A NONLINEAR ADAPTIVE CONTROLLER FOR A FED-BATCH FERMENTATION PROCESS

Emil PETRE

Department of Automatic Control, University of Craiova
A.I. Cuza Str. No. 13, RO-200585 Craiova, Romania, E-mail: epetre@automation.ucv.ro

Abstract: *A nonlinear adaptive controller for controlling a nonlinear and time-varying fed-batch fermentation process involving one limiting substrate for biomass growth and product synthesis is proposed. More exactly, in the context of a large class of fed-batch bioreactors for which dynamical kinetics are not exactly known and time-varying and not all the state variables are measurable, for the regulation of the substrate concentration, based on input-output state feedback linearization technique, a nonlinear adaptive control algorithm is proposed. A recursive least-squares algorithm with forgetting factor for the on-line estimation of the unknown parameters is proposed. The stability and convergence properties of this algorithm are proved provided that the regressor matrix is persistently exciting. Computer simulations performed under identical circumstances are included in order to demonstrate the performances of this controller compared to an exactly nonlinear linearizing controller.*

Keywords: *Fed-batch bioreactors, Nonlinear systems, Nonlinear linearizing control, Nonlinear adaptive control, On-line estimation.*

1. INTRODUCTION

The control problem of biotechnological processes is an important practical problem attracting wide attention. The main engineering motivation in applying control methods to such living systems is to improve operational stability and production efficiency.

Fermentation processes that are carried out in perfectely stirred tank reactors are commonly described by a set of ordinary differential equations expressing mass and energy balances. A basic difficulty for the application of modern control techniques to these processes lies in the fact that, in many cases, the models include kinetic parameters, which are highly uncertain and time varying. For these reasons, chemical and biological reactors are often good candidates for the application of adaptive nonlinear control techniques in order to improve system performances in spite of parametric uncertainty [1].

In contrast with continuous stirred tank reactors, which continuously operate in steady state, fed-batch reactors are permanently in a transient regime and therefore offer challenging problems to the control engineer. A fed-batch reactor is initially partially filled with an amount of some of the needed reactants. The other reactants are

then progressively added to the reactor as and when required. The process is stopped when enough products have been accumulated.

Industrial fed-batch stirred tank reactors are traditionally operated in open loop using precalculated feeding patterns and dosage schemes [13]. But in recent years, there has been strong interest in the application of modern control theories to fed-batch fermentation processes. So, for optimization of alcoholic fed-batch fermentation process there were used dynamic programming [2] and nonlinear programming schemes [3].

In this paper we are concerned with the control of a fed-batch fermentation process involving one limiting substrate for biomass growth and product synthesis, with respect to the volumetric feed rate of this substrate. More exactly, in the context of a large class of fed-batch bioreactors for which dynamical kinetics are not exactly known and time-varying and not all the state variables are measured, for the regulation of the substrate concentration, a nonlinear adaptive control algorithm is proposed.

The adaptive controller is based on the nonlinear structure of the process and it is obtained based on input-output state feedback linearization technique [6]. If the process model is linearly parameterized, for the estimation of the unknown parameters a recursive least-squares algorithm with forgetting factor is proposed. The stability and convergence properties of this algorithm are proved through Lyapunov's methods provided that the regressor matrix is persistently exciting.

Another objective is to analyze why and how to apply feeding rate profiles that are on-line calculated in a feedback loop with an adaptive nonlinear control strategy in order to obtain a maximum production of alcohol.

Computer simulations performed under identical circumstances are included to demonstrate the performances of the designed controllers.

The rest of this paper is organized as follows. The process model is presented in Section 2. Section 3 describes the proposed adaptive control algorithm for which the stability and convergence properties are provided. The effectiveness of the proposed control algorithms is presented in Section 4. Concluding remarks complete this paper.

2. MATHEMATICAL PROCESS MODEL

Consider the class of fed-batch fermentation processes involving one limiting substrate for biomass growth and product synthesis that are carried out in stirred tank bioreactors. A representative process from this class is the *alcoholic fermentation bioprocess* whose mathematical model obtained from mass balance considerations is given by the following set of nonlinear equations:

$$\dot{X}(t) = \mu(\cdot)X - X(F_{in}/V) \tag{1}$$

$$\dot{S}(t) = -\nu_S(\cdot)X + (S_{in} - S)(F_{in}/V) \tag{2}$$

$$\dot{P}(t) = \nu_P(\cdot)X - P(F_{in}/V) \tag{3}$$

$$\dot{V}(t) = F_{in} \tag{4}$$

with X the biomass concentration (g/l), S the substrate concentration (g/l), S_{in} the influent substrate concentration (g/l), P the product concentration (g/l), V the volume of the culture medium (l), F_{in} the volumetric feed rate (l/h), μ the specific growth rate (h^{-1}), ν_S the specific substrate consumption rate (h^{-1}) and ν_P the specific production rate (h^{-1}).

The parameters appearing in this description are complicated functions of the variables of interest. The challenge for the control engineer arises from the fact that the analytical modelling of these specific rate functions μ, ν and ν_P is highly uncertain and generally not reproductible from one fed-batch to the next one. Several experiments have been carried out by specialists (biochemists) and the following expressions for the bacterial growth rate has been adopted. Thus, the kinetic terms are given by [3]:

$$\mu(\cdot) = \mu_{\max}\left(1 - \frac{P}{P_m}\right)\cdot\frac{S}{K_S + S + S^2/K_I} \tag{5}$$

$$\nu_P(\cdot) = \nu_{\max}\frac{S}{K'_S + S + S^2/K'_I} \tag{6}$$

$$\nu_S(\cdot) = \frac{1}{Y_{X/S}}\mu(\cdot) + \frac{1}{Y_{P/S}}\nu_P(\cdot) \tag{7}$$

with $Y_{X/S}$ the biomass on substrate yield coefficient, $Y_{P/S}$ the product on substrate yield coefficient and P_m the alcohol inhibition factor. This model takes into account substrate and product inhibition on the growth and the fact that growth and production interact.

Fed-batch fermentation processes have been found to be most effective in overcoming such effects as substrate inhibition, catabolite repression, and glucose effects. In other words, whenever the specific rate of growth μ and/or production are non-monotonic functions of the limiting substrate concentration (as in our case), a fed-batch operation may be superior and it is then necessary to determine the optimal feed rate of substrate [13].

3. CONTROL STRATEGIES

For the presented bioprocess, the *control objective* is to get a large production of alcohol. From the above considerations, it follows that the alcohol production process requires regulation of the substrate concentration S inside the bioreactor at a set point S^* corresponding to a desired biomass specific growth rate by acting on the feeding substrate rate F_{in}.

3.1. Nonlinear inverse dynamic controller

Controller design is made by the input-output linearizing technique. Remember that the input-output linearizing principle [6] consists of the calculus of a nonlinear control law such that the behaviour of closed loop system (controller + process) is the same as the behaviour of a linear stable system.

Firstly, we consider the ideal case where maximum prior knowledge concerning the process is available. In particular, we suppose that the functions μ, ν and ν_P in the model (1)-(4) are completely known and all state variables are available for on-line measurements.

It can be seen that equations (2) in the model (1)-(4) have the relative degree equal to 1 [6], [9]. Assume that for the closed loop system we wish to have the following first-order linear stable dynamics:

$$\frac{d}{dt}\left(S^* - S\right) + \lambda_1\left(S^* - S\right) = 0\,, \quad \lambda_1 > 0 \tag{8}$$

Then, from (2) and (8), the above closed-loop dynamics will be achieved by implementing the following exactly linearizing nonlinear control law:

$$F_{in} = \frac{V}{S_{in} - S}\left[\lambda_1 (S^* - S) + (1/Y_{X/S})\,\mu X + (1/Y_{P/S})\;\nu_P X\right] \tag{9}$$

where S^* is the desired value of S and λ_1 is a design parameter.

The control law (9) leads to the following linear error model:

$$\dot{e}_t = -\lambda_1 \cdot e_t \tag{10}$$

where $e_t = S^* - S$ represent the tracking error. It is clear that for $\lambda_1 > 0$, the error model (10) has an exponentialy stable point at $e_t = 0$.

3.2. Nonlinear adaptive controller

Let us suppose now that the substrate concentration S and the product concentration P inside of the bioreactor are the only measurements which are available on-line and that specific reaction rates μ and ν_P and obviously ν_S are time varying and unknown.

The practical implementation of the regulation law (9) requires the knowledge of the states S and X, and of the specific reaction rates μ and ν_P. Since the state X is not measured on-line and the kinetic rates μ and ν_P are unknown, the regulation law (9) becomes an adaptation regulation law by replacing the true unknown values of X, μ and ν_P by their corresponding on-line estimates provided by a suitable state observer and a parameter estimator.

For the estimation of the state X, independent of the unknown specific reaction rates μ and ν_P, we use an asymptotic observer [10], which can be derived here as follows. Let us define the auxiliary state $\hat{Z}$ as:

$$\hat{Z} = (1/Y_{X/S})X + (1/Y_{P/S})P + S \tag{11}$$

whose dynamics is readily deduced from the model (1)-(3) and is expressed by the following linear stable equation:

$$\dot{\hat{Z}}(t) = -(F_{in}/V)\hat{Z} + (F_{in}/V)\,S_{in} \tag{12}$$

Then, the on-line estimate $\hat{X}$ of the biomass concentration X is calculated from values of Z computed from its dynamical equation (12), as:

$$\hat{X} = Y_{X/S}\left(\hat{Z} - (1/Y_{P/S})P - S\right) \tag{13}$$

The unknown kinetic terms $\mu(S,P)$ and $\nu_P(S)$ in (5) and (6) can be written as follows:

$$\mu(S,P) = S \cdot \rho_1; \quad \nu_P(S) = S \cdot \rho_2 \tag{14}$$

with ρ_1 and ρ_2 parameters considered completely unknown and time varying. This simply expresses that S is a limiting substrate of the reactions and that, in consequence $\mu = 0$ and $\nu_P = 0$ if $S = 0$. Note that for our example:

$$\rho_1 = \mu_{\max}\left(1 - \frac{P}{P_m}\right) \cdot \frac{1}{K_S + S + S^2 / K_I} \tag{15a}$$

$$\rho_2 = \nu_{\max} \frac{1}{K'_S + S + S^2 / K'_I} \tag{15b}$$

which are positive functions of S.

The estimation of parameters ρ_1 and ρ_2 can be performed by an appropriate parameter estimator applied only the dynamics of S and P given by (2) and (3), respectively, which under the above assumptions can be written as follows:

$$\frac{d}{dt}\begin{bmatrix} S \\ P \end{bmatrix} = \begin{bmatrix} -\frac{1}{Y_{X/S}} & -\frac{1}{Y_{P/S}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S\hat{X} & 0 \\ 0 & S\hat{X} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} - \begin{bmatrix} S \\ P \end{bmatrix} \frac{F_{in}}{V} + \begin{bmatrix} S_{in} F_{in} / V \\ 0 \end{bmatrix} \tag{16}$$

This model can be written in a more compact matrix form as:

$$\dot{\xi}_s = K_s G(\xi_s)\rho - D\xi_s + F \tag{17}$$

with ξ_s the selected state vector, K_s the yield coefficient matrix, ρ the unknown parameter vector, F the feed flow rate vector and D the dilution rate, where:

$$\xi_s = \begin{bmatrix} S \\ P \end{bmatrix}; \; K_s = \begin{bmatrix} -\frac{1}{Y_{X/S}} & -\frac{1}{Y_{P/S}} \\ 0 & 1 \end{bmatrix};$$

$$G(\xi_s) = \begin{bmatrix} S\hat{X} & 0 \\ 0 & S\hat{X} \end{bmatrix}; \; \rho = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}; \; D = \frac{F_{in}}{V};$$

$$F = \begin{bmatrix} S_{in} F_{in} / V \\ 0 \end{bmatrix} \tag{18}$$

3.2.1. A recursive least-squares parameter estimator

Based on the reformulation of the model (17) in a linear regressive form as in [1], in this Section, for the estimating of the unknown parameter ρ, we propose a recursive least-square algorithm with forgetting factor.

First, we note that the solution $\xi_s(t)$ of the differential equation (17) may be written as [1], [10]:

$$\xi_s(t) = \Psi^T(t)\rho(t) + \Psi_0(t) + \varepsilon(t) \tag{19}$$

where $\Psi^T(t)$, $\Psi_0(t)$ and $\varepsilon(t)$ are the outputs of the following linear filters [1]:

$$\frac{d\Psi^T(t)}{dt} = -\Omega\Psi^T(t) + KG(\xi) \tag{20a}$$

$$\frac{d\Psi_0(t)}{dt} = -\Omega\Psi_0 - (\Omega + D \cdot I)\xi_s + F \tag{20b}$$

$$\frac{d\varepsilon(t)}{dt} = -\Omega\varepsilon - \Psi^T \frac{d\rho}{dt} \tag{20c}$$

where Ω in a $(n \times n)$ symmetrically stable matrix, arbitrarily chosen and I is the n-dimensional unity matrix. If the matrix Ω is stable, the matrices $\| KG(\xi) \|$, $\| F \|$ and $\| \xi_s \|$ are uniformly bounded and ρ is derivable and its derivative is bounded, then all the filters (20) are uniformly asymptotically stable.

If we define

$$y(t) = \xi_s(t) - \Psi_0(t) \tag{21}$$

then the following linear regressive model describes the dynamic of the process (17):

$$y(t) = \Psi^T(t)\rho(t) + \varepsilon(t) \tag{22}$$

where $y(t)$ is the output, $\Psi^T(t)$ is the regressor, $\rho(t)$ is the unknown parameter that must be estimated and $\varepsilon(t)$ can be interpreted that a perturbed input. Note that the output y and the regressor Ψ^T can be on-line calculated by means of equations (20) from the disponible data of ξ_s, D and F.

For the estimation of the unknown parameter ρ we will propose a recursive algorithm with forgetting factor based on the standard least-squares algorithm [12]. Corresponding to this algorithm, the estimate $\hat{\rho}$ of ρ will be obtained by minimizing of the following integral-squared-error

$$J(\hat{\rho}) = \int_0^t e^{-\lambda(t-\tau)} \left\| y(\tau) - \hat{y}(\tau) \right\|^2 d\tau = \int_0^t e^{-\lambda(t-\tau)} \left\| \Psi^T \left(\rho(t) - \hat{\rho}(t)\right) \right\|^2 d\tau \tag{23}$$

where $\lambda > 0$ is the forgetting factor.

Owing to the linearity of error equation (22), the estimate $\hat{\rho}(t)$ may be obtained directly from the condition

$$\frac{\partial}{\partial \hat{\rho}} J(\hat{\rho}) = -2\int_0^t e^{-\lambda(t-\tau)} \left[\Psi\left(y(\tau) - \Psi^T \hat{\rho}(\tau)\right)\right] d\tau = 0 \tag{24}$$

From (24), the least-squares estimate is given by

$$\hat{\rho}(t) = \left[\int_0^t e^{-\lambda(t-\tau)} \Psi(\tau)\Psi^T(\tau) d\tau\right]^{-1} \times \times \left[\int_0^t e^{-\lambda(t-\tau)} \Psi(\tau) y(\tau) d\tau\right] \tag{25}$$

where it was assumed that the inverse in (25) exists.

For adaptive control applications, we are interested in a recursive formulation of (25) where parameters are updated continuously on the basis on input-output data. Such an expression may be obtained by defining:

$$\Gamma(t) = \left[\int_0^t e^{-\lambda(t-\tau)} \Psi(\tau)\Psi^T(\tau) d\tau\right]^{-1} \tag{26}$$

so that

$$\frac{d}{dt}\left[\Gamma^{-1}(t)\right] = -\lambda\Gamma^{-1}(t) + \Psi(t)\Psi^T(t) \tag{27}$$

Since,

$$0 = \frac{d}{dt}(I) = \frac{d}{dt}\left[\Gamma(t)\Gamma^{-1}(t)\right] = \left[\frac{d}{dt}\Gamma(t)\right]\Gamma^{-1}(t) + \Gamma(t)\left[\frac{d}{dt}\Gamma^{-1}(t)\right] \tag{28}$$

it follows that:

$$\frac{d}{dt}\Gamma(t) = -\Gamma(t)\left[\frac{d}{dt}\Gamma^{-1}(t)\right]\Gamma(t) = -\Gamma(t)\Psi(t)\Psi^T(t)\Gamma(t) + \lambda\Gamma(t) \tag{29}$$

Now (25) can be written as

$$\hat{\rho}(t) = \Gamma(t)\left[\int_0^t e^{-\lambda(t-\tau)} \Psi(\tau) y(\tau) d\tau\right] \tag{30}$$

such that, using (29) and (30) its derivative with respect to time becomes

$$\frac{d}{dt}\hat{\rho}(t) = -\Gamma(t)\Psi(t)\Psi^T(t)\hat{\rho}(t) + \Gamma(t)\Psi(t) y(t) = \Gamma(t)\Psi(t)\left[y(t) - \Psi^T(t)\hat{\rho}(t)\right] \tag{31}$$

Note that the recursive algorithm (31), (29) is started with arbitrary initial conditions at $t_0 = 0$.

Finally, a recursive least-square algorithm for on-line estimation of parameter ρ, is given by:

$$\frac{d\Psi^T(t)}{dt} = -\Omega\Psi^T(t) + KG(\xi) \tag{32a}$$

$$\frac{d\Psi_0(t)}{dt} = -\Omega\Psi_0 - (\Omega + D \cdot I)\xi + F \tag{32b}$$

$$\frac{d}{dt}\hat{\rho}(t) = \Gamma(t)\Psi(t)\left[y(t) - \Psi^T(t)\hat{\rho}(t)\right], \quad \hat{\rho}(0) = \rho_0 \tag{32c}$$

$$\frac{d}{dt}\Gamma(t) = -\Gamma(t)\Psi(t)\Psi^T(t)\Gamma(t) + \lambda\Gamma(t), \quad \Gamma(0) = \Gamma_0 > 0 \tag{32d}$$

Usually, $\Gamma(0) = \gamma \cdot I$ with $\gamma > 0$, I being the unity matrix, or $\Gamma(0) = \underset{j=1,\ldots,n}{diag}\{\gamma_j\}, \gamma_j \in \Re_+$. The design parameters, that is the matrix Ω is chosen as:

$$\Omega = \underset{j=1,\ldots,n}{diag}\{-\omega_i\}, \ \omega_i \in \Re_+.$$

and the forgeing factor $\lambda \in [0,1]$.

3.2.2. Stability and tracking properties of the least-square algorithm

Let us define the tracking error $\tilde{\rho}$ as:

$$\tilde{\rho} = \rho - \hat{\rho} \tag{33}$$

Considering the equation (32c) and the definition (33), the convergence of the recursive least-squares algorithm (32) is related to the asymptotic stability of following differential equation:

$$\frac{d\tilde{\rho}(t)}{dt} = -\Gamma(t)\Psi(t)\Psi^T(t)\tilde{\rho}(t), \ \Gamma(t) > 0 \tag{34}$$

Regarding to the system (34) we can present the following convergence result.

Theorem 1: If the matrix $\Psi(t)$ in (34), whose components are piecewise continuous and bounded functions, satisfies the persistency of excitation condition, that is there are the constants $\alpha_1, \alpha_2, T_0 > 0$ such that [12],

$$\alpha_2 I \geq \int_t^{t+T_0} \Psi(\tau)\Psi^T(\tau) d\tau \geq \alpha_1 I, \quad \forall t \geq 0 \tag{35}$$

then the system (34) is globally exponentially stable.

Proof. Consider the following Lyapunov function:

$$V(t) = \tilde{\rho}^T(t)\Gamma^{-1}(t)\tilde{\rho}(t) \tag{36}$$

where $\Gamma(t)$ is a $(n \times n)$-symmetrically and positive definite matrix that satisfies the equation (32d). Using (27), the derivative of (36) along the trajectories of (34) takes the form

$$\dot{V}(t) = -\lambda\tilde{\rho}^T(t)\left(\int_0^t e^{-\lambda(t-\tau)}\Psi(\tau)\Psi^T(\tau)d\tau\right)\tilde{\rho}(t) - \tilde{\rho}^T(t)\Psi(\tau)\Psi^T(\tau)\tilde{\rho}(t) \tag{37}$$

where for $\Gamma^{-1}(t)$ it was used its expression from (26). From (37) one obtains the following inequality

$$\begin{aligned}\dot{V}(t) &= \tilde{\rho}^T(t)\left(\frac{d}{dt}\int_0^t e^{-\lambda(t-\tau)}\Psi(\tau)\Psi^T(\tau)d\tau\right)\tilde{\rho}(t) \\ &\quad - 2\tilde{\rho}^T(t)\Psi(t)\Psi^T(t)\tilde{\rho}(t) \\ &\le \tilde{\rho}^T(t)\left(\frac{d}{dt}\int_0^t \Psi(\tau)\Psi^T(\tau)d\tau\right)\tilde{\rho}(t) \\ &\quad - 2\tilde{\rho}^T(t)\Psi(t)\Psi^T(t)\tilde{\rho}(t)\end{aligned} \tag{38}$$

from which

$$\dot{V}(t) \le -\tilde{\rho}^T(t)\Psi(t)\Psi^T(t)\tilde{\rho}(t) \tag{39}$$

with the condition that the matrix $\Psi(t)\Psi^T(t)$ to be uniformly positive. In fact, this is possible only if the persistency of excitation condition (35) is satisfied. Since $V(t)$ is a positive definite and non-increasing function and its time derivative is negative semi-definite, then $\tilde{\rho}(0) = 0$ is an uniformly stable equilibrium point of the system (34). By integrating of (39) from t to $t + T_0$, one obtains:

$$\int_t^{t+T_0}\dot{V}(\tau)d\tau \le -\int_t^{t+T_0}\tilde{\rho}^T(\tau)\Psi(\tau)\Psi^T(\tau)\tilde{\rho}(\tau)d\tau \tag{40}$$

From the condition (35) we have

$$\begin{aligned}\alpha_1\left\|\tilde{\rho}(t)\right\|^2 &\le \int_t^{t+T_0}\tilde{\rho}^T(\tau)\Psi(\tau)\Psi^T(\tau)\tilde{\rho}(\tau)d\tau \\ &\le \alpha_2\left\|\tilde{\rho}(t)\right\|^2, \quad \forall t \ge 0\end{aligned} \tag{41}$$

Using (41), the condition (40) becomes

$$\int_t^{t+T_0}\dot{V}(\tau)d\tau \le -\alpha_1\left\|\tilde{\rho}(t)\right\|^2 \tag{42}$$

Using now a theorem of exponentially stability from [5], [12], it follows that $\tilde{\rho} = 0$ is a globally exponentially stable equilibrium point of the system (35). Then it follows that $\hat{\rho}(t)$ will converge to $\rho(t)$.

3.2.3. The adaptive linearizing algorithm

Considering the notations in the previous sections, the algorithm for on-line computation of the unknown parameter ρ in (17), (18) is particularized as follows:

$$\frac{d\Psi^T}{dt} = -\omega\Psi^T + KG(\xi) \tag{43a}$$

$$\frac{d\Psi_0}{dt} = -\omega\Psi_0 + (\omega - D)\xi + F \tag{43b}$$

$$\frac{d\hat{\rho}}{dt} = \Gamma\Psi\left(\xi - \Psi_0 - \Psi^T\hat{\rho}\right) \tag{43c}$$

$$\frac{d\Gamma}{dt} = -\Gamma\Psi\Psi^T\Gamma + \lambda\Gamma, \qquad \Gamma(0) > 0 \tag{43d}$$

where Ψ^T is the regressor matrix, Γ is the gain matrix of the updating law (44c), and $\omega > 0$ and λ (forgetting factor) are design parameters at the user's disposal to control the stability and the tracking properties of the estimator.

In our case the regressor matrix in (43a-43d) is given by:

$$\Psi^T = \begin{bmatrix} -\frac{1}{Y_{X/S}} & -\frac{1}{Y_{P/S}} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix} \tag{43e}$$

Finally, the full adaptive linearizing controller is made up of the combination of (12), (13) and (43) with the control law (9) rewritten as (44):

$$F_{in} = \frac{V}{S_{in} - S}\left[\lambda_1(S^* - S) + \frac{1}{Y_{X/S}}S\hat{X}\hat{\rho}_1 + \frac{1}{Y_{P/S}}S\hat{X}\hat{\rho}_2\right] \tag{44}$$

and is schematized in Fig. 1.

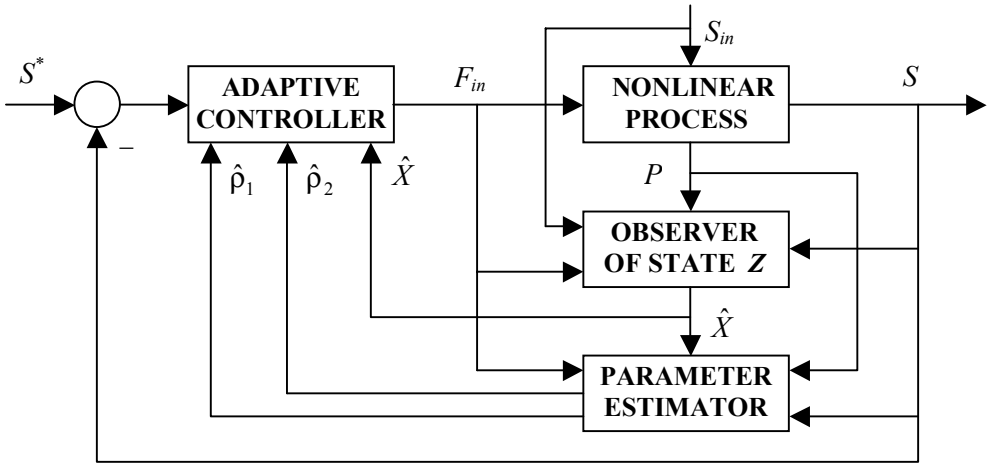

Fig. 1. Schematic view of the adaptive controller

4. SIMULATION RESULTS

The performances of the above nonlinear adaptive controller have been tested by performing extensive simulation experiments.

Simulations of the alcoholic fermentation process where carried out by numerically integrating the complete model described in Section 2 with the following set yield and kinetic coefficient values [3]:

$$\mu_{\max} = 0.54\ h^{-1}, K_S = 5\ g/l,\ K_I = 201\ g/l,$$
$$\nu_{\max} = 2.1\ h^{-1}, K_S^{'} = 9\ g/l, K_I^{'} = 297\ g/l,$$
$$P_m = 70\ g/l, Y_{X/S} = 1.5, Y_{P/S} = 0.43,$$
$$S_{in} = 160\ g/l, V \in [V_0, V_{\max}] = [2, 16]\ l,$$
$$F_{in} \in [0, 2]\ l/h.$$

The equations (1)-(4) were integrated under the following initial conditions:

$$X(0) = 1.5\ g/l,\ S(0) = 3.0\ g/l,$$
$$P(0) = 0.0\ g/l,\ V(0) = V_0 = 4.0\ g/l.$$

From equations (1), (2) and (4) we see that the accumulations of the biomass quantity (XV) and of the product quantity (PV) in the reactor are governed by the following differential equations:

$$\frac{d}{dt}(XV) = \mu \cdot (XV);\ \frac{d}{dt}(PV) = \nu_P \cdot (PV) \quad (45)$$

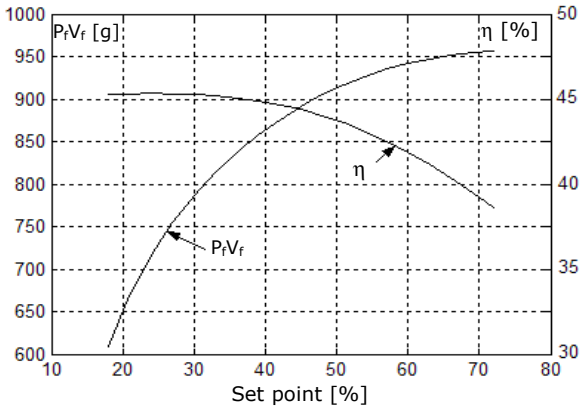

Fig. 2. Final product quantity $P_f V_f$ and yield η vs. set point

It appears that the biomass production rate and the production rate are maximum when the specific rate functions μ and ν_P themselves are maximum. It follows that the optimization of the process efficiency, i.e. the maximization of the biomass production and of the production of alcohol, clearly requires maintaining the substrate concentration S at a set point S^* that maximizes the biomass growth rate and the product growth rate respectively. We have thus formulated an optimization objective in terms of a regulation problem.

In fact, the substrate regulation in fed-batch processes can constitute an efficient tool to manage the yield-production conflict which occurs in many practical applications, particularly in alcoholic fermentation process.

Thus, if we suppose that the final volume of culture medium is fixed, i.e. $V(t_f) = V_{\max}$, where t_f is the final time of the fed-batch operation, we can define the yield η as the ratio

of the final product quantity and the total amount of substrate, which has been consumed:

$$\eta = \frac{P(t_f)V(t_f)}{S(0)V(0) + \int_0^{t_f} S_{in}F_{in}(t)dt} \qquad (46)$$

The parameters $P_f V_f = P(t_f)V(t_f)$ and η are plotted in Fig. 2 with respect to the set point S^*, for a series of simulations, performed with the model (1)-(4) under the above adaptive control when $t_f = 10.5\,h$.

From this figure, it appears clearly that the set point S^* (and obviously the associated adaptive controller) can be viewed as a mean to modulate the process between production maximization (but with a low yield) obtained with $S^* = 72\ g/l$, and the yield maximization (with a low production) obtained with $S^* = 18\ g/l$. It is clear that the set point S^* must be chosen in a such mode, which corresponds to a compromise between the values, which optimize the specific growth and production rates.

The graphs of Fig. 3a-3c show the performances of the adaptive nonlinear control given by (12), (13), (43) and (44) by comparison to the behavior of closed-loop system when all elements (state variables and kinetics) are assumed to be known and the control law is given by (9). So, Fig. 3a shows the evolution of state variables (biomass X and estimated biomass $\hat{X}$, substrate S, alcohol P), and Fig. 3b shows the profile of the control variable F_{in}, when the set point is $S^* = 44\ g/l$. (*Observation*. Note that in Fig. 3a, to be represented in the same figure, the values of X and $\hat{X}$ were multiplied by 4).

From these figures it is clear that the two algorithms (9) and (44) lead to a similar behavior of the state variables.

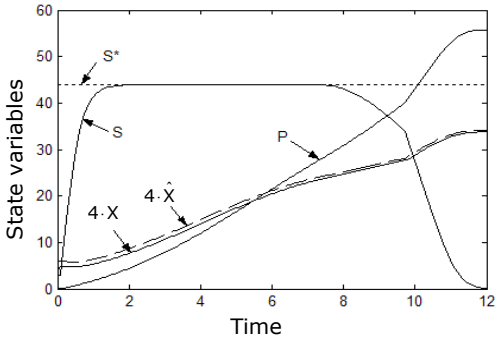

Fig. 3a. Evolution of state variables

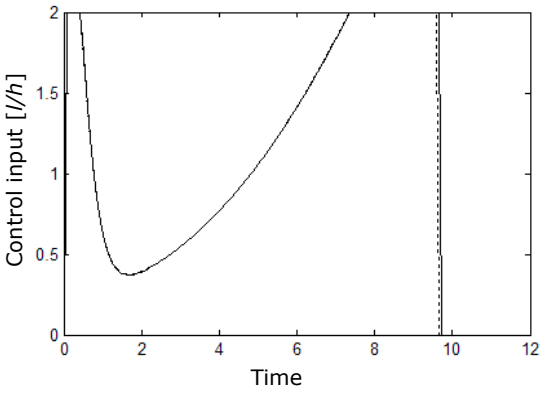

Fig. 3b. Profile of control variable

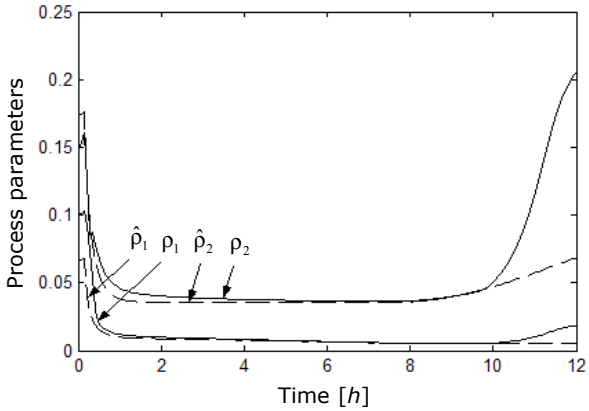

Fig. 3c. Evolution of process parameters

Furthermore the estimated parameters $\hat{\rho}_1$ and $\hat{\rho}_2$ track their true values ρ_1 and ρ_2, as it is shown in Fig. 3c.

In Fig. 3a-3c the adaptive controller has been implemented under the following initial conditions and design parameters:

$\hat{Z}(0) = 3.8\ g/l$; $\hat{\rho}_1(0) = 0.1$; $\hat{\rho}_2(0) = 0.15$; $\lambda = 0.55$; $\Gamma(0) = 5 \cdot I_2$; $\lambda_1 = 2.5$; $\omega = 1/T$ where T is the sample interval.

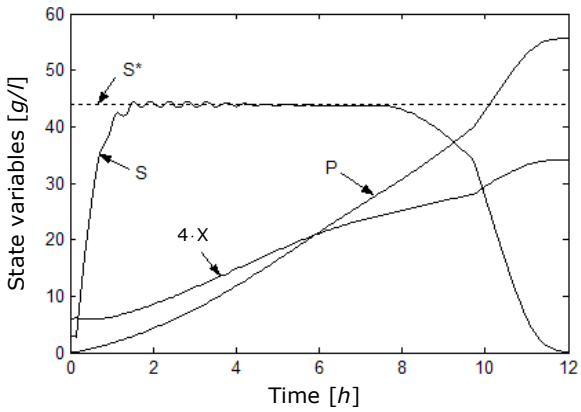

Fig. 4a. Evolution of state variables

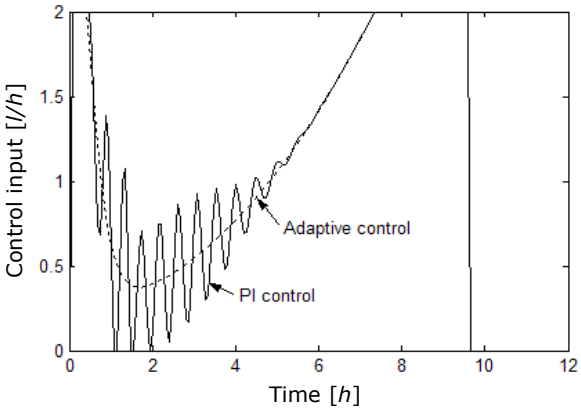

Fig. 4b. Profile of control variable

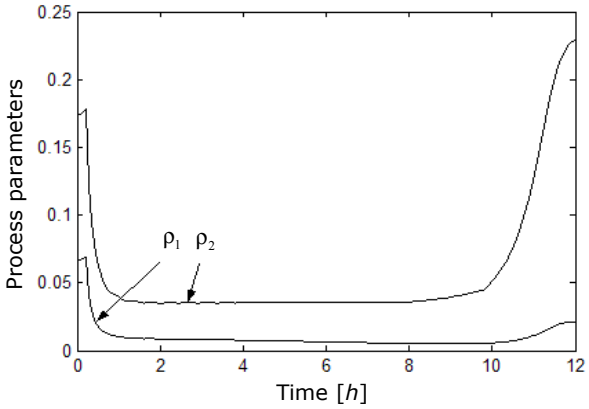

Fig. 4c. Evolution of process parameters

We have also compared our adaptive control algorithm with a simple classical *PI* regulator, which computes the value of the influent flow rate F_{in} from the regulation error $\left(S^* - S\right)$ as:

$$F_{in}(t) = K_R\left(S^* - S\right) + \frac{1}{T_I}\int_0^t \left(S^* - S\right)d\tau \qquad (47)$$

Figures 4a-4c show the evolution of the state variables (biomass, substrate, product (alcohol)) and the control variable (feed rate) when the control law is given by (47) with $K_R = 0.7$ and $T_I = 0.4$ and the set point $S^* = 44\ g/l$.

We have noted that, although reasonable set of design parameters have been considered, the performances of the *PI* controller are not so good, it may be degrading quite easily.

5. CONCLUSIONS

In this paper an adaptive scheme has been presented for the control of the substrate concentration in a fed-batch fermentation bioreactor.

The algorithm consists of two estimation steps: the estimation of the unmeasured state and then the estimation of the unknown specific reaction rates. For the estimation of the unknown parameters a recursive least-squares algorithm with forgetting factor was proposed. The stability and convergence properties of this algorithm were proved provided that the regressor matrix is persistently exciting.

Because the goal of a fed-batch operation is generally to accumulate some reaction products that are harvested at the end of the operation (the product P, in our example), the goal of the feedback control is clearly not to stabilize the process globally but rather to keep unstable trajectories under control. We have thus formulated an optimization objective in terms of a regulation problem and discussed why and how must to apply the feed rate that is calculated on-line with the adaptive nonlinear scheme.

The results of the application of the proposed algorithm in numerical simulation confirm the efficiency of the control scheme.

REFERENCES

[1] Bastin, G., Dochain, D., *On-line Estimation and Adaptive Control of Bioreactors*, Elsevier, Amsterdam, 1990.

[2] Duvivier, E., Sévely, Y., Simulation de la comande en temps minimal d'un procédé de la fermentation semicontinu: programmation dynamique et transformation de Kelley, *RAIRO APII*, vol. 22, no. 4, pp. 363-380, 1988.

[3] Queinnec, I., Dahhou, B., Sévely, Y., Theoretical and experimental results in optimization of fedbatch fermentation processes, *Journal of Systems Engineering*, vol. 1, pp. 31-40, 1991.

[4] Dahhou, B., Roux, G., Cheruy, A., Linear and non-linear adaptive control of alcoholic fermentation process: experimental results, *Inter. Journal of Adaptive Control and Signal Processing*, vol. 7, pp. 213-223, 1993.

[5] Isidori, A., *Nonlinear Control Systems. Third edition*, Springer-Verlag, Berlin, 1995.

[6] Lin, C.-F., *Advanced Control Systems Design*, PTR Prentice Hall, Englewood Cliffs, New Jersey, 1994.

[7] Marino, R., Tomei, P., *Nonlinear Control Design*, Prentice Hall Inter., London, 1995.

[8] Petre, E., Popescu, D., On adaptive non-linear control for fed-batch alcoholic fermentation process, *10-th Int. Conf. on Control Systems and Computer Science* -

CSCS'10, Bucharest, May 24-26, 1995, vol. 3, pp. 184-189.

[9] Petre, E., *Nonlinear Control Systems - Applications in Biotechnology (in Romanian)*, Universitaria, Craiova, Romania, 2002.

[10] Petre, E., Adaptive Control Strategies for a Class of Time Delay Nonlinear Bio-processes, *Rev. Roum. Sci. Techn.-Électrotechn. et Énerg.*, **48**, 4, pp. 567-582, Bucarest, Romania, 2003.

[11] Petre, E., Selisteanu, D., *Modelling and Identification of the Depollution Bio-processes (in Romanian)*, Universitaria, Craiova, Romania, 2005.

[12] Sastry, S., Isidori, A., Adaptive Control of Linearizable Systems, *IEEE Trans. Aut. Control*, vol. 34, no. 11, pp. 1123-1131, Nov. 1989.

[13] Sastry, S., Bodson, M., *Adaptive Control: Stability, Convergence and Robustness*, Englewood Cliffs, NJ: Prentice-Hall, 1989.

[14] Van Impe, J.F., Claes, J.E., Ryckaert, V.G., Bastin, G., Characterization of optimal feed rate profiles for fed-batch fermentation processes, *3-th IEEE Conference on control applications*, vol. 3, pp. 1947-1952, 1994.