Effects of Time Delay Statistical Parameters on the Most Likely Regions of Stability in an NCS

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Abstract: In view of the fact that real networked control systems (NCSs) with uncertain events and traffic are subject to violation of the nominal conditions used in the design and analysis of their behavior, it is essential to consider (somehow) the probability of these conditions being (or not) fully satisfied. This paper focuses on practical issues and discusses the effect of the distributions of delays on the determination of the most likely stability regions to be considered in the design of a NCS. An alternative methodology for the analysis of NCSs is proposed here for implementation in real cases. The methodology is based on the Monte Carlo method coupled with a sorting algorithm and a gradient search using the analysis of time responses. The proposed method is compared with a traditional technique based on the polytopic overapproximation method to provide the maximum possible delays. The obtained results yielded less conservative stability limits, aligned with the tendency to use adaptive control techniques appropriate for practical NCS applications. Experimental results show that the upper bound of the delays that preserves the stability may be higher than the corresponding bound for the constant delay. Case studies presented in the paper show that the most likely region of stability of the considered NCSs is heavily dependent on the time-delays distributions. The obtained reduction of conservativeness warrants further investigation on method reliability in other cases.

Keywords: Monte Carlo method, Networked Control Systems, NCS stability, NCS modeling, probability distribution, variable delays.

1. INTRODUCTION

In networked control systems, NCSs, the field elements (sensors and actuators) are connected to the controllers by means of a data communication network. As shown in Fig. 1, an NCS can present, besides the dead-time of the plant (τ_p), delays in the sensor data acquisition (τ_s), in the execution of the control algorithm (τ_c), in the actuation (τ_a) and in the message transmissions between the sensor and the controller (τ_{sc}) and between the controller and the actuator (τ_{ca}). A control loop closed over a data communication network has to cope with time-delays, delay variations, sampling irregularities, corrupted data and packet losses. In NCSs, there is, also, the difficulty in establishing a time reference due to synchronization problems between the clocks of each network element.

Further, in a real NCS, the delays vary over a very wide range, appearing in both control signal and feedback measurement links, i.e., in the input and output of the plant. Being a time-dependent function it is of paramount importance the evaluation of how the dynamic variation of the delays interferes in the behaviour of the closed-loop. Depending on the traffic conditions, the NCS delays can exhibit large variations with very different statistical characteristics. The delays can present different distributions with time-varying parameters, e.g., presenting time-dependent mean and variance. The occurrence of variable delays causes uncertainties in the representation of the loop dynamic. These uncertainties increase the complexity of the analysis and synthesis of the controllers, as well as the difficulty in simulating the real conditions by means of computational algorithms.

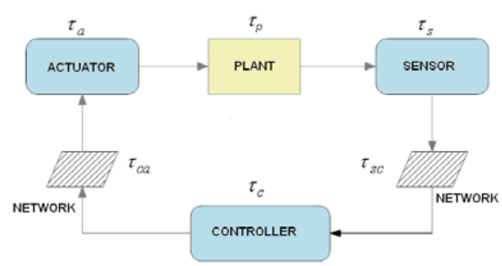

Fig. 1. NCS Architecture.

An important issue in NCS is the way that the delays and the corresponding dynamic behaviours are modelled. The great difficulty is to establish a realistic model with an integrated incorporation of the delay dynamic behaviours and all the phenomena present in a real NCS.

The model should be feasible for purposes of controller synthesis and also analyzable by the available stability methods. Some modelling of NCS has been performed elsewhere based on continuous time (Gao *et al.,* 2008), discrete time (Heemels *et al.,* 2010; Stefan *et al.,* 2011) or hybrid models (Chen and Zheng, 2011).

Recently, the implementations of control strategies more appropriate for NCSs are calling an increasing attention of many researchers. Among the most frequently mentioned control strategies, the following groupings could be highlighted: (i) classical techniques, such as PID and its derivations (P, PI, PD) (Khan *et al.,* 2008) coupled or not with Smith predictors with some sort of adaptive feature (Du and Du, 2009) (ii) advanced strategies such as predictive/adaptive control laws (Jota, 1987; Pin and Parisini, 2011; Caruntu and Lazar, 2011; Tejado *et al.,* 2010; Tian *et al.,* 2012), robust H_2 and H_∞ (Wu *et al.,* 2010; Gao *et al.,* 2008), with explicit compensation of the control signal (Martins and Jota, 2009), event-based control (Wang and Lemmon, 2011), using switched systems theory (Donkers *et al.,* 2011; Ma *et al.,* 2009), Youla parameterization (Goodwin *et al.,* 2008), (iii) artificial intelligence schemes using neural networks theory (Zhang and Wang, 2010), controllers based on fuzzy logic (Hanchevici *et al.,* 2012) or genetic algorithms to tune the PID parameters (Hanchevici and I. Dumitrache, 2012), (iv) stochastic control using optimal stochastic control (Nilson, 1998), hybrid Kalman filter (Stoica, 2012), to cite a few. Under realistic conditions, to guarantee satisfactory performance, it is imperative to consider the adoption of advanced control strategies (with some sort of adaptive mechanism, compensation techniques for the delays effects and mitigation of packet losses, or even a proper switching control scheme). Nevertheless, these sophistications add more complexity to the representation and stability analysis.

Another challenge in NCS is how to obtain more realistic stability regions, due to the conservatism and the limitations of the analysis tools for real applications. It is important that the analysis tool be flexible concerning the adopted controller structure and feasibility (in terms of execution time and numerical conditioning), especially, when the number of uncertainties (inherent to real NCSs) increases.

Several stability analysis studies of NCSs published in the literature (Heemels *et al.,* 2010; Dritsas and Tzes, 2010; Gao *et al.,* 2008 and internal references) are based on Lyapunov theory. The major limitations of this approach for application in real NCS (besides the difficulty in finding the candidate function) are the inherent conservatism. Especially if the candidate functions do not consider the specific characteristics of the delay dynamic behaviour and the probability of uncertain events (present in real NCSs, such as, varying delays, packet loss, unpredicted disconnections, etc) occurring or not. In the traditional Lyapunov-based analysis of systems with uncertain and bounded time-varying delays, stability cannot be guaranteed for delays with the upper bound greater than the analytical maximum constant delay (Liu and Fridman, 2009). However, in many particular systems (e.g. NCS with varying delays) the upper bound that preserves the stability may be higher than the corresponding bound for the constant delay (Liu and Fridman, 2009) (see discussions on quenching phenomena in (Louissel, 2001; Papachristodoulou *et al.,* 2007 and internal references).

Probabilistic approaches for the analysis of NCS stability was proposed in (Peng *et al.,* 2009; Yue *et al.,* 2009). In (*Peng et al.,* 2009) a criterion based on a robust formulation was derived considering the probability that the delays occur in different time scales. Also in (Peng *et al.,* 2009), the synthesis condition was derived and gains in order to 10^{-6} were obtained. The small gains may allow an increase in the maximum delay as the expense of a significant performance restriction especially in real applications. Another problem is how to represent a real NCS with a relatively large number of uncertainties, switching structures and other advanced control structures (with, e.g., adaptive compensation technique), in a viable manner to implement these methods.

Generally, in the analysis and synthesis methods presented so far in the literature, stability has been formally guaranteed for relatively limited delay variations and packet losses. However, in practical applications of NCS, with uncertain events and traffic conditions, it is not possible to have an absolute guarantee that the delays and packet losses are, at all times, within the a priori established maximum and minimum limits. Thus there is always a probability that the real system will become unstable. In practice, NCSs are subject to possible violations of the nominal conditions used in the design. Thus, for a realistic analysis, it should be considered (somehow) that these conditions may (or may not) be fully met.

This paper intends to shed some light on the aforementioned issues by providing an alternative approach for the analysis of networked control systems for applications in practical cases. The proposed methodology is based on the well known Monte Carlo method (Metropolis and Ulam, 1949) and, under those conditions, provides the most likely stability regions. The intended task of this work is to evidence the importance of considering the probability distribution function of the delays in the NCS design. In this paper the effect of the variable delays (on the determination of the stability regions used for design of the NCS) is considered. The proposed method has great flexibility for the analysis of the effects, such as fluctuations in the sampling period and packet losses, as well as the control strategy adopted. This method has additional advantages for the operation of the real process, since higher delay limits (compared to those obtained by traditional methods based on polytopic approaches) can be applied without destabilizing the system, depending on the form in which delays occur.

The paper shows that the most likely region of stability strongly relies on the time-delay distributions and consequently on time-delay statistical parameters. Depending on the type of the delay distributions (Gaussian, exponential, uniform, etc.) and their corresponding statistical parameters (mean, variance standard deviation, etc.), different regions of stability are achieved. In this paper, the maximum delay, that the NCS can cope with, has been used as a metric to evaluate the region of stability because the delay is one of the most critical variables in control loop closed over a network. The delays are considered uncertain and time varying (as observed in real NCSs).

2. PROPOSED METHOD

The goal of the proposed method is to systematically evaluate the possible behaviours of the closed-loop system subject to uncertainties by analyzing its time response. This will permit evaluating the applicability of the various control techniques for solving the problems commonly encountered in NCS. The methodology, based on the Monte Carlo method coupled with a sorting algorithm (Sedgewick, 1978) and a gradient search (Yuan, 2008), intends to provide less conservative limits of stability or, equivalently, the most likely ones. As the accuracy of Monte Carlo algorithms depends on representativeness of random samplings, the best results are achieved by increasing the number of samples. The most likely stability regions can be probabilistically guaranteed with a finite (relatively large) number of samples.

In this proposed method, one of the uncertain (or variables) parameters present in a NCS is represented as varying randomly (with possible abrupt changes) and the others vary with relatively slow variations within predefined ranges. All combinations of the parameters with slow variations (within the predefined ranges) are considered. The uncertain parameter, with possible abrupt changes, randomly generated, is applied in the model of interest (for all combinations of the other uncertain or variable parameters with slow variations) and the corresponding time response of the closed loop system is analyzed. The analyses of the time responses start by searching the first two peaks of the time response of the closed-loop system. However, it should be similarly expanded, whenever necessary, to three, four or more periods so as to improve the accuracy of the obtained value.

Fig. 2 shows a flowchart of the core of the proposed method, for a given combination of the uncertain (or variables) parameters with slow variations. In this flowchart, each element of the uncertain parameter vector **θ** represents one of the possible uncertainties (or variables parameters) present in the NCS (θ_j, with j=1,..., m), $\theta_1(l)$ is the value of the random uncertain parameter evaluated in the l-th sample (l=1,…, n), $\overline{\theta}_1(i)$ represents the maximum achieved value of θ_1 in the i-th iteration (i=1,…, i_{max}), $\theta_{1f}(i)$ is the filtered value of $\overline{\theta}_1(i)$ and α_1 is the chosen resolution. At each i-th iteration, n samples (l=1,…, n) of the closed loop time response subject to the uncertainties is evaluated. In the case studied in this paper, the time delay is considered a random uncertain parameter with abrupt variations and the controller gains are considered adjustable (adaptive) parameters with slow variations. The uncertain parameter (θ_1) is randomly generated with a given distribution (such that $\theta_{1min} < \theta_1(l) < \theta_{1max}$) and the other parameters ($\theta_2,\ldots,\theta_m$) are swept, with slow variations, in the considered ranges, for all allowable combinations. The initial value of $\overline{\theta}_1(1)$ is also randomly generated. The value of $\overline{\theta}_1(i)$ is incremented or decremented in order to converge towards the limits of stability, first, using a sorting (Quicksort) algorithm (Sedgewick, 1978) and, later, a gradient search (Yuan, 2008). Initially, the Quicksort algorithm is used to quickly come close to the stability border region; then an algorithm based on the gradient method (Yuan, 2008), is used to achieve better accuracy. The Quicksort algorithm ends when the value of $\overline{\theta}_1(i)$ returns stable and unstable responses sequentially for four consecutive iterations, indicating that the stability border has been found. Subsequently, an algorithm based on the gradient method is applied to obtain the necessary convergence. In the algorithm based on the gradient method, it is considered that the derivative of the function of variation of $\overline{\theta}_1(i)$ is constant and is scaled by a predefined factor, δ.

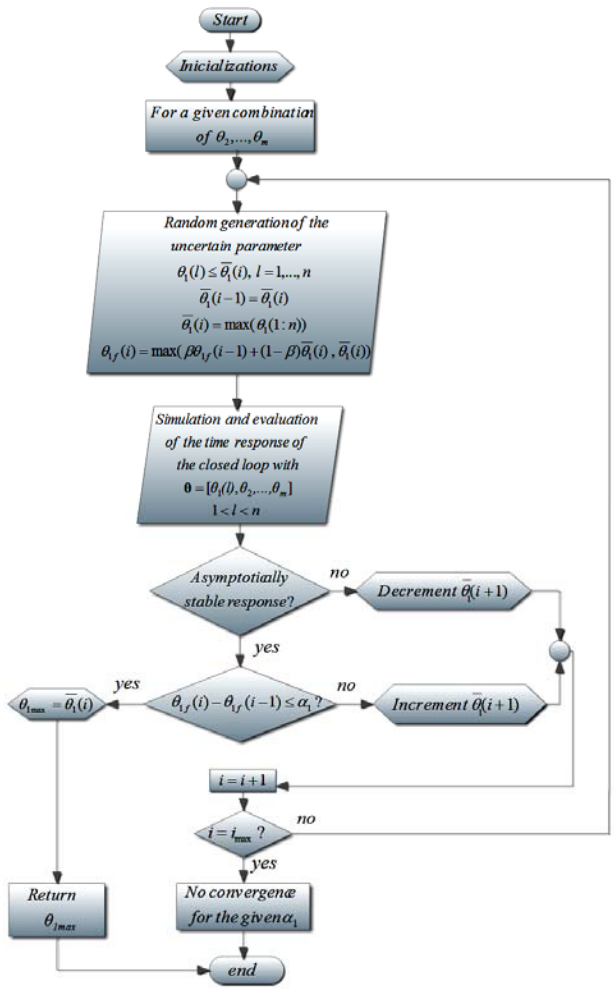

Fig. 2. Flowchart of the proposed algorithm core (starting with a given combination values of the slow varying parameters).

The amplitudes of the first and second peaks are initially considered in the analysis of the time response of the closed loop. If the amplitude of the second peak is higher than that of the first one (indicating a possible unstable response), the value of the upper limit random uncertain parameter $\overline{\theta}_1(i)$ is decreased in the next iteration ($\overline{\theta}_1(i+1) = \overline{\theta}_1(i) - \delta_1\overline{\theta}_1(i)$). If the analysis of the first peak indicates a damped response, the upper bound of the random uncertain parameter $\overline{\theta}_1(i)$ is incremented in the next iteration ($\overline{\theta}_1(i+1) = \overline{\theta}_1(i) + \delta_1\overline{\theta}_1(i)$). It is expected that, at least asymptotically, the simulations tend to indicate the threshold of the random uncertain parameters that will lead to stable responses (for all possible combinations of the other uncertain or variable parameters with slow variations in the considered ranges). The proposed algorithm introduces a forgetting factor with an appropriately chosen time-window (in this paper, it is considered 10% of the maximum number of iterations). Thus, whenever the maximum achieved value, $\bar{\theta}_1(i)$, exceeds the "filtered value", $\theta_{1f}(i)$, the filter is restarted with $\theta_{1f}(i)=\bar{\theta}_1(i)$; otherwise $\theta_{1f}(i)=\beta*\theta_{1f}(i-1)+(1-\beta)*\bar{\theta}_1(i)$.

The stopping criterion is established based on the difference between the maximum achieved filtered value up to the current iteration, $\theta_{1f}(i)$, and the one calculated at the previous iteration $\theta_{1f}(i-1)$. If, after a certain number of iterations (according to the chosen time-window), the difference $\theta_{1f}(i)-\theta_{1f}(i-1)$ is not greater than the chosen resolution α_1 then the algorithm is terminated. It is important that the value of α_1 be compatible with the magnitude of $\bar{\theta}_1$. Resolution α_1 is chosen proportional to the sampling period, since this is a good metric to relate the magnitude of the delay. At the exit, the upper limit of the maximum uncertainties for the most likely stability region of the NCS in question is determined. The criterion allows a systematic analysis of possible behaviours of the NCS subject to uncertainties. Thus, it is expected that it be more suited for controller design in practical applications.

Since delay is the most important uncertain parameter in an NCS, a numerical example with randomly generated uncertain delays will be presented in Section 3. A comparison is made with other methods available in the literature.

3. CASE STUDIES

3.1. Simulated case study

In the next analysis, the model extracted from (Heemels *et al.,* 2010) is used. This model is a well-known benchmark example frequently used in the NCS literature. It has been chosen to give a basis for comparison with other methods. The plant is represented by:

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}0\\ 0.1\end{bmatrix}\mathbf{u}(t-\tau(t)) \qquad (1)$$
$$\mathbf{y}(t)=[1 \quad 0]\mathbf{x}(t)$$

The loop is closed through a state feedback controller as in (2).

$$\mathbf{u}(t)= [K_1 \quad K_2]\mathbf{x}(t), \qquad (2)$$

K_1 and K_2 beings the gains of state feedback. In this case study, the uncertain parameter vector $\boldsymbol{\theta}=[\tau, K_1, K_2]^T$, where τ is a random uncertain parameter, K_1 and K_2 are adjustable parameters. K_2 is varied in the range of $1 < K_2 < 20$ and K_1 is fixed at 3.75 (as in Heemels *et al.,* 2010). The variable delays τ are randomly generated with gaussian and uniform distributions (such that $\tau_{min} < \tau(i) < \tau_{max}$). The resolution used was α_1=0.005 and the value of the factor δ_1=0.5%.

An NCS architecture, as depicted in Fig.1, is considered to analyze the effects of the delay statistics and to determine stability regions. In this architecture it is assumed that the network lies between the controller and the actuator (that is, $\tau_{ca}=\tau$ and $\tau_{sc}=0$).

In order to emulate the (variable) delay, an oversampled discrete-time model (with the main sample period h=1s and the secondary sample $h_2=h/200$) has been used to generate the random delays at each time instant of intersampling, $t=k*h_2$ $(h_2<<h)$, considering the delays as integer multiples of h_2. The equivalent discrete model (with h_2) used in the simulations is given by:

$$\mathbf{x}(k+1)=\begin{bmatrix}1 & 0.005\\ 0 & 0.995\end{bmatrix}\mathbf{x}(k)+\begin{bmatrix}0.0000012\\ 0.0004999\end{bmatrix}\mathbf{u}(k-\tau(k)) \qquad (3)$$
$$\mathbf{y}(k)=[1 \quad 0]\mathbf{x}(k),$$

where the delays $\tau(k) \in [0, \tau_{max}]$. The discretization has been accomplished using a zero-order hold (ZOH) and the initial condition of the state vector used in the simulations was $x_0=[1\ 0]^T$.

Fig. 3 shows the values of maximum variable delays as a function of speed gain, K_2, obtained using the proposed method subject to delays with gaussian and uniform distributions. The results are compared to those given by a method called "Gridding Method" presented in (Heemels *et al.,* 2010) which is based on polytopic overapproximation using the gridding idea with bounded norm. For completeness, the outcomes of the Nyquist method are also superimposed in Fig. 3.

It is worth noting that the maximum delay to which this NCS can probabilistically be submitted is greater than the maximum fixed delays. This seemingly surprising result can be explained by the "compensation" effect caused by the variations of the delay at each sample (a positive delay variation can be followed by a negative one, and so on). Due to variations of the delays, the duration of the effective control signal applied to the plant is, in fact, variable (and, on average, much shorter than the maximum one). As a consequence, when designing networked controllers for real applications, if only fixed (maximum) delays are taken as actual conditions, the results will be far more conservative than necessary. This implies a significant reduction of the performance or may even indicate the non-existence of a feasible controller for the (nominal) considered conditions.

Fig. 3 also highlights, comparatively, the delay values obtained by each of the aforementioned methods. The run times of the methods were: Nyquist 0.3s, Gridding 952s, proposed method with uniform delays 691s and with gaussian delays 699s. These tests confirm the viability of the proposed method in terms of runtime. As shown in Fig 3, the maximum limits obtained by the proposed method with gaussian distribution delays were higher than those obtained with uniform distribution delays. This reflects the importance of analyzing the effects of delay distribution in the closed loop response of the system for an appropriate design.

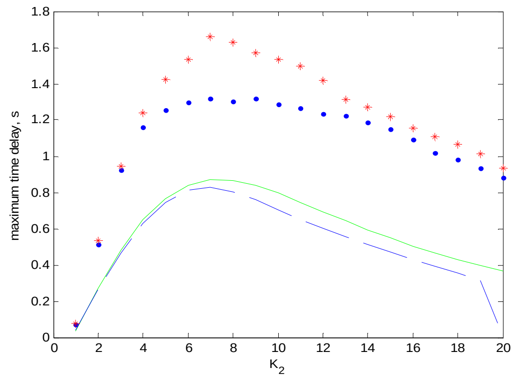

Fig. 3. Comparison among the most likely stability regions: Gridding (dashed line), proposed method with gaussian variable delays (stars), proposed method with uniform variable delays (dots) and Nyquist with constant delays (continuous line).

To analyze the effects of the delay statistics in the closed loop behaviour and so to demonstrate one of the possible application of the proposed method, system (3) was used, with the state feedback controller (2), as in Heemels et al. (2010). Two extreme values of one of the uncertain (variable) parameters were chosen, namely $\boldsymbol{\theta}=[\tau, 3.75, 1]^T$ and $\boldsymbol{\theta}=[\tau, 3.75, 8]$.

Fig. 4 illustrates the simulated results for $\boldsymbol{\theta}=[\tau, 3.75, 1]^T$ with 200 samples where the maximum delay obtained is τ_{max}=0.075s. Fig. 4 (a) presents the profile of the simulated delays with uniform distribution as a function of the sample number. Fig. 4 (b) shows the histogram with the number of occurrences of delays between 0 and 0.075s. Fig. 4 (c) illustrates a typical time response of the closed-loop system simulated with the delays depicted in Fig. 4 (a). The (critically damped) oscillatory response indicates that the system is operating at the stability threshold as predicted by the algorithm. By using the Nyquist criterion (with a given fixed delay), for one of these conditions, i.e., $\boldsymbol{\theta}=[\tau, 3.75, 1]^T$), the maximum achieved value, τ_{max} , is equal to 0.033s.

Fig. 5 shows the simulated results for $\boldsymbol{\theta}=[\tau, 3.75, 8]^T$ with gaussian distribution. Although the value of the maximum delay returned (τ_{max}=1.655s) was greater than that obtained from the Nyquist criterion (τ_{max}=0.865s) and from the Gridding method with variable delays (τ_{max}=0.803s), the result of Fig 5 is also conservative, since the response is well damped in this condition. To obtain a less conservative result it is important to expand the evaluation beyond the second peak in the time response, since all responses with increasing initial oscillation are classified by the method as unstable (even if the convergence occurs afterwards).

Table 1 highlights the values of maximum delay, average delay, variance of the delays and the time spent repeating the same algorithm 10 times for $\boldsymbol{\theta}=[\tau, 3.75, 8]^T$ (runs #1 to #10) with uniformly distributed random delays. Table 2 presents the same parameters for delays with gaussian distribution. Note that, in all tests, the obtained average delay is shorter than the maximum delay obtained by the Nyquist criterion (τ_{max}=0.865s).

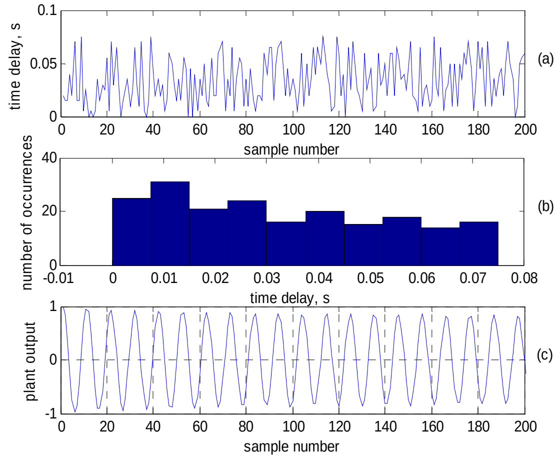

Fig. 4. Stability threshold to (3) with controller (2) ($\boldsymbol{\theta}=[\tau, 3.75, 1]^T$): (a) delays applied with uniform distribution (b) number of occurrences of delays and τ_{max}=0.075s (c) time response of the closed-loop.

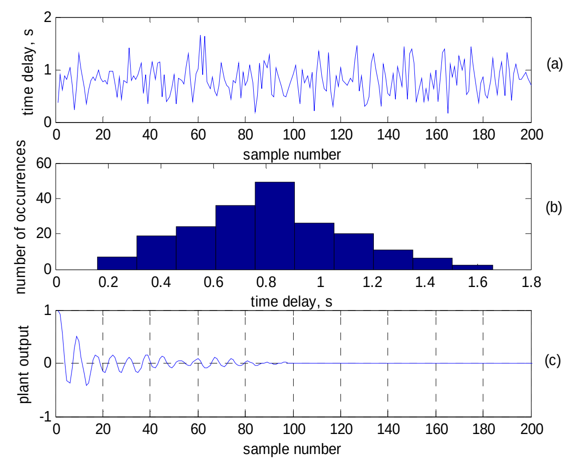

Fig. 5. Stability threshold to (3) with controller (2) ($\boldsymbol{\theta}=[\tau, 3.75, 1]^T$): (a) delays applied with gaussian distribution (b) number of occurrences of delays and τ_{max}=1.655s (c) time response of the closed-loop.

Fig. 6 presents a surface plot of the region of stability obtained for the simulated example with three uncertain (variable) parameters $\boldsymbol{\theta}=[\tau, K_1, K_2]^T$, where τ represents the random uncertain parameter (with uniform distribution) and K_1 and K_2 the variable (adjustable) gains (in the range of 1 to 20). The surface shows the limits of the variable delays such that the system remains stable as a function of the parameters K_1 e K_2 (which can be adjusted, for example, according to the delay variations to ensure stability or performance conditions using an adaptive controller). The maximum value obtained for the random delays with uniform distribution was 1.775s for the parameters K_1=1 and K_2=5. This surface can be useful, for instance, as a metric in adaptive control strategies.

Table 1. Numerical simulation results with θ=[τ, 3.75, 8]^T and uniform distribution.

Run #	Run time (s)	Initial $\bar{\theta}_1$	Average Delay	Variance	τ_{max}
1	27.676	6.649	0.623	0.132	1.300
2	19.092	5.266	0.641	0.135	1.290
3	18.068	1.302	0.663	0.132	1.265
4	42.885	2.492	0.697	0.130	1.270
5	21.033	2.826	0.689	0.146	1.370
6	40.927	4.798	0.685	0.137	1.305
7	30.456	7.401	0.659	0.149	1.325
8	27.110	1.666	0.633	0.133	1.300
9	23.391	5.953	0.640	0.147	1.305
10	43.960	2.599	0.644	0.149	1.320
Ave_ rage	29.460	4.095	0.657	0.139	1.305

Table 2. Numerical simulation results with θ=[τ, 3.75, 8]^T and Gaussian distribution.

Run #	Run time (s)	Initial $\bar{\theta}_1$	Average Delay	Variance	τ_{max}
1	31.023	8.607	0.807	0.087	1.655
2	25.884	3.908	0.782	0.104	1.675
3	33.152	9.270	0.783	0.096	1.665
4	45.409	1.538	0.755	0.091	1.625
5	36.140	5.104	0.781	0.086	1.570
6	32.866	9.564	0.817	0.095	1.700
7	46.016	6.499	0.815	0.088	1.650
8	42.087	5.595	0.723	0.093	1.565
9	36.975	5.512	0.779	0.106	1.505
10	30.834	8.688	0.773	0.090	1.660
Ave_ rage	36.039	6.429	0.782	0.094	1.627

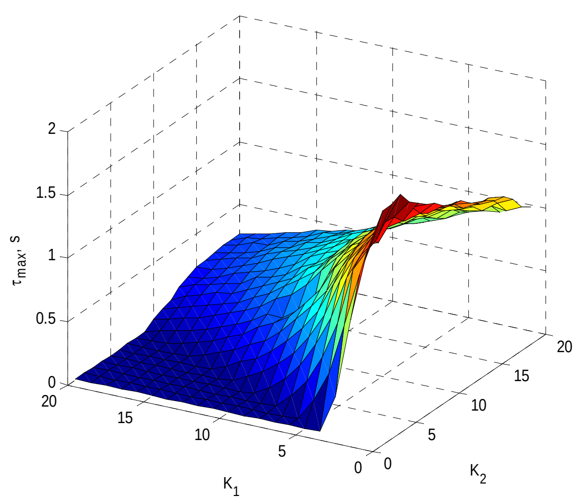

Fig. 6. Stability region with **θ**=[τ, K_1, K_2]^T, being the τ random uncertain parameter, K_1 and K_2 being varied in the range of 1 to 20.

3.2. Real case study

The proposed method has been applied in a real case study using an NCS implemented with a control and monitoring platform called NCS-CMUF (Jota *et al.,* 2011). Typical characteristics of NCS, such as maximum delays and the number of packet droupts, can impose severe limitations on performance of the control loops (Peserchini and Jota, 2013; Baillieul and Antsaklis, 2007). When a real time system is implemented, it is essential that the hard time constraints, where the computer system must react within specific time bounds (Baillieul and Antsaklis, 2007), be evaluated and properly tackled. In the proposed platform the algorithms are capable of dealing well with the time constraints and guarantee the performance of the closed loop control for systems sampled in order of tenths of seconds.

A first order plant (with time constant equal to 32 seconds) was controlled using a PI Controller (h=5s, T_i=5.1s and K_c=1). It is considered that the sensor is time-driven and the controller and the actuator are event-driven. In this platform, the messages have timestamps and the control loop is closed with four levels of the networks: CAN (Controller Area Networked) network; RS-232 serial and two levels of Internet, as shown in Fig. 7. Table 3 presents the values returned for the proposed method repeating the same algorithm 10 times (with exponential distributed random delays, this distribution is most similar to the measured real delays (Batista, 2011).

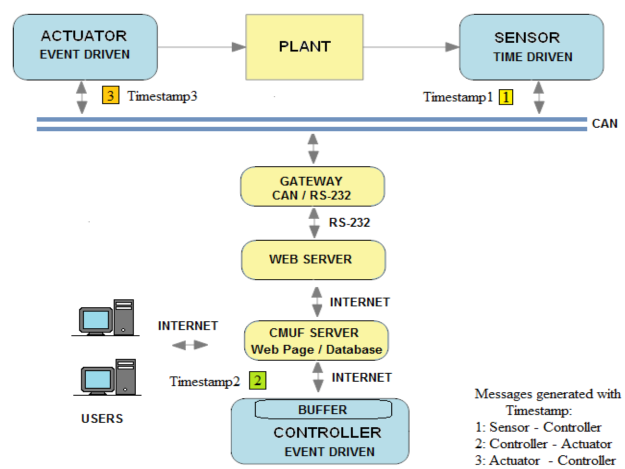

Fig. 7. CMUF architecture.

Fig. 8 shows the actual measured data (at an interval of more than 100 hours and controlling in real time) of the plant output and input, the time delays and histogram with number of occurrences of the delays. The time delays were measured between sensor and actuator messages (τ= timestamp3 – timestamp1 = τ_{sc}+ τ_c+ τ_{ca}+τ_a). The values of the maximum delay (equal to 236ms) and the medium delay (equal to 14.23ms) are very smaller than the delays listed in Table 3 for the stability limit. As an alternative to obtain longer delays (in the order of seconds, as listed in Table 3), an extra lag has been introduced. A delay buffer has been used in the controller to hold the messages before they are sent to the actuator, thus generating a longer delay (τ_{Buffer}).

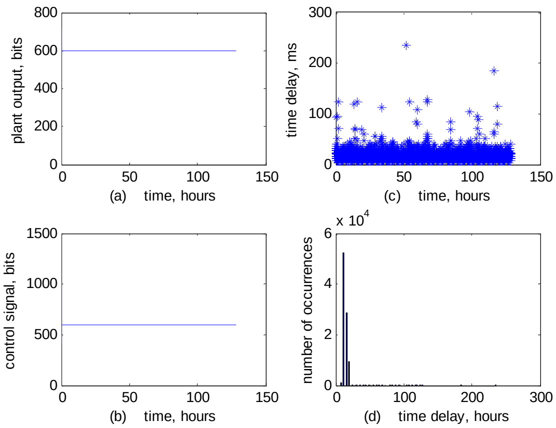

Fig. 8. Real case study: (a) plant output (b) control signal (c) time delays (d) histogram with number of occurrences of the delays.

Fig. 9 shows the measured real data of the plant output and input, the time delays (with buffer of delays generated with exponential distribution) and histogram with number of occurrences of the delays. The values of the maximum delay (equal to 22.580s) and the medium delay (equal to 4.419s) are similar to the medium values returned by the method proposed. The maximum delay (22.580s) applied in the real NCS is greater than the obtained by the Nyquist criterion with a given fixed delay (7.160s in this case) and the measured plant output confirms the stable response.

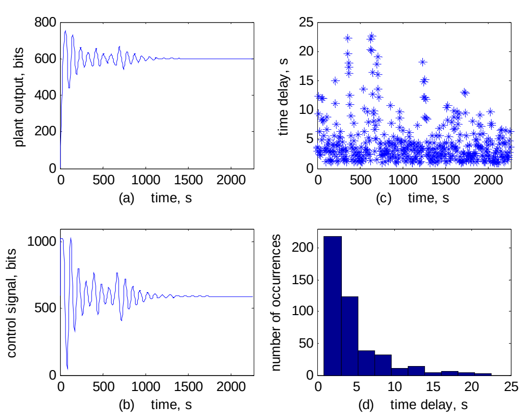

Fig. 9. Real case study with buffer to increase the delays: (a) plant output (b) control signal (c) time delays (d) histogram with number of occurrences of the delays. The time delays were also measured between sensor and actuator messages (τ= timestamp3 – timestamp1 = $\tau_{sc}+\tau_c+\tau_{Buffer}+\tau_{ca}+\tau_a$).

The proposed method results in values closer to the instability limit (e.g., $\mathbf{\theta}=[\tau, 3.75, 1]^T$, Fig. 4), and in some conditions (e.g., with higher gain $\mathbf{\theta}=[\tau, 3.75, 8]^T$, Fig. 5) it can present a degree of conservatism. However, even with its inherent conservatism, the proposed method provides a stability region (Fig. 3) greater than the current methods and hence may be more suitable for applications in which adaptive control techniques are employed. Furthermore, it allows showing the impact of the delay statistics (Tables 1 and 2) in the determination of the stability regions. The consideration of the statistics of the delays has not yet received due attention in the literature and how this information could be effectively used in the analysis and synthesis of NCSs is an open problem. These issues were recently addressed in (Peng *et al.,* 2009). In future work, it is intended to validate the proposed methodology in a real case using an NCS-CMUF platform (Jota *et al.,* 2011), which is being installed in the Blood Center of Belo Horizonte for monitoring and control of blood component refrigerators.

Table 3. Numerical simulation results with exponential distribution for the real case study.

Run #	Run time (s)	Initial $\bar{\theta}_1$	Average Delay	Variance	τ_{max}
1	17.216	82.150	3.968	12.252	22.575
2	12.931	33.300	4.126	12.061	21.150
3	19.620	67.900	3.777	10.757	19.475
4	9.304	112.350	3.797	12.411	23.575
5	12.480	15.225	3.667	12.245	22.100
6	8.251	44.725	4.054	12.986	21.575
7	6.925	108.575	4.097	13.189	22.650
8	10.604	27.700	3.846	11.155	24.000
9	7.437	27.850	4.136	14.499	21.600
10	14.124	41.300	4.325	15.566	25.750
Ave_ rage	11.889	56.108	3.979	12.712	22.445

4. CONCLUSION

From the results presented in the paper, it can be concluded that broader stability regions of NCSs can be obtained when compared to those obtained from the currently most used methods. The proposed methodology provides some insight towards the determination of more realistic limits of stability for practical applications. It is expected that it could be useful in the evaluation of the applicability of the various control techniques for solving NCS problems. Using the proposed method, it is possible to analyze the behaviour of NCSs subject to different uncertain (or variable) parameters in an integrated way. It also provides information to evaluate the effects of uncertain parameter statistics on the determination of the most likely regions of stability, with an increase, under certain conditions, of more than 100% in the maximum delay. The use of numerical simulations, based on a Monte Carlo scheme, appears to be an interesting alternative and deserves to be explored for application in NCSs, since the practical limits obtained can be used as a metric in the design of the networked control. Larger sample number and longer time simulations should be used for more precise and reliable results.

The study presented in the paper has made it clear that the stability limits of NCSs depend on the way in which delays occur. In the case of variables delays, values beyond the limits obtained for fixed delays can be supported by NCSs without destabilizing the system. The knowledge of the delay statistical parameters is of paramount importance, since they affect significantly the behaviour of the closed loop response. The delay statistics should, then, be evaluated in real-time and can be used even in adaptive strategies to obtain a better compensation strategy.

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