THE USE OF THE MATRIX (M_{dpx}) ASSOCIATED TO THE TAYLOR SERIES, FOR NUMERICAL MODELING AND SIMULATION OF THE PROCESSES WITH DISTRIBUTED PARAMETERS

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Abstract: *The paper presents a possible variant of a systemic approach, for the numerical modeling and simulation of some usual categories of processes with distributed parameters, by "the matrix of the partial derivatives of the state vector", noted with M_{dpx} and associated to the Taylor series. The definition and use of the matrix M_{dpx} in this paper could be considered as being original in this domain.*

Keywords: *partial derivative equations, state variables, Taylor series, numerical integration*

1. THE CATEGORY OF PROCESSES WITH DISTRIBUTED PARAMETERS

These processes are defined by equations, or equation systems with partial derivatives (pde) [eventually ordinary differential equations (ode)] whose (independent or dependent) state variables fulfill the continuity conditions in the Cauchy way.

An example of pdeII.4 (of the IInd order, with four independent variables) can be presented on the complete form (1).
The next coefficients ($a_{...}$) can be constant or $a_{...}=a_{...}(t, p, q, r)$ wit the fulfillment of the continuity conditions.

$$\begin{aligned}
&a_{0000}y + a_{1000}\frac{\partial y}{\partial t} + a_{0100}\frac{\partial y}{\partial p} + a_{0010}\frac{\partial y}{\partial q} + a_{0001}\frac{\partial y}{\partial r} + \\
&a_{2000}\frac{\partial^2 y}{\partial t^2} + a_{1100}\frac{\partial^2 y}{\partial t\partial p} + a_{0200}\frac{\partial^2 y}{\partial p^2} + a_{0110}\frac{\partial^2 y}{\partial p\partial q} + \\
&+a_{0020}\frac{\partial^2 y}{\partial q^2} + a_{0011}\frac{\partial^2 y}{\partial q\partial r} + a_{0002}\frac{\partial^2 y}{\partial r^2} + a_{1001}\frac{\partial^2 y}{\partial t\partial r} + \\
&+a_{1010}\frac{\partial^2 y}{\partial q\partial t} + a_{0101}\frac{\partial^2 y}{\partial p\partial r} = \varphi(t,p,q,r)
\end{aligned} \tag{1}$$

The independent variables (t), (p), (q) and (r) represent the time (t) and (p), (q) and (r) can be, for instance, space variables in different coordinates: Cartesian, polar, etc.

Introducing the notation:

$$x_{TPQR}=\frac{\partial^{T+P+Q+R}y}{\partial t^{T}\partial p^{P}\partial q^{Q}\partial r^{R}} \tag{2}$$

which in particular can become

$$x_{T}=\frac{\partial^{T}y}{\partial t^{T}}=\frac{d^{T}y}{dt^{T}} \tag{3}$$

pde II.4 from (1) can be rewritten in a simpler form

$$\begin{aligned}&a_{0000}x_{0000}+a_{1000}x_{1000}+a_{0100}x_{0100}+a_{0010}x_{0010}+\\&+a_{0001}x_{0001}+a_{2000}x_{2000}+a_{1100}x_{1100}+a_{0200}x_{0200}+\\&+a_{0110}x_{0110}+a_{0020}x_{0020}+a_{0011}x_{0011}+a_{0002}x_{0002}+\\&+a_{1001}x_{1001}+a_{1010}x_{1010}+a_{0101}x_{0101}=\varphi_{0000}\end{aligned} \tag{4}$$

Further on, the numerical integration will be considered in accordance to the time (t) for the entire paper.

2. THE DEFINITION OF "THE MATRIX OF PARTIAL DERIVATIVES OF THE STATE VECTOR" ($\mathbf{M}_{dpx}$)

"The matrix of partial derivatives of the state vector", defined as:

$$\mathbf{M}_{dpx}\underset{n\times N}{=}\begin{array}{c|c} & 1\times M\\ \hline \mathbf{x} & \mathbf{x}_{PQR}\\ \mathbf{x}_{T} & \mathbf{x}_{TPQR}\end{array}\underset{n\times N}{=}\begin{array}{c|c} & 1\times M\\ \hline x_{0000} & \mathbf{x}_{0PQR}\\ x_{1000} & \mathbf{x}_{1PQR}\\ x_{2000} & \mathbf{x}_{2PQR}\\ \dots & \dots\\ x_{n-1,000} & \mathbf{x}_{n-1,PQR}\\ x_{n,000} & \mathbf{x}_{n,PQR}\\ x_{n+1,000} & \mathbf{x}_{n+1,PQR}\\ x_{n+2,000} & \mathbf{x}_{n+2,PQR}\\ \dots & \dots\\ x_{n-1+N,000} & \mathbf{x}_{n-1+N,PQR}\end{array} \tag{5}$$

noted with $\mathbf{M}_{dpx}$, is the complex matrix (5) formed of:

- the state vector $\mathbf{x}(n\text{x}1)$, having a number of (n) lines, equal to the number of state variables and implicitly to the order (pde) with respect to time (t), in accordance to which the integration operates;
- the state vector derived N times with respect to time $\mathbf{x}_{T}(N\times 1)=\frac{d^{N}}{dt^{N}}(\mathbf{x})$, where usually N≥4;
- the state vector, partially derived with respect to (p), (q) and (r), noted with $\mathbf{x}_{PQR}(n\text{x}M)$, where (M) corresponds to the total number of the partial derivatives that were operated;
- the vector partially derived with respect to (p), (q) and (r), noted with $\mathbf{x}_{TPQR}(N\text{x}M)$, the total number of these partial derivatives being (M).

As a result,

$$\mathbf{x}_{PQR}=\frac{\partial^{P+Q+R}}{\partial^{P}p\partial^{Q}q\partial^{R}r}(\mathbf{x});_{(P+Q+R=M)} \tag{6}$$

$$\mathbf{x}_{TPQR}=\frac{\partial^{P+Q+R}}{\partial^{P}p\partial^{Q}q\partial^{R}r}(\mathbf{x}_{T});_{(P+Q+R=M)} \tag{7}$$

where usually M>N. All these matrices ($\mathbf{x}_{PQR}$) and ($\mathbf{x}_{TPQR}$) have a great number of elements. With the above four independent variables, the elements of the line matrix ($\mathbf{x}_{\tau PQR}$) for τ = 0, 1, 2, …, n-1+N, correspond the following partial derivatives, progressively increasing: $\mathbf{x}_{\tau 100}$, $\mathbf{x}_{\tau 010}$, $\mathbf{x}_{\tau 001}$, $\mathbf{x}_{\tau 200}$, $\mathbf{x}_{\tau 110}$, $\mathbf{x}_{\tau 020}$, $\mathbf{x}_{\tau 011}$, $\mathbf{x}_{\tau 002}$, $\mathbf{x}_{\tau 101}$, $\mathbf{x}_{\tau 300}$, $\mathbf{x}_{\tau 210}$, $\mathbf{x}_{\tau 120}$, $\mathbf{x}_{\tau 030}$, $\mathbf{x}_{\tau 021}$, $\mathbf{x}_{\tau 012}$, $\mathbf{x}_{\tau 003}$, $\mathbf{x}_{\tau 102}$, $\mathbf{x}_{\tau 201}$, $\mathbf{x}_{\tau 111}$, $\mathbf{x}_{\tau 400}$, $\mathbf{x}_{\tau 310}$, $\mathbf{x}_{\tau 220}$, $\mathbf{x}_{\tau 130}$, $\mathbf{x}_{\tau 040}$, $\mathbf{x}_{\tau 031}$, $\mathbf{x}_{\tau 022}$, $\mathbf{x}_{\tau 013}$, $\mathbf{x}_{\tau 004}$, $\mathbf{x}_{\tau 103}$, $\mathbf{x}_{\tau 202}$, $\mathbf{x}_{\tau 301}$, $\mathbf{x}_{\tau 211}$, $\mathbf{x}_{\tau 121}$, $\mathbf{x}_{\tau 112}$, and so on, where the index (τ) underlines the order of the partial derivative with respect to time (t). For the above example, M=3 if only the derivatives of the Ist order are considered; M=9 if we consider all the derivatives of the Ist and IInd order; M=19 if we consider all the derivatives of the Ist, IInd and IIIrd order, →M=34 if we limit to all the derivatives of the Ist, IInd, IIIrd and IVth order, etc.

Due to some reasons to which we will return, for the element ($x_{n,000}$) from the general form (5), we will use the name of "pivot element", which result from the explicit use of the partial derivative of a maximum order with respect to time (t), from the general form (pde). In the example (4), this "pivot element" is (x_{2000}).

Returning to pde (4), where n=2 and the state vector $\mathbf{x}(2\text{x}1)$ we have ($\mathbf{M}_{dpx}$) of the particular form:

$$\mathbf{M}_{dpx} = \begin{array}{c} \\ n{=}2 \\ \times \\ N \end{array} \begin{array}{|c|c|} \multicolumn{2}{c}{1{\times}M} \\ \hline \mathbf{x} & \mathbf{x}_{PQR} \\ \hline \mathbf{x}_{T} & \mathbf{x}_{TPQR} \\ \hline \end{array} = \begin{array}{c} \\ n{=}2 \\ \times \\ N \end{array} \begin{array}{|c|c|} \multicolumn{2}{c}{1{\times}M} \\ \hline x_{0000} & \mathbf{x}_{0PQR} \\ \hline x_{1000} & \mathbf{x}_{1PQR} \\ \hline x_{2000} & \mathbf{x}_{2PQR} \\ \hline x_{3000} & \mathbf{x}_{3PQR} \\ \hline \dots & \dots \\ \hline x_{1+N,000} & \mathbf{x}_{1+N,PQR} \\ \hline \end{array} \tag{8}$$

with $N \geq 4$, and for the elements of the matrix ($\mathbf{x}_{TPQR}$), the indexes $\tau = 0, 1, 2, \dots 1+N$. The dimensions M=3; M=9; M=19 or M=34 correspond to the limitations to partial derivatives (with respect to p, q, r) of the Ist order, Ist and IInd order, Ist, IInd and IIIrd order, respectively Ist, IInd, IIIrd and IVth order. Thus, for each (pde) we can associate a model expressed by "the matrix of partial derivatives of the state vector" with the dimension $\mathbf{M}_{dpx}[(n+N)x(1+M)]$, where (n) represents the number of lines of the state vector $\mathbf{x}(nx1)$. The choice of the dimensions $N \geq 4$ and M=3, 9, 19 or 34, conditions the error of the approximation for the numerical solution, which becomes smaller as N and M become bigger.

3. STATE VARIABLES, INITIAL CONDITIONS, BORDER CONDITIONS, FINAL CONDITIONS

Formally identical with (1) and (4) we can write a big diversity of (pde) for which the Ist, IInd, IIIrd or IVth order are associated to the derivatives with respect to time (t), and the number (2), (3) or (4) of the independent variables contains the variable (t). The number of state variables corresponds to the order of these (pde) with respect to time (t), and the indexes underline the number of independent variables that define (pde), emplified in Table1.

Table. 1.

edp	I.2	I.3	I.4	II.2	II.3	II.4
x	x_{00}	x_{000}	x_{0000}	x_{00}	x_{000}	x_{0000}
				x_{10}	x_{100}	x_{1000}

edp	III.2	III.3	III.4	IV.2	IV.3	IV.4
x	x_{00}	x_{000}	x_{0000}	x_{00}	x_{000}	x_{0000}
	x_{10}	x_{100}	x_{1000}	x_{10}	x_{100}	x_{1000}
	x_{20}	x_{200}	x_{2000}	x_{20}	x_{200}	x_{2000}
				x_{30}	x_{300}	x_{3000}

For a number of (2), (3) or (4) independent variables the following variables correspond: (t, p), (t, p, q), respectively (t, p, q, r).
Table 2 presents the state vector for the initial conditions ($\mathbf{x}_{IC}$) and the state vector for some possible border conditions ($\mathbf{x}_{BC}$), respectively final conditions ($\mathbf{x}_{FC}$), where the indexes (o) and (f) underline the initial values, respectively the final values.

Table 2.

x_{IC}	x_{BC}		x_{FC}
$\mathbf{x}(t_0, p)$	$\mathbf{x}(t, p_0)$	$\mathbf{x}(t, p_0, q, r)$	$\mathbf{x}(t_f, p)$
$\mathbf{x}(t_0, p, q)$	$\mathbf{x}(t, p_0, q)$	$\mathbf{x}(t, p_0, q_f, r)$	$\mathbf{x}(t_f, p, q)$
$\mathbf{x}(t_0, p, q, r)$	$\mathbf{x}(t, p_0, q_f)$	$\mathbf{x}(t, p, q_0, r)$	$\mathbf{x}(t_f, p, q,r)$

4. NUMERICAL INTEGRATION OF (pde) BY TAYLOR SERIES AND ($\mathbf{M_{dpx}}$)

If the numerical integration, in order to obtain the state vector ($\mathbf{x}$) operates by Taylor series, the well known vector relations are being used:

$$\mathbf{x}_k \cong \mathbf{x}_{k-1} + \sum_{\tau=1}^{T} \frac{\Delta t^{\tau}}{\tau!} \mathbf{x}_{\tau,k-1} \tag{9}$$

$$\mathbf{x}_{PQR,k} \cong \mathbf{x}_{PQR,k-1} + \sum_{\tau=1}^{T} \frac{\Delta t^{\tau}}{\tau!} \mathbf{x}_{\tau PQR,k-1} \tag{10}$$

The indexes (k) and (k-1) represent the current and regressive sequence from the moments $t_k = k \cdot \Delta t$, respectively $t_{k-1} = (k-1) \cdot \Delta t$, where the integration step (Δt) is considered to be small enough. From (5) it can be seen that (9) results from the components of the vector ($\mathbf{x}_{T,\ k-1}$), and (10) is obtained from the components of the matrix ($\mathbf{x}_{TPQR,k-1}$). Thus, for the right members belonging to (9) and (10) we have:

$$\mathbf{M}_{dpx,k-1} = \begin{array}{|c|c|} \hline \mathbf{x}_{k-1} & \mathbf{x}_{PQR,k-1} \\ \hline \mathbf{x}_{T,k-1} & \mathbf{x}_{TPQR,k-1} \\ \hline \end{array} \tag{11}$$

everything being considered at the sequence (k-1).
The sequence (k-1) at the moment $t_{k-1} = (k-1) \cdot \Delta t$ could also correspond to the beginning of the calculations [at $t_{k-1} = t_0$ and initial conditions (IC) known], for which

$$\mathbf{x}_{k-1} = \mathbf{x}_{IC} = \mathbf{x}(t_{k-1}, p, q, r) \quad (12)$$

which represents the known state vector for the initial conditions (IC). As a result, the matrix

$$\mathbf{x}_{PQR,k-1} = \mathbf{x}_{PQR,CI} = \frac{\partial^{P+Q+R}}{\partial p^P \cdot \partial q^Q \cdot \partial r^R}(\mathbf{x}_{k-1}) \quad (13)$$

where the partial derivatives will operate successively, with respect to (p), (q) and (r) of the Ist order and then Ist and IInd order, Ist, IInd and IIIrd order, etc.
The calculus of the matrices ($\mathbf{x}_{PQR, k-1}$) from (13) begins in the first line of the vector ($\mathbf{x}_{k-1}$), and then continues successively to the last line of this vector.
The elements of this vector ($\mathbf{x}_{T, k-1}$) and the matrix ($\mathbf{x}_{TPQR, k-1}$) are established in the following succession:

a) the pivot element ($x_{n,000,k-1}$) is calculated from (5), which also represents the element in the first line of the vector ($\mathbf{x}_{T,k-1}$). This element is a polynomial function with respect to the existing elements in ($\mathbf{x}_{k-1}$) and ($\mathbf{x}_{PQR, k-1}$), defined in (12) and (13).
b) The first line of the matrix ($\mathbf{x}_{TPQR, k-1}$), respectively ($\mathbf{x}_{nPQR, k-1}$) is obtained by the partial derivation of the pivot element ($x_{n,000, k-1}$) with respect to (p), (q), and (r), progressively from the Ist order, then Ist and IInd, then Ist, IInd and IIIrd order, etc. All these elements are polynomial functions, with respect to the existing elements in ($\mathbf{x}_{k-1}$) and ($\mathbf{x}_{PQR, k-1}$) defined in (12) and (13).
c) The following element ($x_{n+1,000, k-1}$) from the second line of the vector ($\mathbf{x}_{T, k-1}$) is obtained by the analytic derivation with respect to time of the pivot element ($x_{n,000, k-1}$).
d) The second line of the matrix ($\mathbf{x}_{TPQR, k-1}$), that is ($\mathbf{x}_{n+1, PQR, k-1}$) is obtained by the partial derivation of the element ($x_{n+1,000, k-1}$) previously calculated, with respect to (p), (q) and (r), progressively for the Ist order, then Ist and IInd, then Ist, IInd and IIIrd order, etc. All these elements are also polynomial functions previously calculated with respect to the existing elements in ($\mathbf{x}_{k-1}$), ($\mathbf{x}_{PQR, k-1}$), ($x_{n,000}$) and in the first line of the matrix ($\mathbf{x}_{TPQR, k-1}$), respectively ($\mathbf{x}_{nPQR, k-1}$).
e) The following element ($x_{n+2,000, k-1}$) in the third line of the vector ($\mathbf{x}_{T,,k-1}$) is obtained by the analytical derivation with respect to time of the vector ($\mathbf{x}_{n+1, 000, k-1}$).
f) The third line of the matrix ($\mathbf{x}_{TPQR, k-1}$), respectively ($\mathbf{x}_{n+2, PQR, k-1}$) is obtained by the partial derivation of the element ($x_{n+2, 000, k-1}$), previously calculated, with respect to (p), (q) and (r) progressively for the Ist order, then Ist and IInd, then Ist, IInd and IIIrd order, etc. All these elements are also polynomial functions previously calculated with respect to the existing elements in ($\mathbf{x}_{k-1}$), ($\mathbf{x}_{PQR, k-1}$), ($x_{n,000}$) ($x_{n+1,000}$) as well as in the first two lines of the matrix ($\mathbf{x}_{TPQR, k-1}$), respectively ($\mathbf{x}_{nPQR, k-1}$) and ($\mathbf{x}_{n+1, PQR, k-1}$).

This algorithm continues to the last line of the vector ($\mathbf{x}_{T, k-1}$) and the matrix ($\mathbf{x}_{TPQR, k-1}$), respectively ($\mathbf{x}_{1+N,000, k-1}$) and ($\mathbf{x}_{1+N, PQR, k-1}$).
Finally, the components of the matrix ($\mathbf{M}_{dpx,k-1}$) in (11) necessary for (9) and (10) have resulted this way: ($\mathbf{x}_{k-1}$) in (12); ($\mathbf{x}_{PQR, k-1}$) in (13); ($\mathbf{x}_{T, k-1}$) and ($\mathbf{x}_{TPQR, k-1}$) from the stages of calculations a), b),…f). Thus from (9) and (10) we calculate ($\mathbf{x}_k$), respectively ($\mathbf{x}_{PQR, k}$) having for each integration step a number of Taylor series of

$$NST = n \cdot (1+M) \quad (14)$$

according to (n) and (M) from (5).

The total number (M) of the partial derivatives which corresponds to the number of the columns of the matrix ($\mathbf{x}_{PQR}$) or ($\mathbf{x}_{TPQR}$) from (5) depends on the number of independent variables (t, p, q, r), noted with (NIV), as well as on the maximum order of the partial derivatives (MOPD) that are taken into consideration. For instance, if we limit to MOPD =4 and NIV = 4, the number of these partial derivatives is included in the row presented after formula (7).
If we keep MOPD =4 but NIV = 3, we will have the following row of partial derivatives: $x_{\tau 10}$; $x_{\tau 01}$; $x_{\tau 20}$; $x_{\tau 11}$ $x_{\tau 02}$ $x_{\tau 30}$; $x_{\tau 21}$; $x_{\tau 12}$ $x_{\tau 03}$ $x_{\tau 40}$; $x_{\tau 31}$; $x_{\tau 22}$; $x_{\tau 13}$;$x_{\tau 04}$.

Finally, if we maintain MOPD =4 and NIV= 2, then the row of these partial derivatives becomes: $x_{\tau 1}$; $x_{\tau 2}$; $x_{\tau 3}$; $x_{\tau 4}$.

As a result, according to (14) and to those present as follows in Table 3 we present the number of Taylor series (NTS) for each integration step (Δt). We denote with (n) the number of state variables (which also corresponds to the order (pde) with respect to time (t), limited here to 4 and with (NIV) the number of independent variables (formed out of t, p, q, r) limited here also to 4.

Table 3.

NIV=2	n	1				2			
	M	1	2	3	4	1	2	3	4
	NTS	2	3	4	5	4	6	8	10
NIV=3	n	1				2			
	M	2	5	9	14	2	5	9	14
	NTS	3	6	10	15	6	12	20	30
NIV=4	n	1				2			
	M	3	9	19	34	3	9	19	34
	NTS	4	10	20	35	8	20	40	70

NIV=2	n	3				4			
	M	1	2	3	4	1	2	3	4
	NTS	6	9	12	15	8	12	16	20
NIV=3	n	3				4			
	M	2	5	9	14	2	5	9	14
	NTS	9	18	30	45	12	24	40	60
NIV=4	n	3				4			
	M	3	9	19	34	3	9	19	34
	NTS	12	30	60	105	16	40	80	140

With the stages covered above, the matrix ($\mathbf{M}_{dpx,k-1}$) is calculated completely, and the sequence (k-1) for the moment (t_{k-1}) is considered to be ended.

The sequence (k) at the moment $t_k = k \cdot \Delta t$ consists in the establishment of:

$$\mathbf{M}_{dpx,k} = \begin{array}{|c|c|} \hline \mathbf{x}_k & \mathbf{x}_{PQR,k} \\ \hline \mathbf{x}_{T,k} & \mathbf{x}_{TPQR,k} \\ \hline \end{array} \qquad (15)$$

where the state vector ($\mathbf{x}_k$) and the matrix ($\mathbf{x}_{PQRk}$) are calculated according to (9), respectively (10), for which the development of Taylor series of the line (τ) results as:

$$\mathbf{x}_{\tau 000k} = \mathbf{x}_{\tau 000k-1} + \sum_{T=1+\tau}^{\omega} \frac{\Delta t^{T-\tau}}{(T-\tau)!} \cdot \mathbf{x}_{T000,k-1} \qquad (16)$$

respectively

$$\mathbf{x}_{\tau PQR,k} = \mathbf{x}_{\tau PQR,k-1} + \sum_{T=1+\tau}^{\omega} \frac{\Delta t^{T-\tau}}{(T-\tau)!} \cdot \mathbf{x}_{TPQR,k-1} \qquad (17)$$

The above (τ) lines correspond to τ = 0, 1, 2, … (n-1), and the last considered derivative is ω = n-1+N, as it can be observed in (5).

As a result, the Taylor series for the first line (τ = 0) in (16) and (17) contain the maximum number of (n-1+N) derivatives with respect to time, and the Taylor series for the last line (ω=n-1) contain the minimum number of (N) derivatives with respect to time.

After finishing the calculations for ($\mathbf{x}_k$) and ($\mathbf{x}_{PQRk}$) in (15) we calculate ($\mathbf{x}_{Tk}$) and ($\mathbf{x}_{TPQRk}$), according to the stages a), b), …f) from the previous sequence (k-1), which now becomes the current sequence (k). Finally we obtain ($\mathbf{M}_{dpxk}$) from (15) with the important observation that all these details of calculus, for (15) from the sequence (k) now operate automatically through the program.

The above sequence (k) ended this way will be considered again the sequence (k-1) for (15), which now becomes (11), and then we restart the stages of automatic calculations through the program, presented at the sequence (k-1). It will thus insure the incrementation of the moment $t_k = t_{k-1} + \Delta t$ and implicitly the iterative advance with the integration step (Δt), from (t_0) to (t_f).

5. STAGES OF NUMERICAL INTEGRATION, FOR AN EXAMPLE OF (PDE) II.4

We revert to the example of (pde) II.4 from (1) and (4), from which we extrapolate the pivot variant of the form:

$$\begin{aligned} x_{2000} = \frac{1}{a_{2000}} [\varphi_{0000} - (a_{1000} \cdot x_{1000} + a_{0100} \cdot x_{0100} + \\ a_{0010} \cdot x_{0010} + a_{0001} \cdot x_{0001} + a_{1100} \cdot x_{1100} + a_{0200} \cdot x_{0200} + \\ + a_{0110} \cdot x_{0110} + a_{0020} \cdot x_{0020} + a_{0011} \cdot x_{0011} + a_{0002} \cdot x_{0002} + \\ + a_{1001} \cdot x_{1001} + a_{1010} \cdot x_{1010} + a_{0101} \cdot x_{0101})] \end{aligned} \qquad (18)$$

We operate the following partial derivatives:

$$\begin{aligned} X_{2+T,PQR} = \frac{1}{a_{2000}} [\varphi_{TPQR} - (a_{1000} \cdot X_{1+T,PQR} + \\ + a_{0100} \cdot X_{T,P+1,QR} + a_{0010} \cdot X_{T,P,1+Q,R} + \\ + a_{0001} \cdot X_{TPQ,1+R} + a_{1100} \cdot X_{1+T,1+P,Q,R} + \\ + a_{0200} \cdot X_{T,2+P,Q,R} + a_{0110} \cdot X_{T,1+P,.1+Q,R} + \\ + a_{0020} \cdot X_{TP,2+Q,R} + a_{0011} \cdot X_{TP,1+Q,1+R} + \\ + a_{0002} \cdot X_{TPQ,2+R} + a_{1001} \cdot X_{1+T,PQ,1+R} + \\ + a_{1010} \cdot X_{1+T,P,1+Q,R} + a_{0101} \cdot X_{T,1+P,Q,1+R})]; \end{aligned} \qquad (19)$$

for T = 1, 2, …5; P = 0, 1, …6; Q = 0, 1, …6; R = 0, 1, …6.

We calculate:

$$\mathbf{M}_{dpx} = \overset{n=2}{\underset{N=5}{\times}} \overset{1\times 18}{\left[\begin{array}{c|c} \mathbf{x} & \mathbf{x}_{PQR} \\ \hline \mathbf{x}_T & \mathbf{x}_{TPQR} \end{array}\right]} =$$

$$= \overset{n=2}{\underset{N=5}{\times}} \overset{1\times 18}{\left[\begin{array}{c:c|c|c|c|c} x_{0000} & x_{0100} & x_{0010} & x_{0001} & \cdots & x_{0666} \\ \hline x_{1000} & x_{1100} & x_{1010} & x_{1001} & \cdots & x_{1666} \\ \hdashline x_{2000} & x_{2100} & x_{2010} & x_{2001} & \cdots & x_{2666} \\ \hline x_{3000} & x_{3100} & x_{3010} & x_{3001} & \cdots & x_{3666} \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline x_{6000} & x_{6100} & x_{6010} & x_{6001} & \cdots & x_{6666} \end{array}\right]} \tag{20}$$

resulting the dimensions $\mathbf{x}(2x1)$; $\mathbf{x}_{PQR}(2x18)$; $\mathbf{x}_T(5x1)$; $\mathbf{x}_{TPQR}(5x18)$ and $\mathbf{M}_{dpx}(7x19)$.

Further on, the details of calculus from the sequence (k-1) correspond to those presented at (11), (12), (13) and the stages a), b), …f), and for the sequence (k) we operate according to (15), (16) and (17).

In order to insure the start of the calculations for checking the performance of the numerical integration, we have used a particular solution, common in technique, of the form:

$$\begin{aligned} y_{AN}(t,p,q,r) &= y_{oooo} + \\ &+(J_{0T} + J_{1T}\cdot\varepsilon^{-\frac{t}{T_1}} + J_{2T}\cdot\varepsilon^{\frac{-t}{T_2}})\cdot \\ &\cdot(J_{0P} + J_{1P}\cdot\varepsilon^{\frac{-p}{P_1}} + J_{2P}\cdot\varepsilon^{\frac{-p}{P_2}})\cdot \\ &\cdot(J_{0Q} + J_{1Q}\cdot\varepsilon^{\frac{-q}{Q_1}} + J_{2Q}\cdot\varepsilon^{\frac{-q}{Q_2}})\cdot \\ &\cdot(J_{0R} + J_{1R}\cdot\varepsilon^{\frac{-r}{R_1}} + J_{2R}\cdot\varepsilon^{\frac{-r}{R_2}})\cdot K_u\cdot u \end{aligned} \tag{21}$$

where:

$y_{0000}=1$; $t_0=0$; $t_f=1$; $p_0=0$; $p_f=1$; $q_0=0$; $q_f=1$; $r_0=0$; $r_f=1$; $T_1= 0.15\cdot t_f$; $T_2= 0.1\cdot t_f$;
$P_1= 0.15\cdot p_f$; $P_2 = 0.1\cdot p_f$; $Q_1= 0.15\cdot q_f$; $Q_2 = 0.1\cdot q_f$; $R_1= 0.15\cdot r_f$; $R_2= 0.1\cdot r_f$; $K_u=1$;
$u=1$; $J_{0T} = J_{0P} = J_{0Q} = J_{0R} =1$;

$$J_{1T} = -\frac{T_1}{T_1-T_2};\ J_{2T} = -\frac{T_2}{T_2-T_1};\ J_{1P} = -\frac{P_1}{P_1-P_2};$$

$$J_{2P} = -\frac{P_2}{P_2-P_1};\ J_{1Q} = -\frac{Q_1}{Q_1-Q_2};\ J_{2Q} = -\frac{Q_2}{Q_2-Q_1};$$

$$J_{1R} = -\frac{R_1}{R_1-R_2};\ J_{2R} = -\frac{R_2}{R_2-R_1};$$

So, numerical integration frames in a hypercube with unitary sides. The indicator of the performance of the numerical integration is defined by the "cumulative relative error in percentages" (crepy), respectively

$$crep = crepx_{0000} = 100\cdot\frac{\sum_{K_0}^{K_f}\left|\Delta x_{0000K}\right|}{\sum_{K_0}^{K_f}\left|y_{ANK}\right|}, \tag{22}$$

where y_{ANk} is the particular analytical solution (21), and $\Delta x_{0000k}=x_{0000k} - y_{Ank}$ is the error of the solution numerically approximated (x_{0000k}) against the particular analytical solution (y_{ANk}). The numerator and denominator of (22) are considered in absolute values. The notation $\sum_{K_0}^{K_f}$ is a symbol of the iterative sum of all sequences of calculus, from $K_0 = {}^{t_0}\!/_{\Delta t}$ to $K_f = {}^{(t_f - t_0)}\!/_{\Delta t}$ where (t_0) and (t_f) correspond to the initial moments, respectively final, and (Δt) is the integration step, considered small enough.

Table 4

t_k	0.01	0.1	0.2	0.3	0.4
x_{0000k}	1.0015	1.124	1.297	1.478	1.622
$crepy_k$	0	$1.6\cdot10^{-5}$	10^{-4}	$2.6\cdot10^{-4}$	$3\cdot10^{-4}$

t_k	0.5	0.6	0.7	0.8	0.9	1
x_{0000k}	1.726	1.794	1.845	1.876	1.897	1.912
$crepy_k$	$1.1\cdot10^{-3}$	$3.5\cdot10^{-3}$	$8\cdot10^{-3}$	$1.4\cdot10^{-2}$	$1.8\cdot10^{-2}$	$1.9\cdot10^{-5}$

Table 4 presents the results of the numerical integration for all coefficients ($a_{\dots}$) in (18) of unitary value, and $p = p_f$; $q=q_f$; $r=r_f$, for the integration step Δt=0,01, according to the program RBPD3(4). Even if the order of partial derivatives with respect to (t), (p), (q) and (r) from the Taylor series has been limited to 6, the values ($crepy_k$) maintain low enough, that is under 0,02%. Comparable results for ($crepy_k$) have been obtained for multiple other values of (p), (q) and (r) for the same integration step Δt=0.01.

6. EXAMPLES OF MODELING-SIMULATION FOR (pde), BY TAYLOR SERIES AND THE MATRIX (M_{dpx})

The method of numerical modeling-simulation of processes with distributed parameters presented in this paper has been verified on a great and diversified number of cases, out of which we have extracted (15) examples. For each example we present: the type of (pde), the dimensions (M_{dpx}), the particular analytical solution $y_{AN}(t, \ldots)$, the initial and final limits of the integration ($t_0, t_f, p_0, p_f, \ldots$) as well as (crepy) to which we complete with further details.

Pde I.2. The program EDPTL32(33)

$$a_{00}\,y + a_{10}\frac{\partial y}{\partial t} + a_{01}\frac{\partial y}{\partial p} = \varphi(t,p) \quad (23)$$

$$\mathbf{M}_{dpx}(7\times 30) = \begin{array}{|c|c|}\hline \mathbf{x}(1\times1) & \mathbf{x}_P(1\times30) \\ \hline \mathbf{x}_T(6\times1) & \mathbf{x}_{TP}(6\times30) \\ \hline \end{array} \quad (24)$$

$$y_{AN}(t,p) = y_{00} + (J_{0T} + J_{1T}\cdot\varepsilon^{-\frac{t}{T_1}} + J_{2T}\cdot\varepsilon^{-\frac{t}{T_2}})\cdot$$
$$\cdot(J_{0P} + J_{1P}\cdot\varepsilon^{-\frac{p}{P_1}} + J_{2P}\cdot\varepsilon^{-\frac{p}{P_2}})\cdot K_u\cdot u \quad (25)$$

$t_0=0$; $t_f=10$; $p_0=0$; $p_f=10$; $\Delta t=0{,}1$; crepy$\le 6.2\cdot10^{-5}$.

Pde I.3. The program EDPTL35(36)

$$a_{000}y + a_{100}\cdot\frac{\partial y}{\partial t} + a_{010}\cdot\frac{\partial y}{\partial p} + a_{001}\cdot\frac{\partial y}{\partial q} = \varphi(t,p,q) \quad (26)$$

$$\mathbf{M}_{dpx}(7\times 30) = \begin{array}{|c|c|}\hline \mathbf{x}(1\times1) & \mathbf{x}_{PQ}(1\times30) \\ \hline \mathbf{x}_T(6\times1) & \mathbf{x}_{TPQ}(6\times30) \\ \hline \end{array} \quad (27)$$

$$y_{AN}(t,p,q) = y_{000} + J\cdot\varepsilon^{-\frac{t}{T}}\cdot\varepsilon^{-\frac{p}{P}}\cdot\varepsilon^{-\frac{q}{Q}} \quad (28)$$

$t_0=0$; $t_f=10$; $p_0=0$; $p_f=10$; $q_0=0$; $q_f=10$; $\Delta t=0{,}5$; crepy = $(2\cdot 10^{-6}\div 2\cdot10^{-5})$.

Pde I.4. The program RBPD1(2)

$$a_{0000}y + a_{1000}\cdot\frac{\partial y}{\partial t} + a_{0100}\cdot\frac{\partial y}{\partial p} + a_{0010}\cdot\frac{\partial y}{\partial q} + a_{0001}\cdot\frac{\partial y}{\partial r} = \varphi(t,p,q,r) \quad (29)$$

$$\mathbf{M}_{dpx}(7\times 344) = \begin{array}{|c|c|}\hline \mathbf{x}(1\times1) & \mathbf{x}_{PQR}(1\times343) \\ \hline \mathbf{x}_T(6\times1) & \mathbf{x}_{TPQR}(6\times343) \\ \hline \end{array} \quad (30)$$

$$y_{AN}(t,p,q,r) = y_{0000} + J\cdot\varepsilon^{-(\frac{t}{T}+\frac{p}{P}+\frac{q}{Q}+\frac{r}{R})}\cdot K_u\cdot u \quad (31)$$

$t_0=0$; $t_f=1$; $p_0=0$; $p_f=1$; $q_0=0$; $q_f=1$; $r_0=0$; $r_f=1$; $\Delta t=0{,}01$; crepy = $(5\cdot 10^{-6}\div 2\cdot10^{-2})$.

a.

Pde II2. The program EDPTL90(91)

$$a_{00}y + a_{10}\cdot\frac{\partial y}{\partial t} + a_{01}\cdot\frac{\partial y}{\partial p} + a_{20}\cdot\frac{\partial^2 y}{\partial t^2} + a_{11}\cdot\frac{\partial^2 y}{\partial t\cdot\partial p} + a_{02}\cdot\frac{\partial^2 y}{\partial p^2} = \varphi(t,p) \quad (32)$$

$$\mathbf{M}_{dpx}(9\times 31) = \begin{array}{|c|c|}\hline \mathbf{x}(2\times1) & \mathbf{x}_P(2\times30) \\ \hline \mathbf{x}_T(7\times1) & \mathbf{x}_{TP}(7\times30) \\ \hline \end{array} \quad (33)$$

$$y_{AN}(t,p) = y_{00} + (J_{0T} + J_{1T}\cdot\varepsilon^{-\frac{t}{T_1}} + J_{2T}\cdot\varepsilon^{-\frac{t}{T_2}})\cdot$$
$$\cdot(J_{0P} + J_{1P}\cdot\varepsilon^{-\frac{p}{P_1}} + J_{2P}\cdot\varepsilon^{-\frac{p}{P_2}})\cdot$$
$$\cdot K_u\cdot u\cdot(u_0 + u_A\sin\omega t) \quad (34)$$

$t_0=0$; $t_f=10$; $p_0=0$; $p_f=10$; $\Delta t=0{,}1$; crepy = $(3\cdot 10^{-5}\div 5\cdot10^{-3})$.

b.

Pde II3. The program EDPTL48(49)

$$a_{000}y + a_{200}\cdot\frac{\partial^2 y}{\partial t^2} + a_{020}\cdot\frac{\partial^2 y}{\partial p^2} + a_{002}\cdot\frac{\partial^2 y}{\partial q^2} = \varphi(t,p,q) \quad (35)$$

$$\mathbf{M}_{dpx}(8\times 64) = \begin{array}{|c|c|}\hline \mathbf{x}(2\times1) & \mathbf{x}_{PQ}(2\times63) \\ \hline \mathbf{x}_T(6\times1) & \mathbf{x}_{TPQ}(6\times63) \\ \hline \end{array} \quad (36)$$

$$y_{AN}(t,p,q) = y_{000} + J\cdot\varepsilon^{-(\frac{t}{T}+\frac{p}{P}+\frac{q}{Q})}\cdot K_u \quad (37)$$

$t_0=0$; $t_f=10$; $p_0=0$; $p_f=10$; $q_0=0$; $q_f=10$; $\Delta t=0{,}1$; crepy = $(6\cdot 10^{-6}\div 9\cdot10^{-4})$.

Pde II4. The program RBPD3(4). Example presented in detail in [5], respectively [1], [4].

c.

Pde III.2. The program EDPTL55(56)

$$a_{00}y + a_{30}\frac{\partial^3 y}{\partial t^3} + a_{03}\frac{\partial^3 y}{\partial p^3} = \varphi(t,p) \quad (38)$$

$$\mathbf{M}_{dpx}(9\times 20) = \begin{array}{|c|c|}\hline \mathbf{x}(3\times 1) & \mathbf{x}_P(3\times 19) \\ \hline \mathbf{x}_T(6\times 1) & \mathbf{x}_{TP}(5\times 19) \\ \hline \end{array} \quad (39)$$

$$\begin{aligned} y_{AN}(t,p) &= y_{00} + \\ &+ (J_{0T} + J_{1T}\varepsilon^{-t/T_1} + J_{2T}\varepsilon^{-t/T_2})\cdot \\ &(J_{OP} + J_{1P}\varepsilon^{-p/P_1} + J_{2P}\varepsilon^{-p/P_2})K_u u \end{aligned} \quad (40)$$

t_0=0; t_f=10; p_0=0; p_f=10; Δt=0,1; crepy = ($5\cdot 10^{-5} \div 5\cdot 10^{-3}$).

d.

Pde III.3. The program EDPTL62(63)

$$a_{000}y + a_{300}\frac{\partial^3 y}{\partial t^3} + a_{030}\frac{\partial^3 y}{\partial p^3} + a_{003}\frac{\partial^3 y}{\partial q^3} = \varphi(t,p,q) \quad (41)$$

$$\mathbf{M}_{dpx}(8\times 64) = \begin{array}{|c|c|}\hline \mathbf{x}(3\times 1) & \mathbf{x}_P(3\times 63) \\ \hline \mathbf{x}_T(5\times 1) & \mathbf{x}_{TP}(5\times 63) \\ \hline \end{array} \quad (42)$$

$$y_{AN}(t,p,q) = y_{000} + J\cdot\varepsilon^{-(\frac{t}{T}+\frac{p}{P}+\frac{q}{Q})}\cdot K_u\cdot u \quad (43)$$

t_0=0; t_f=10; p_0=0; p_f=10; q_0=0 ; q_f=10 ; Δt=0,1; crepy = ($10^{-6} \div 10^{-2}$).

Pde IV.2. The program EDPTL60(61)

$$a_{00}y + a_{40}\frac{\partial^4 y}{\partial t^4} + a_{04}\frac{\partial^4 y}{\partial p^4} = \varphi(t,p) \quad (44)$$

$$\mathbf{M}_{dpx}(10\times 21) = \begin{array}{|c|c|}\hline \mathbf{x}(4\times 1) & \mathbf{x}_P(4\times 20) \\ \hline \mathbf{x}_T(6\times 1) & \mathbf{x}_{TP}(4\times 20) \\ \hline \end{array} \quad (45)$$

$$\begin{aligned} y_{AN}(t,p) &= y_{00} + (J_{0T} + J_{1T}\cdot\varepsilon^{-t/T_1} + J_{2T}\cdot\varepsilon^{-t/T_2}) \\ &(J_{0P} + J_{1P}\cdot\varepsilon^{-p/P_1} + J_{2P}\cdot\varepsilon^{-p/P_2})\cdot K_u\cdot u \end{aligned} \quad (46)$$

t_0=0; t_f=10; p_0=0; p_f=10; Δt=0,1; crepy = ($10^{-5} \div 10^{-1}$).

Pde IV.3. The program EDPTL64(65)

$$a_{000}y + a_{400}\frac{\partial^4 y}{\partial t^4} + a_{040}\frac{\partial^4 y}{\partial p^4} + a_{004}\frac{\partial^4 y}{\partial q^4} = \varphi(t,p,q) \quad (47)$$

$$\mathbf{M}_{dpx}(8\times 64) = \begin{array}{|c|c|}\hline \mathbf{x}(4\times 1) & \mathbf{x}_{PQ}(4\times 63) \\ \hline \mathbf{x}_T(4\times 1) & \mathbf{x}_{TPQ}(4\times 63) \\ \hline \end{array} \quad (48)$$

$$y_{AN}(t,p,q) = y_{000} + J\cdot\varepsilon^{-t/T}\cdot\varepsilon^{-p/P}\cdot\varepsilon^{-q/Q}\cdot K_u\cdot u \quad (49)$$

t_0=0; t_f=10; p_0=0; p_f=10; q_0=0 ; q_f=10 ; Δt=0,1; crepy = ($10^{-5} \div 10^{-2}$).

Pde IV.4. The program EDP44.1(.2)

$$a_{4000}\frac{\partial^4 y}{\partial t^4} + a_{1111}\frac{\partial^4 y}{\partial t\partial p\partial q\partial r} = \varphi(t,p,q,r) \quad (50)$$

$$\mathbf{M}_{dpx}(8\times 8) = \begin{array}{|c|c|}\hline \mathbf{x}(4\times 1) & \mathbf{x}_{PQR}(4\times 7) \\ \hline \mathbf{x}_T(4\times 1) & \mathbf{x}_{TPQR}(4\times 7) \\ \hline \end{array} \quad (51)$$

$$y_{AN} = t^4\cdot p\cdot q\cdot r \quad (52)$$

t_0=0; t_f=10; p_0=0; p_f=10; q_0=0 ; q_f=10 ;r_0=0 ; r_f=10 ; Δt=0,01; crepy = $10^{-5} \div 5\cdot 10^{-3}$.

Pde II.2, non linear. The program RBPD5(6)

$$(a_{00} + a_{01}y)\frac{\partial y}{\partial p} + a_{10}\frac{\partial y}{\partial t} + a_{02}\frac{\partial^2 y}{\partial p^2} = \varphi(t,p) \quad (53)$$

$$\mathbf{M}_{dpx}(6\times 10) = \begin{array}{|c|c|}\hline \mathbf{x}(1\times 1) & \mathbf{x}_P(1\times 9) \\ \hline \mathbf{x}_T(5\times 1) & \mathbf{x}_{TP}(5\times 9) \\ \hline \end{array} \quad (54)$$

$$\begin{aligned} y_{AN}(t,p) &= y_{00} + (J_{0T} + J_{1T}\cdot\varepsilon^{-\frac{t}{T_1}} + J_{2T}\cdot\varepsilon^{\frac{-t}{T_2}}) \\ &(J_{0P} + J_{1P}\cdot\varepsilon^{\frac{-p}{P_1}} + J_{2P}\cdot\varepsilon^{\frac{-p}{P_2}})(y_{ff} - y_{00}) \end{aligned} \quad (55)$$

t_0=0; t_f=14; p_0=0; p_f=7; Δt=($10^{-4} \div 10^{-1}$) ; crepy = ($8\cdot 10^{-4} \div 0{,}8$).

The nonlinear (pde) (53) approximates a modified variant of the Cochen equation from the theory and practice of the columns of isotopic separation for (N^{15}). The particular solution (55) has proved to be very close to the experimental results of the concentration y(t, p) in accordance to which we have also calculated (crepy) for different integration steps (Δt).

e.

<u>System of two pde II.2. The program S2EDP 22(P).</u>

$$a_1 \cdot \frac{\partial y_1}{\partial t} + a_2 \cdot \frac{\partial^2 y_1}{\partial t^2} + a_3 \cdot \frac{\partial^2 y_2}{\partial t \cdot \partial p} = \varphi_1(t,p) \tag{56}$$

$$a_4 \cdot \frac{\partial^2 y_2}{\partial t^2} + a_5 \cdot \frac{\partial^2 y_1}{\partial t \cdot \partial p} + a_6 \cdot \frac{\partial^2 y_2}{\partial p^2} = \varphi_2(t,p) \tag{57}$$

$$\mathbf{M}_{dpx}(12 \times 4) = \boxed{\begin{array}{c|c} \mathbf{x}(4\times1) & \mathbf{x}_P(4\times3) \\ \hline \mathbf{x}_T(8\times1) & \mathbf{x}_{TP}(8\times3) \end{array}} \tag{58}$$

$$y_1 = t^2 \cdot p^3, \tag{59}$$

$$y_2 = t^3 \cdot p^2, \tag{60}$$

$t_0=0$; $t_f=10$; $p_0=0$; $p_f=10$; $\Delta t=0{,}001$; crepy = ($2 \cdot 10^{-3} \div 6 \cdot 10^{-2}$).

For the system of (pde) (56) and (57) we have considered the state variables: $x_{1.00}=y_1$; $x_{2.00}=y_2$; $x_{1.10} = \frac{\partial y_1}{\partial t}$ and $x_{2.10} = \frac{\partial y_2}{\partial t}$ so that the state vector of this system is:

$$\mathbf{x}(4\times1) = \begin{array}{|c|} \hline x_{1.00} \\ \hline x_{2.00} \\ \hline x_{1.10} \\ \hline x_{2.10} \\ \hline \end{array} \tag{61}$$

As a result, the use of ($\mathbf{M}_{dpx}$) can be extended for systems of (pde).

<u>Control system of a process, defined by pde II.2. The program 94(95).</u>

The control scheme in Fig. 1

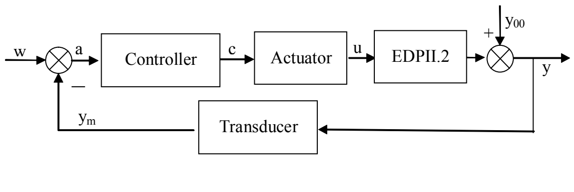

Fig. 1.

is defined by the following equation system:

$$a = w - y_M, \tag{62}$$

$$c = K_R \cdot a + K_I \cdot \int a dt + K_D \frac{da}{dt}, \tag{63}$$

$$u = K_v \cdot c, \tag{64}$$

$$\begin{aligned} a_{00} \cdot y + a_{10} \cdot \frac{\partial y}{\partial t} + a_{01} \cdot \frac{\partial y}{\partial p} + a_{20} \cdot \frac{\partial^2 y}{\partial t^2} + \\ + a_{11} \cdot \frac{\partial^2 y}{\partial t \cdot \partial p} + a_{02} \cdot \frac{\partial^2 y}{\partial p^2} = \varphi(t,p), \end{aligned} \tag{65}$$

$$y_{CM} = K_M \cdot y. \tag{66}$$

The succession of the above equations corresponds to: the comparison element, the PID controller, the flow-rate, the process with distributed parameters (pde II.2) designed for control and respectively the transducer of chemical concentration (y). For (65), the particular solution is expressed by:

$$\begin{aligned} y = y_{00} + (J_{0T} + J_{1T} \cdot \varepsilon^{\frac{-t}{T_1}} + J_{2T} \cdot \varepsilon^{\frac{-t}{T_2}}) \\ (J_{0P} + J_{1P} \cdot \varepsilon^{\frac{-p}{P_1}} + J_{2P} \cdot \varepsilon^{\frac{-p}{P_2}}) K_u \cdot u, \end{aligned} \tag{67}$$

where (T_1; T_2) and (P_1; P_2) are time constants, respectively length constants, associated to a column of isotopic separation. The system contains four state variables, respectively two for the controller (PID) and two for the process with distributed parameters (pde II.2).
Finally it has been operated with

$$\mathbf{M}_{dpx}(11 \times 31) = \boxed{\begin{array}{c|c} \mathbf{x}(4\times1) & \mathbf{x}_P(4\times30) \\ \hline \mathbf{x}_T(7\times1) & \mathbf{x}_{TP}(7\times30) \end{array}} \tag{68}$$

which for the reference signal

$$w = 100 + 30 \cdot \sin(\frac{2\pi}{T_1 + T_2} \cdot t) \tag{69}$$

and for $K_u=1$; $K_v=1$; $K_M=0{,}5$; $K_R=1{,}85$; $t_0=0$; $t_f=15$; $p_0=0$; $p_f=10$; $\Delta t=0{,}01$ we have crepy = ($3 \cdot 10^{-4}$-10^{-3}).
The results are diverse for different weights of the effects of (PID) in the controller, as well as for other parameters of structure of the system.

f.

Control system of a process, defined by pde II.3. The program EDPTL96P(97P)

The control scheme corresponds to the one in Fig. 1, the only change being the replacement of the process PDEII.2 with the new process pde II.3, defined by:

$$a_{000}\cdot y + a_{100}\frac{\partial y}{\partial t} + a_{010}\frac{\partial y}{\partial p} + a_{001}\frac{\partial y}{\partial q} + a_{200}\frac{\partial^2 y}{\partial t^2} + a_{110}\frac{\partial^2 y}{\partial t\cdot\partial p} +$$
$$+a_{020}\frac{\partial^2 y}{\partial p^2} + a_{011}\frac{\partial^2 y}{\partial p\cdot\partial q} + a_{002}\frac{\partial^2 y}{\partial q^2} + a_{101}\frac{\partial^2 y}{\partial t\cdot\partial q} = \varphi(t,p,q) \tag{70}$$

The equation system (62), (63), …(66) remains unchanged, but (65) is replaced with (70), whose particular solution is considered of a polynomial form of the third degree with respect to (t, p and q), respectively:

$$y = y_{000} + (J_{0T} + J_{1T}\cdot t + J_{2T}\cdot t^2 + J_{3T}\cdot t^3)(J_{0P} + J_{1P}\cdot p +$$
$$+J_{2P}\cdot p^2 + J_{3P}\cdot p^3)(J_{0Q} + J_{1Q}\cdot q + J_{2Q}\cdot q^2 + J_{3Q}\cdot q^3)\cdot$$
$$\cdot K_u\cdot u \tag{71}$$

The system also contains four state variables, out of which two belong to the (PID) controller, and two to the process with distributed parameters (pdeII.3).

$$\mathbf{M}_{dpx}(9\times16) = \begin{array}{|c|c|}\hline \mathbf{x}(4\times1) & \mathbf{x}_P(4\times15) \\ \hline \mathbf{x}_T(5\times1) & \mathbf{x}_{TP}(5\times15) \\ \hline\end{array} \tag{72}$$

with the observation that the relatively low degree of the polynomial form (71) has allowed the limitation of the number of columns in (72).
For the reference signal:

$$w=100+30\cdot\sin(\frac{2\pi}{t_f}\cdot t) \tag{73}$$

and for: K_u=1; K_v=1; K_M=1; K_R=1,85; t_0=0; t_f=10; p_0=0; p_f=10;q_0=0; q_f=10; Δt=0,01 we have crepy = $(10^{-5}\div4\cdot10^{-4})$.

In this example also the results have been diverse, for different weights of the effects of (PID) controller, as well as for other parameters of structure of the system.

7. CONCLUSIONS

The paper defines and uses "the matrix of partial derivatives of the state variables" ($\mathbf{M}_{dpx}$) associated to the method of integration by Taylor series.
Using ($\mathbf{M}_{dpx}$) we complete:

- the modeling of (pde) or the systems of (pde) [eventually also having (ode)], by establishing the elements and dimensions (n, N, M) of this matrix;
- numerical simulation, by elaborating a logical scheme, with a specific architecture, imposed to the iterations of calculus, for the elements that compose ($\mathbf{M}_{dpx}$);
- the analyses of propagation of trunk errors and the appreciation of the performances of numerical integration, with respect to the dimensions(n, N, M) and the elements of this matrix.

Using ($\mathbf{M}_{dpx}$), associated to Taylor series, for the numerical integration of (pde), or systems of (pde) with or without (ode), presents:

- the disadvantage of a relatively great volume for preparation, in order to insure the beginning of calculation;
- the advantage of a method of numerical integration with a quite general, unitary, performing and well systemized character, applied for wide categories of processes with distributed parameters.

The 15 examples, from 6.1, 6.2, … 6.15 have been succinctly presented, the eventual details concerning the structure parameters ($a_{...}$, $J_{...}$, $T_{...}$, $Q_{...}$, $R_{...}$, K_u, u, K_R, K_I, K_D, K_v, K_M, etc.) constructively functional (for 6.14; 6.15, etc.) or phenomenological interpretation resulting from the correspondent programs for each example. It has been dwelled on the value (crepy) which resulted from the running the programs on the computer, values that have proved to be extremely small, even for the dimensions (N, M) of $\mathbf{M}$[(n+N)x(1+M)] of not too great values.

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