MOBILE ROBOT EXPERIMENTAL MODELING AND CONTROL STRATEGIES USING SENSOR FUSION

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Abstract: This research work deals with the problem of modeling and design of low level speed controller for the mobile robot PRIM. The main objective is to develop an effective educational, and research tool. On one hand, the interests in using the open mobile platform PRIM consist in integrating several highly related subjects to the automatic control theory in an educational context, by embracing the subjects of communications, signal processing, sensor fusion and hardware design, amongst others. On the other hand, the idea is to implement useful navigation strategies such that the robot can be served as a mobile multimedia information point. It is in this context, when navigation strategies are oriented to goal achievement, that a local model predictive control is attained. Hence, such studies are presented as a very interesting control strategy in order to develop the future capabilities of the system. In this context the research developed includes the visual information as a meaningful source that allows detecting the obstacle position coordinates as well as planning the free obstacle trajectory that should be reached by the robot.

Keywords: Control education, model based control, predictive control, robot vision, sensor fusion

1. INTRODUCTION

Nowadays, the majority of mobile platforms for educational community cannot be used as opened platforms in a broad sense. In order to overcome this problem, the main purpose of this work is to propose an open mobile platform in order to achieve for all users the transparency of different hardware, sensors, communication systems, computer and control algorithms through a relatively easy understanding. The integration of the above subjects results in an instructional tool with a great flexibility and multiple application fields. From beginning, the idea of making use of the mobile robot platform PRIM has not been restricted only to the interests of educational community. Its philosophy can also be used as a mobile multimedia information point in the commercial applications, in which the navigation strategies are oriented to achieve the goal.

Within this context, the present research work has been developed as an educational set of laboratory experiments that can help the students to achieve the reinforcement of their knowledge learned from the textbooks and computer simulations, without forgetting the research objectives. In this sense, the sensor fusion using visual information should be integrated in the proposed control methods in order to obtain proper navigation strategies, which is one of the main research objectives.

This paper is organized as follows:

Section 1 gives a brief presentation about the aim of the present work. The use of a robot as an open mobile platform is of sufficient educational, commercial, and research interests.

In the Section 2, the platform is introduced as an electromechanical system. A general description of the platform as a flexible multipurpose tool is given.

Section 3 describes the experiments to be realized in order to find the parametric model of robot suitable for designing and implementing the low level speed control law. The analysis of coupling effects between two dc motors and different models is done.

Section 4 presents the control strategies used for achieving the path following of reference trajectories, which are mainly heuristic. In

Section 5, a local model predictive control strategy is presented as an important clue in order to achieve the final goal, using the local information provided by the robot sensors.

Section 6 presents the visual information, to accomplish with the navigation objectives, within a sensor fusion context. Finally, in the

Section 7, some conclusions are made with special attention paid into the future research works with their orientation to the improvement of the obtained results in a wide sense.

2. ELECTROMECHANICAL AND SENSORIAL SYSTEMS

The mechanical structure of the robot PRIM is made of aluminum, with two independent wheels of 16cm diameters actuated by two dc motors. The distance between two wheels is 56.4cm. A third spherical omni-directional wheel is used to guarantee the stability of system. The maximum continuous torque of each motor is 131mNm. The gear reduction proportion for each motor is 86:1 and thus the total force actuating on the robot is near 141N. Shaft encoders with 500 counts/rev are placed at the motor axes, which provide 43000 counts for each turn of the wheel. A set of PLD (programmable logic device) boards connected to the digital outputs of the shaft encoders. The printed circuits boards (PCB) are used to measure the speed of each motor at every 25ms. An absolute counter provides the counts in order to measure the robot position by the odometer system. Another objective of these boards is to generate a signal of 23khz PWM for each motor. The communication between the central digital computer and the boards is made through the parallel port. The speed is commanded by a byte and thus it can generate from 0 to 127 advancing speed commands. The maximal speed is near 0.5m/s. A set of microcontroller boards (MCS-51) is used to read information available from different connected sensors. The distance between objects is provided by an array of 8 sonar sensors, which are based on ultrasound sensors within a range of measurement from 3cm to 6m. The data provided by these boards is gathered through the serial port in the central computer. The rate of communication with these boards is 9600 b/s. Fig. 1 shows the electronic and sensorial system blocks. The data gathering and control by digital computer is set to 100ms.

The proposed educational open hardware has its advantages in many aspects. First, the use of a structure similar to that employed by students at the laboratories can enable their easy

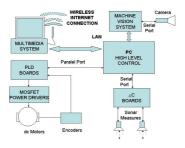


Figure 1. The sensorial and electronic system blocs

understanding and prototyping of new low level hardware. Furthermore, the reinforcement of the teaching activities can be achieved through the knowledge integration of different subjects. The system flexibility is increased with the possibility of connecting with other computer systems through a local LAN. The connected computers will increase the capabilities of the system and can be used as a multimedia point of information or as a machine vision system. The machine vision system controls a color camera EVI-D70P-PAL from Sony. This camera has motorized pan, tilt, zoom, focus, iris, shutter, through the VISCA RS232-C control protocol. The machine vision system computer unit is composed by a PC desktop, where it is used a Meteor-II as frame grabber, in union with the MIL libraries. The configuration of this work uses a horizontal field of view of 48°, and a vertical of 37°. The focus, pan and tilt remains fixed under present configuration. Hence, the camera pose is set to 109cm from the floor with a tilt angle of 58°. The obstacle perception information is transmitted to the control unit connecting the USB port to the LAN. A more general description of these characteristics can be found in [1].

3. EXPERIMENTAL MODEL AND LOW LEVEL CONTROL

The model identification presented in this section is described by using a useful methodology that provides a simplified dynamic model of the system. The model is obtained trough the approach of a set of lineal transfer functions that include the nonlinearities of the whole system. The parametric identification process is based on black box models [2]-[4]. Thus, the transfer functions are related to a set of polynomials that allow the use of analytic methods in order to deal with the problem of controller design. The nonholonomic system

dealt with in this work is considered initially as a MIMO (multiple input multiple output) system, as shown in Fig. 2, due to the dynamic influence

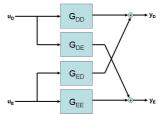


Figure 2. Structure of MIMO system between two dc motors. This MIMO system is composed of a set of SISO subsystems with coupled connection.

The approach of multiple transfer functions consists in making the experiments with three different (slow, medium and fast) speeds. In order to find a reduced-order model for the design of low level controllers, several studies and experiments have been done through the system identification, model simplification and controller design.

3.1 System Identification

The parameter estimation is done by using a PRBS (Pseudo Random Binary Signal) as excitation input signal. It guarantees the correct excitation of all dynamic sensible modes of the system along the whole spectral range and thus results in an accurate precision of parameter estimation. The experiments to be realized consist in exciting the two dc motors in different (low, medium, and high) ranges of speed.

The ARX (auto-regressive with external input) structure has been used to identify the parameters of the system. The problem consists in finding a model that minimizes the error between the real and estimated data. By expressing the ARX equation as a lineal regression, the estimated output can be written as:

$$\hat{\mathbf{y}} = \boldsymbol{\theta} \boldsymbol{\varphi} \tag{1}$$

with \hat{y} being the estimated output vector, θ the vector of estimated parameters, and φ the vector of measured input and output variables. By using the coupled system structure, the transfer function of the robot can be expressed as follows.

$$\begin{pmatrix} Y_D \\ Y_E \end{pmatrix} = \begin{pmatrix} G_{DD} & G_{ED} \\ G_{DE} & G_{EE} \end{pmatrix} \begin{pmatrix} U_D \\ U_E \end{pmatrix}$$
 (2)

where Y_D, and Y_E represent the speeds of right

and left wheels, and U_D and U_E the corresponding speed commands, respectively. In order to know the coupled system, the matrix of transfer function should be identified.

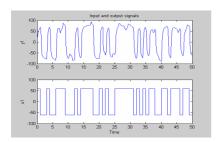


Figure 3. Left speed Output for a left PRBS input signal

Fig. 3 shows the speed response of the left wheel corresponding to a left PBRS input signal. The filtered data, that represent the average value of five different experiments with the same input signal, is used for identification. The system is identified by using the identification toolbox "ident" of Matlab for second order models. After the frequency filtering and tendency suppression, the following continuous transfer function matrix is obtained:

It is shown by simulation results that the obtained model fits well with the experimental data.

3.2 Simplified model of the System

This section studies the coupling effects and the way for obtaining a reduced-order dynamic model. It is seen from (3) that the dynamics of two dc motors are different and the steady gains of coupling terms are relatively small (less than 20% of the gains of main diagonal terms). Thus, it is reasonable to neglect the coupling dynamics so as to obtain a simplified model.

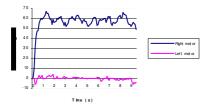


Figure 4. Coupling effects at the left wheel for medium speeds

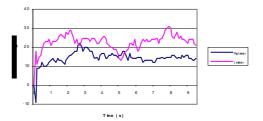


Figure 5. Different gain for slow speeds for the same consign

In order to verify the above facts from real results, a set of experiments have been done by sending a zero speed command to one motor and different non-zero speed commands to the other motor.

In Fig. 4, it is shown a response obtained on the left wheel, when a medium speed command is sent to the right wheel. The experimental result confirms that the coupled dynamics can be neglected.

The existence of different gains in steady state is also verified experimentally. As shown in Fig. 5, the gain of right dc motor is greater than that of left motor in the range of low speed. Finally, the order reduction of system model is carried out trough the analysis of pole positions by using the method of root locus. It reveals the existence of a dominant pole and consequently the model order can be reduced from second order to the first order. Within the range of medium speeds, the following first order transfer functions are obtained:

$$\begin{pmatrix} V_D \\ V_E \end{pmatrix} = \begin{pmatrix} DD & ED \\ DE & EE \end{pmatrix} = \begin{pmatrix} \frac{0.9001}{0.352s+1} & 0 \\ 0 & \frac{1,007}{0.5017s+1} \end{pmatrix} \begin{pmatrix} C_D \\ C_E \end{pmatrix}$$
 (4)

Afterwards, as shown in Fig. 6, the system models are validated through the experimental data by using the PBRS input signal.

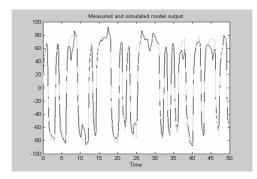


Figure 6. Experimental and model data for medium speeds.

3.3 Speed controller design

The low level speed control is performed by using PID controllers. Due to the existence of an integrator in the transfer functions, an integrating control law is adopted for the position servo control. The closed-loop transfer function is obtained as follows:

$$TF = \frac{vel(s)}{Cons(s)} = \frac{\frac{k_I k}{s(s\tau+1)}}{1 + \frac{k_I k}{s(s\tau+1)}} = \frac{k_I k}{s(s\tau+1) + k_I k}$$
 (5)

The controller design is done by using the method of pole placement for different system models. Then, the frequency response of the real system is compared with that of the models after order reduction, in which similar responses are obtained. The smoothness of the controlled responses is also analyzed by experiments.

3.4 Odometer system design

Denote (x, y, θ) as the coordinates of position and orientation, respectively. Fig. 7 describes the positioning of robot as a function of the radius of left and right wheels (R_e, R_d) , and the angular incremental positioning (θ_e, θ_d) , with E being the distance between two wheels and dS the incremental displacement of the robot.

The position and angular incremental displacements are expressed as:

$$dS = \frac{R_d d\theta_d + R_e d\theta_e}{2} \qquad d\theta = \frac{R_d d\theta_d - R_e d\theta_e}{E}$$
 (6)

The coordinates (x, y, θ) can be expressed:

$$x_{n} = x_{n-1} + dS \cos(\theta_{n-1} + d\theta)$$

$$y_{n} = y_{n-1} + dS \sin(\theta_{n-1} + d\theta)$$

$$\theta_{n} = \theta_{n-1} + d\theta$$
(7)

Thus, the incremental position of the robot can be obtained by the odometer system through the available encoder information obtained from (6) and (7).

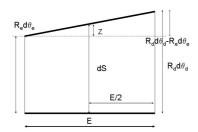


Figure 7. Positioning of robot as function of the angular movement of each wheel.

4. HEURISTIC CONTROL STRATEGIES FOR NAVIGATION

In this Section, different control strategies are presented for realizing the path following and collision avoidance during the navigation.

4.1 Path following

The speed control of robot is made through the reference consignment in the same sense, in order to achieve the smooth path following of given trajectories.

The navigation strategies are based on the configuration space, where the path is related to a set of points joined by straight lines [5]. In order to reduce the risk of collisions, the configuration space is increased with the wide path motion [6]. In the robot systems subject to the nonholonomic constrains, it is usually difficult to achieve a stabilized tracking of trajectory points by using lineal feedback laws [7]. In the research results presented by Hindman and Hauser, with a nonholonomic systems similar to that presented in this work, it is demonstrated by Lyapunov stability theory that the asymptotic stability exists in the control system with respect to the desired trajectory [8]. Hence, the employed control strategies consist in minimizing the distance error of orientation and position, as it is shown in Fig. 8.

A set of heuristic control strategies is proposed for the path following of robot starting from a given position and orientation. By using a low level controller, it is established previously the robot speed during the trajectory tracking. Simultaneously, the odometer system provides the robot position (x, y, θ) . The first straight line starts the trajectory to be followed, and the distance from the robot position (x,y) to the line can be computed.

If the distance is greater than a heuristic threshold, the speed commands sent to the wheels are modified. When the distance to the line is less than a threshold, the angular error correction is used for both speeds instead of using the distance decreasing error. Once the robot is near to the end of straight line (depending on the speed), another new line is considered as a trajectory to follow, along which the process will continue. This heuristic rule has given a good trajectory tracking, with

positioning errors less than 5cm along the straight lines, as shown in Fig. 9. In this heuristic strategy, the following parameters should be taken into account: the allowable speed on each line

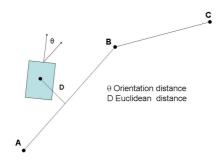


Figure 8. Orientation and position distance of the robot during the trajectory following

segment, the distance or orientation errors to be corrected with respect to the line, the differential of command speeds for each wheel in order to reduce the distance to the line, the constant values applied to the command speeds in order to reduce the orientation errors and the distance of robot to the end of the line when the trajectory line should be changed. In the present work, the differential of command speeds are set to 10% of robot speed and the change of trajectory points are related to the robot inertia.

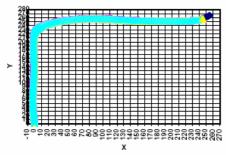


Figure 9. Trajectory following using heuristic rules

4.2 Collision avoidance in navigation

The sonar and camera sensors are used to perform the navigation without having maps to avoid the risk of collisions. This research is motivated by different necessities in navigation, such as the tracking of walls and the equidistant navigation of obstacles, etc. A more detailed explanation about the obstacle detection using sensor fusion and visual information is presented in Section 6.

5. LOCAL MODEL PREDICTIVE CONTROL

The model predictive control, MPC, has many interesting aspects for its application to mobile robot control. The MPC is the most effective advanced control technique, as compared to the standard PID control, that has made a significant impact to the industrial process control [11]. The philosophy of present research arises in strategies navigation oriented achievement. However, the navigation strategies presented until now cannot achieve the objective when navigation is based on maps. This problem can be solved using global sensor systems that provide real time information about the trajectories to be followed in order to achieve the goal. Since the sensorial system of some robots is just local, the global trajectory planning becomes unfeasible. By using a MPC, the idea of the receding horizon can deal with the local sensor information. In this way it is proposed a local model predictive control, LMPC, in order to use the technique in the strategies navigation oriented achievement. Another interesting point related to the use of the technique of MPC is the continuously decreasing prices of the computers, and their increasing capabilities.

The MPC is based on minimizing a cost function, related to the objectives, through the selection of the optimal inputs. In this case, the cost function can be expressed as follows:

$$J(n,m) = \min_{\left\{U(k+i|k)^{j=m-1}\atop j=0\right\}} \left\{ \left[X\right]^{T} P\left[X(k+n|k) - X_{d}\right] + \sum_{i=1}^{n-1} \left[X(k+i|k) - X_{d}\right]^{T} Q\left[X(k+i|k) - X_{d}\right] \right\} + \sum_{i=0}^{m-1} U^{T}(k+i|k) RU(k+i|k)$$

The first term of (8) is referred to the desired coordinates achievement, X_d =(x_d , y_d , θ_d), the second to the trajectory that can be followed, and the last one to the input signals. The parameters P, Q, and R are weighting parameters that express the importance of each term. The system constrains are also considered:

$$\begin{cases}
|U(k+i|k)| \le G_1 & \alpha \in [0,1) \\
|[x_{k+i}, y_{k+i}] - [x_o, y_o]| \ge G_2 \\
|[x_{k+n}, y_{k+n}] - [x_d, y_d]| \le \alpha |[x_k, y_k] - [x_d, y_d]|
\end{cases} (9)$$

The limitation of the input signal is taken into account in the first constraint. The second constraint is related to the obstacle points where the robot should avoid the collision. The last one is just a convergence criterion.

The LMPC algorithm is run in following steps:

- 1) To read the actual position
- 2) To minimize the cost function, and to obtain a series of optimal input signals.
- 3) To choose the first obtained input signal as command signal
- 4) Go back to the step one in the next sampling period.

The minimization of the cost function is a nonlinear problem in which the following equation should be verified:

$$f(\alpha x + \beta y) \le \alpha f(x) + \beta f(y) \tag{10}$$

It is a convex optimization problem [12] caused by the trigonometric functions used in (7). The use of interior point methods can solve the above problem [13]. Among many algorithms that can solve the optimization, the descent methods are used, such as the gradient descent method, steepest descent method, or the Newton's method, among others, [14], [15].

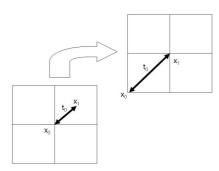


Figure 10. Optimal interval search

The gradient descent algorithm has been implemented in this work. In order to obtain the optimal solution, some constraints over the inputs are taken into account:

- 1. There is kept fixed signal increment during part of prediction horizon.
- 2. The input signals remain constant during the remaining interval of time.

The input constraints present advantages such like the reduction in the computation time and

the smooth behavior of the robot during the prediction horizon. Thus, the set of available input is reduced to one value. In order to reduce the optimal signal value search, the possible input sets are considered as a bidimensional array, as shown in Fig. 10. Then, the array is decomposed into four zones, and the search is just located to analyze the center points of each zone. It is considered the region that offers better optimization, where the algorithm is repeated for each sub-zone, until no sub-interval can be found. Once the algorithm is proposed, several simulations have been carried out in order to test the effectiveness, and then to make the improvements. Several considerations about the cost function are taken into account:

- 1. When only the desired coordinates are considered the robot could not arrive in the final point. Fig. 11 shows that the inputs can minimize the cost function by shifting the robot position to the left. The reason can be found at (4), where the left motor has more gain than the right. This problem can be easily solved by considering a straight line trajectory from the actual point of the robot to the final desired point. Thus, the trajectory should be included into the cost function.
- 2. When the robot orientation is reversed to the goal point. The robot cannot find solution, so initial points bring far away from goal and no action is done. This problem has been solved, just considering the orientation as one of the parameters to be minimized, when several orientation discrepancies are found.
- 3. When obstacles are presented across the goal straight line trajectory the robot stop, no point can approach the robot to the objective point.

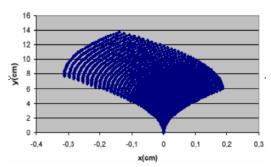


Figure 11. The left deviation is due by the greater left gain of the robot

The third consideration may be solved by increasing the predictive horizon. However, with

unknown size of obstacles, this strategy does not guarantee a final solution. Another possibility is to use global sensors, and consequently just use a set point tracking into the MPC algorithms. However, the use of local sensor systems makes suitable the use of alternative navigation strategies. Thus, navigation strategies presented in section 4, such as the collision avoidance methods based on sonar, or machine vision system, can overcome obstacle collisions until that the optimal input signals can approach again the robot to the final goal.

The computation time, for each LMPC steep, was just of 20ms, running under 2.7GHz PC. The results presented in this work, make suitable the use of LMPC in the robot where the control period is set to 100ms. The selected prediction horizon is composed of five sample periods, in which the last two ones are just steady states. Finally, the prediction horizons between 0.5s and 1s are considered appropriate by taking into account the dynamic of the robot.

6. LOCAL VISUAL INFORMATION AND SENSOR FUSION

This section presents briefly the sensor fusion uses and their relationship with the robot navigation strategies. The odometer, machine vision and sonar sensor systems should provide enough information in order to obtain proper navigation strategies. In this sense the research developed is based on obstacle detection and path planning using a local map based on sensor information where obstacle avoidance should be planned. Thus, the robot position is known using odometer system, while obstacle locations are provided by camera as well as sonar information. Consequently, the different control navigation strategies presented can be planned using the robot position coordinates and the robot relative local map coordinates.

The machine vision system can provide a local map coordinates referred to the robot position. Thus, the local map provided by the camera configuration of this work, presented at Section 2, can be computed using trigonometric expressions and perspective projection [16].

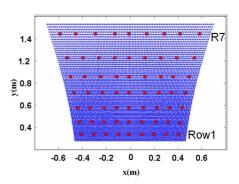


Figure 12. Local perception grid coordinates relative to the robot position, at a 96x72 and 9x7 image size resolution. At the 9x7 size level it is possible to distinguish 7 different rows (Row 1 being the closest to the robot).

The Fig. 12 shows the local visual map obtained in this work. Using computer vision methods obstacle coordinates can be found, as i.e. actually the authors are testing cooperative DFF (depth from focus), OFT (optical flow techniques), and odometer system information, that allow a 3D scenario recovery [1]. Once the local map is known the trajectory can be planned using the different navigation strategies presented in Section 4, or the MPC techniques presented in Section 5. In situations with local minimal failures, obstacle contour tracking is proposed until a new path that minimizes the final desired coordinate distance is found.

7. CONCLUSIONS AND FUTURE WORK

The open mobile platform presented in this paper has provided with a broad understanding of results, and it has been served as a useful tool for the teaching reinforcement. This issue has been mainly focused on the experience of control theory but with a set of other topics involved in the experiments. The transparency of the proposed platform has achieved the improvement of results in an integrated context.

Concerning about the directions of future research, the improvement of control strategies presented in this work will be done mainly by applying the LMPC presented in this work. The main objective is the simultaneous achievement of goal point and collision avoidance. However, as it has been seen in the section 5 the local perception provided by the sensor system, and specially the machine vision system, can fall in local minimization. This can be solved by using the methods of heuristic obstacle collision

avoidance until a new trajectory, in order to reach the objective point through the minimization of the function cost.

Studies involving new robot behaviors, i.e., turning around itself, will also be carried out.

The sensor fusion and environment understanding are another research topic. In this case, the accumulative errors, provided by the dead reckoning, should be set to zero periodically. A feasible way to do it is the natural landmarks detection, where the machine vision system can become a very useful tool [1].

Some comparison study (heuristics will done predictive) be through the experimentation of path following. Despite of the multiple works that should be carried out to improve the obtained results, it is also possible to make the commercial application of some proposed strategies as a multimedia information mobile point and thus to develop other research works. In this sense, the integration of highly related subjects in a multi-sensorial rich information environment is an interesting goal of the proposed open mobile platform. It involves a lot of teaching and research activities as well as some increasing commercial and social interests.

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