More relaxed stability conditions for fuzzy TS control systems by optimal determination of membership function information

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Abstract: In this paper, a systematic and optimal approach is presented for more relaxed stability analysis conditions and controller design for Takagi-Sugeno fuzzy systems. The approach is based on the idea of non-quadratic Lyapunov function. The non-quadratic Lyapunov function is a fuzzy blending of multiple quadratic Lyapunov function. The weak point of non-quadratic Lyapunov function is that upper bounds of time derivatives of membership functions is considered known or selected by trial and error. In this paper, the upper bounds are determined based on the concept of decay rate and control input constraint. In contrast to the existing work based on non-quadratic Lyapunov function, the proposed method leads to more relaxed stability analysis conditions and wider stability region by optimal calculation of the upper bounds of the time derivatives of membership functions. The proposed approach provides a stable closed-loop control system with faster response and less control effort along with wider stability region in terms of system parameters variations. Several numeric examples and comparisons illustrate the effectiveness and superiority of the proposed method.

Keywords: Stability conditions, Relaxed conservativeness, Linear matrix inequality, Parallel distributed compensation, Takagi-Sugeno fuzzy model.

1. INTRODUCTION

Many practical systems have nonlinear dynamic. Conventional linear controllers may lead to local stability in the case of nonlinear systems, and design of nonlinear controllers can be difficult in general. Today, fuzzy controllers are used in the control of nonlinear systems since fuzzy models are universal approximators (Tanaka, 2001) and can represent a nonlinear system efficiently. One of the proposed fuzzy models is Takagi-Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985). In TS fuzzy model, the local dynamics of each fuzzy rule are represented by linearizing the nonlinear system at each operation point. The overall model of the system is obtained by fuzzy blending of these local models.

The fuzzy control design problem is formulated by using Parallel Distributed Compensation (PDC) scheme. The PDC offers a procedure to design a fuzzy controller from a given TS fuzzy model to calculate state feedback gains. The idea of the PDC design is to derive each control rule based on the corresponding TS fuzzy rule. The main advantage of the PDC controller is to provide a systematic approach to cope with stabilization and performance issues. It is well known that sufficient conditions for the stability and performance of a system can be restated in terms of Linear Matrix Inequalities (LMI) (Scherer and Weiland, 2004) and can be solved by using numerical convex optimization. Many papers have been concerned about stability and stabilization conditions using Lyapunov functions.

In (Tanaka and Sugeno, 1992) it has been proven that the stability of TS model can be obtained if there exists a common constant positive definite matrix of a conventional quadratic Lyapunov function satisfying the stability conditions of all subsystems, simultaneously. Several researches have been conducted based on this approach (Chun-Hsiung Fang *et al.,* 2006; Euntai Kim and Heejin Lee, 2000; Korba *et al.,* 2003; Tanaka, 2001; Tanaka *et al.,* 2001a, 1996; Teixeira *et al.,* 2003; Yong-Yan Cao and Zongli Lin, 2003). Using quadratic Lyapunov function leads to conservative stability analysis conditions, and perhaps no positive definite matrix exists to satisfy the stability conditions (Tanaka, 2001).

To solve the conservativeness problem, piecewise Lyapunov function was proposed (Borne and Dieulot, 2005; Gang Feng, 2003; Gang Feng *et al.,* 2005; Johansson *et al.,* 1999). The piecewise Lyapunov function is a blending of separate conventional quadratic Lyapunov functions, each of which is suitable for a single partition. To obtain the global stability, some boundary conditions are applied due to the discontinuities of the function across the subspace boundaries. One of the main disadvantages of piecewise Lyapunov functions is that the controller design leads to non-convex optimization problems in general (Tanaka *et al.,* 2003).

Another type of proposed Lyapunov function is non-quadratic Lyapunov function (Johansson, 1999). This function is a fuzzy blending of multiple conventional quadratic Lyapunov function. It is also named fuzzy Lyapunov function (Tanaka *et al.,* 2001b) or parameterized Lyapunov function (PLF) (Mozelli *et al.,* 2009). Non-quadratic Lyapunov function is a proper alternate since it is smooth, contrary to a piecewise Lyapunov function, thus avoiding the boundary conditions problem (Tanaka, 2001). Several researches have been ascertained based on the non-quadratic Lyapunov function approach (Bernal and Hušek, 2005; Chadli *et al.,* 2002; Chien-Hung Liu *et al.,* 2005; Manai and Benrejeb, 2011; Mozelli *et al.,* 2009; Tanaka *et al.,* 2007, 2003, 2001b; Wu and Zhang, 2007; Zhou *et al.,* 2007).

The main disadvantage of using the non-quadratic Lyapunov function approach is presence of time derivative of membership functions in the stability conditions. In order to convert these conditions into LMIs, upper bounds for the time derivatives of the membership functions must be considered (Bernal and Hušek, 2005; Chadli *et al.,* 2002; Chien-Hung Liu *et al.,* 2005, 2005; Manai and Benrejeb, 2011; Mozelli *et al.,* 2009; Tanaka *et al.,* 2007, 2003, 2001b; Wu and Zhang, 2007; Zhou *et al.,* 2007). The upper bound of time derivative of membership function is chosen by trial and error (Abdelmalek *et al.,* 2007; Ariño *et al.,* 2010; Mozelli *et al.,* 2009). In other words there is no systematic approach for choosing the upper bounds. Few of the literature are concerned about systematic approaches for choosing upper bounds. In (Abdelmalek *et al.,* 2007; Manai and Benrejeb, 2011; Tanaka *et al.,* 2001a) an approach was proposed based on the derivation of membership function with respect to states. Regardless, time derivation cannot be defined for some membership functions. In (Bernal *et al.*, 2010; Guerra *et al.,* 2012), global stabilization is reduced to local stabilization to overcome the problem of time derivate of membership function. Hence, an attempt was made to find an estimation of region of attraction. This approach has a restrictive assumption due to local stabilization and also requires too many conditions to be satisfied. In (Mozelli *et al.,* 2009; Rhee and Won, 2006), a line-integral Lyapunov function is proposed in which the time derivate of membership function does not appear. But its applicability significantly decreases because the line-integral Lyapunov function must be path-independent (Guelton *et al.,* 2010).

In this paper a new systematic approach is proposed for choosing the upper bounds of the time derivative of the membership functions based on the concept of decay rate and input constraint to achieve more relaxed stability conditions. It is shown that the upper bounds have a direct effect on the speed of response or equivalently on the decay rate or the largest Lyapunov exponent. Increasing the speed of response and physical restrictions on control inputs requires that the maximum upper bounds of the membership functions be determined in an optimal manner rather than by trial and error. To do this, the maximizing problem is formulated in Generalized Eigen Value problem (GEVP). To show the effectiveness of the proposed approach, several examples are presented and compared to the existing results.

This paper is organized as follows: In Section 2 TS fuzzy model is presented. Stability conditions are discussed in Section 3. Existing results and the new stability conditions and the relation of upper bound of time derivative of membership function with decay rate are also explored in Section 3. Constraint on the control input is investigated in Section 4. Simulation and comparison results are given in Section 5 and finally conclusions are presented in Section 6.

2. TS FUZZY MODEL

A TS fuzzy system is described by a set of fuzzy IF-THEN rules that represent locally linear input-output relations of a system. The overall fuzzy model of the system is obtained by fuzzy blending of linear system models. The i -th rule of a general r -rules TS fuzzy model can be written as follows:

$$\text{if } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip},$$

$$\text{then}\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases}, i = 1,2,\ldots,r \tag{1}$$

where, $z(t) = [z_1(t),\ldots,z_p(t)]^T \in \mathcal{R}^{p\times 1}$ is the premise variable vector whose elements are the function of states in general, $x(t) \in \mathcal{R}^{n\times 1}$ is the state vector and M_{ij} is the j -th fuzzy set related to the i -th premise variable. The overall fuzzy system with rule (1), singleton fuzzifier and center average defuzzifier is of the form:

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{r} W_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} W_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \end{aligned} \tag{2}$$

$$\begin{aligned} y(t) &= \frac{\sum_{i=1}^{r} W_i(z(t))\{C_i x(t)\}}{\sum_{i=1}^{r} W_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\{C_i x(t)\} \end{aligned} \tag{3}$$

$$W_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \tag{4}$$

where, $h_i(z(t))$ is normalized membership function in relation to the i -th rule such that:

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \tag{5}$$

3. STABILITY CONDITIONS

To obtain stability conditions, the i -th rule of non-quadratic Lyapunov function candidate and PDC controller is defined as:

$$\text{if } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip}$$

$$\text{then}\begin{cases} V(x(t)) = x(t)^T P_i x(t) \\ u(t) = -F_i x(t) \end{cases} \tag{6}$$

The PDC controller, non-quadratic Lyapunov function and the TS fuzzy model share the same fuzzy sets in premise parts. The overall fuzzy controller system and non-quadratic Lyapunov function is of the form:

$$V\left(x\left(t\right)\right)=\sum_{i=1}^{r}h_i\left(z\left(t\right)\right)x\left(t\right)^T P_i x\left(t\right) \tag{7}$$

$$u\left(t\right)=-\sum_{i=1}^{r}h_i\left(z\left(t\right)\right)F_i x\left(t\right) \tag{8}$$

By substituting (8) into (2) the closed loop system is written as:

$$\dot{x}\left(\mathrm{t}\right)=\sum_{i=1}^{r}\sum_{i=1}^{r}h_i\left(z\left(t\right)\right)h_j\left(z\left(t\right)\right) \left\{A_i-B_iF_i\right\}x\left(t\right) \tag{9}$$

Based on the non-quadratic Lyapunov function (7) and following two assumptions, stability conditions can be derived (Abdelmalek et al., 2007):

Assumption 1: The time derivation of membership function has an upper bound.

$$\left|\dot{h}_\rho\right|<\phi_\rho \qquad \rho=1,\ldots,r \tag{10}$$

Assumption 2: The local positive definite symmetric matrices P_i for $i=1,\ldots,r$ are proportionally related, such that:

$$P_i=\alpha_{ji}P_j \qquad i,j=1,\ldots,r \tag{11}$$

$$\alpha_{ij}=\begin{cases}1/\alpha_{ji}>0 & i\neq j\\ 1 & i=j\end{cases} \tag{12}$$

Lemma 1: $X_i=\alpha_{ij}X_j$ where $X_i=P_i^{-1}$.

Proof: From (12), one has $P_i=\alpha_{ji}P_j$

$$P_i^{-1}=\alpha_{ji}^{-1}P_j^{-1} \Rightarrow \quad X_i=\alpha_{ij}X_j. \tag{13}$$

Lemma 2: $\sum_{\rho=1}^{r}\phi_\rho P_\rho=\gamma_i P_i$ where $\gamma_i=\sum_{\rho=1}^{r}\phi_\rho\alpha_{i\rho}$.

Proof: By using Assumption 2 ($P_\rho=\alpha_{i\rho}P_i$), one has:

$$\sum_{\rho=1}^{r}\phi_\rho P_\rho=\sum_{\rho=1}^{r}\left(\phi_\rho\alpha_{i\rho}P_i\right)=\sum_{\rho=1}^{r}\left(\phi_\rho\alpha_{i\rho}\right)P_i=\gamma_iP_i \tag{14}$$

where $\gamma_i=\sum_{\rho=1}^{r}\phi_\rho\alpha_{i\rho}$.

Lemma 3 (Schur complement): Suppose an affine partitioned matrix

$$F=\begin{bmatrix}F_1 & F_2\\ F_3 & F_4\end{bmatrix} \tag{15}$$

where, F_1 is square. $F\leq 0$, if and only if

$$\begin{cases}F_4\leq 0\\ F_1-F_2F_4^{-1}F_3\leq 0\end{cases} \tag{16}$$

Proof is presented in (Scherer and Weiland, 2004).

Theorem 1: Under Assumptions 1 and 2, the fuzzy system can be stabilized via the PDC fuzzy controller, if there exist positive constants φ_ρ, α_{ij} and matrices $X_i=P_i^{-1}$, $M_i=F_iP_i^{-1}$ for $i,j,\rho=1,\ldots,r$ such that (Abdelmalek et al., 2007):

$$X_i=X_i^T>0 \qquad i=1,\ldots,r \tag{17}$$

$$\sum_{\rho=1}^{r}\phi_\rho X_\rho+X_iA_j^T+A_jX_i-\alpha_{ij}M_j^TB_j^T -\alpha_{ij}B_jM_j<0 \tag{18}$$

$$\begin{gathered}X_iA_j^T+X_iA_k^T+A_jX_i+A_kX_i-\alpha_{ij}M_j^TB_k^T\\ -\alpha_{ik}M_k^TB_j^T-\alpha_{ij}B_kM_j-\alpha_{ik}B_jM_k<0\\ for \quad i,j,k=1,\ldots,r \quad \& \ j<k\end{gathered} \tag{19}$$

Proof of LMI (18): By using (7)

$$V=\sum_{i=1}^{r}h_ix^TP_ix \tag{20}$$

$$\dot{V}=\dot{x}^T\left(\sum_{i=1}^{r}h_iP_i\right)x+x^T\left(\sum_{\rho=1}^{r}\dot{h}_\rho P_\rho\right)x+ x^T\left(\sum_{i=1}^{r}h_iP_i\right)\dot{x} \tag{21}$$

By substituting (9) into (21):

$$\begin{aligned}\dot{V}=x^T&\left\{\begin{array}{l}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j=1}^{r}h_ih_jh_j\left(G_{jj}^TP_i+P_iG_{jj}\right)\\ +\sum_{\rho=1}^{r}\dot{h}_\rho P_\rho\end{array}\right\}x\\ +x^T&\left\{\begin{array}{l}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j<k}^{r}h_ih_jh_k\left(\frac{G_{jk}+G_{kj}}{2}\right)^TP_i\\ +P_i\left(\frac{G_{jk}+G_{kj}}{2}\right)\end{array}\right\}x\end{aligned} \tag{22}$$

$$\begin{aligned}<x^T&\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j=1}^{r}h_ih_jh_j\left\{\begin{array}{l}\left(G_{jj}^TP_i+P_iG_{jj}\right)\\ +\sum_{\rho=1}^{r}\phi_\rho P_\rho\end{array}\right\}\right\}x\\ +x^T&\left\{\begin{array}{l}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j<k}^{r}h_ih_jh_k\left(\frac{G_{jk}+G_{kj}}{2}\right)^TP_i\\ +P_i\left(\frac{G_{jk}+G_{kj}}{2}\right)\end{array}\right\}x\end{aligned} \tag{23}$$

where, $G_{ij}=A_i-B_iF_j$. The time derivative of non-quadratic Lyapunov function is negative, if the following inequalities hold:

$$\sum_{\rho=1}^{r}\phi_\rho P_\rho+G_{jj}^TP_i+P_iG_{jj}<0 \tag{24}$$

$$\left(\frac{G_{jk}+G_{kj}}{2}\right)^T P_i + P_i\left(\frac{G_{jk}+G_{kj}}{2}\right) < 0 \tag{25}$$

By pre- and post-multiplying (24) by P_i^{-1} and using Lemma 2, (16), one has:

$$P_i^{-1}\left(\sum_{\rho=1}^{r}\phi_\rho\left(\alpha_{i\rho}P_i\right)\right)P_i^{-1} + P_i^{-1}\left(A_j - B_jF_j\right)^T \tag{26}$$
$$P_iP_i^{-1} + P_i^{-1}P_i\left(A_j - B_jF_j\right)P_i^{-1}$$

$$= P_i^{-1}\sum_{\rho=1}^{r}\phi_\rho(\alpha_{i\rho}) + P_i^{-1}\left(A_j - B_jF_j\right)^T \tag{27}$$
$$+\left(A_j - B_jF_j\right)P_i^{-1}$$

$$= \sum_{\rho=1}^{r}\phi_\rho(\alpha_{i\rho})P_i^{-1} + P_i^{-1}A_j^{\;T} - P_i^{-1}F_j^TB_j^T \tag{28}$$
$$+A_jP_i^{-1} - B_jF_jP_i^{-1}$$

By defining $X_i = P_i^{-1}$, (28) will be continued as:

$$= \sum_{\rho=1}^{r}\phi_\rho(\alpha_{i\rho})X_i + X_iA_j^{\;T} - X_iF_j^TB_i^T + A_jX_i \tag{29}$$
$$-B_jF_jX_i$$

Using Lemma 1 ($X_i = \alpha_{ij}X_j$), (29) will be continued as:

$$= \sum_{\rho=1}^{r}\phi_\rho(\alpha_{i\rho})X_i + X_iA_j^{\;T} - \alpha_{ij}X_jF_j^TB_i^T \tag{30}$$
$$+A_jX_i - B_jF_j\alpha_{ij}X_j$$

By defining $M_j = F_jX_j$ LMI (18) is obtained.

Proof of LMI (19): Following the same procedure of the proof of LMI (18), LMI (19) will be obtained.

3.1 Relation Of Upper Bound Of Time Derivative Of Membership Function To Decay Rate

Definition of decay rate: If a Lyapunov function satisfies the following inequality (Slotine and Li, 1991)

$$\dot V\left(x\left(t\right)\right) + 2\alpha V\left(x\left(t\right)\right) < 0 \tag{31}$$

where $\alpha \geq 0$ is called the decay rate, then Lyapunov function exponentially converges to zero and the system is exponentially stable.

Since membership function derivative depends on state vector derivative (Abdelmalek *et al.,* 2007) in a derivation chain rule, upper bounds of membership functions cannot be determined accurately. So, in (Abdelmalek *et al.,* 2007), the upper bounds of membership function were selected by trial and error. On the other hand, the upper bounds of membership function affect the system response speed. The larger upper bounds lead to higher response speed. Also, the upper bounds of membership function affect the decay rate of exponential stability. So, optimal determination of the upper bounds of membership functions is formulated in a generalized eigenvalue problem.

The speed of response is related to the decay rate, that is, the largest Lyapunov exponent. It is mentioned that choosing the smaller ϕ_ρ leads to less conservative conditions (Tanaka *et al.*, 2001a) but decreases the speed of response. To study the relation of ϕ_ρ to decay rate, the non-quadratic Lyapunov function (7) is substituted into (31).

$$\begin{aligned}
&\dot V\left(x\left(t\right)\right) + 2\alpha V\left(x\left(t\right)\right)\\
&< x^T\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j=1}^{r}h_ih_jh_j\begin{pmatrix}G_{jj}^TP_i + P_iG_{jj}\\ +\sum_{\rho=1}^{r}\phi_\rho P_\rho + 2\alpha P_i\end{pmatrix}\right\}x\\
&+x^T\left\{\begin{matrix}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j<k}^{r}h_ih_jh_k\left(\frac{G_{jk}+G_{kj}}{2}\right)^T P_i\\ +P_i\left(\frac{G_{jk}+G_{kj}}{2}\right)\end{matrix}\right\}x
\end{aligned} \tag{32}$$

Using Lemma 2, (32) will be:

$$\begin{aligned}
&= x^T\left\{\begin{matrix}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{j=1}^{r}h_ih_jh_j\left(G_{jj}^TP_i + P_iG_{jj}\right)\\ +\left(\alpha_i + 2\alpha\right)P_i\end{matrix}\right\}x\\
&+x^T\left\{\begin{matrix}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k>j}^{r}h_ih_jh_k\left(\frac{G_{jk}+G_{kj}}{2}\right)^T P_i\\ +P_i\left(\frac{G_{jk}+G_{kj}}{2}\right)\end{matrix}\right\}x
\end{aligned} \tag{33}$$

In practice, It is interested in maximizing α to increase the speed of the response in the presence of α_i. Maximizing α can be equivalently replaced by the problem of maximizing γ_i in the presence of α. Without loss of generality, one can maximize α_i when $\alpha = 0$. For instance, $\gamma_i = 1$ and the maximized $\alpha = 4$ can be replaced by maximized $\gamma_i = 5$ and $\alpha = 0$. Hence, the problem of maximizing the speed of the response can be handled as in Theorem 2.

Theorem 2: The TS fuzzy system (9) is stable in large if there exist symmetric matrices $X_i = P_i^{-1}$, matrices $M_i = F_iP_i^{-1}$ and positive scalars ϕ_i for $i = 1,\ldots,r$ such that:

$$\underset{X_1,\ldots,X_r,M_1,\ldots,M_r}{\text{Maximize}}\ \phi_1,\ldots,\phi_r \tag{34}$$

Subject to :

$$X_i = X_i^T > 0 \qquad i = 1,\ldots,r \tag{35}$$

$$\sum_{\rho=1}^{r}\phi_\rho X_\rho + X_iA_j^T + A_jX_i - \alpha_{ij}M_j^TB_j^T \tag{36}$$
$$-\alpha_{ij}B_jM_j < 0 \qquad i,j = 1,\ldots,r$$

$$X_i A_j^T + X_i A_k^T + A_j X_i + A_k X_i - \alpha_{ij} M_j^T B_k^T$$
$$-\alpha_{ik} M_k^T B_j^T - \alpha_{ij} B_k M_j - \alpha_{ik} B_j M_k < 0 \quad (37)$$
$$for\ i,j,k = 1,\dots,r\ \&\ j<k$$

Proof: Following the same procedure of the proof of Theorem 1 and the relation of upper bounds of time derivative of membership functions to the decay rate, the proof of Theorem 2 is straightforward.

Theorem 2 is hard to use directly due to maximizing r scalar constants ($\phi_1,\dots,\phi_r$), simultaneously. Also, inequality (36) is BMI due to the variables ϕ_ρ and X_ρ for $\rho = 1,\dots,r$. To overcome these difficulties, Theorem 3 is proposed.

Theorem 3: The TS fuzzy system (9) is stable in large if there exist symmetric matrices $X_i = P_i^{-1}$, matrices $M_i = F_i P_i^{-1}$ for $i = 1,\dots,r$ and positive scalar β such that:

$$\text{Minimize}\ \beta \quad (38)$$

Subject to :

$$X_i = X_i^T > 0 \qquad i = 1,\dots,r \quad (39)$$

$$\begin{bmatrix} \begin{Bmatrix} X_i A_j^T + A_j X_i \\ -\alpha_{ij} M_j^T B_j^T - \alpha_{ij} B_j M_j \end{Bmatrix} & (X_i + I) \\ (X_i + I)^T & -\beta\tau_i I \end{bmatrix} \le 0 \quad (40)$$

$$X_i A_j^T + X_i A_k^T + A_j X_i + A_k X_i - \alpha_{ij} M_j^T B_k^T$$
$$-\alpha_{ik} M_k^T B_j^T - \alpha_{ij} B_k M_j - \alpha_{ik} B_j M_k < 0 \quad (41)$$
$$for\ \ i,j,k = 1,\dots r\ \&j<k$$

where $\tau_i = \left(\sum_{\rho=1}^{r} \alpha_{\rho i}\right)^{-1}$.

Proof: By defining

$$\Phi = \max_{\rho=1,\dots,r} \{\phi_\rho\} \quad (42)$$

and by substituting (42) into (36), one has:

$$\Phi \sum_{\rho=1}^{r} X_\rho + X_i A_j^T + A_j X_i - \alpha_{ij} M_j^T B_j^T$$
$$-\alpha_{ij} B_j M_j < 0 \quad (43)$$

If (43) holds, then (36) will hold. Maximizing Φ leads to maximizing the upper bounds of the time derivative of the membership functions and hence maximizing the response speed. By using Lemma 2, and (16), inequality (43) can be rewritten as:

$$\Phi\left(\sum_{\rho=1}^{r} \alpha_{\rho i} X_i\right) + X_i A_j^T + A_j X_i - \alpha_{ij} M_j^T B_j^T - \alpha_{ij} B_j M_j \quad (44)$$

$$= \frac{\Phi}{2} \sum_{\rho=1}^{r} \alpha_{\rho i} (X_i + X_i^T) + X_i A_j^T + A_j X_i - \alpha_{ij} M_j^T B_j^T$$
$$-\alpha_{ij} B_j M_j \quad (45)$$

$$= \frac{\Phi}{2} \sum_{\rho=1}^{r} \alpha_{\rho i} \{(X_i + I)(X_i + I)^T - X_i X_i^T - I\} + X_i A_j^T$$
$$+A_j X_i - \alpha_{ij} M_j^T B_j^T - \alpha_{ij} B_j M_j \quad (46)$$

By defining $\beta = \frac{2}{\Phi}$, and $\tau_i = \left(\sum_{\rho=1}^{r} \alpha_{\rho i}\right)^{-1}$, (46) will be continued as:

$$= X_i A_j^T + A_j X_i - \alpha_{ij} M_j^T B_j^T - \alpha_{ij} B_j M_j$$
$$+\beta^{-1}\tau_i^{-1}(X_i + I)(X_i + I)^T - \beta^{-1}\tau_i^{-1} X_i X_i^T \quad (47)$$
$$-\beta^{-1}\tau_i^{-1} I$$

Since $\beta^{-1}\tau_i^{-1} X_i X_i^T \ge 0$, and $\beta^{-1}\tau_i^{-1} I \ge 0$, (47) will be continued as:

$$\le X_i A_j^T + A_j X_i - \alpha_{ij} M_j^T B_j^T - \alpha_{ij} B_j M_j$$
$$+\beta^{-1}\tau_i^{-1}(X_i + I)(X_i + I)^T \quad (48)$$

By employing Schur complement, (15) and (16) are as follow:

$$F_1 = X_i A_j^T + A_j X_i - \alpha_{ij} M_j^T B_j^T - \alpha_{ij} B_j M_j$$
$$F_2 = (X_i + I)$$
$$F_3 = (X_i + I)^T \quad (49)$$
$$F_4^{-1} = -\beta^{-1}\tau_i^{-1} I$$

LMI (40) is obtained. Proof of (41) is the same as (40).

4. CONTROL INPUT CONSTRAINT

Maximizing speed of response leads to maximizing the amplitude of the control input. In practice, there are limitations in exerting high amplitude control input and many processes are subjected to constraints on input (Namazov *et al*., 2011). In this section, LMI conditions for input constraint are derived.

Theorem 4: Assume that the initial condition $x(0)$ of the fuzzy system (9) is known. The constraint $\|u\|_2 < \mu$ is enforced at all times, if the LMIs (50) and (51) hold.

$$\begin{bmatrix} 1 & x(0)^T \\ x(0) & X_i \end{bmatrix} \ge 0 \ \ , for\ h_i(z(0)) \ne 0 \quad (50)$$

$$\begin{bmatrix} X_i & M_i^T \\ M_i & \mu^2 I \end{bmatrix} \ge 0 \quad (51)$$

Proof of LMI (50): Assume that for the non-quadratic Lyapunov functions, the inequality (52) holds (Tanaka, 2001):

$$V(x(t)) \le V(x(0)) \le 1 \quad (52)$$

$$1 - V(x(0)) \ge 0 \quad (53)$$

$$1 - \sum_{i=1}^{r} h_i(z(0)) x(0)^T P_i x(0) \ge 0 \quad (54)$$

$$\sum_{i=1}^{r} h_i\left(z\left(0\right)\right) - \sum_{i=1}^{r} h_i\left(z\left(0\right)\right) x\left(0\right)^T P_i x\left(0\right) \geq 0 \tag{55}$$

Inequality (55) is implied by the following inequality:

$$h_i\left(z\left(0\right)\right) - h_i\left(z\left(0\right)\right) x\left(0\right)^T P_i x\left(0\right) \geq 0 \tag{56}$$

Assume that $h_i\left(z\left(0\right)\right) > 0$. Dividing inequality (39) by $h_i\left(z\left(0\right)\right)$, one has:

$$1 - x\left(0\right)^T P_i x\left(0\right) \geq 0 \tag{57}$$

By employing Schur complement, Eqs (15-16) are as follows:

$$\begin{aligned} &F_1 = 1 \\ &F_2 = x\left(0\right)^T \\ &F_3 = x\left(0\right) \\ &F_4^{-1} = P_i \end{aligned} \tag{58}$$

One has:

$$\begin{bmatrix} 1 & x\left(0\right)^T \\ x\left(0\right) & P_i^{-1} \end{bmatrix} \geq 0 \tag{59}$$

By defining $X_i = P_i^{-1}$, the LMI (50) is achieved.

Remark 1: (57) is obtained by assuming $h_i\left(z\left(0\right)\right) \neq 0$. If $h_j\left(z\left(0\right)\right) = 0$ for $j \in \{1, 2, \ldots, r\}$ then (56) is equal to zero and so is greater than or equal to zero. Hence, inequality (56) holds and it is not necessary to transform it into LMI (57).

Proof of LMI (51): The input constraint can be reformulated as follows:

$$\|\mathrm{u}\|_2 < \mu \Rightarrow u^T\left(t\right) u\left(t\right) \leq \mu^2 \tag{60}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T F_i^T F_j x \leq \mu^2 \tag{61}$$

$$\frac{1}{\mu^2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T F_i^T F_j x \leq 1 \tag{62}$$

Comparing (62) and (52) shows a similarity in the right hand side of both inequalities. Since (52) holds and it is trying to prove that the left side of (62) is less that one, one applies an assumption on control input constraints to proceed with our formulations in LMI. Therefore, it is assumed that the left side of (62) is less than $V\left(x\left(t\right)\right)$. This assumption implies (62). Then, one obtains:

$$\begin{aligned} &\frac{1}{\mu^2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T F_i^T F_j x \\ &\leq V\left(x\left(t\right)\right) = \sum_{i=1}^{r} h_i x^T P_i x \end{aligned} \tag{63}$$

holds, (62) can be obtained. Inequality (63) is implied as follows:

$$\begin{aligned} &\frac{1}{\mu^2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T F_i^T F_j x \\ &-\frac{1}{2}\left(\sum_{i=1}^{r} h_i x^T P_i x + \sum_{j=1}^{r} h_j x^T P_j x\right) \leq 0 \end{aligned} \tag{64}$$

$$\begin{aligned} &\frac{1}{2\mu^2}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j x^T \{F_i^T F_j + F_j^T F_i\} x \\ &-\frac{1}{2}\left(\sum_{i=1}^{r} h_i x^T P_i x + \sum_{j=1}^{r} h_j x^T P_j x\right) \leq 0 \end{aligned} \tag{65}$$

$$x^T \left\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left\{\frac{F_i^T F_j + F_j^T F_i}{2\mu^2} - \frac{1}{2}\left(P_i + P_j\right)\right\}\right\} x \leq 0 \tag{66}$$

$$\frac{1}{2\mu^2}\left\{F_i^T F_j + F_j^T F_i\right\} - \frac{1}{2}\left(P_i + P_j\right) \leq 0 \tag{67}$$

Multiplying (67) by 2, left side of the inequality (67) will be:

$$\frac{1}{\mu^2}\left\{F_i^T F_j + F_j^T F_i\right\} - \left(P_i + P_j\right) \tag{68}$$

$$\begin{aligned} &= \frac{1}{\mu^2}\left\{F_i^T F_i + F_j^T F_j - \left(F_i - F_j\right)^T \left(F_i - F_j\right)\right\} \\ &-\left(P_i + P_j\right) \end{aligned} \tag{69}$$

Since $\left(F_i - F_j\right)^T \left(F_i - F_j\right) \geq 0$, (69) will be continued as:

$$\begin{aligned} &\frac{1}{\mu^2}\left\{F_i^T F_i + F_j^T F_j - \left(F_i - F_j\right)^T \left(F_i - F_j\right)\right\} \\ &-\left(P_i + P_j\right) \leq \frac{1}{\mu^2}\left\{F_i^T F_i + F_j^T F_j\right\} - \left(P_i + P_j\right) \end{aligned} \tag{70}$$

If

$$\frac{1}{\mu^2}\left\{F_i^T F_i + F_j^T F_j\right\} - \left(P_i + P_j\right) \leq 0 \tag{71}$$

holds, (67) can be obtained. Inequality (71) is implied as follows:

$$\frac{1}{\mu^2} F_i^T F_i - P_i \leq 0 \tag{72}$$

$$P_i - \frac{1}{\mu^2} F_i^T F_i \geq 0 \tag{73}$$

By pre- and post-multiplying (73) to P_i^{-1}, one has:

$$P_i^{-1} P_i P_i^{-1} - \frac{1}{\mu^2} P_i^{-1} F_i^T F_i P_i^{-1} \geq 0 \tag{74}$$

By defining $X_i = P_i^{-1}, M_i = F_i X_i$ and using Schur complement,

$$\begin{aligned} &F_1 = X_i \\ &F_2 = M_i^T \\ &F_3 = M_i \\ &F_4^{-1} = \frac{1}{\mu^2} I \end{aligned} \tag{75}$$

Finally, (74) leads to (51).

5. SIMULATION EXAMPLES AND COMPARISONS RESULTS

This section presents some examples that illustrate the effectiveness of the new proposed approach.

Example 1. (An inverted pendulum on a cart) Consider the problem of balancing and swinging-up an inverted pendulum on a cart using the proposed approach (Tanaka et al., 2001a). The system is modeled by two fuzzy rules (Abdelmalek et al., 2007):

$$if\ x_1(t) is\ about\ 0$$

$$then\ \begin{cases} \dot{x}(t) = A_1x(t) + B_1u(t) \\ y(t) = C_1x(t) \end{cases}$$

$$if\ x_1(t) is\ about\ \pm\frac{\pi}{2}$$

$$then\ \begin{cases} \dot{x}(t) = A_2x(t) + B_2u(t) \\ y(t) = C_2x(t) \end{cases}$$

where,

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\dfrac{4l}{3} - aml} & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi\left(\dfrac{4l}{3} - aml\beta^2\right)} & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ -\dfrac{a}{\dfrac{4l}{3} - aml} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -\dfrac{a\beta}{\dfrac{4l}{3} - aml\beta^2} \end{bmatrix}$$

and $g = 9.8m/s^2$ is the gravity constant, $m = 2Kg$ is the mass of the pendulum, $2l = 1m$ is the length of the pendulum, $M = 8Kg$ is the mass of the cart, $a = 1/(m+M)$ and $\beta = \cos(88^\circ)$. The membership functions are:

$$h_1(x_1(t)) = \begin{cases} \dfrac{\sin(x_1(t))}{x_1(t)}, & x_1(t) \neq 0 \\ 1, & x_1(t) = 0 \end{cases}$$

$$h_2(x_1(t)) = 1 - h_1(x_1(t))$$

For $a_{12} = 1.3$, $a_{21} = 1/a_{12}$, $x(0) = [\pi/6\ \ 0]^T$ and heuristically defining $\varphi_1 = \varphi_2 = \pi/1.5$ in (Abdelmalek et al., 2007), the local feedback gains of PDC controller are obtained as follows:

$$F_1 = [-630.7446\ \ -164.6591],$$

$$F_2 = 10^{-3}[-1.2396\ \ -0.2958]$$

By using the proposed approach in this paper and choosing $\mu = 282.5$ (Tanaka et al., 2001a), one has:

$$P_1 = \begin{bmatrix} 3.6472 & 0.5375 \\ 0.5375 & 0.1031 \end{bmatrix} > 0$$

$$P_2 = \begin{bmatrix} 3.6472 & 0.5466 \\ 0.5466 & 0.1066 \end{bmatrix} > 0$$

$$F_1 = [-538.4464\ \ -81.9253]$$

$$F_2 = [-379.3624\ \ -88.3836]$$

$$\phi = 5.1184$$

Fig. 1 shows the state responses of the system obtained from the proposed approach by solid line and the solution from (Abdelmalek et al., 2007) is shown by dashed line. Fig. 2 shows the control input trend. Comparing the results shows the priority of the proposed approach to achieve faster response and less energy consumption.

Example 2. Consider the following fuzzy system (Abdelmalek *et al.,* 2007; Tanaka *et al.,* 2001b):

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{A_ix(t) + B_iu(t)\}$$

$$h_1(x_1(t)) = \frac{1 + \sin(x_1(t))}{2}$$

$$h_2(x_1(t)) = 1 - h_1(x_1(t))$$

$$A_1 = \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & -4 \\ 20 & -2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

For $a_{12} = 0.2$, $a_{21} = 1/a_{12}$, $x(0) = [1\ \ 1]^T$ and heuristically defining $\varphi_1 = \varphi_2 = 0.5$ in (Abdelmalek et al., 2007), the local feedback gains of PDC controller are obtained:

$$F_1 = [0.0262\ \ 0.1232]$$

$$F_2 = [-3.4925\ 1.9967]$$

By using the proposed approach and choosing $\mu = 4.5$, one has:

$$\phi = 3.1136$$

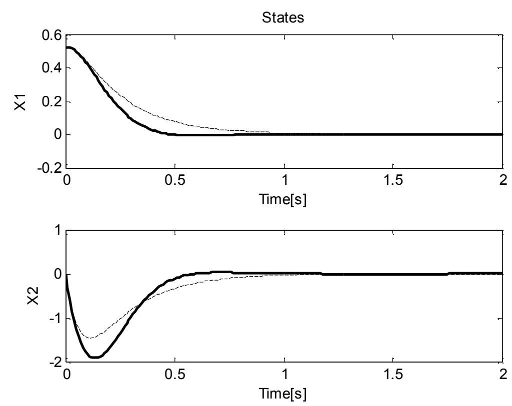

Fig. 1. States responses of closed loop system based on the approach of (Abdelmalek *et al.*, 2007) shown by dashed line and the proposed approach shown by solid line.

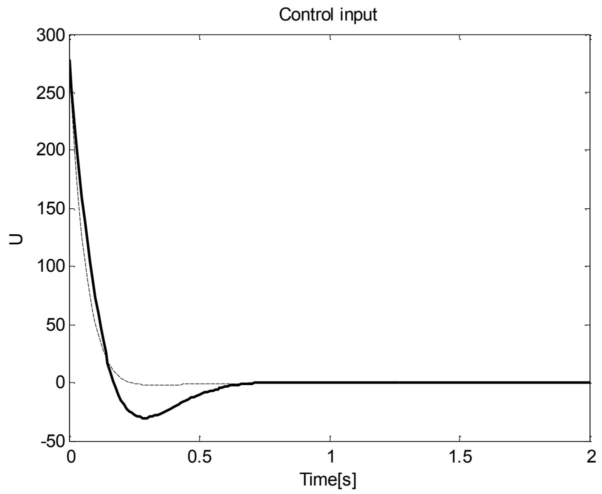

Fig. 2. Control law trend based on the approach of (Abdelmalek *et al.,* 2007) shown by dashed line and the proposed approach shown by solid line.

$$P_1 = \begin{bmatrix} 0.4941 & -0.0084 \\ -0.0084 & 0.0959 \end{bmatrix} > 0$$
$$P_2 = \begin{bmatrix} 1.0795 & -0.1702 \\ -0.1702 & 0.2605 \end{bmatrix} > 0$$
$$F_1 = \begin{bmatrix} -0.5315 & 1.1330 \end{bmatrix}$$
$$F_2 = \begin{bmatrix} -0.3237 & 2.2212 \end{bmatrix}.$$

Fig. 3 shows the states evolution of the closed loop system. The increasing system response is evident for the control system designed by the proposed approach compared with the method proposed by (Abdelmalek *et al.,* 2007).

Fig. 4 shows the control input variation imposed on the system. Less energy consumption and less control input amplitude are obtained by the proposed approach compared with (Abdelmalek *et al.*, 2007).

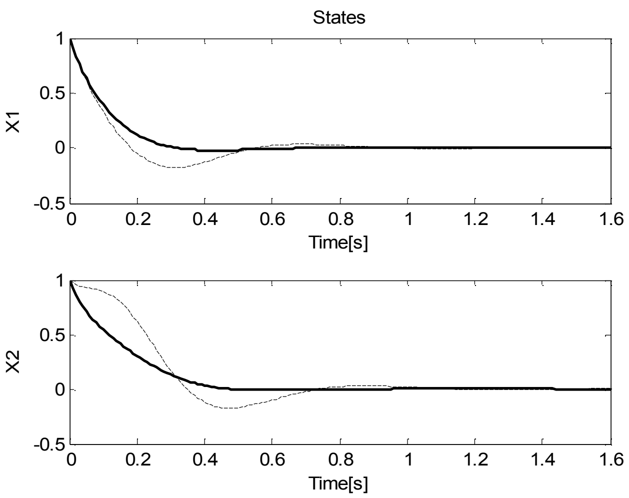

Fig. 3. The states variations of the closed loop system based on the approach of (Abdelmalek et al., 2007) shown by dashed line and the proposed approach shown by solid line.

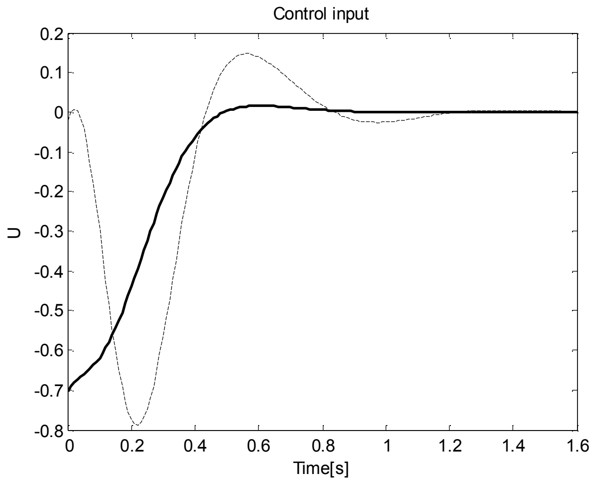

Fig. 4. Control input variations obtained by the approach of (Abdelmalek *et al.,* 2007) shown by dashed line and the proposed approach shown by solid line.

The next example investigates the feasibility of the associated fuzzy control synthesis.

Example 3. Consider the following fuzzy system (Guerra et al., 2012; Rhee and Won, 2006):

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left\{A_i x(t) + B_i u(t)\right\}, |x_i| < \pi/2$$
$$h_1\left(x_1(t)\right) = \frac{1-\sin\left(x_1(t)\right)}{2}$$
$$h_2\left(x_1(t)\right) = \frac{1+\sin\left(x_1(t)\right)}{2}$$
$$A_1 = \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} a & -5 \\ 1 & 2 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} b \\ 2 \end{bmatrix}, -22 \le a \le 10, -4 \le b \le 27$$

The parameters a and b are set in a prescribed grid in order to check the feasibility of the different design methods. Fig. 5 shows the parameters region where the stability of the fuzzy control system is guaranteed by using the controller design proposed in different references (Guerra *et al.,* 2012; Rhee and Won, 2006) and in this paper. In this figure, the '×' and 'o' marks indicate the existence of the feasible stabilizing controller ensured by (Guerra *et al.,* 2012) and (Rhee and Won, 2006), respectively. The '+' mark shows the parameters values for which stability region is guaranteed by the proposed method in this paper. It is obvious that our stability region is wider than the results obtained in (Guerra *et al.,* 2012; Rhee and Won, 2006).

Example 4. Consider the system introduced in example 3. For $a = 4$ and $b = 2$, neither the local stabilization method (Guerra et al., 2012) nor line integral Lyapunov function approach (Rhee and Won, 2006) can guarantee the stabilization of the closed loop system. By employing the proposed approach and using $a_{12} = 0.5$, $a_{21} = 1/a_{12}$,

$x(0)=\begin{bmatrix}0.2 & 0.2\end{bmatrix}^T, \mu=10$ the local gains are obtained as follows:

$$\phi = 0.4628$$

$$P_1 = \begin{bmatrix} 114.0496 & -118.1630 \\ -118.1630 & 131.1275 \end{bmatrix} > 0$$

$$P_2 = \begin{bmatrix} 117.1053 & -119.2458 \\ -119.2458 & 128.9077 \end{bmatrix} > 0$$

$$F_1 = \begin{bmatrix} -88.6731 & 104.6815 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -74.9715 & 83.7171 \end{bmatrix}$$

Fig. 6 shows the states responses of the closed loop and Fig. 7 indicates the control effort for the control system.

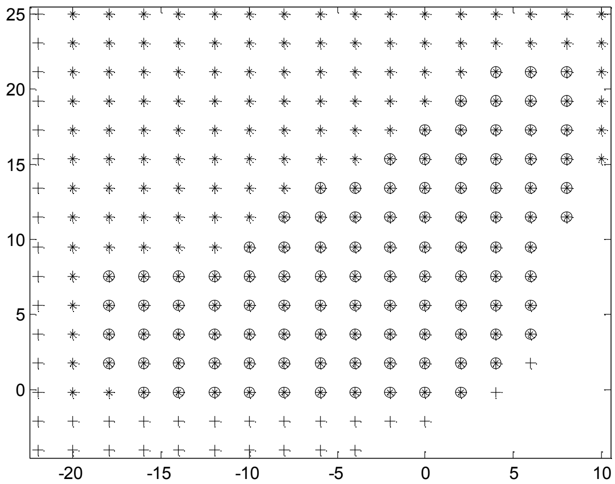

Fig. 5. Stabilization region based on theorems in (Guerra et al., 2012) (o), in (Rhee and Won, 2006) (×) and in this paper (+).

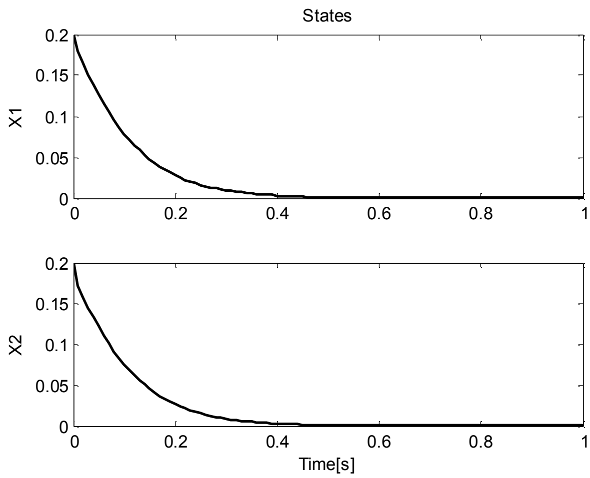

Fig. 6. States evolution of the closed loop system.

6. CONCLUSIONS

A systematic procedure was proposed to improve stability and stabilization conditions of Takagi-Sugeno control systems. The novel procedure was dedicated to optimal design of upper bound of time derivatives of membership functions. The optimal upper bound and local feedback gains of the controller are determined to satisfy the stability condition and input constraint. In previous research, knowledge of the upper bounds was mandatory for the LMI formulation of stability conditions, whereas employing the new approach leads to the optimal selection of the upper bounds in a generalized eigenvalue problem and therefore less conservative conditions. Fig. 8 shows the controller design procedure of the proposed approach. Fig. 8 indicates that, first the nonlinear dynamic equations and the equivalent TS fuzzy model are derived to exactly represent the original nonlinear system behavior. Also, an upper bound of the H_2 norm of control input is measured due to the practical restrictions. After that, considering the TS fuzzy model and control input information and utilizing theorems 3 and 4, proposed in this paper, provide the feedback gains of PDC controller and the non-quadratic Lyapunov function. Simulation examples and comparison results demonstrated the advantages of the proposed approach.

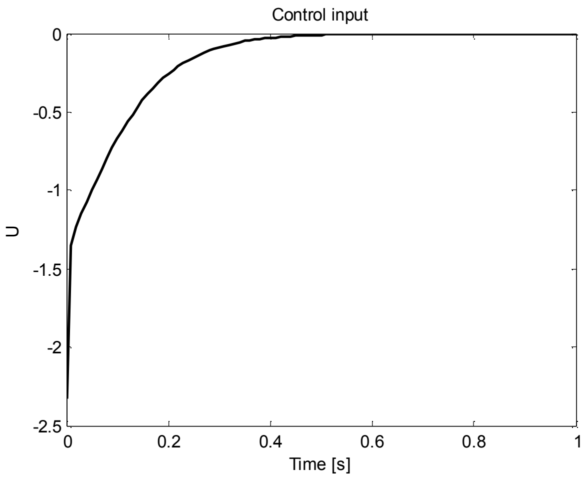

Fig. 7. Control effort.

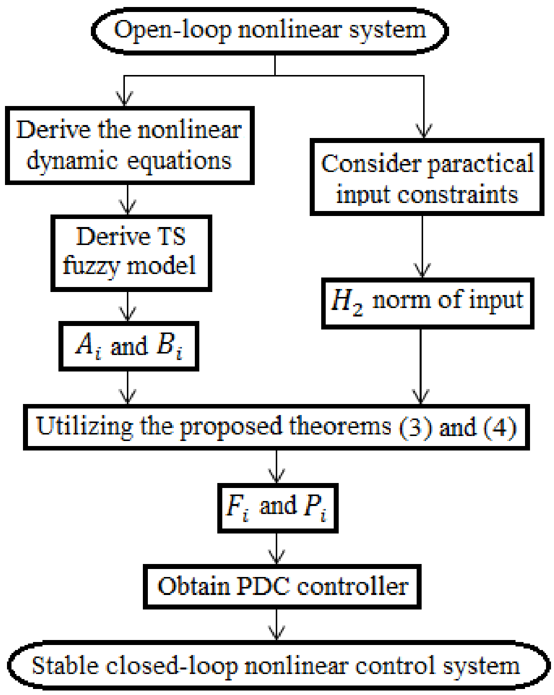

Fig. 8. Flowchart of the controller design procedure.

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