systems can be interpreted as digital real-time systems which are embedded in analog environments. Continuous (analog) variables take the values from the set of real numbers and the discrete variables take the values from a finite set of symbols. Analog part of the hybrid system is described with differential or difference equation [11] and discrete part of hybrid system is a event driven dynamics which can be described using concept from discrete event system such as timed automata, max-plus algebra or Petry nets [5]. For hybrid discrete systems whose components are dominantly discrete events analysis and design are based on tools from computer sciences [1].

From the clasical control theory point of view hybrid systems can be considered as a switching control between analog feedback loops. This area of research now is a very reech [23], [27].

Now we have a different approaches for control of hybrid dynamical systems. In the [3] mixed logical dynamical model for hybrid system is proposed. This model is described by linear dynamical equations subject to linear mixed-integer inequalites. The complementarity class of hybrid systems are systems with inequality constraints [13]. Concept of picewise linear system is introducet in [19]. Equivalence between above classes of hybrid systems is established in [14].

Recently, the stochastic frame for hybrid dynamical systems is introduced [17], [18]. Stochastic hybrid systems (SHS) arise in numerous applications of systems with multiple models: air traffic management, flexibile manufacturing systems, fault tolerant systems etc. Very general class of SHS are diffusion processes with Markovian switching parameters.

In the last years new paradigm for adaptive control is developed {20], [24]. If conventional adaptive control is used, experience shows that the presence of large parameter errors will, generally, results in a slow convergence with large transient errors. Because, it is proposed concept of multiple models to identifay the unknown plant. That is foundation for higher level of adaptive control. Similar approach is taken, also, in [28] where mapping of hybrid states to hybrid control is based on system performance.

The theory of hybrid control systems is very powerfull. Now is clear that such kinde of system can describe the system with quantization [22], TCP congestion control [16] and control of wireless communication networks [21].

The field of time-delay systems had its origins in the 18th centery. Systematic and comprehinseve coverage of modern control theory for time-delay dynamic system is presented in [26]. The problem of the time-delay system stability, from the LMI point of view, is considered in [12].The recent developments on the class of uncertain deterministic and stochastic dynamical systems with time-delay are deeply covered in [4]. In [7] the concept of hybryd control is used for systems with input delay. Using suitable transformation the models with input delay is converted in delay free models and then, using LMI tool, the robust controller is derived. The networked control systems with time-varyng transmission times and unmodeled dynamics is considered in [8]. The system is described with functional differential equation and stability is proved using Lyapunov- Razumikhin theory.

In this paper we will adapt the multi model approach for control of time-delay systems using hybrid control methodology. The analog part of system will be described by difference equation. In the paper is considered general case when time-delay of real physical system is not equal to integer multiple of sampling period. The presence of time-delay will increases the dimension of equivalent model, in state space form, without the delay. The discrete event part is determined with the index of performance what is a natural criterion for optimal control.

In this paper the next results are established

(i) Equilibrium point $x = 0$ of the hybrid system is exponetially stable in Lyapunov sense

(ii) Performance of hybrid control systems is no worse then the best non-switching strategy

2. DISCRETE MODEL FOR PROCESS WITH TIME-DELAY

We will consider continuous-time state variable model that includes a delay in control action.The state equation is

$$\dot{x}_1(t) = Fx_1(t) + Gu(t-\tau), \tag{1}$$

where τ is a delay in the system. In this paper we will consider the discrete-time version of system (1). It is supposed that τ is not equal to integer multiple of sampling periods. Instead that we will separate the system delay τ into an integral number of sampling periods l and a positive number m less then one such that

$$\tau = lT - mT \tag{2}$$

Wher T is a sampling period and

$$l > 1,\quad 0 \le m < 1 \tag{3}$$

For such case discrete model is [11]

$$x(k+1) = Ax(k) + bu(k) \tag{4}$$

where

$$x(k) = \begin{bmatrix} x_1(k) \\ x_{n+1}(k) \\ x_{n+2}(k) \\ \vdots \\ x_{n+l}(k) \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A=\begin{bmatrix}\Phi & \Gamma_1 & \Gamma_2 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 0 & 1 & 0 & \cdots & 0\\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1\\ 0 & 0 & 0 & 0 & 0 & \cdots & 0\end{bmatrix}$$

$x_{n+1}(k)=u(k-l)$,

$x_{n+2}(k)=u(k-l+1)$, …,

$x_{n+l}(k)=u(k-1)$

$\Phi(a)=e^{Fa}$

$$\psi(a)=\frac{1}{a}\int_0^a e^{F\sigma}d\sigma$$

$\Gamma_1=(T-mT)\Phi(mT)\psi(mT)$

$\Gamma_2=mT\psi(mT)$

Remark 1. The discrete model (4) of the process is very general in ratio to description of process time delay. One can see that time delay increases the dimensions of vecrors and matrices in the model. When fraction part of the delay equal to zero then matrix $\Gamma_2=0$

3. DESCRIPTION OF TIME-DELAY SYSTEM BY MULTIPLE MODELS

In this part of paper we consider multiple model description of processes with time-delay. It will be assumed that the process model is a member of admissible process models

$$\mathsf{F} = \underset{p\in\mathsf{P}}{U}\,\mathsf{F}_p \tag{5}$$

where P is matrix index set which represents the range of parametric uncertainty so that for each fixed $p\in\mathsf{P}$ the subfamily F_p accaunts for unmodeled dynamics. Usually, P is compact subset of a finite-dimensional normed linear vector space [15].

In this paper we will suppose that process with time delays for large class of structured uncertainty can be described with collection of linear time invariant systems which have the form of relations (4)

$$x\,(k+1)=A_p x(k)+b_p u \tag{6}$$
$$p=1,2,....,s$$

where $x\in R^{n+l}$ and $u\in R^1$ are state and control signal of the system respectively.

Relation (6) describes the continuous part of system. The event driven part can be dscribed in the next form

$$p^+(t)=\varphi(p(t),\sigma(t)) \tag{7}$$

where $p(t)$ is discrete event variable, $\sigma(t)$ is a discrete input and $\varphi(\cdot,\cdot)$ is a function which describes behaviour of $m(t)$. It is important to note that

$$p^+(t)=(t_{n+1})\ ,\ p(t)=p(t_n)\ ,\ t_n<t_{n+1} \tag{8}$$

Specific form of switching sequence will be described in the next part of the paper.

Remark 2. In [6] continuous-time model with unmodeled dynamic is considered

$$\dot{x}(t)=(A_m+\Delta A_m(\omega(t)))x(t)+(B_m+\Delta B_m(\omega(t)))u(t)$$

where uncertainty vector $\omega(t)$ is Lebesque measurable and within an allowable bounding set $\Omega\in R^p$ for all $t\in[0,\infty]$.The switching sequence has a more complex form then in this paper.

Remark 3. Generally, logic part of the system can be described as a automata or Petry net [5]. If the analysis and design of the hybrid system is based on theory of discrete event system the main tool is: representation theory, supervisory control, computer simulation and verification. In this paper we will consider hibryd system from the clasical control theory point of view and in that case hybrid dynamic system can be interpreted as a switching control between analog feedback loops.

4. THE SWITCHING CONTROLLERS

Generally, no single controller is capable of solving the regulation problem for the entire set of process models (5). Owing that we will use the family of controllers [24]

$$\{C_q : q \in D\} \tag{9}$$

where D is index set. It supposed that this family is sufficiently rich so that every admissible process model can be stabilized by controller C_q for some index $q \in D$. In this paper will be considered the case

$$F = D \tag{10}$$

The members of family C_q are LQ controllers with prescribed degree of stability. Performance index for such controllers is

$$J_i = \sum_{k=i}^{\infty} \lambda^{-2k} \left[x^T(k) Q x(k) + r u^2(k) \right] \tag{11}$$

where $\lambda \in (0,1]$ is degree of stability. For fixed p optimal controller is dermined with the solutione of the next equation [2]

$$S_p = \frac{A_p^T}{\lambda^2} \left[S_p - \frac{S_p b b^T S_p}{b^T S_p b + r} \right] A_p + Q \tag{12}$$

Optimal analog control law is

$$u(k) = -\frac{b^T S_p A_p}{\lambda \left(b^T S_p b + r \right)} x(k) \tag{13}$$

$p = 1,2, \ldots, s$

Optimal performance for criterion (11) and control law (13) is

$$\lambda^{-2i} x^T(i) \left(S_p - Q \right) x(i) \tag{14}$$

The discrete feedback is

$$p^+(k) = \arg\min \left\{ \lambda^{-2k} x^T(k) \left(S_p - Q \right) x(k) \right\} \tag{15}$$

Remark 4. The control law (15) is more complex when in the continuous part of the system exists unmodeled dynamics. In that case switching from event p to event p_1 will be decided only if the worst case performance for the system p_1 is better then the best performance for system p [6]

Remark 5. Control law which is determined with relations (13) and (15) can be interpreted as supervisory control [24]. Instead of multi estimstor in original formulation of supervisory control, in this paper is used matrix S_p which is relevant for index performance calculation.

Remark 6. The control of uncertain dynamic system, in the presence of input saturation, is considered in [10]. The uncertaintes satisfay the matching conditions. Robust controller is based on combination of switching (picewise linear LQ controller), high-gain (which is incorporated by multiplying the gains of switching contrrollers with a scaling factor) and oversaturation. It is shown, using picewise quadratic Lyapunov functions, that the uncertain system can be exponentially stabilized.

4. ASYMPTOTIC LYAPUNOV STABILITY OF SWITCHED SYSTEM

It this part of the paper we will prove that multi model system (6) and (8) under the feedback (13) and (15) is asymptotically stable in the Lyapunov sense. Results are formulated in the form of theorem.

Theorem 1. Let us suppose that for hybrid systems (6) and (8) and hybrid controller (13) and (15) is satisfied

1° Matrix Q is positive definite and $r > 0$

2° For fixed p-th subsystem, couple

$$\left[\lambda^{-1} A_p \,,\ \mathrm{b} \right]$$

is completely stabilizable

3° For fixed p-th subsystem, couple

$$\left[\lambda^{-1} A_p \,,\ D_p \right] \ ,\ D_p D_p^T = Q$$

is completely detectable

4° For arbitrary switching sequence index of performance is bounded, i.e. for $\forall$ t

$$J_i \le \eta \ ,\ \eta \in (0, \infty)$$

Then

(i) The equilibrium point $x = 0$ of hybrid system is exponentially stable in Lyaponov sense

(ii) The performance of hybrid system J_0 is never worse then the performance of non-switching LQ controller, i.e.

$$J_0 \le \eta_0$$

whereby for fixed p-th subsystem is

$$\eta_0 = \min\{x^T(0)(S_p - Q)x(0)\} \quad ▪$$

Proof: Let us introduce the next transformation

$$\hat{x}(k) = \lambda^{-k} x(k) \tag{16}$$

$$\hat{u}(k) = \lambda^{-k} u(k) \tag{17}$$

Relation (6), (11) and (13) will take the form

$$\dot{\hat{x}}(k+1) = \lambda^{-1} A_p \hat{x}(k) + b\hat{u}(k) \tag{18}$$

$$\hat{u}(k) = -\frac{b^T S_p A_p}{\lambda(b^T S_p b + r)} \hat{x}(k) \tag{19}$$

$$J_i = \sum_{k=i}^{\infty} \left[\hat{x}^T(k) Q \hat{x}(k) + r\hat{u}^2(k)\right] \tag{20}$$

From condition 4° of theorem follows

$$\sum_{k=i}^{\infty} \hat{x}^T(k) Q \hat{x}(k) \le \eta \tag{21}$$

and using condition 1° of theorem we have

$$\sum_{k=0}^{\infty} \|\hat{x}(k)\|^2 \le \frac{\eta}{\lambda_{\min}\{Q\}} = \eta_1 \tag{22}$$

From relation (18) and (19), for p-th closed-loop subsystem, we have

$$\dot{\hat{x}}(k+1) = \tilde{A}_p \hat{x}(k) \tag{23}$$

where

$$\tilde{A}_p = \frac{1}{\lambda} A_p - \frac{bb^T S_p A_p}{\lambda(b^T S_p b + r)} \tag{24}$$

and using condition 3° of theorem one can conclude that subsystem (23) is asimptotically stable.
Hence, all eigenvalues $\lambda_j\{\tilde{A}_p\}$ of $\tilde{A}_p$ are inside unit circle.

From relation (23) follows

$$\dot{\hat{x}}(k) = (\tilde{A}_p)^k \hat{x}(0) \tag{25}$$

Let us introduce

$$\sigma_p = \max_{p=1,\dots,s} \left| \lambda_j\{\tilde{A}_p\} \right| \tag{26}$$

It is possible to finde a constant $c_p > 0$ such that

$$\|\hat{x}(k)\| \ge (c_p \sigma_p)^k \|x(0)\| \tag{27}$$

Let us define

$$\sigma = \max_{p=1,\dots,s} \quad \sigma_p > 0 \tag{28}$$

$$c = \min_{p=1,\dots,s} \quad c_p > 0 \tag{29}$$

Then

$$\|\hat{x}(k)\|^2 \ge (c\sigma)^k \|\hat{x}(0)\| \tag{30}$$

From (23) we have

$$\dot{\hat{x}}(k) = (\tilde{A}_p)^{k-i} \hat{x}(i) \tag{31}$$

and then

$$\|\hat{x}(k)\|^2 \ge (c\sigma)^{2(k-i)} \|\hat{x}(i)\|^2 \tag{32}$$

Suppose that for some $k > 0$

$$\|\hat{x}(k)\| = M > \sqrt{\frac{\eta_1}{1 + (c\sigma)^2}} \tag{33}$$

We have according with the (30) and (33)

$$\sum_{k=i}^{i+1}\|\hat{x}(k)\|^2 = \|\hat{x}(i)\|^2 + \|\hat{x}(i+1)\|^2 \geq$$
$$\|\hat{x}(i)\|^2 + (c\sigma)^2\|\hat{x}(i)\|^2 = M^2 + M^2(c\sigma)^2 \tag{34}$$

From (22) and (34) follows

$$M \leq \sqrt{\frac{\eta_1}{1+(c\sigma)^2}} \tag{35}$$

This contradicts the hypothesis (33) and we conclude that

$$\|\hat{x}(k)\| \leq \sqrt{\frac{\eta_1}{1+(c\sigma)^2}} \tag{36}$$

Since the right hand side, in relation (36), is independent of k, we have

$$\|x(k)\|_\infty \leq \sqrt{\frac{\eta_1}{1+(c\sigma)^2}} \tag{37}$$

For LQ problem η_1 is bounded by

$$\eta_1 \leq \frac{c_1\|\hat{x}(0)\|^2}{\lambda_{\min}\{Q\}} \quad , \ \exists c_1 > 0 \tag{38}$$

and we have from last two relations

$$\|\hat{x}(k)\|_\infty \leq \sqrt{\frac{c_1}{\lambda_{\min}\{Q\}\left[1+(c\sigma)^2\right]}}\|\hat{x}(0)\| \tag{39}$$

and according with last relation

$$\lim_{k\to\infty}\hat{x}(k) = 0 \tag{40}$$

Having in mind relation (16) we can write

$$\lim_{k\to\infty}\lambda^{-k}x(k) = 0 \tag{41}$$

and so the first statement of theorem is proven.

Let us introduce

$$T(k) = x^T(k)Qx(k) + ru^2(k) \tag{42}$$

Let us suppose that the sequence of discrete state

$$p = \{p(t_j)\ ,\ j = 0,1\ldots\}\ ,\ t_0 = 0 \tag{43}$$

where t_j is integer multiple of sampling period. The fixed switching sequence is

$$p_N = \{p(0), p(t_1)\ldots\ ,\ p(t_N)\}\ ,\ \exists N > 0 \tag{44}$$

and index of performance, according with (11) and (42)

$$J_N = \sum_{k=0}^{t_1}\lambda^{-2k}T(k) + \ldots + \sum_{k=t_{N-1}}^{t_N}\lambda^{-2k}T(k) +$$
$$+\sum_{k=t_N}^{\infty}\lambda^{-2k}T(k) = \sum_{k=0}^{t_1}\lambda^{-2k}T(k) + \ldots + \tag{45}$$
$$+\sum_{k=t_{N-1}}^{t_N}\lambda^{-2k}T(k) + \lambda^{-2t_N}x^T(t_N)\left(S_{p(t_N)} - Q\right)x(t_N)$$

From switching criterion (15) follows

$$\lambda^{-2t_N}x^T(t_N)\left(S_{p(t_N)} - Q\right)x(t_N) \leq$$
$$\leq \lambda^{-2t_N}x^T(t_N)\left(S_{p(t_{N-1})} - Q\right)x(t_N) \tag{46}$$

From last two relations we have

$$J_N \leq \sum_{k=0}^{t_1}\lambda^{-2k}T(k) + \ldots + \sum_{k=t_{N-1}}^{t_N}\lambda^{-2k}T(k) + \lambda^{-2t_N}$$
$$\cdot x^T(t_N)\left(S_{p(t_{N-1})} - Q\right)x(t_N) = \sum_{k=0}^{t_1}\lambda^{-2k}T(k) + \tag{47}$$
$$\cdots + \sum_{k=t_{N-2}}^{t_{N-1}}\lambda^{-2k}T(k) + \lambda^{-2t_{N-1}}x^T(t_{N-1})\cdot$$
$$\cdot\left(S_{p(t_{N-1})} - Q\right)x(t_{N-1})$$

Using same procedure we finally have

$$J_N \leq x^T(0)\left(S_{p(0)} - Q\right)x(0) = \eta_0 \tag{48}$$

and from that follows

$$J = \lim_{N\to\infty}J_N \leq \eta_0 \tag{49}$$

Theorem is proved ▪

Remark 7. In [6] in the model description the analog uncertainty is considered. The proof of hybrid system stability, for that case, is different from above theorem. The proof is based on performance dominant condition

$$A_p x(k) + b_p u(k) \le k_1 \lambda^{-2k}\left(x^T(k) Q x(k) + r u^2(k)\right) +$$

$$+ c\|x(k)\| \le k_2 \lambda^{-2k}\left(x^T(k) Q x(k) + r u^2(k)\right)$$

The alternative approach, based on LMI tool, is proposed in [9].

5. CONCLUSION

In this paper the problem of design of hybrid LQ contrroller for systems with time-delay is considered. It is separeted time-delay into an integral number of sampling periods and fraction part of sampling period. Such model covers the systems with arbitrary time-delay. Switching strategy is determined using index of performance. For uniformly bounded index of performance the switching controllers garantee stability of feedback sytem. Also, it is shown that performance of hybrid control systems is no worse then the best non-switching strategy.

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