ON THE ROBUSTNESS OF MODIFIED SMITH PREDICTOR CONTROLLER FOR TIME DELAY PROCESSES

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Abstract: *One of the main problems of Smith predictor control of time-delay processes is, that it does not handle the disturbances, especially in case the process has an integrator. In this case, the Smith predictor is not able to reset the steady state error. In this paper a modified Smith predictor control based on a variable structure algorithms for nominal controller and an approximate model for process is proposed. It is shown that the proposed method can assure the robustness of system even in presence of dead-time uncertainty.*

Keywords: *PID controller, time delay uncertainty, Smith predictor, robustness.*

1. INTRODUCTION

A time delay system is a special case of infinite dimensional system which has an infinite number of poles. The easiest way to control time-delay system is to remove the effects of delay elements so that the well-developed control techniques for finite dimensional systems can be applied.

A predictor that generates future outputs of the process is usually employed in the feedback loop of delay compensation in order to cancel the effects of delay. The Smith predictor is one of the most widely-used delay compensation methods based on the concept (Marshall, 1979). In this case, the model of the process is incorporated in the controller to predict the effects of the actual process output. As a result, time-delay is eliminated from the characteristic equation of closed-loop system and thus the controller can be designed without considering time-delay.

Conventional design methods for delay-free systems are directly applicable and the output responses after the delay duration can be adjusted as desired.

In practical control systems, there exists inevitable disturbances and modeling error. They affect the prediction of the effect of the current control actions. Though the Smith predictor offers potential improvement in the closed loop performances over the conventional controllers, it suffers from a sensitivity problem. In the face of inevitable mismatches between the model and actual process, the closed loop performances can be very poor (Marshall, 1992). A lot of work has been done in relations to the robustness issues of the Smith predictor system. For example, (Santacesaria, 1993) presented a simple criterion for the tuning of Smith predictor when the time delay is not precisely known; (Dumitrache *et al.*, 1998) developed a compensation procedure of time-delay and time-constant uncertainties based on mismatched model.

A further problem is, that Smith predictor does not handle the disturbances, especially in case the process has an integrator. In this case, the Smith predictor is not able to accommodate a load disturbance on the process input. In fact, it can be proved that in steady state the ratio between the process value and the load disturbances is proportional to the model gain and time delay. This means that the integral of the load disturbance will not be compensated; or in other word, that the controller is not able to reset the steady state error. To improve the steady state characteristics of Smith predictor control, several modified Smith predictor control have been introduced (Watanabe, 1981, Weidong, 1996).

This paper is organized as follows. In the following section the assessment of the achievable performance of classical Smith predictor is formulated. In the third section the proposed structure is presented. The robustness is discussed in section 4, and concluding remarks are presented in section 5.

2. SMITH PREDICTOR CONTROLLER

The most usual models for slow and very slow processes are:

$$H_1(s) = \frac{k_P e^{-ts}}{T_P s + 1} \tag{1}$$

$$H_2(s) = \frac{k_v e^{-ts}}{s(T_v s + 1)} \tag{2}$$

$$H_3(s) = \frac{k_v}{s} e^{-ts} \tag{3}$$

Fig. 1 shows a control block diagram using Smith predictor method.

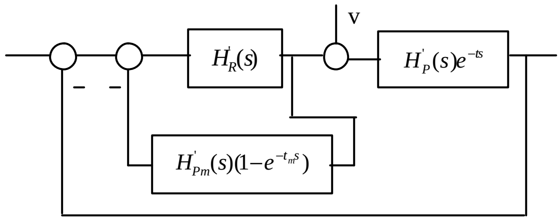

Fig.1. Classical Smith predictor control structure

If we consider the transfer function model of the process:

$$H_P(s) = H_p'(s)e^{-ts} \tag{4}$$

where $H_p'(s)$ is strictly proper, stable, and rational function, the Smith predictor controller transfer function is

$$H_R(s) = \frac{H_R'(s)}{1 + H_R'(s)H_p'(s)(1 - e^{-ts})} \tag{5}$$

where $H_R'(s)$ represents the controller designed for $H_{pm}'(s)$ only (i.e. the process without time-delay).

If the model and the actual process are identical, i.e.

$$H_p'(s) = H_{pm}'(s)\ ,\ t_m = t$$

the control structured illustrated in fig.1 leads to the closed-loop transfer function:

$$H_{0m}(s) = \frac{H_R'(s)}{1 + H_p'(s)H_R'(s)} H_p'(s)e^{-ts} \tag{6}$$

which shows that the time-delay is decoupled from the control-loop.

For the controller $H_R'(s)$ a conventional design methods (Wang, *et al.*, 1997) based on delay-free part of process model is used:

$$H_R'(s) = \frac{1}{H_p'(s)} \frac{H_{0m}'(s)}{1 - H_{0m}'(s)} = \frac{k_0}{sH_p'(s)} \tag{7}$$

where

$$H_{0m}'(s) = \frac{1}{T_0 s + 1} \tag{8}$$

The corresponding control algorithms for the models taken into account are:

- for process 1 a PI controller with $k_R = \frac{k_0 T_p}{k_p}$ and $T_i = T_p$;
- for process 2 a PD controller with $k_R = \frac{k_0}{k_v}$ and $T_d = T_v$;
- for process 3 a P controller with $k_R = \frac{k_0}{k_v}$.

Assuming that disturbance signal affect on the process input the closed loop function for exact model matching is as follows:

$$H_{0v}(s) = \frac{1}{1 + H_p^{'}(s) H_R^{'}(s)} H_p^{'}(s) e^{-ts} \tag{9}$$

Note that the transfer functions $H_{om}(s)$ and $H_{ov}(s)$ have the same characteristic equations. If the plant is asymptotically stable (process type (1)) and the controller has an integrator (PI) with a good process model ($H_P(s) = H_{Pm}(s)$), the PI controller can be tuned as it would be for a process without a time delay. Even in presence of parametric uncertainty, with methods presented in (Dumintrache and Mihu, 2000) the robustness is preserved. In Fig. 2 it is shown the setpoint and disturbances step responses for process (1) and PI controller tuning according procedure presented in (Dumitrache and Mihu, 2000).

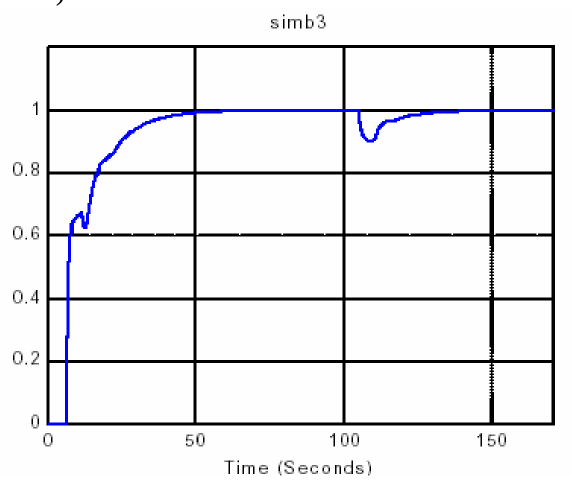

Fig. 2. The step setpoint and disturbance response of system 1

As we can see the robustness is preserved in presence of dead-time uncertainty.

It must be emphasized that in case the process has an integrator the steady state error is not zero, even the controller has an integrator.

The equation (9) is rewritten as:

$$H_{0v}(s) = \frac{H_P^{'}(s) e^{-ts} + H_R^{'}(s) H_P^{'}(s) e^{-ts} \left[H_P^{'}(s) - H_P^{'}(s) e^{-ts} \right]}{1 + H_P^{'}(s) H_R^{'}(s)} \tag{10}$$

For the plant described by (3) and a PI controller, the term $\left[H_P^{'}(s) - H_P^{'}(s) e^{-ts} \right]$ from equation (10) when $s \to 0$ becomes:

$$\lim_{s \to 0} \left[\frac{k_v}{s} - \frac{k_v}{s} e^{-ts} \right] = k_v t \neq 0 .$$

From this results:

$$y_{sv} = \lim_{s \to 0} H_{ov}(s) = k_v t . \tag{11}$$

That implies the disturbance steady state error is nonzero. More, it could be demonstrated (Watanabe, 1990) that the feedback control system is internally instable.

3. THE MODIFIED SMITH PREDICTOR CONTROLLER

In order to stabilize the control system and to obtained disturbances steady state error an approximate model is adopted (Astrom, 1993), i.e.

$$H_{Pm}^{'}(s) = \frac{k_m}{s + a} \tag{12}$$

The transfer function $H_{0v}(s)$ (from (10)) becomes:

$$H_{0v}(s) = \frac{\frac{k_v}{s} e^{-ts} + H_R^{'}(s) \frac{k_m}{s+a} e^{-ts} \left[\frac{k_v}{s} - \frac{k_v}{s} e^{-ts} \right]}{1 + H_R^{'}(s) \frac{k_m}{s+a} + H_R^{'}(s) e^{-ts} \left[\frac{k_v}{s} - \frac{k_m}{s+a} \right]} \tag{13}$$

With a PI nominal controller ($H_R^{'}(s)$), when $s \to 0$:

$$y_{sv} = \lim_{s \to 0} H_{ov}(s) = 0 .$$

The step setpoint and disturbance responses are presented in Fig. 3.

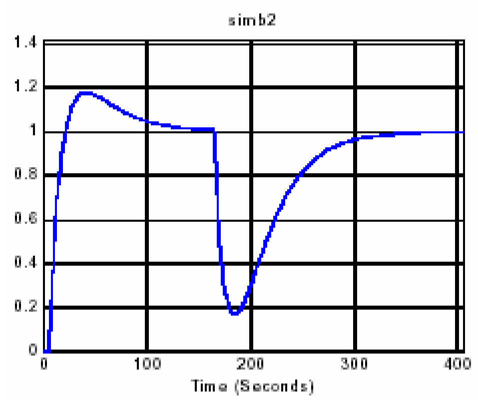

Fig. 3. The step setpoint and disturbances response

The steady state error is zero, but the transient performances are very poor.

In order to improve performances, both to the references and disturbances, we consider for nominal controller a variable structure, as shown in Fig. 4.

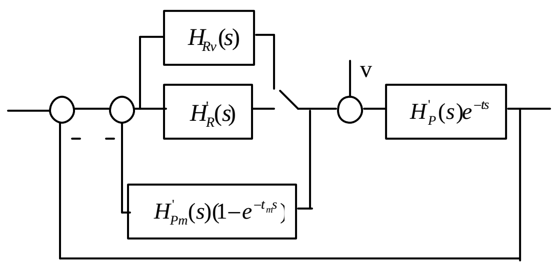

Fig. 4. The variable control structure

The two controllers in the control structure are considered as follows:

$$H_{Rv}(s)=\frac{k_{Rv}(1+T_{iv}s)}{T_{iv}s} \tag{14}$$

and

$$H_R'(s)=k_R\,. \tag{15}$$

The variable control structure was designed considering that the dominant exogen variable is the disturbance.

The variable control structure works as follows:

- the P controller ($H_R'(s)$) is connected until the system reaches a nominal regime;
- when the nominal regime is reached, the PI controller ($H_{Rv}(s)$) is connected to assure the disturbance rejection.

In Fig. 5 is presented the step setpoint and disturbance response for the proposed variable control structure.

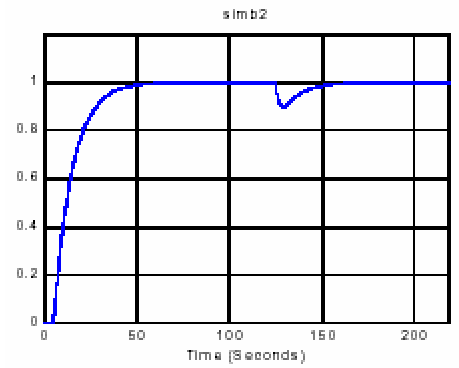

Fig. 5. The time domain response of the proposed control structure

We can see a considerable improvement of the performances comparing with modified Smith predictor with approximate model proposed in (Astrom *et al.*, 1993).

Taking into account the possibility to develop application programs, it is not a problem to realize software such a variable control structure and to assure no wind-up when switching between the two controllers.

4. THE ROBUSTNESS OF THE CONTROL SYSTEM

First we consider the robustness with regard to the setpoint, i.e. the Smith controller structure with nominal controller of proportional type.

In order to provide robust stability, the necessary and sufficient condition is

$$\left|L_M(jw)h(jw)\right|<1 \tag{16}$$

where $L_M(s)$ represent the multiplicative uncertainty and $h(wj)$ is named the complementary of the sensitivity function. In the case of modified Smith predictor control system:

$$h(wj)=H_{0m}(wj)\,.$$

For process (3), $L_M(s)$ is:

$$\left|L_M(jw)\right| = \left|\frac{k_v}{k_{vm}} e^{-(t-t_m)jw} - 1\right|$$

and

$$\left|H_{0m}(jw)\right| = \left|\frac{k_R k_m (jw+a)}{jw(jw+a) + k_R k_m jw + k_R k_m a e^{-t_m jw}}\right|$$

As shown in Fig. 6, the condition (16) is fulfilled, not only for the time delay matching (--), but also for dead time uncertainties (++). The numerical values are: $k_R = 0.1$, $k_v = k_{vm} = 1$, $d = 2$, $t_m = 5$, $t = 7$, $a = 5$.

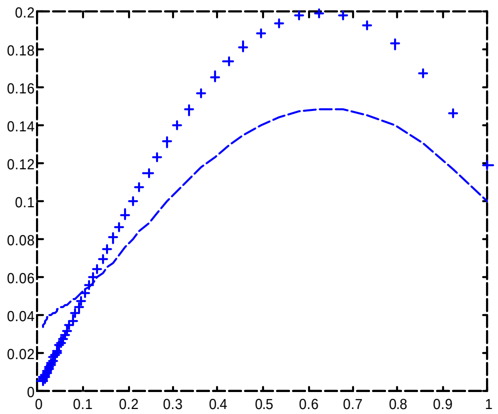

Fig. 6. The robustness characteristic

For robust stability with regard to disturbances in presence of dead time uncertainties, we consider the Nyquist plot of the open loop transfer function:

$$H_d(jw) = \frac{H_{Rv}(jw) H_p^{'}(jw) e^{-tjw}}{H_{Rv}(jw) H_m(jw)\left(1 - e^{-t_m jw}\right)}$$

with $H_{Rv}(s)$ from relation (14) and $H_p^{'}(s)$ from relation (3).

In Fig. 7 is presented the Nyquist plot of the above open loop transfer function.

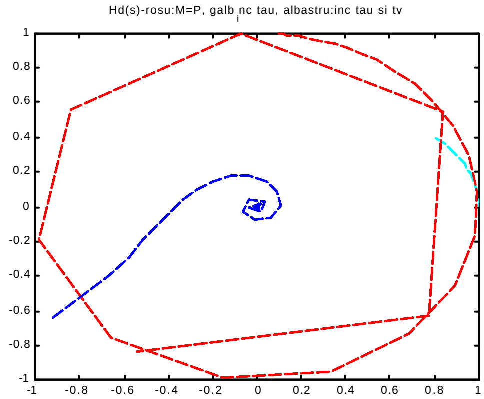

Fig. 7. The Nyquist plot for dead time model match

The numerical values of the parameters are: with the following numerical values for the parameters: $k_{Rv} = 0.1$, $T_{iv} = 20$ $k_v = 1$, $k_{vm} = 0.05$, $d = 0$, $t = t_m = 5$, $a = 5$.

The Nyquist plot of the open loop transfer function in presence of dead time uncertainties ($d = t - t_m = 2$) and for the same numerical values of the parameters as before is presented in Fig. 8.

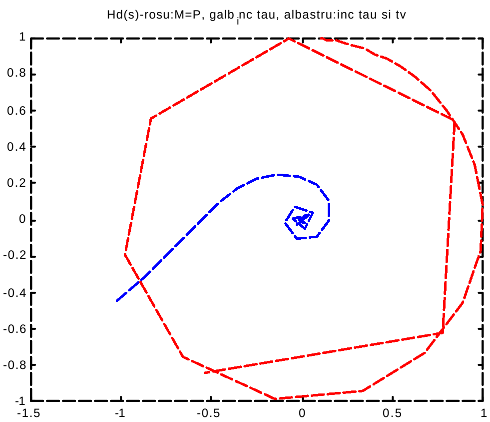

Fig. 8. The Nyquist plot with dead time uncertainties

We can see that in the presence of dead time uncertainties the stability reserve, i.e. M_A and M_Φ, is preserved.

From time domain response of the control system shown in Fig. 9, it can be seen that in presence of dead time uncertainties the dynamic performances are slightly different compared with the response in the case of dead time match (Fig. 5).

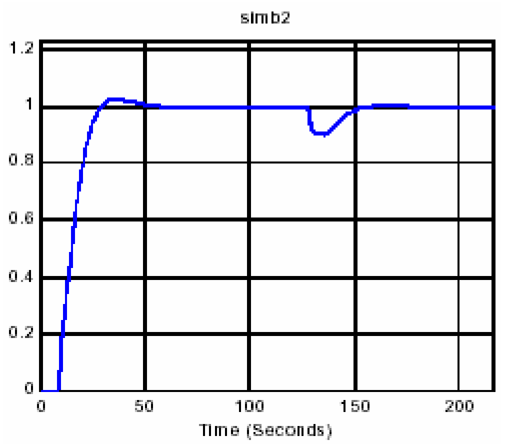

Fig. 9. The time domain response of the proposed control system in presence of dead time incertainties

5. CONCLUSIONS

This paper proposes a variable control structure for time-delay process based on modified Smith predictor controller. The proposed structure offers substantial improvement in the closed loop performances over the conventional Smith predictor.

It was shown that the proposed method can assure the robustness of the control system with regard to disturbances, even in the case when the process has an integrator. It should be mentioned that the program application must realize the control signal equilibration when switching from a controller to another.

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