Energy-aware Stochastic Control of Two-hop Routing in Networked Cyber Physical System

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Abstract: To enable efficiently use of the distributed resources, Networked Cyber Physical System (NCPS) needs to allow the composition of state messages generated by different nodes. Consider the nodes' limited energy and the limited lifetime of these messages, how to transmit them to other designated nodes fast with constraint is very important in NCPS. Because the expenditure of energy in data dissemination is originated from transmission process mostly, this paper adopts probability Two-hop routing method and the objective is to select proper probability to maximize the number of satisfied destinations. This paper uses the Edge-markovian graph to model NCPS and use discrete time Markov process to study the evolving rule of Two-hop routing method, and then studied mainly the *static*, *stochastic* and *threshold* control policies, theoretical and numerical results show that the optimal policy is the *threshold* form.

Keywords: Networked Cyber Physical System, Stochastic Control, Limited Energy, Two-hop Routing, Edge-markovian Graph.

1. INTRODUCTION

Traditional embedded and control systems typically close loops around important physical phenomena. The key goal of Cyber Physical System (CPS) (E. A. Lee, 2008) is to understand how to couple the cyber and physical realms effectively. So there exists interaction between the cyber and physical fields and the decision of the physical realm needs the messages from cyber realm. For the large-scale application of CPS, Networked CPS (NCPS) is needed (Minyoung Kim, 2010). An NCPS can provide complex, situation-ware, and often critical services in applications such as distributed surveillance and control, crisis response, or networked space/satellite missions, and so on. Because NCPS is a distributed system, the state of one node may have an effect on another node's physical field, that is, to enable efficient use of the distributed resources, NCPS needs to allow the composition of state messages generated by different nodes. Obviously, the dissemination of nodes' state is very important in NCPS. Also, the methods from cyber fields such as our data diffusion method must consider the actual situation of the physical field. Many peers of NCPS are working in the harsh, wireless and dynamic environment, so their energy is limited and the data dissemination methods must consider its impact. On the other hand, the maximal lifetime of the message is also limited and the messages out of date may cause worse effect than the case that didn't get message at all (etc., in war field). So the data dissemination methods should finished with limited energy before the deadline of the messages. Further, because the energy is limited and the destination nodes may be very large, not all of the destinations can be satisfied. In this situation, what can be done is to maximize the satisfied destinations with limited energy before the deadline of the message.

In order to diffuse message effectively, first, it should understand the system's topology structure. NCPS often working in harsh environment, the system should be able to take advantage of opportunities for communication and must be robust against delays and disruptions due to, e.g., mobility failures. Information diffusion and optimization in NCPS should take place locally at any node, though a certain degree of global awareness may be needed. Because of those problems, traditional connected graph is hard to model NCPS, and a loosely application framework based on Delay Tolerant Networks (DTN) (Fall K, 2003) is proposed in paper (Minyoung Kim, 2010). In this paper, the data dissemination method is also oriented to the loosely coupled system. Routing (by abuse of language, routing and data dissemination indicate the same thing in this paper) in traditional networks work on the assumption that there exists at least one path between endpoints, so these routing methods can not be used in DTN directly.

In order to overcome the network partitions, nodes of the DTN communicate through a "store-carry-forward" mode. Due to node mobility, different links come up and down. If the sequence of connectivity graphs over a time interval is overlapped, then an end-to-end path might exist, so the message should be forwarded over the existing link, stored and carried at the next hop until the next link comes up, and so on and so forth, until it reaches the destination. The basic routing method in DTN is epidemic routing (ER) (A. Vahdat, 2000) in which each node receiving the message carries it as it moves, and then forwarding it to all new nodes it encounters which does not have the message yet. Obviously, this would consume large energy, in order to use resource efficiently some economic methods such as Two-hop (M. Grossglauser, 2002) is proposed. In Two-hop method, the source node forwards message to every new encountered node and it also waste energy, so to use it more efficiently is necessary. Authors in paper (E. Altman, 2009a) study the optimal control problem of two-hop in DTN and (E. Altman, 2009b) study the problem with heterogeneous nodes, a more recent paper (Yong Li, 2010) consider the impact of node's selfish on ER method. But all of them are based on the exponentially model which failed to capture the strong dependence between the existence (and the absence) of a link. On the other hand, all of them consider only one destination and their object is to maximize the delivery ratio. For multi-destinations, due to limited energy, the delivery ratio may be 0 all the time, so this objective function is not proper for multi-destinations situation. To denote the dependence in link, Edge-markovian graph (C. Avin, 2008) is proposed to model the fast changing world. But the works in Edge-markovian graph are oriented to analysis of the performance with some basic methods, to our best knowledge, none of the works considered how to control these methods to get much better performance. The main contributions of this paper can be summarized as follows:

a. It uses discrete time Markov process to model the probability Two-hop routing process with multi-destinations under Edge-markovian graph;
b. It studies three control policies based above analysis, and proves that the optimal policy is the *threshold* form, that is, an policy which is not *threshold* form can not be optimal;
c. It gives the numerical results and shows that the optimal threshold policy is better.

The rest of the paper is organized as follows: in section 2, some related works are introduced, and then in section3 it studies the Two-hop method using discrete time Markov process, and explores some control policies based on it. The numerical results are given in section 4, at last, it summaries the main work.

2. RELATED WORK

For NCPS, paper (A. Benveniste, 2010) proposed a loosely time-triggered architectures, and then paper (Minyoung Kim, 2010) proposed a loosely application framework based on DTN. These papers show that the topology of NCPS is loosely coupled. So traditional connected graph is not proper to model NCPS, and it poses some challenges to guarantee the quality of service (Qos) (F. Xia, 2008) in the system. For those challenges, first and foremost is how to guarantee the quality of communication. Because communication depends on the topology of the system, how to model the topology of NCPS should be studied first. At present, DTN is often used to describe the intermittently-connected networks and paper (Minyoung Kim, 2010) has proposed the architecture based on it. DTN was first proposed in paper (Fall K, 2003). Because the phenomenon described in DTN emerged in many situations, examples include interplanetary Internet (IPN) (Burleigh S, 2003), military networks (Krishnan R, 2007), wildlife tracking and habitat monitoring sensor networks (Juang P, 2002) etc, it is a very hot topic in recent years. Most of the search lies in the routing field with different application environment, such as (Wei Gao, 2011), (Mohammad Arif, 2011) and (E. Bulut, 2010). But most of these methods need some prior knowledge of the network, so it is hard to use in many situations and ER is often used in the zero-knowledge environment. Further, because ER would consume large energy, Two-hop is used more often. Another research direction of DTN is in theoretical field, Zhang studied the performance of ER in paper (X. Zhang, 2007), and then many other papers (R.Bakhshi, 2010) and (Klein D. J, 2010) are proposed using different methods. But all of them are only explored the performance of ER, and they failed to study how to control the algorithm for more efficiently use. Authors in paper (E. Altman, 2009a) study the optimal control problem of two-hop in DTN and (E. Altman, 2009b) study the problem with heterogeneous nodes, a more recent paper (Yong Li, 2010) consider the impact of node's selfish on two-hop routing and paper (M. Khouzani, 2011) studied the optimal control problem of ER in more common case. But all of them are based on the exponentially model which failed to capture the strong dependence between the existence (and the absence) of a link. To overcome this problem, a more advantaged model called Edge-markovian model is proposed in paper (C. Avin, 2008), but the fruits based on the model are rare. Authors in paper (A. E. Clementi, 2008) studied finished time of ER in the model and only given some approximate results. Paper (John Whitbeck, 2011) studied ER under different message size, but it still only got the approximate results. To our best knowledge, none of the works considered the optimal control problem in Edge-Markovian Dynamic Graphs. So our work is different from existing works.

3. EVOLVING PROCESS OF TWO-HOP METHOD

3.1 Network Model

Suppose there are N+L nodes in the NCPS. The last L nodes are called the destinations, which only receive message without forwarding, and one of the first N nodes is called the source node indicated as S, the other N-1 nodes are called relay nodes. This paper adopts a discrete time model, considering time slot duration $\Delta>0$. The t-th slot corresponds to interval $[t\Delta, (t+1)\Delta]$. The source S created message m at time 0 with the maximal lifetime $T\Delta$, so the maximal number of slots is equal to $T>0$. For abuse of language, this paper will use T to indicate the maximal lifetime of m. Further the paper assumes that a node that receives a copy during a time slot can forward it starting from the following time slot. If a node received m, it can be seen as infected. Given any two nodes i and j, the symbol $e_{ij}(t)$ indicates state of the link (or called edge) between them at t-th slot. If $e_{ij}(t)=1$, it can be seen that i and j encountered with each other at t-th slot, and $e_{ij}(t)=0$ indicates the link is absent. Every link changes its state at the beginning of a slot according to a two-state Markov process and keep invariance in the same slot, that is, if $e_{ij}(t)$ =0, $e_{ij}(t+1)$ =1 with probability $0\leq\alpha\leq 1$ and keep invariance with probability 1-α, similarly, if $e_{ij}(t)$ =1, $e_{ij}(t+1)$ =0 with probability $0\leq\beta\leq 1$ and keep invariance with probability 1-β. Suppose that $\alpha+\beta<1$ and the transitive process is shown in Fig.1 as follows,

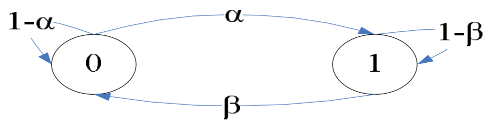

Fig. 1. State transition diagram of the link

This is a Bernoulli process and there exists the stationary distribution. Let π_0 and π_1 indicate the probability a link is in state 0 and 1 when the system went into stationary state, separately, they are shown as follows

$$\begin{cases}\pi_0=\beta/(\alpha+\beta)\\ \pi_1=\alpha/(\alpha+\beta)\end{cases} \tag{1}$$

This paper uses the probabilistic Two-hop method, that is, if *S* meets another relay node without message *m* at *t*-th slot, *S* forwards *m* to this relay node with probability *p(t)*. The problem is that how to control the probability to maximize the infected destinations under limited energy, and this paper proposed three probabilities (also called policies), that is, *static* policy, *threshold* policy and *stochastic* policy.

In the system, there will be many different messages. However, this paper only considers one message for certain duration in this paper, and the results can be extended to multi-messages easily.

3.2 Evolving Process

Because it has assumed that a node that receives a copy during a time slot can forward it only from the next slot, the forwarding process is started at the beginning of a slot, and the receiving process is appeared when the slot is finished. So the number of infected nodes keeps invariance during one slot. Let *X(t)* indicates the number of infected relay nodes at the starting of the *t*-th slot, also *Y(t)* indicates the number of infected destination nodes at the starting of the *t*-th slot. Now the objective is to solve the optimization problem as follows,

$$\begin{cases}Maximize\ \ E(Y(T))\\ Subject\ to\ \ E(X(T))+E(Y(T))\leq\sigma\end{cases} \tag{2}$$

E(X(T)) and *E(Y(T))* indicate the expectation of *X(t)* and *Y(t)* at the starting of the *T*-th slot, separately. Symbol $\sigma>0$ is called the maximal energy of the system. Obviously, the objective is to maximize the expectation of the number of the satisfied destinations. *E(X(T))* + *E(Y(T))* indicates the expectation of transmission times of the message. This paper only consider the energy consumption used for data dissemination, because the expenditure of energy in this process is due to transmission mostly, it can be seen that the total energy consumption is proportional to the total transmission times during the message's lifetime. The energy consumption of one time transmission includes both the reception energy at the receiving node and the sending energy at the transmitting node. Therefore, the expectation of total energy consumption can be expressed as follows,

$$\varepsilon(E(X(T))+E(Y(T))),\varepsilon>0 \tag{3}$$

For simplicity, let ε=1 and get the formula (2).

The relay nodes receive message only from the source node, and the process is independent, so the evolving rule of *X(t)* is,

$$X(t+1)=X(t)+\sum_{j=1}^{N-X(t)}\delta_{t+1}(j) \tag{4}$$

Above formula means that the number of infected relay nodes at the beginning of the (*t*+1)-slot equals to the number of infected relay nodes at the beginning of the *t*-slot added by the number of infected relay nodes during time interval [$t\Delta$, $(t+1)\Delta$]. $\delta_{t+1}(j)=1$ indicates the event that node *j* without message before receives *m* during [$t\Delta$, $(t+1)\Delta$], and it satisfies,

$$p(\delta_{t+1}(j)=1)=p(e_{Sj}(t)=1)p(t) \tag{5}$$

Symbol $p(\delta_{t+1}(j)=1)$ means the probability of node *j* is infected at the *t*-th slot, and the symbol $p(e_{Sj}(t)=1)$ indicates the probability of the link between node *j* and *S* is in state 1 at the *t*-th slot. Now the problem is how to get the probability $p(e_{Sj}(t)=1)$. In fact, from the event that $\delta_{t+1}(j)=1$, it can easily know that node *j* is not infected before time $t\Delta$. So at the current time,

$$\begin{aligned}p(e_{Sj}(t)=1)&=p(e_{Sj}(t-1)=0|\delta_t(j)=0)\alpha\\&+p(e_{Sj}(t-1)=1|\delta_t(j)=0)(1-\beta)\\&=p(e_{Sj}(t-1)=1|\delta_t(j)=0)(1-\alpha-\beta)+\alpha\end{aligned} \tag{6}$$

According to the Bayesian formula, the next formula is right,

$$\begin{aligned}&p(e_{Sj}(t-1)=1|\delta_t(j)=0)\\&=\frac{p(\delta_t(j)=0|e_{Sj}(t-1)=1)p(e_{Sj}(t-1)=1)}{\sum_{b=0}^{1}p(\delta_t(j)=0|e_{Sj}(t-1)=b)p(e_{Sj}(t-1)=b)}\\&=\frac{p(e_{Sj}(t-1)=1)(1-p(t-1))}{1-p(e_{Sj}(t-1)=1)p(t-1)}\end{aligned} \tag{7}$$

Combined with (6) and (7) the evolving process of $p(e_{Sj}(t)=1)$ can be got with different time *t*. Now, further combined with (4) and (5), the expectation of *X(t)* is,

$$E(X(t))=N-(N-1)\prod_{i=0}^{t-1}(1-p(e_{Sj}(i)=1)p(i)) \tag{8}$$

The initial status does not have any impact on the analysis process, so this paper simply supposes that the system started from the stationary state, that is, at the 0-th slot, any two nodes connected with probability π_1.

For *Y(t)*, it has similar formula as *X(t)*, that is,

$$Y(t+1)=Y(t)+\sum_{j=1}^{L-Y(t)}\varphi_{t+1}(j) \tag{9}$$

Symbol $\varphi_{t+1}(j)=1$ represents the event that destination j received message in the time interval $[t\Delta, (t+1)\Delta]$.

Because only one message can be transferred in a slot, at the t-th slot, destinations can be satisfied only by nodes that got m before t-slot, that is nodes in $X(t)$. Specially, at $(t-1)$-slot, destinations (not infected at current time) must disconnect with any relay node already infected, or they would be infected, so at the t-th slot, if the destination is still not infected, the state of its link with nodes in $X(t-1)$ is from 0 to 0, but the link between the destinations and any node infected at the $(t-1)$-th slot (nodes in $X(t)- X(t-1)$) may be in any state during time $[(t-1)\Delta, t\Delta]$, so the state of these links is 0 with probability π_0. According to above analysis, if node j is not satisfied at t-th slot, formula (10) can be got (by abuse of language, $X(t)$ indicates the set of infected relay nodes at t-slot, not only indicates its number),

$$\begin{cases} e_{ij}(t-1)=0 & ,i\in X(t-1) \\ e_{ij}(t-1)=0 or 1, & i\in X(t)-X(t-1) \end{cases} \tag{10}$$

So the probability of the event that node j is not satisfied at t-th slot is,

$$p(\varphi_{t+1}(j)=1)=1-(1-\alpha)^{X(t-1)}\pi_0^{\,X(t)-X(t-1)} \tag{11}$$

According to (9) and (11), the expectation of $Y(T)$ is shown as follows,

$$E(Y(t+1))=L-L(1-\alpha)^{\sum_{i=0}^{t-1}E(X(i))}\pi_0^{\,E(X(t))} \tag{12}$$

From formulas above, the optimization problem in formula (2) can be solved.

3.3 Optimal Control

This section will explore the optimal control problem with three special forwarding polices, that is, *static*, *threshold* and *stochastic* policy defined as follows,

Definition 1: The forwarding policy is called *static* if the relay nodes infected with the same probability p ($0\le p\le 1$ is a constant) at every slot, that is $p(\delta_k(j)=1)=p$, for every relay node j and $0\le k\le T$. Policy is called *threshold* if there exists a constant $0\le h\le T$, and the forwarding probability $p(t)$ satisfies, $t\le h$, $p(t)=1$, $t>h$, $p(t)=0$. If $p(t)$ is selected randomly from [0, 1] at t-th slot, the policy can be called *stochastic* policy.

Next this paper will prove that the front two polices have the optimal value of p and h, separately, and the *stochastic* policy is only used to compare with the other policies and this comparison can show that without some control, the performance of the two-hop method is poor in the energy limited environment. First, lemma 1 is given as follows,

Lemma 1: $E(Y(T))$ is increasing with $E(X(t))$, $0\le t<T$.

Proof: In fact, $E(X(t))$ can be seen as a stochastic order (M. Shaked, 1994). Given two stochastic orders $E1(X(t))$ and $E2(X(t))$, if they satisfy, $E1(X(t))\le E2(X(t))$, for all t, $0\le t<T$, and there exists at least one constant $0\le c\le T$, and $E1(X(c))<E2(X(c))$, according to the definition of stochastic order in paper (M. Shaked, 1994), $E1(X(t))$ is smaller than $E2(X(t))$. Further, according to formula (11), under these stochastic orders, the inequality $E1(Y(T))<E2(Y(T))$ is right. So the lemma is proved. □

Because at 0-th slot every link is in state 1 with probability π_1, according to formula (7), $p(e_{Sj}(t)=1)\le\pi_1$, for every $0\le t<T$. Obviously $p\le\pi_1$.

For the *static* policy, formula (8) can be converted to the following form.

$$E(X(t))=N-(N-1)(1-p)^t \tag{13}$$

So $E(X(T))$ is increasing with the parameter p, and the paper has lemma 2 as follows,

Lemma 2: The optimal *static* policy p^* satisfies, $p^*=\pi_1$ or p^* saturates the constraint in (2).

Proof: Because $E(X(T))$ is increasing with the parameter p, combined with Lemma 1, it can be easily seen that $E(Y(T))$ is increasing with the parameter p.

When $N+L\le\sigma$, $p=1$ is a feasible solution of the formula (2). From above analysis, $p\le\pi_1$, so $p^*=\pi_1$ is the optimal policy.

When $N+L>\sigma$, there exists a solution of p^* to saturate the constraint in (2), obviously. It need to prove that p^* is optimal. Given another constant $p1$, if $p1< p^*$, let $E(X(T))_{con}$ denotes the value under condition *con*(in next part of the paper, all symbols related to $E(X(T))$ or $E(Y(T))$ have similar means), so $E(X(T))_{p1}$ denotes the value under probability $p1$. It has shown that $E(X(T))_{p1}+ E(Y(T))_{p1}<\sigma$, and $E(Y(T))_{p1}< E(Y(T))_{p^*}$, so people can increase $E(Y(T))_{p1}$ through increasing the value of $p1$ until reaches to p^*. If $p1> p^*$, it has $E(X(T))_{p1}+ E(Y(T))_{p1}>\sigma$, so $p1$ is not a feasible solution and p^* is the optimal solution. □

Remark: In fact, it can not keep $p=\pi_1$ in every slot, for example, if at t-th slot $p=\pi_1$, the forwarding probability $p(t)$ is 1, so if the message is failed to transmit, the link in this slot is in state 0 surely, and changed to 1 in the next slot with probability α. So the probability of the transmission finished successfully in the $(t+1)$-th slot is at most $\alpha<\pi_1$. In general way, if p is small, the policy is easily to realize, if p is bigger, the value of the objective function in (2) is bigger than the real value. Suppose two nodes i and j, if the value of p in the t-th slot is small, even if $p(e_{ij}(t)=1)$ is small, it can make $p(\delta_{t+1}(j)=1)=p$ through increasing the value of $p(t)$, but if p is bigger, for example $p\ge\pi_1$, even if $p(t)=1$, $p(\delta_{t+1}(j)=1)$ is still hard to reach p. On the other hand, if $p(t)$ is increased in the t-slot, $p(e_{ij}(t)=1)$ will become smaller in the next slot and $p(\delta_{t+2}(j)=1)$ is hard to reach p, further. So the optimal *static* policy is an upper bound of the realized *static* policy. In the next section, it will see through theoretical and numerical results that the optimal *threshold* policy is better than optimal *static* policy, even though it is an upper bound, there is no need to study the problem exactly.

Now the paper starts to explore the optimal *threshold* policy, and according to its definition, the expectation of $X(t)$ is shown as follows,

$$E(X(t))=\begin{cases}N-(N-1)\pi_1(1-\alpha)^{t-1}, t\le h\\ N-(N-1)\pi_1(1-\alpha)^{h-1}, t>h\end{cases} \quad (14)$$

The optimal *threshold* policy has similar properties as the optimal *static* policy, that is,

Lemma 3: The optimal *threshold* policy h^* satisfies, $h^*=T$ or h^*saturates the constraint in (2).

Proof: The proof process is also like Lemma 2. First, this paper can get the result that $E(X(T))$ and $E(Y(T))$ are increasing with the parameter h.

When $N+L\le\sigma$, $h^*=T$ is a feasible solution of the formula (2). For any value $h1< h^*$, it has $E(Y(T)_{h1}< E(Y(T))_{h^*}$. So $h^*=T$ is the optimal value.

When $N+L>\sigma$, and h^* saturates the constraint in (2). Given another constant $h1$, if $h1< p^*$, it can get $E(X(T))_{h1}+ E(Y(T))_{h1}<\sigma$, and $E(Y(T))_{h1}< E(Y(T))_{h^*}$. if $h1> h^*$, it has $E(X(T))_{h1}+ E(Y(T))_{h1}>\sigma$, and $h1$ is not a feasible solution, so h^* is optimal. □

In fact, as long as $E(X(T))$ is increasing with the ordinary forwarding probability $p(t)$ (also stochastic order), the optimal *threshold* policy is the optimal policy. The lemma is described as follows,

Lemma 4: Suppose $E(X(T))$ is increasing with stochastic order $p(t)$, optimal *threshold* policy is the optimal policy.

Proof: Suppose that the optimal *threshold* policy $u1$ with forwarding probability $p(t)$ and optimal value h^*, that is, when $t\le h^*$, $p(t)=1$, $h^*<t\le T$, $p(t)=0$. When $h^*=T$, $p(t)=1$ in all the time duration, obviously, this is the optimal policy. Now, only consider the case $h^*<T$, according to Lemma 3, h^* saturates the constraint in (2). Given another policy $u2$ different from $u1$, suppose the forwarding probability of $u2$ at t-th slot is $p2(t)$, there exists a constant $0<c< h^*$, and $p2(t)$ satisfies, $p2(t)\le p(t)$, $t\le h^*$, $p2(c)\le p(c)$, if the constant c does not exist, $p2(t) = p(t)$, $t\le h^*$. Because $u2$ is different from $u1$, there exists another constant $h^*<s<T$ and $p2(s)>0 = p(s)$, so the stochastic $p(t)$ is smaller than $p2(t)$. According to the hypothesis and Lemma 1, $E(X(T))_{u2}+ E(Y(T))_{u2}> E(X(T))_{u1}+ E(Y(T))_{u1}=\sigma$, so $u2$ is not a feasible solution. Now it can be seen that the constant c exists, so $E(Y(t))_{u2}\le E(X(t))_{u1}$, $t\le c$, $E(Y(c))_{u2}< E(X(c))_{u1}$, $E(Y(t))_{u2}\le E(X(t))_{u1}$, $c<t\le h^*$. When $t > h^*$, $E(X(t))_{u1}$ has reached to the maximal value, so $E(Y(t))_{u2}\le E(X(t))_{u1}$ is still right. That is, the stochastic $E(X(t))_{u2}$ is smaller than $E(X(t))_{u1}$, further according to Lemma 1, $u1$ is the optimal policy. □

In fact, $E(X(T))$ is really increasing with $p(t)$ which is shown in Lemma 5 as follows:

Lemma 5: $E(X(T))$ is increasing with stochastic order $p(t)$.

Proof: Suppose the forwarding probability of policy $u1$ in t-th slot is $p1(t)$. Given another policy $u2$, and its forwarding probability $p2(t)$ satisfies: $t\ne h$, $p1(t)= p2(t)$, $t=h$, $p1(t)> p2(t)$, $0<h\le T$ is a constant. According to the definition of the stochastic order, the forwarding probability of $u1$ is bigger.

In fact, the mobility rule of nodes in the network is not related to the forwarding policy, so in every slot, for example, in t-th slot, the topologies in different policies are the same. Let $\zeta_i(t)$, $0<t\le T$ denotes the probability of node i received a copy in t-th slot, and it is not related to whether node i has received message before, so it is not the real probability. Whether node i meets S in every slot is uncertain in our model, which means that the probability $p_{iS}(t)$ that they meet with each other is smaller than 1. So $\zeta_i(t)= p_{iS}(t)p(t)$ ($p(t)$ is the forwarding probability in t-th slot), and it is easily to have, $0\le\zeta_i(t)<1$.Let $P_i(t)$ denotes the real probability of node i received message till t-th slot, it satisfies,

$$\begin{aligned}P_i(t)&=P_i(t-1)+(1-P_i(t-1))\zeta_i(t)\\&=P_i(t-1)(1-\zeta_i(t))+\zeta_i(t)\end{aligned} \quad (15)$$

For node i, under policy $u1$, suppose that the probability $\zeta_i(t)$ is $\eta1(t)$, similarly, under policy $u2$, the probability is $\eta2(t)$. The real probability under policy $u1$ is $\rho1(t)$ and under policy $u1$ is $\rho2(t)$. So they satisfy: $t\ne h$, $\eta1(t)= \eta2(t)$, $t=h$, $\eta1(t)> \eta2(t)$, $0<h\le T$. Further according to Eq.(15), people can get: $t<h$, $\rho1(t)= \rho2(t)$. When $t=h$, because $\eta1(h)> \eta2(h)$, they have, $\rho1(t)> \rho2(t)$, when $t=h+1$, because $\eta1(t)= \eta2(t)$ and $\rho1(t)> \rho2(t)$, they have $\rho1(t)> \rho2(t)$. When $t>h$, because $\eta1(t)= \eta2(t)$, they have $\rho1(t)> \rho2(t)$ which can be proved in the same manner as above by Eq.(15). So it can easily get $\rho1(T)> \rho2(T)$. In fact, $E(X(T))$ can be described as follows,

$$E(X(T))=E(\sum_{i=1}^{N}P_i(T))=NP_i(T)$$

So they satisfy, $E(X(T))_{u1}=N\rho1(T)$, $E(X(T))_{u2}=N\rho2(T)$. From analysis above, $E(X(T))_{u1}> E(X(T))_{u2}$ is right. So $E(X(T))$ is really increasing with stochastic order $p(t)$. □

4. NUMERICAL RESULTS

Numerical results have been obtained by simulating the discrete-time system with Matlab. This paper gives the network parameters the same as paper (John Whitbeck, 2011), that is, $\alpha=0.05$, $\beta=0.58$ and the slot duration $\Delta=15$s, the number of nodes $N=50$, $L=10$ and $T=10$. For the *stochastic* policy, at t-th slot, infected nodes forwarding with probability $p(t)$ selected from [0, 1] randomly, so $p(\delta_{t+1}(j)=1)$ can be seen as a random value, too. For simplicity, because $p(\delta_{t+1}(j)=1)$ is smaller than π_1, this paper randomly selects $p(\delta_{t+1}(j)=1)$ from $[0, \pi_1]$.

4.1 Impact of the Maximal Energy

In this section, the maximal energy σ is increased from 5 to 50, and other parameters equal to the default ones and the numerical results is shown in Fig.2.

Random 1 to 3 in Fig.2 indicate the three operational results of the *stochastic* policy. Obviously, the performance of the optimal *threshold* policy is better than other policies. The satisfied destinations in all policies are increasing with the increasing of the maximal energy, and if the energy is sufficient, all of them can infect nearly all the destinations, but if the maximal energy is less than 25, the advantage of the optimal *threshold* policy is obvious. Fig.2 also shows that the *stochastic* policy is worse than the optimal *static* policy, and this shows that some control methods is necessary of Two-hop routing in NCPS.

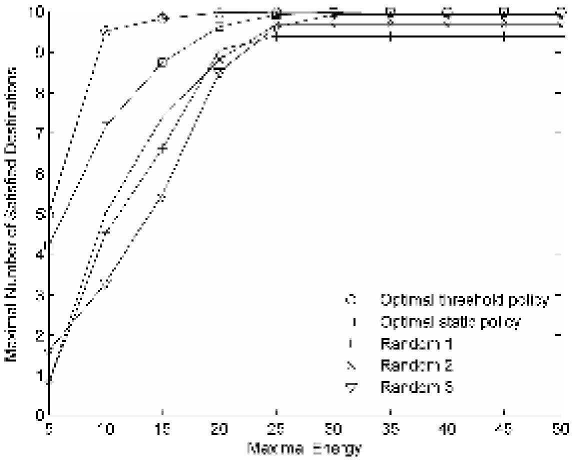

Fig. 2. Performance with different maximal energy

4.2 Impact of the Maximal Lifetime

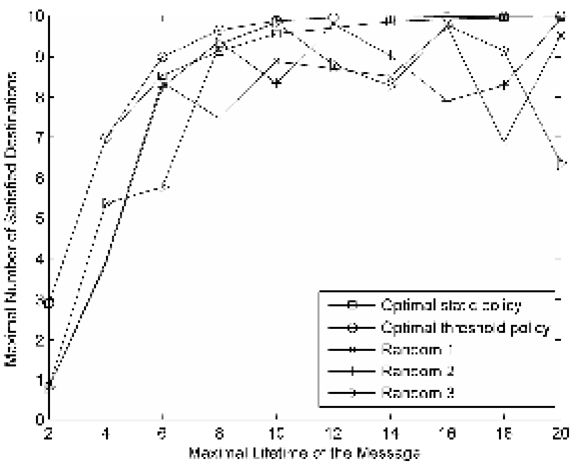

Fig. 3. Performance with different maximal lifetime

Now set σ=20 and increase T from 2 to 20. The result is shown in Fig.3. This result also shows that the optimal *threshold* policy is the best, but the *stochastic* policy fluctuates with different maximal lifetime, this is because the maximal energy is fixed, given two stochastic policies *s1* and *s2* with different maximal lifetime of *T1* and *T2*, separately. Even if *T1>T2*, the performance of *s1* may be still worse than *s2*. For example, the forwarding probability *p(t)* is very big in the front part, the limited energy may be exhausted soon, though there is much time left, there is no energy to run, so the policy may be less powerful, and the result of Fig.3 shows the analysis is right.

4.3 Impact of the Number of Destinations

Here, let the parameter L increase from 1 to 10, σ=20, T=10 and other parameters keep invariance. The result is shown in Fig.4, and it indicates that when L is small, all of the policies can achieve good performance, including the stochastic policy. That is to say, when the number of destination nodes is small, it is not necessary to control the algorithm in this setting. But when L reaches to 8, the performance of the stochastic policy is reduced, and the optimal *threshold* policy is also the best.

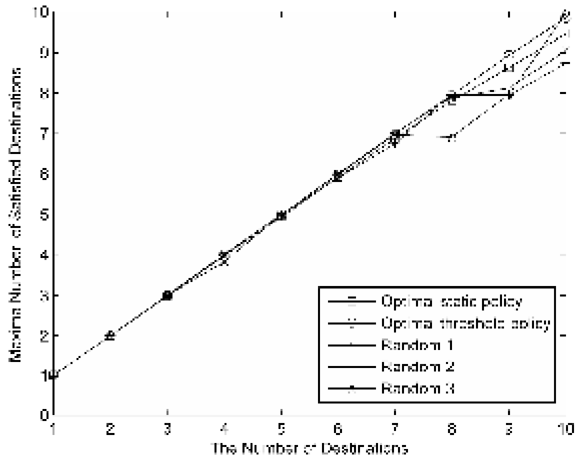

Fig. 4. Performance with different destination number

4.4 Impact of Different Requests

In this section, the paper only considers the optimal *threshold* and *static* policies. Let the parameter L =10, T=10 and other parameters keep invariance, and the minimal number of destinations that must be satisfied is increased from 1 to L, with the different request, this paper explores the minimal energy consumption to finish the task. Fig.5 shows that the optimal *threshold* policy used less energy.

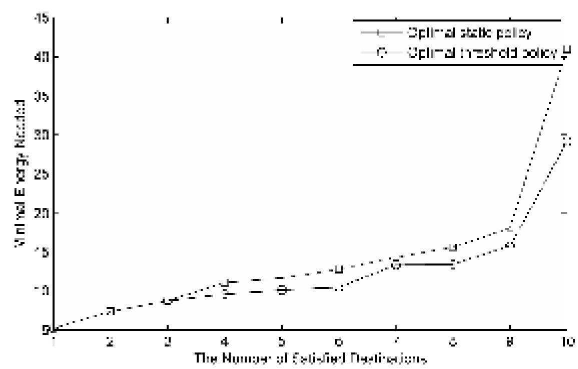

Fig. 5. Minimal energy consumption with different requests

From above numerical results it can easily see that the optimal *threshold* policy is the best.

4.5 Impact of the Forwarding Probability

This section will explore the monotone increasing property of *E(X(T))* with *p(t)*. Because there are infinite stochastic orders of *p(t)*, it only considers some special cases, that is, *p(t)* keeps invariance in the same policy (called *equal probability* policy), for example, $p(t)=c1(0\le c1\le 1$ is a constant). It is worth mentioning that this policy is different from the *static* policy above, because in *static* policy, $p(\delta_t(j)=1) = p(t)p(e_{ij}(t)=1)= p$, but in *equal probability* policy though *p(t)* keeps invariance, $p(e_{ij}(t)=1)$ is different in different slots, because $p(\delta_t(j)=1)$ is changing all the time.

c1 is increased from 0 to 1 and *T* from 1 to 20. Because here the objective is to study the monotone increasing property of *E(X(T))*, there is no need to constraint the maximal energy, and other parameters are the same as the default ones. According to (6) and (7), Eq.(16) can be got,

$$\begin{cases} p(e_{sj}(t)=1)=\dfrac{p(e_{sj}(t-1)=1)(1-p(t-1))(1-\alpha-\beta)}{1-p(e_{sj}(t-1)=1)p(t-1)}+\alpha \\ p(e_{sj}(0)=1)=\pi_1 \end{cases} \quad (16)$$

Combined with (8), the numerical result can be got and is shown in Fig.6.

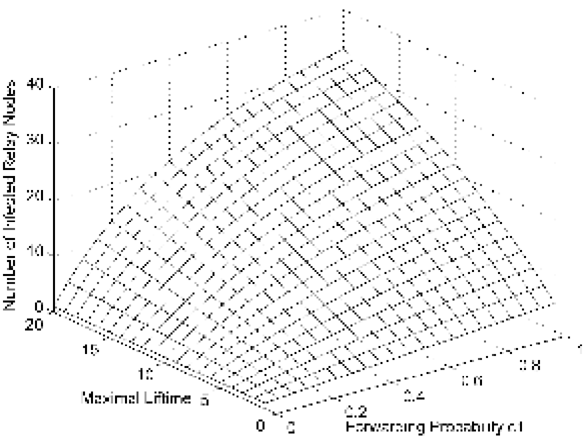

Fig. 6. Number of Infected Relay Nodes with Different Forwarding Probability and Maximal Lifetime

From the figure people can see that *E(X(T)))* increases with the increasing of the forwarding probability under fixed *T* and this matches our expectation. Fig.6 also shows that under fixed forwarding probability the bigger of the maximal lifetime of message, the more nodes will be infected. Now set *T*=10, the paper gets Fig.7.

Obviously, *E(Y(T))* increases with forwarding probability, and this proves Lemma 5, further. When *c1* reaches to about 0.16, *E(Y(T))* has already reached to its maximal value *L*, in this situation, if forwarding probability is keep on increasing , it only wastes energy foolishly, so the optimal *equal probability* policy exists and is near this value. Next, this paper will check the model through simulation using (The Network Simulator NS-2), and it uses the Rollernet trace (P. – U. Tournoux, 2009) where the parameters of the graph are originated from.

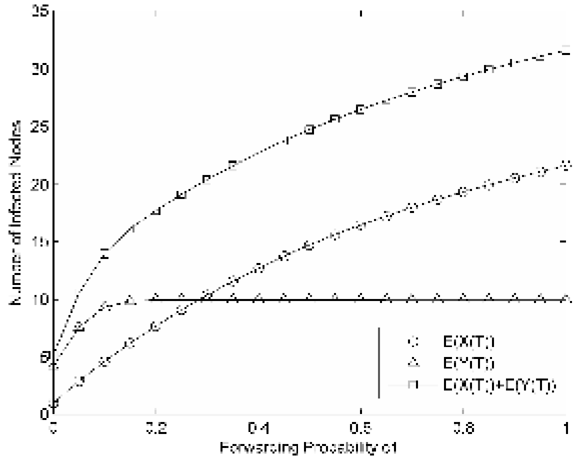

Fig. 7. Performance with different forwarding probability

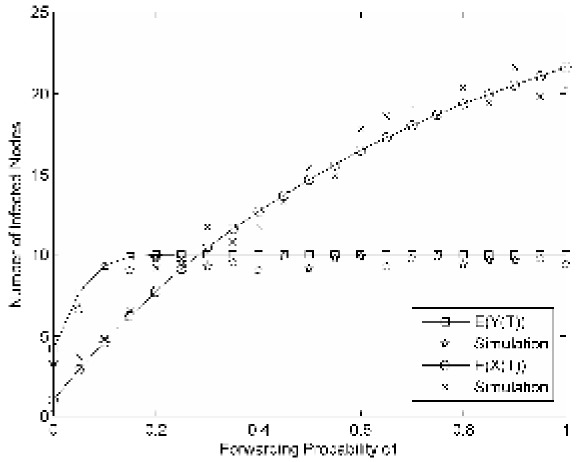

Fig. 8. Theoretic and Simulation Results

Because the number of polices is infinite, here, the simulation only adopts the *equal probability* policy, the result is shown in Fig.8. So the theoretical model fits the simulation result very well, and this proves that this model is right. In the future, more simulations will be carried out with different data sets and different policies.

5. CONCLUSIONS

This paper studied the stochastic control problem of Two-hop routing in NCPS. The NCPS was modelled as an Edge-markovian graph, and based on it, this paper studied the evolving process of Two-hop using the discrete time Markov process, then it explored three control polices, theoretical result shows that the optimal policy is *threshold* form. At last, it introduced the numerical results and checked the model through some simulation.

In many applications, most of the nodes in NCPS are heterogeneous, and it is interesting to explore this optimization problem with multi-class nodes in the future.

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