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DESIGN PROCEDURES OF SOME INTERPOLATIVE CONTROL STRUCTURES WITH ROBUSTNESS PROPERTIES AND LIMITATIONS

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Abstract: *The advantages using interpolative blocks in control structures are, mainly, based on simplifying the solutions (easier implementation and reduced calculus time) and the possibility to ensure for the control systems, in a relative easy way, some robustness properties. The goal of the paper is to underline the possibilities offered by the interpolative-type control and to illustrate this through a case study regarding to a positioning system. The case study suggests different issues from which the problem can be viewed, from the control, robustness properties and also, from the stability point of view, in Lyapunov's acception.*

Keywords: *interpolation, interpolative controller, robustness and limitations.*

1. INTRODUCTION

Interpolation means a generalization form usually used in human reasoning. Mainly, through interpolation can be adopted decisions – an un-limited number of them - for current cases, based on accumulated experience from a finite number of situations and on a generalization mechanism, experience synthesised as causal correlations. Unlike the rigorous generalization met in classical math, interpolation generalization – even in the cases in which is followed by a mathematical approach -, represents a non-rigorous generalization, meaning that it works in unpredictable situations, achieving correctable effects.

In engineering interpolation is used from a long time, the "engineering tables" represents one of the most popular form. In automatic control also interpolation was used from some period of time in order to realize some correction elements. Interpolation usage was growing with the fuzzy and neural controller apparition. Those generalization mechanism is based on an "interpolative reasoning" [1], [2], [7]. Because the mathematical substrate they are based on, they have the "ability to hide the interpolative character", so the practitioners through the interpolative fundaments do not perceive those kinds of controllers and their hybrids.

As an alternative to the fuzzy and neural controllers, in 1996 the so-called "interpolative controllers" are proposed. They refer on explicit interpolation as presented in [3], [4]. The object of the present paper is to presents some aspects (design, robustness and limitations aspects) related to such controllers.

Generally, the interpolative-type controllers implement the control algorithms through support points defined in interpolative blocks, which are used either alone (Drechsel in [3]), or integrated in dynamic structures (as in [5], [6]). This offers the possibility to start from an already existing initial solution ([5], [6]), and to correct the initial dependency in a quite simple manner. The changes operated in the tables, within the design operation of the interpolative controller, are made to improve performances for the control system they are started from. The improvement techniques used are different, according to each and every application. The improvement criterion assures, step by step, a structure with better performances than the initial one. In essence, the interpolative controllers used in [5] and [6] and also in this paper, are interpolation tables, which contain a finite number of support values, collected from support points. The difference between these researches is the manner in which the support values are collected [8].

The second part of the paper underlines some theoretical aspects concerning the methodology used to obtain interpolative controllers based on fuzzy controllers, leading to more robust control system than the fuzzy ones. A study case considering also limitations is presented in the 3rd part. Finally a few remarks conclude the presentation and an appendix resumes some of the theoretical aspects used in section 3.

2. DESIGN PROCEDURES FOR INTERPOLATIVE CONTROLLERS WITH ROBUSTNESS PROPERTIES

In this paper, as start point for the design (*Step 0*) is considered the simple fuzzy structure with a fuzzy controller with synthetic input (*RG_1F*) represented in Figure 1, based on some relations between the control signal *sd* (the signed distance to the switching line in the state

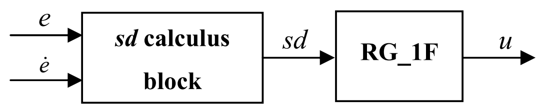


Fig 1. Signed distance *sd* calculus and the associated *RG_1F* controller.

plane of calculus block), the error *e* and it's first derivative $\dot{e}$, developed in [9]. The design procedure is described in [5], [8] and [14]; it is not an issue for the present paper.

To obtain a simpler control structure with better dynamic performances, the *RG_1F* is replaced with a single input interpolative-type controller (*RG-1I*) like in Figure 2 and two design steps are covered.

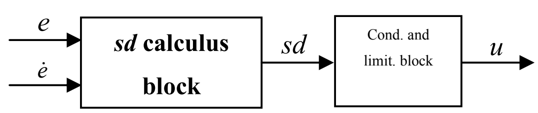


Fig 2. Signed distance *sd* calculus and the associated *RG_1I* controller.

Step 1: The *RG_1I* is developed in order to replace the *RG_1F*. The purpose is to achieve an interpolative single-input controller with, at least, the same behaviour of the control system.

The development run as follow: based on the single-input fuzzy controller a set of significant support points are extracted from the input-output control dependence of the *RG_1F* and as a result an almost identical interpolative reproduction of the fuzzy characteristic will be obtained. Such interpolative blocks can be implemented as "look-up tables" in application tools as MATLAB-SIMULINK or through dedicated numerical programs. Finally, a simple single-input interpolative controller *RG_1I* (developed in two ways) is obtained, with performances at least equal (but improvable) to those of the initial structure, but with less calculus time and with simpler implementation possibilities.

Step2. An interpolative controller, robust to changes in reference signal is developed. The purpose is to achieve a simpler single-input controller with improved behaviour of the control system. The development consists in an iterative tuning of the interpolative block.

Step3. Real processes are non-linear. It's difficult to consider the non-linearity from the beginning, in the first design steps. From this reason, they can be treated after the linear structures are designed. The problem can be viewed in different ways; in the present paper one of them is presented. Usually, the non-linearity issue is omitted or is considered in a superficial manner and so a significant amount of studies have only academic value without an engineering, applicative one. In this context, in the last step of the study presented in section 3 two categories of limitations are considered: on the states variables level and on the command level. The solution presented in the paper has a quasi-analytical character.

3. STUDY CASE

The objective of this case study is to control a linear positioning system with the transfer function [10]:

$$H(s)=\frac{1}{s\cdot(s+1)} \qquad (1)$$

Step 0. The fuzzy control scheme in Figure 3 with a synthetic input *sd* for the fuzzy bloc FB is used as a starting point.

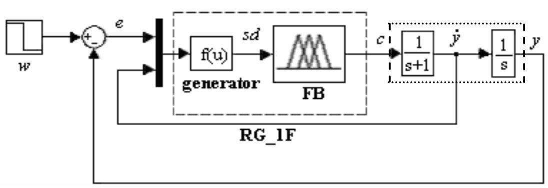
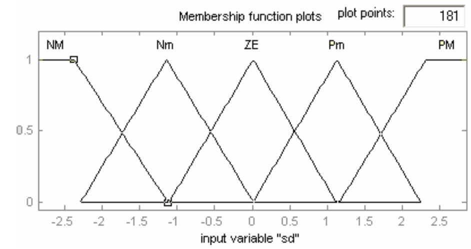
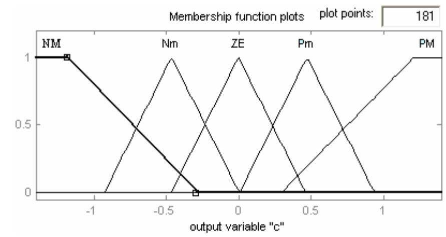


Fig 3. Fuzzy control scheme with *RG_1F* controller with synthetic input for the positioning system

The domains considered for implementation of FB are D_{sd} = [-2.85, 2.85] and D_c = [-1.4, 1.4], with D_{sd} determined by geometric means [10] and D_c validated by considering the necessity of obtaining zero steady state error. The associated fuzzy variables for *sd* and *c*, described by 5 linguistic terms as in Figure 4a and in Figure 4b are related according to the rule base presented in table 1.



a)



b)

Fig 4. Form and distribution of membership functions for *RG_1F* controller: *sd* (a) and *c* (b)

The time response of the system with *RG_1F* for the following scenario is depicted in Figure7 in order to illustrate how the system is stabilized in a certain state, different from the origin; the pulse reference signal

$$w(t)=\sigma(t)-\sigma(t-10) \qquad (2)$$

is considered. The pulse is applied in zero initial conditions.

Table 1 – Rule base for the RG_1F

sd	NB	NM	ZE	PM	PB
c	NB	NM	ZE	PM	PB

Step 1: In order to obtain a simple controller, easier to implement and, in the same time, with a reduced calculus time, the *RG_1F* will be replaced with an interpolative controller *RG_1I*, with the same sd-generator (synthetic input – generator), like in Figure 5. The first operation for obtaining of *IB* consists in determining the input-output transfer characteristic of the fuzzy block *FB*. The result is illustrated in Figure 6 for $sd \in [-1.75, 1.75] \subset D_{sd}$.

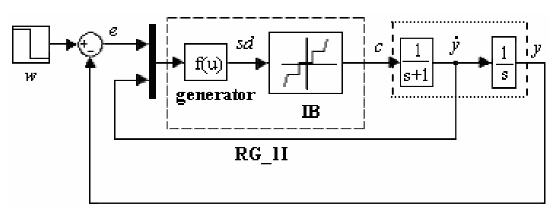


Fig 5. Interpolative-based control scheme with *RG_1I* for the positioning system

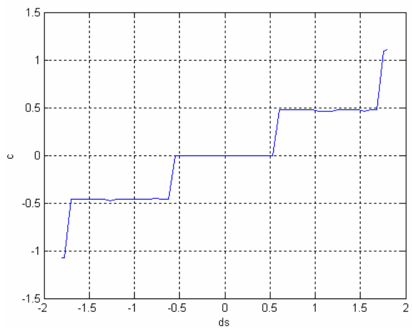


Fig 6. Transfer characteristic of the fuzzy block FB

In order to reproduce the characteristic in Figure 6, as significant support points for the interpolative block *IB* the 6 „breaking" points from the characteristic in the command domain and another two intermediate points ((-1.76, -1) and (1.76, 1)) from the extreme segments of the characteristic are used. The interpolation table 2 is obtained.

Table 2 - Interpolation table for the RG_1I(F) controller

sd	-1.76	-1.62	-1.62	-0.5	0.5	1.62	1.62	1.76
c	-1	-0.55	-0.55	0	0	0.55	0.55	1

Based on table 2, in SIMULINK is implemented a first version of the interpolative block *IB*, denoted *I(F)*, as a one-dimensional „look-up table". The response of the control system with the interpolative block *I(F)* at reference signal (2) (curve *RG-1I* in Figure 7) reproduces the behaviour of control system with the fuzzy controller *FB* (curve *RG-1F* in Figure 7).

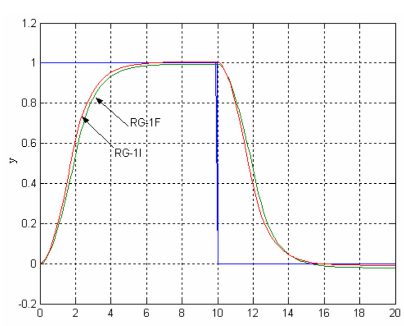


Fig 7. Comparative responses for the systems with *RG_1F* and *RG_1I(F)* at reference signal (2)

Step 2. In this step for the tuning of the control system in Figure5, the ladder signal (3) is considered as reference signal, instead of the signal (2).

$$w(t)=0.25\sigma(t)+0.5\sigma(t-10)+0.25\sigma(t-20)-$$
$$-0.25\sigma(t-40)-0.25\sigma(t-50) \qquad (3)$$

Because the output didn't follow the reference, the behaviour of both fuzzy and interpolative systems is unsatisfactory. The situation can be corrected by considering the jump of the reference Δw also in the control strategy, as a new input of the blocks *RG_1F, a*nd *RG_1I.* In this situation the control structure from Figure 8 will be considered instead the one in Fig. 5, and the new interpolative block, with two inputs is denoted as *IRB*.

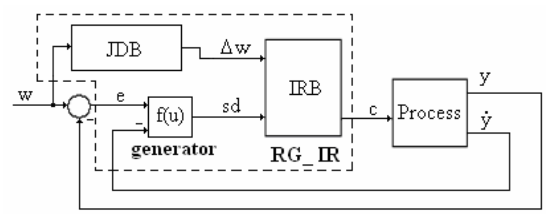


Fig 8. Control structure with interpolative robust controller *RG_IR*

To develop *IRB*, for Δw variable a 9 values granularity in the universe of discourse is adopted: {-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, 1} $\subset$ [-1 1]. At the beginning, for each value of Δw a 1-dimensional interpolation table is built and finally the 2-dimensional table 3 results:

Table 3 - Interpolation support points for IRB

Δw \ sd	-2.25	-0.9	-0.48	0	0.48	0.9	2.25
-1	-1	-0.62	-0.04	0	0.04	0.62	1
-0.75	-1	-0.62	-0.11	0	0.11	0.62	1
-0.5	-1	-0.62	-0.14	0	0.14	0.62	1
-0.25	-1	-0.62	-0.14	0	0.14	0.62	1
0	-1	-0.62	-0.16	0	0.16	0.62	1
0.25	-1	-0.62	-0.14	0	0.14	0.62	1
0.5	-1	-0.62	-0.14	0	0.14	0.62	1
0.75	-1	-0.62	-0.11	0	0.11	0.62	1
1	-1	-0.62	-0.04	0	0.04	0.62	1

The results obtained using this interpolative robust controller is presented in Figure 9.

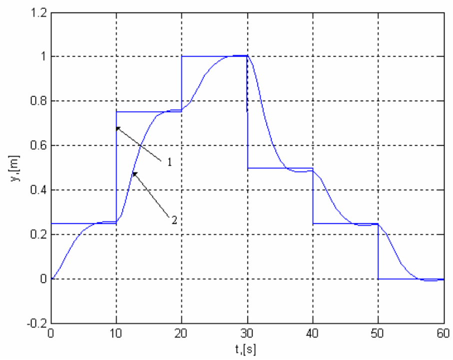


Fig 9. Response in position y (curve 2) of the control system in Figure 8 at reference signal (3) (curve 1)

By analysing table 3 from a simplification perspective, finnaly it can be minimized, with the same results, reducing *sd* excursion to 3 values as in interpolation table 4, with only 12 support points.

Table 4 - Interpolation upport point simplified table for IRB

Δw \ sd	-0.48	0	0.48
0	-0.16	0	0.16
[0.25, 0.5]	-0.14	0	0.14
0.75	-0.11	0	0.11
1	-0.04	0	0.04

'*Step3*. In real positioning systems the allowed variations in position and speed are limited. As a consequence, for the next part of the study, the following limitations for the states of the plant given by transfer function (5), respectively by the model $\dot{x}_1 = x_2$, $\dot{x}_2 = -x_2 + c$, $y = x_1$ are considered:

$$x_1 \in [x_{1\min}, x_{1\max}] = [-2, 2],$$
$$x_2 \in [x_{2\min}, x_{2\max}] = [-3, 3]. \quad (4)$$

In these circumstances the nonlinear one replaces the linear model of the plant:

$$\begin{cases} \dot{x}_1 = x_2 \cdot \sigma[(x_2 - x_{2\min}) \cdot (x_{2\max} - x_2)] \\ \dot{x}_2 = -x_2 \cdot \sigma[(x_2 - x_{2\min}) \cdot (x_{2\max} - x_2)] + c \\ y = x_1 \cdot \sigma[(x_1 - x_{1\min}) \cdot (x_{1\max} - x_1)] \end{cases} \quad (5)$$

and the control scheme in Figure 8 is replaced with the scheme in Figure 10. The states x_1, x_2 correspond to y, $\dot{y}$ from Figure 8, respectively.

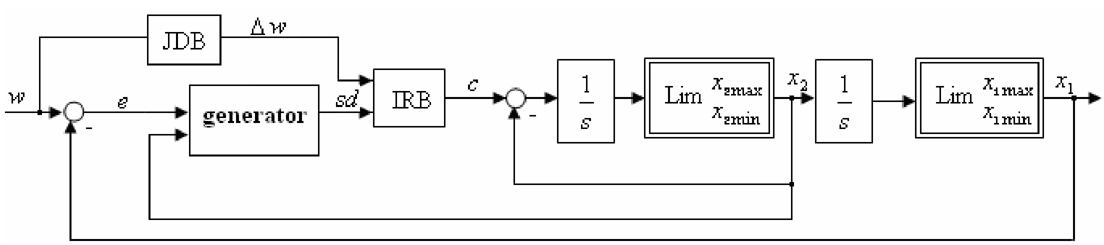


Fig 10. Control scheme with IRB controller considering limitations at the states level

Next, in this section the control problem is developed in order to provide Lyapunov stability according with those presented in the appendix. We denote $\tilde{x}_1 = x_1 - w$ and $\tilde{x}_2 = x_2 - 0$. The input in IRB block, *sd* is calculated in generator block, according to the last expression from the equalities:

$$sd = \frac{\dot{e} + \lambda \cdot e}{\sqrt{1+\lambda^2}} = \frac{-\tilde{x}_2 - \lambda \cdot \tilde{x}_1}{\sqrt{1+\lambda^2}} = = \frac{-x_2 - \lambda \cdot (x_1 - w)}{\sqrt{1+\lambda^2}} = \frac{-x_2 + \lambda \cdot u}{\sqrt{1+\lambda^2}} \quad (6)$$

In order to design the conditioning and limitation block $U_{V\lim} - L$ (see Figure A-1 in appendix), the candidate function

$$V(\tilde{x}) = \tilde{x}^T \cdot \underbrace{diag(p_{11}, p_{22})}_{P} \cdot \tilde{x} = p_{11} \cdot \tilde{x}_1^2 + + p_{22} \cdot \tilde{x}_2^2 = p_{11} \cdot (x_1 - w)^2 + p_{22} \cdot x_2^2 \quad (7)$$

is adopted. Its first time-derivative is:

$$\dot{V}(\tilde{x}) = 2 \cdot p_{11} \cdot \tilde{x}_1 \cdot \dot{\tilde{x}}_1 + 2 \cdot p_{22} \cdot \tilde{x}_2 \cdot \dot{\tilde{x}}_2 = = 2 \cdot p_{11} \cdot (x_1 - w) \cdot \dot{x}_1 + 2 \cdot p_{22} \cdot x_2 \cdot \dot{x}_2, \quad (8)$$

where $\dot{x}_1$ and $\dot{x}_2$ have the expressions in (5).

Considering the admissible domain $\widetilde{X}_a = [\widetilde{x}_{1\min}, \widetilde{x}_{1\max}] \times [\widetilde{x}_{2\min}, \widetilde{x}_{2\max}] = [-2-w, 2-w] \times [-3, 3]$, associated with the limitations (4), the expression of the first derivative becomes:

$$\dot{V}(\widetilde{x}) = 2 \cdot p_{11} \cdot \widetilde{x}_2 \cdot [\widetilde{x}_1 + \beta \cdot (-\widetilde{x}_2 + c)] \tag{9}$$

where $\widetilde{x} \in \widetilde{X}_a$ and $\beta = \dfrac{p_{22}}{p_{11}}$. Imposing the restriction $\dot{V} < 0$, it follows, as necessary, the condition:

$$2 \cdot p_{11} \cdot \widetilde{x}_2[\widetilde{x}_1 + \beta(-\widetilde{x}_2 + u)] < 0, \widetilde{\mathrm{x}} \in \widetilde{\mathrm{X}}_{\mathrm{a}} \tag{10}$$

Analysing this inequality, it is easy to conclude that:

- In each area where $\widetilde{x}_2 < 0, \widetilde{x}_1 > 0$ and $\widetilde{x}_2 > 0, \widetilde{x}_1 < 0$, always exists $\beta \geq 0$ such that condition (10) is satisfied; this means that the areas can be covered in any situation with concatenable stable sub-domains.
- If conditions $c > \widetilde{x}_2$ and $c < \widetilde{x}_2$ respectively are fulfilled in the areas $\widetilde{x}_2 < 0, \widetilde{x}_1 \leq 0$, and $\widetilde{x}_2 > 0, \widetilde{x}_1 \geq 0$ respectively, then there always exists $\beta \geq 0$ such as (10) to be satisfied and, as a result, each area can be covered with concatenable sub-domains. If conditions $c > \widetilde{x}_2$ and $c < \widetilde{x}_2$ respectively are not fulfilled for $\widetilde{\mathrm{x}} \in \widetilde{\mathrm{X}}_{\mathrm{a}}$ then, without modifying the Lyapunov function, inferior limitation of *c* is imposed, such as $c > x_2$ and $c < x_2$ respectively; the re-configuration of the system (by adding an inferior or superior limitation bock for *c*) will be also imposed.
- For $\widetilde{x}_2 = 0$ and $\widetilde{x}_1 \neq 0$, the blocking of the system, due to the fact that $\dot{V} = 0$, must be prevent by imposing a command with the following form: $c = -\varepsilon \cdot \widetilde{x}_1$, $\varepsilon > 0$.
- In the point $\widetilde{x}_2 = 0$ and $\widetilde{x}_1 = 0$, no conditioning is needed.

The results of this above discussion (Lyapunov conditionings) can be transposed graphically as in Figure 11, where $\widetilde{x}_{1\min} = x_{1\min} - w_{\max}$ and $\widetilde{x}_{1\max} = x_{1\max} - w_{\min}$. In the discussion the idea of a Lyapunov function defined on distinct sections by different numerical expressions appears from the specifications referring to β.

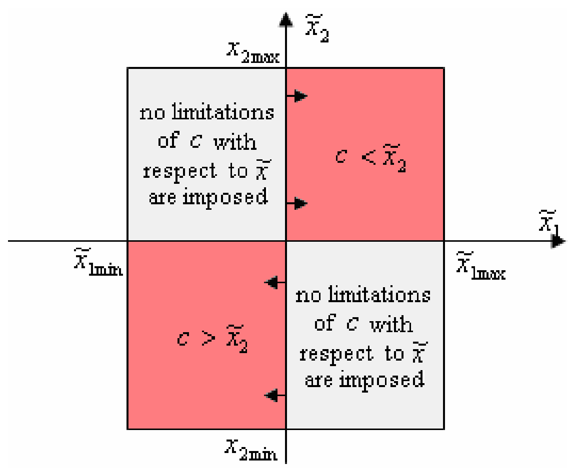


Fig 11. Lyapunov conditionings

In order to implement the Lyapunov conditionings, the SIMULINK scheme in Figure 12, in „continuous time" variant, is given.

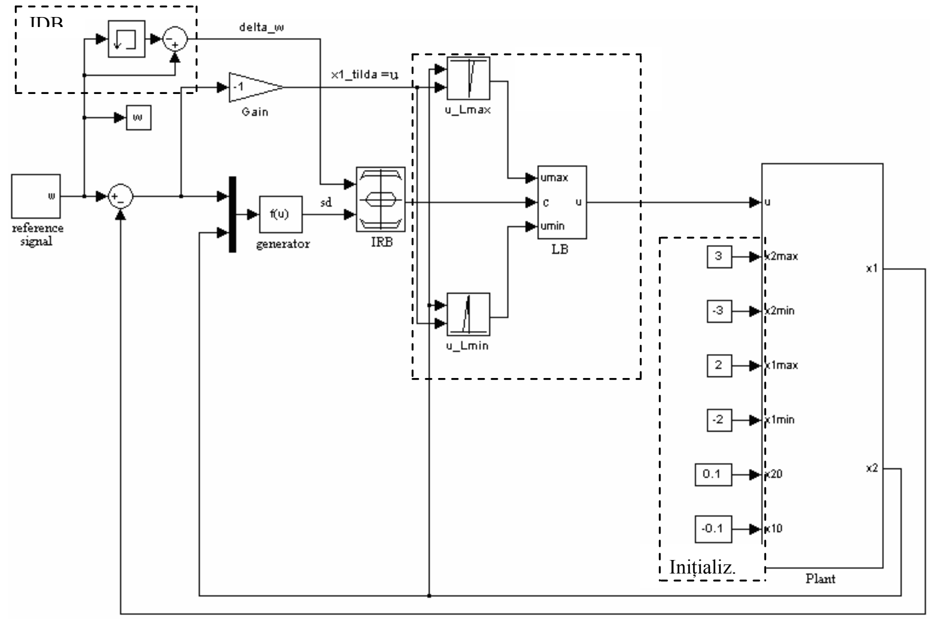


Fig 12. Control scheme with IRB block and interpolative limitation of the command

The block "*Plant*" represents the system S and is modelled by the equation (5); the "*Cond. and limit. Block*" corresponds to the conditioning and limitation ensemble $U_{V\lim}$-L; the rest of the blocks from the scheme correspond to the regulator block (*IRB-JDB-*generator) in Figure 10. Is important to observe that only the Plant is an inertial system (that means: the plant has states).

In order to study the behaviour of the control system, several steps were followed:

a) At the beginning, the fulfilling of the conditions in Figure 11 must be analysed. for this purpose, the SIMULINK scheme in Figure13 was used.

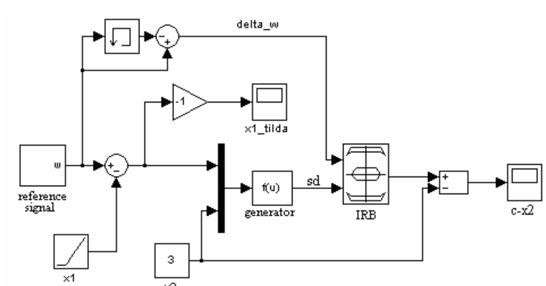


Fig 13. SIMULINK scheme, helping to verify the Lyapunov stability conditions

To exam the performance, a simulation scenario was adopted, characterized by the followings:

- Exploring with x_1, using a ramp type signal, its maximum admissible domain $[x_{1\min}, x_{1\max}] = [-2, 2]$, during the simulation time (60seconds). For this time interval, $\tilde{x}_1 \in [-2.3, 2]$ (see the variations in Figure 14).
- Usage of the ladder type signal in (3) as reference signal. The same input was used in the above paragraph in the initial designing procedure of the *IRB*, to determine its action with respect of both *sd* and Δw.
- Considering of $\tilde{x}_2 = x_2$ as parameter, with values in its maximum admissible domain $\tilde{x}_2 \in [x_{2\min}, x_{2\max}] = [-3, 3]$.

In Figure 14 the variations of $\tilde{x}_1(t)$ and $c - \tilde{x}_2 = f(t)$ are presented for 4 values of the parameter, (see the condition in quadrant I and III in Figure 11). It can be observed that in each case, the conditions from Figure 11 are fulfilled. For $\tilde{x}_2$, only positive values were considered, because according to Figure 11, the whole calculus scheme represents an odd function with respect to $\tilde{x}_2$.

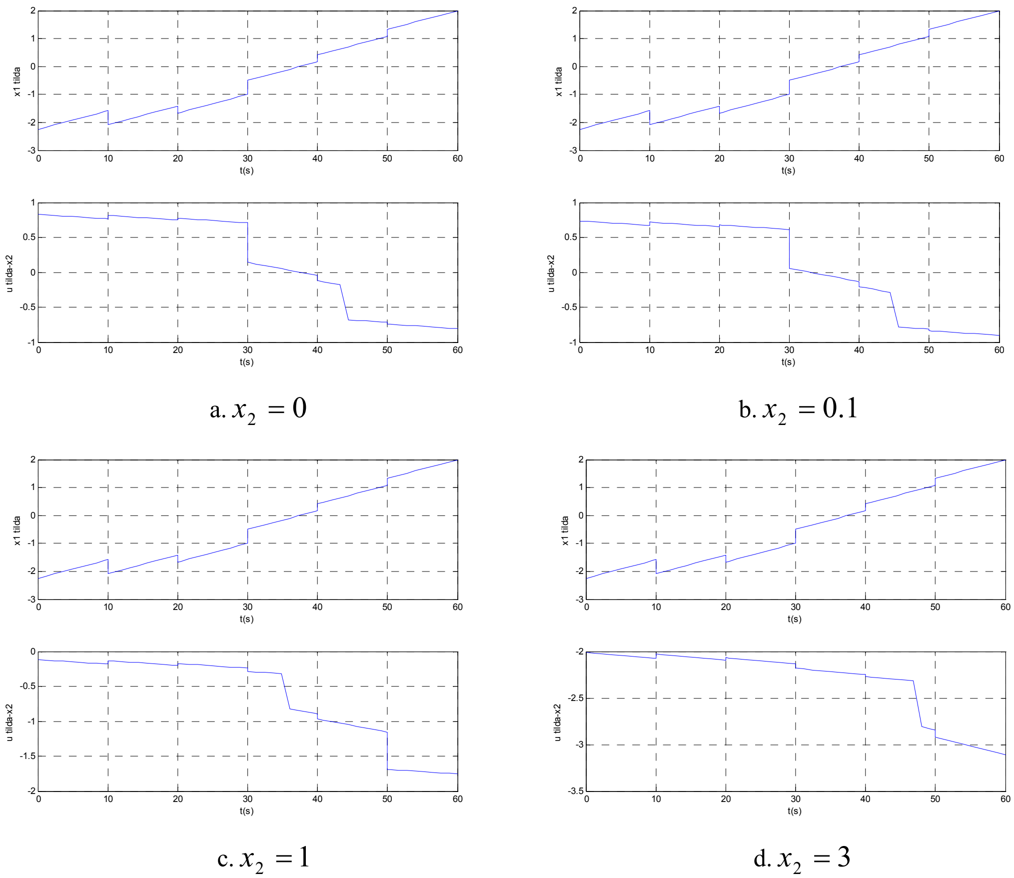


Fig 14. The results of the stability analysis for different values of x_2

These results did not illustrate the need of some command limitation blocks as in Figure13, because the conditions in odd quadrants in Fig.11 are always fulfilled. In the same time, we observe that the conclusion, also, is not very rigorous because the system is highly non-linear, analytically un-approachable, and the study done under accepting of Figure 14 is incomplete and did not guarantee the stable behaviour of the system.

As a result, the first step must be continued with a second one, by investigating the necessity to consider a limitation block; this will impose sufficiency conditions from the $\dot{V} < 0$ perspective.

b) As for IRB, we will consider also for the limitation and conditioning system an interpolative-type solution. The corresponding support points, obtained through the analysis based on the results suggested by Figure 11, are given in tables 5 and 6. The support values of the support points associated to even quadrants in Figure 11 are the extreme values of the $[u_{\min}, u_{\max}] = [-1,1]$ domain. In the other two quadrants the support values were chosen in such a way as to respect the conditions depicted in Figure 11.

Table 5 - Support points for the generator block for the lover limit u_{Lmin}

$x_2/\tilde{x}_1$	-2	-1	0	2
3	-1	-1	-1	-1
0	-1	-1	-1	-1
-0.002	-1	-1	-0.001	-1
-1	-1	-1	-0.99	-1
-3	-1	-1	-1	-1

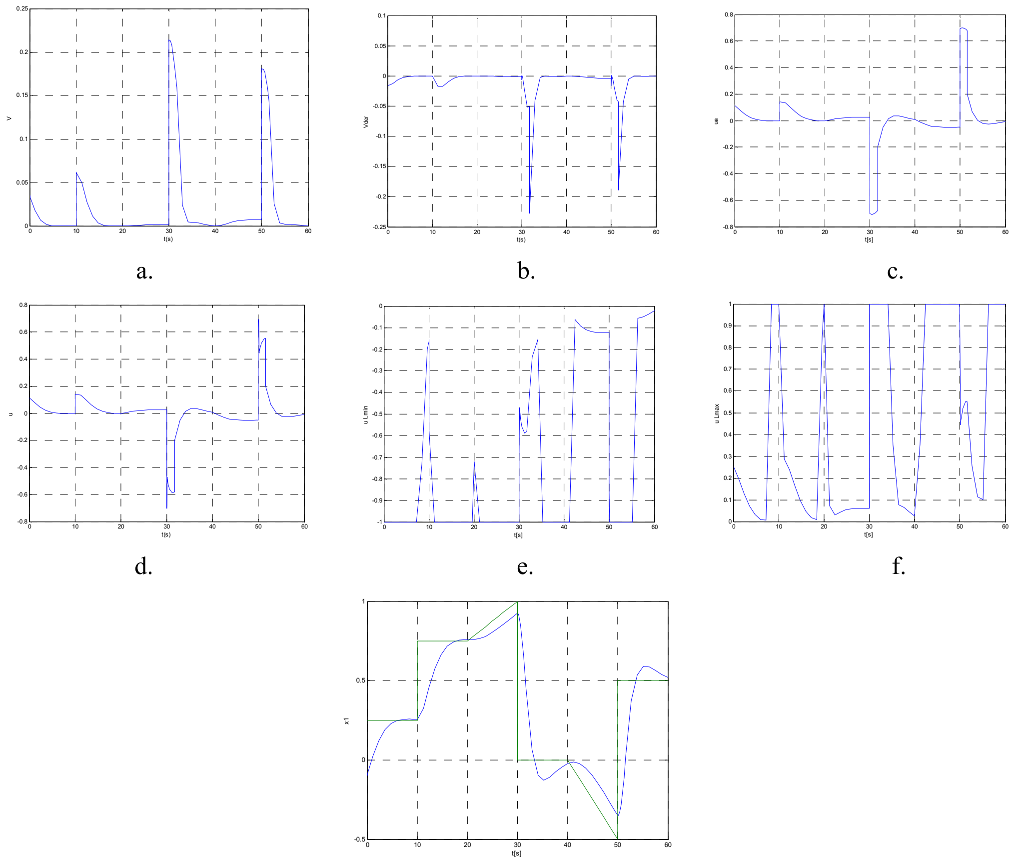
Table 6 - Support points for the generator block for the upper limit u_{Lmax}

$x_2/\tilde{x}_1$	-2	0	1	2
3	1	1	1	1
1	1	0.99	1	1
0.002	1	0.001	1	1
0	1	1	1	1
-3	1	1	1	1

In order to test the working manner of the conditioning and limitation block, we will take another simulation scenario, considering the reference signal:

$$w(t) = 0.25\sigma(t) + 0.5\sigma(t-10) + 0.025 \cdot \\ \cdot (t-20) - 0.75\sigma(t-30) - 0.05(t-40) + \\ + 0.5\sigma(t-50) \quad (11)$$

In Figure 15 the results are depicted. Figure 15g illustrate the response $y(t) = x_1(t)$ of the system and in the other figures the evolution of the following variables is given: V (15a), $\dot{V}$ (15b), c (15c), u (15d), u_{Lmin} (15e), u_{Lmax} (15f).



a. b. c. d. e. f. g.

Fig 15. Results simulation based on the scheme in figure 13 at reference signal (11)

Different from the previous scenario, in the present case the command limitation block acts

indeed. It can be observed that even in the situation of a tougher scenario from the amplitude reference modifications point of view, the stability conditions are fulfilled (see figure 15a and b) and the system's performances are satisfactory.

4. CONCLUSIONS

Real world problems are usually much more demanding than the hypothetic issues solved by most of the researches in studies in intelligent control field. Not only the method is important, but also the real application possibilities. Real control systems have to face with limitations and different operating regimes. An appropriate control solution has to be also simple to implement, not time consuming, stable and robust. In the present study one was tried to demonstrate that interpolative control structures can fulfil such demands.

Interpolative controllers based on tables represent an important class of non-linear controllers that can lead to high performances control systems. For their design it can be considered as starting points other controller structures. In particular, in the present paper the starting point was a control structure with fuzzy controller.

In the paper was also made some studies on the robustness of an interpolative control structure, at step changes in reference, with limitations both at the states and command level. The command limitations are also made through interpolation and are based on Lyapunov's second method. The presented solution did not guarantee the finding of a matrix P appropriate for any regime; the described procedure offers, from this reason, alternatives to other procedures used to analyse the stability based on direct Lyapunov method [11], [12], [13].

REFERENCES

[1] Åström, K.J. and B. Wittenmark, - „Computer controlled systems – analysis and design", Prentice Hall, 1997.

[2] Choi, B.-J., Kwak, S.-W., Kim, B.K. – "Design and stability analysis of single-input fuzzy controller", IEEE Transactions on Systems, Man and Cybernetics, part B: Cybernetics, vol.30/2, pp. 303-309, 2000.

[3] Dale, S., - „Contribuții la studiul sistemelor de reglare cu regulatoare de tip interpolativ", Editura Politehnica, Timişoara, 2006.

[4] Dale, S., Dragomir, T.L., - "Interpolative Control versus Fuzzy Control for a fourth order System", CSCS'16, pp.419-426, Bucuresti, 2007.

[5] Dragomir, T.L., Dale, S. – "Cases of using interpolator type blocks in control structures", Proceedings of SACCS'01, Iaşi, CD, 2001.

[6] Dragomir, T.L., Dale, S., Bălaş, M.M. – "Some aspects regarding interpolative structures", Proceedings of IWICS'01, Bucureşti, pp. 6-19, 2001.

[7] Drechsel, D. – „RIP control", CEAI 1/1999, Bucureşti, pp. 7-17.

[8] Drechsel, D.– „Regelbasierte Interpolation und fuzzy control", Vieweg, 1996.

[9] Dubois, D., Prade, H. and Grabisch, M. – „Gradual rules and the aproximation of control laws", John Wiley & Sons, pp.147-183, 1995.

[10]Dumitrache, I., - „Ingineria reglării automate", Editura Politehnica Press, 2005.

[11]Kóczy, L.T. and Hirota, K.– „Approximate reasoning by linear rule interpolation and general approximation", Internat. J. Approx. Reason. 1993/9, pp. 197-225, 1993.

[12]Răsvan, V., - „Teoria stabilității", Editura ştiințifică şi enciclopedică, Bucureşti, 1987.

[13]Slotine, J-J.E. and Li, W., - „Applied nonlinear control", Prentice Hall, 1991.

[14]Zadeh, L.A., – „Interpolative reasoning as a common basis for inference in fuzzy logic, neural network theory and the calculus of fuzzy If/Then rules", Opening Talk, 2nd International conference on fuzzy logic and neural networks, Iizuka, pp. XIII-XIV, 1992.

APPENDIX: On a non-linear Lyapunov stabilisation structure

Let be *S* an invariant system, with the state vector *x* and the scalar input *u*:

$$\dot{x} = f(x,u). \tag{A -1}$$

We consider that x_w is a desired equilibrium state corresponding to a value u_w of *u* ($f(x_w, u_w) = 0$). Particularly, the equilibrium state $x_w = 0$ can be of interest. In order to reduce the stability study in x_w to that of a relative repose state, we need to define new state variables:

$$\widetilde{x} = x - x_w. \tag{A-2}$$

Hence, the equations (A-1) take the form:

$$\dot{\widetilde{x}} = \widetilde{f}(\widetilde{x},u), \quad \widetilde{f}(\widetilde{x},u) = f(\widetilde{x} + x_w, u) . \tag{A-3}$$

We assume that $\widetilde{x} \in X_a$, with X_a the allowed domain of $\widetilde{x}$, limited in respect with every component of $\widetilde{x}$.

Let be a Lyapunov candidate function $V(\tilde{x})$, derivable on X_a, and with closed equipotential lines. Then,

$$\dot{V}(\tilde{x}) = V_{\tilde{x}}^T \cdot \dot{\tilde{x}} = V_{\tilde{x}}^T \cdot \tilde{f}(\tilde{x}, u). \tag{A-4}$$

For a given state x, allowed and fixed, and a variable u, $\dot{V}(\tilde{x})$ can be seen as a function depending only on u:

$$\dot{V}(\tilde{x}) = V_{\tilde{x}}^T \Big|_{\tilde{x}=x-x_w} \cdot f(x,u) = W_x(u) \quad . \tag{A-5}$$

We denote with $U_a = [u_{\min}, u_{\max}]$ the set of practical values for u, with $U_V(x)$ the values of u which ensures $\dot{V}(\tilde{x}) < 0$ for the fixed value of the state x

$$U_V(x) = \{u \in U_a | W_x(u) < 0\}, \tag{A-6}$$

and with

$$U_V = \bigcup_{x-x_w \in X_a} U_V(x) \tag{A-7}$$

the values of u which ensure $\dot{V}(\tilde{x}) < 0$ when x-x_w covers X_a completely.

Practically, the set $U_V(x) = [u_{L\min}(x), u_{L\max}(x)] \subseteq U_a$, corresponding to a given x, is determined by exploring with the command u the whole domain U_a.

The structure in figure A-1 can be used to respect the condition $W_x(u) < 0$. The command signal u of the system S represents the output of a *Conditioning and limitation block*. It acts with respect of the external control signal c, the current state x and the desired equilibrium state x_w of the system:

$$u = \begin{cases} c, \text{ daca } c \in [u_{L\min}(x), u_{L\max}(x)] \\ u_L \in \{u_{L\min}(x), u_{L\max}(x)\}, \text{ daca } c \notin [u_{L\min}(x), u_{L\max}(x)] \end{cases} . \tag{A-8}$$

Here u_L represents the limit values: $u_L = u_{L\min}(x)$ if $c \le u_{L\min}(x)$ and $u_L = u_{L\max}(x)$ if $c \ge u_{L\max}(x)$.

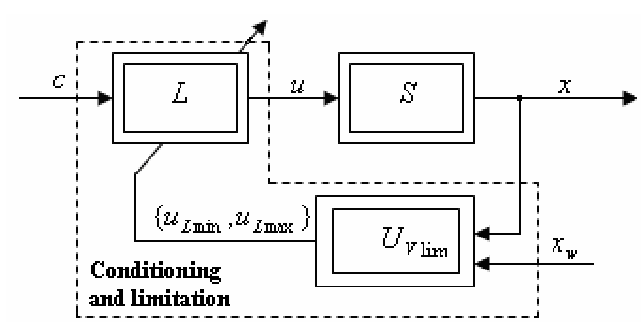


Figure A-1. The structure of a system with command limitation and conditioning block

In the presence of Conditioning ($U_{V\lim}$) and limitation block (L) the system S will evolve permanently such as:

$$\dot{V}(\tilde{x}) \le 0, \tag{A-9}$$

and the candidate function $V(\tilde{x})$ will become a Lyapunov function. From the causality point of view the evolution of the structure is the result of the external command c and initial conditions of the system S. In so far as the limitation in (A-8) acts, the restriction domain introduced trough the adopted

Lyapunov function V will constrain the evolution of S. When the block $U_{V\lim}$ can be designed, the structure assures the asymptotical evolution condition $\tilde{x} \xrightarrow[t\to\infty]{} 0$.

In order to become a Lyapunov function, $V(\tilde{x})$ must fulfil the following conditions on the limited domain X_a:

$$V(\tilde{x}) > 0\,,\ \tilde{x} \neq 0\,, \tag{A-10}$$

$$V(0) = 0\,, \tag{A-11}$$

$$\dot{V}(\tilde{x}) < 0\,, \tilde{x} \neq 0 \tag{A-12}$$

Usually, one looks to operate with a single mathematical expression for $V(\tilde{x})$, on the entire domain X_a. It's well known that it is not always possible to find such an expression as well as that is possible to find more than one. In the same context we admit that is possible to find a candidate function $V(\tilde{x})$ defined on two domains by two different expressions $V_1(\tilde{x})$ and $V_2(\tilde{x})$, such that the ensemble $V(\tilde{x})$ has the properties of a Lyapunov function. We assume that $V(\tilde{x})$ is defined by $V = V_1$ on a sub-domain of X_a and by $V = V_2$ on the rest of X_a, and that the condition $V_1(0) = V_2(0) = 0$ is fulfilled. In this case, denoting $D_{\dot{V}_1} = \left\{\tilde{x} \in X_a \middle| \dot{V}_1(\tilde{x}) < 0\right\}$ and $D_{\dot{V}_2} = \left\{\tilde{x} \in X_a \middle| \dot{V}_2(\tilde{x}) < 0\right\}$, we have $D_{\dot{V}_1} \cup D_{\dot{V}_2} = X_a$. Let $X_{x_0} \subseteq X_a$ be the domain of initial allowed values for $\tilde{x}$. If $x(0) \in X_{x_0} \subset D_{\dot{V}_1}$ and for $t \to \infty$ the state $\tilde{x}$ don't leave the domain $D_{\dot{V}_1}$, then $\tilde{x}$ tends asymptotically toward the equilibrium state $\tilde{x} = 0$. If $\tilde{x}$ leaves the domain $D_{\dot{V}_1}$, passing trough $D_{\dot{V}_2}$ and then remains there, the state $\tilde{x}$ will tend also asymptotically to the equilibrium state. In this second case, the temporal evolution of the system S is followed at the beginning through V_1, and after $\tilde{x}$ enters in $D_{\dot{V}_2}$ domain, through V_2. Because $D_{\dot{V}_1} \cup D_{\dot{V}_2} = X_a$, the reasoning is viable for both cases $D_{\dot{V}} = D_{\dot{V}_1} \cap D_{\dot{V}_2}$ is the void set or a non-void set. When the characteristic point $\tilde{x}$ passes from a domain to another $V(\tilde{x})$ will continue monotonically to decrease, in the same time with the variation of $\tilde{x}$ to the equilibrium state. The discussion holds on for other situations also (for example for the situation of multiple passing from a domain to the other). Moreover, the above reasoning can be generalised: $V(\tilde{x})$ can be defined on a non-restricted large number of sections by the restrictions V_1, V_2,... of some real functions with real variables, with the concatenable domains $D_{\dot{V}_1}$, $D_{\dot{V}_2}$,... (also $D_{\dot{V}_1} \cup D_{\dot{V}_2} \cup ... = X_a$). Than, by passing of the characteristic point $\tilde{x}$ from a domain to another $V(\tilde{x})$ will monotonically decrease.

It's important to underline the fact that the entire discussion is unfold strictly in the availability frame of Lyapunov stability theorem and that the additional aspect developed here is only a methodological one. It regards the operation with Lyapunov functions with different expressions on sub-domains witch covers by their reunion the allowed space X_a of $\tilde{x}$, and which have the property to become zero for $\tilde{x} = 0$. According to above reasoning, this condition will be sufficient. As in the usual case of the candidate functions defined by a single expression, there is no guaranteed procedure to obtain V_1, V_2... with concatenable domains $D_{\dot{V}_1}$, $D_{\dot{V}_2}$, ... In order to determine these expressions matters the analyzed system, the restrictions imposed to the signals, the ability to guess a candidate function etc.