INTERNAL MODEL CONTROL STRUCTURES USING DELTA DOMAIN REPRESENTATION

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Abstract: *In case of sampled data, the shift operator representation is very sensitive to parameter changes for fast sampling. In this case the use of delta models is advantageous. When controlling plants with long dead-time, the delta representation of dead-time deteriorates its superiority. The paper introduces a hybrid handling of control algorithms using IMC (Internal Model Control) structure, handling the dead-time in z-domain and the model dynamics in delta domain. The hybrid algorithms are given for dead-beat control and for Smith-predictor control.*

Keywords: *Delta models, IMC structure, dead-beat control, Smith predictor.*

1. INTRODUCTION

During the past years, as the digital technology advanced in speed and performance, fast sampling and its problems have become more and more actual. Sampled systems generally are represented in shift operator form, which has some disadvantages at fast sampling: loss of information may occur because of finite word length and arithmetical operations [1]. One possibility to overcome this drawback is to use the *delta operator representation* (delta representation) when implementing the controller [2],[3].

The delta operator can be defined as follows (it was introduced in this form by [2]):

$$\delta = \frac{q-1}{h} \qquad \text{or} \qquad \gamma = \frac{z-1}{h} \tag{1}$$

where h is the sampling period, q and (later) q^{-1} are the forward and backward shift operator in time domain, and z (and z^{-1}) is the associated complex variable. Also in [3] the variable γ is introduced to represent the delta operator.

It can be noticed that if the sampling period tends to zero $h \rightarrow 0$, the continuous derivative of the function is obtained:

$$\lim_{h \to 0} \delta x(t) = \frac{d}{dt} x(t) \tag{2}$$

If the delta transformation is applied to a given *x(t)* continuous function, the first order difference results in form of [4]:

$$\delta x_k = \frac{x_{k+1} - x_k}{h} = \frac{x(kh+h) - x(kh)}{h} \qquad (3)$$

One of the main advantages of the delta representation lies in the implementation of a transfer function (t.f), which should be based on feedback of the integrators represented in delta form (see Fig. 1).

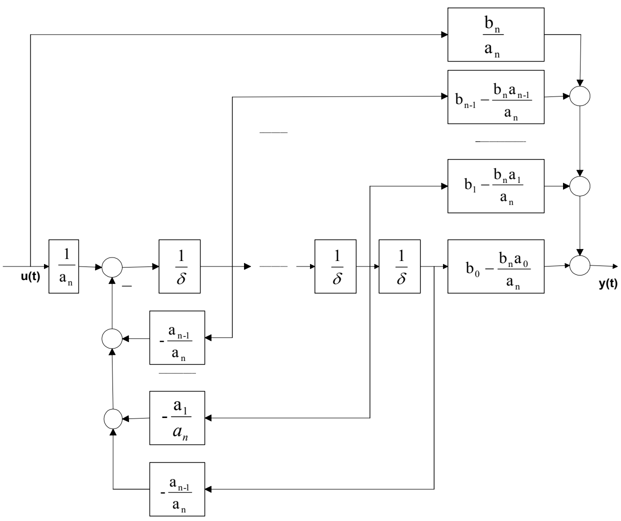

Fig.1. Implementation of delta transfer functions

The values of the parameters are the coefficients of the delta t.f. represented as follows:

$$H(\gamma) = \frac{b_n\gamma^n + \cdots + b_1\gamma + b_0}{a_n\gamma^n + \cdots a_1\gamma + a_0} \qquad (4)$$

The delta t.f. can be obtained either from the pulse transfer function applying equation (1) or from the continuous transfer function using the following calculus relation:

$$H(\gamma) = \frac{\gamma}{1+h\gamma} T[L^{-1}\{\frac{1}{s}H(s)\}] \quad , \qquad (5)$$

where, $T\{.\}$ is called the generalized operational transform in [3]. For small sampling time the parameters of the delta t.f. $H(\gamma)$ approach the parameters of the continuous t.f. $H(s)$.

Two properties of the delta transformation are very useful in controller design:

- One that reflects the fact that in the case of the delta transformation the stability domain expands as the sampling period gets smaller and finally - if the sampling period tends to 0 - it is a circle centered in (-1/h, 0) and with a radius of 1/h, the boundary being

$$|1+h\,\gamma|<1 \qquad (6)$$

- A second one that characterizes the way of pole transposition – particularly of the dominant complex conjugate poles of a second order continuous system - from *Z*-domain to delta.

For example in case of a second order continuous system with damping coefficient ξ=0.707 when using $Z\{.\}$ transformation the well-known „heart-shaped" territory limited inside the unit circle is obtained (see Fig.1). For the delta representation for *h*=1 it is shifted in the circle centered in (–1,0), and for *h*= 0.1 it becomes 10 times larger. By reducing the sampling period even more the poles of the system in delta domain get closer to the continuous poles, especially the dominant ones. This remark sustaines the idea of controller design techniques in delta domain being close to the design methods from continuous time.

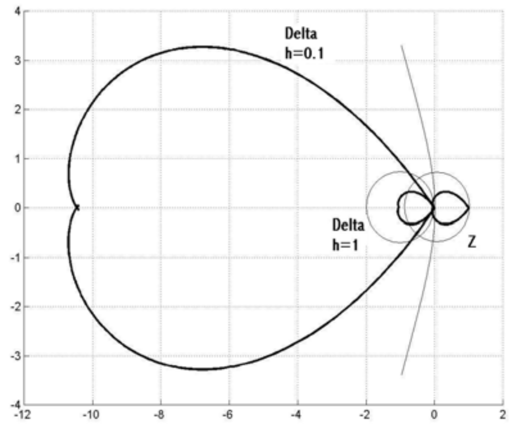

Fig.2. Dominant complex-conjugate poles locus in continuous (ξ=0.707), delta and Z domain

Controller design can be performed in different ways: since for small sampling times the delta model is very close to the continuous time model, one advantageous approach is to design the controller in continuous time and to implement it in delta domain [2],[4],[5]. Another approach is based on designing the delta controller directly to the delta model of the plant, which must be obtained in advance.

2. CONTROLLER DESIGN

2.1. *IMC-based Dead-Beat Controller for Processes with Dead-time*

The control structure which deals with dead-time compensation let be as shown in Fig.3, ZOH denotes zero order hold and the controller, with t.f. $C(*)$, contains the cancellable part of the plant t.f., $(A(*)/B^+(*))$.

In this case the controller part $A(*)/B^{+}(*)$ is in discrete time (both z- domain representation or delta domain representation). The limitation is included in the IMC structure, this way insuring a better handling of limitation. It is known that in case of IMC control, if the plant is open-loop stable ([6],[7],[8]), the controller can be designed as the best approximation of the inverse of the plant model. If there is perfect modelling, that is no mismatch between the plant and model, the control is actually open-loop. If there is mismatch, the feedback signal works against it.

Use of delta transformation is advantageous for fast sampling rates [5]. In the case when relative dead-time $d=T_H/h$ is quite big, delta form looses the essence of this advantage as $(h\gamma+1)^{-d}$ term includes again very small coefficient values. Therefore a hybrid z-delta control structure has been proposed [8], as shown in Fig.4.

The main idea is that the plant's model and inverse model will be represented in delta form, while the dead-time will be expressed in z-domain, combining the advantages of both delta parametrization (lower sensitivity to parameter mismatch) and z-domain representation of the dead-time.

Hence the controller can be adapted to different values of the dead-time by simply adjusting the value of d in the model. Thus the controller is very sensitive to mismatch in the dead-time.

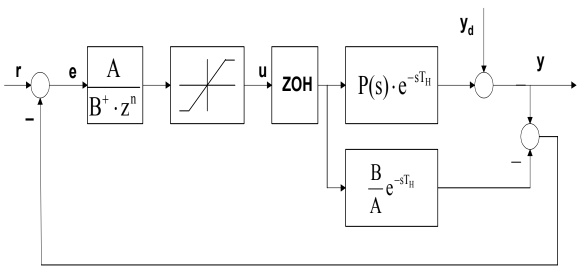

Fig.3. Dead-time compensation

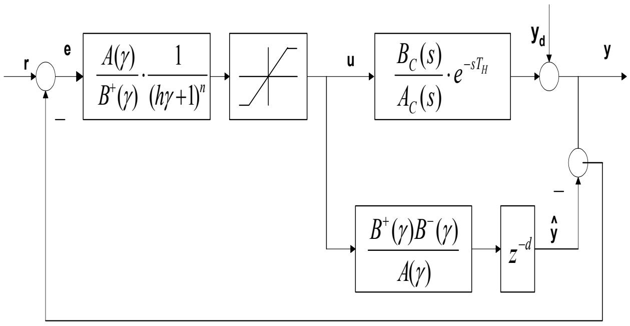

Fig.4. Hybrid dead-time compensating structure

For better explanation let the t.f. of a continuous plant be:

$$P(s) = \frac{B_C(s)}{A_C(s)} \cdot e^{-sT_H} \quad . \tag{7}$$

Sampling it with a ZOH using a sampling time h the discrete t.f. is obtained

$$P(z) = \frac{B_D(z)}{A_D(z)} \cdot z^{-d} \qquad \text{where} \quad d=T_H/h \quad . \tag{8}$$

The corresponding delta t.f. $P(\gamma)$ results as:

$$P(\gamma) = \frac{B(\gamma)}{A(\gamma)} \cdot \frac{1}{(h\gamma+1)^d} \quad . \tag{9}$$

The numerator of the plant can be decomposed into B^+ and B^-, B^+ containing the cancellable and B^- the unstable and poorly damped zeros:

$$B(\gamma) = B^+(\gamma) \cdot B^-(\gamma) \tag{10}$$

Needing to fulfill the requirement of zero steady-state error, the following ocndition has to be ensured:

$$B^-(0) = 1 \tag{11}$$

For the design of a dead-beat controller (DB-controller) in delta domain the closed loop t.f. $H_r(\gamma)$ must be imposed the form (12):

$$H_r(\gamma) = \frac{C(\gamma)\cdot P(\gamma)}{1+C(\gamma)\cdot P(\gamma)} = B^-(\gamma)\cdot\frac{1}{(h\gamma+1)^d}\cdot\frac{1}{(h\gamma+1)^n} \tag{12}$$

where n is:

$$n = 1 + \deg(B^-) \quad . \tag{13}$$

To illustrate the form of the controller, the pure delta representation is given:

$$C(\gamma) = \frac{A(h\gamma+1)^d}{B^+[(h\gamma+1)^{d+n} - B^-]} \tag{14}$$

As far as the hybrid representation is concerned, the model will have the structure in fig.5.

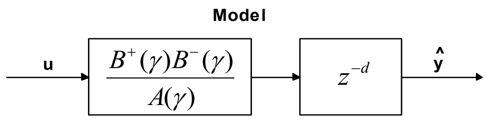

Fig.5. Internal hybrid model of the plant

The controller part then results as in Fig.4:

$$C(\gamma) = \frac{A(\gamma)}{B^+(\gamma)} \cdot \frac{1}{(h\gamma+1)^n} \tag{15}$$

To exemplify the procedure, let a second order plant be considered in continuous form:

$$P(s)=\frac{A_C}{(T_{1S}s+1)(T_{2S}s+1)}\cdot e^{-sT_H} \quad (16)$$

The delta t.f. obtained through sampling results:

$$P(\gamma)=A\frac{\tau\gamma+1}{(T_1\gamma+1)(T_2\gamma+1)}\cdot\frac{1}{(h\gamma+1)^d} \quad (17)$$

The controller is obtained based on the above considerations.

As a numerical example, let the t.f. of the plant with a dead time $T_H = 0.1$ be (17). Having a sampling period of $h=0.05$, it leads to a discrete dead time of $d=2$.

$$P(s)=\frac{1}{(0.67s+1)(0.33s+1)}\cdot e^{-0.1s} \quad (18)$$

According to (5) the delta t.f. will be:

$$P(\gamma)=\frac{0.0259\gamma+1}{(0.6953\gamma+1)(0.3556\gamma+1)}\cdot\frac{1}{(0.05\gamma+1)^2} \quad (19)$$

The numerator must be separated as follows:

$$B^+(\gamma)=1 \quad \text{and} \quad B^-(\gamma)=0.0259\gamma+1 \quad (20)$$

The controller, as part of the hybrid architecture, results as:

$$C(\gamma)=\frac{(0.6953\gamma+1)(0.3556\gamma+1)}{(0.05\gamma+1)^2} \quad (21)$$

The model (see Figs.4 and 5) will be calculated.

If there are no limitations imposed, the behaviour of the system in this case is depicted in Fig.6.

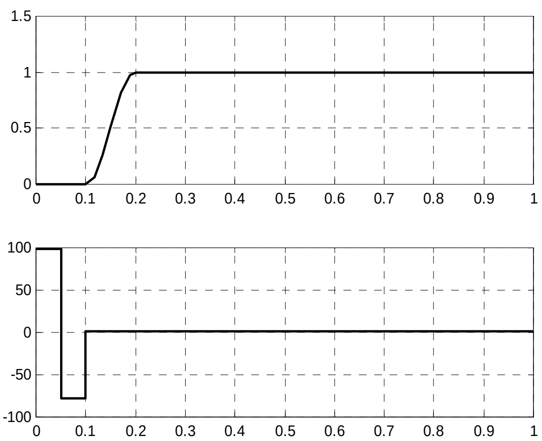

Fig.6. Dead-beat behaviour without limitations

If restrictions in the command signal are imposed, for example $u_{max}=10$ and $u_{min}=-10$, the following behaviour from Fig. 7 is obtained.

Let us consider a longer dead-time, for example $T_H=0.3$ which results in $d=6$. The response of the system is depicted in Fig.7.

In order to give an illustrative comparison between hybrid z-delta domain control structure and pure z-domain representation controller, let be considered the equivalent z-domain controller for continuous t.f. (17), using the same sampling time $h=0.05$.

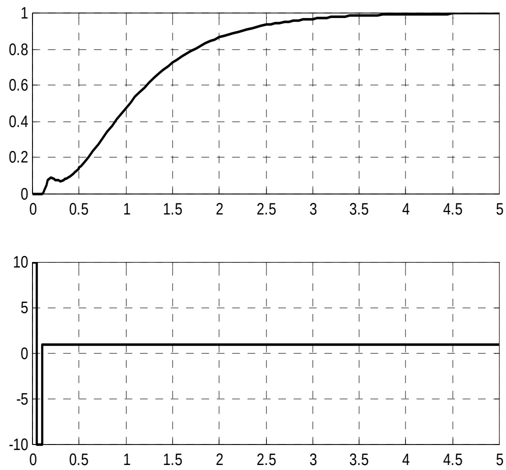

Fig.7. Dead-beat design with limitations

The pulse transfer function of the plant will be:

$$P(z)=\frac{0.0052455\,z+0.00486467}{z^2-1.7874945\,z+0.79766048}\cdot\frac{1}{z^2} \quad (22)$$

The dead-beat controller in this case will be:

$$C(z)=\frac{z^2-1.7875z+0.7976}{0.0101z^2} \quad . \quad (23)$$

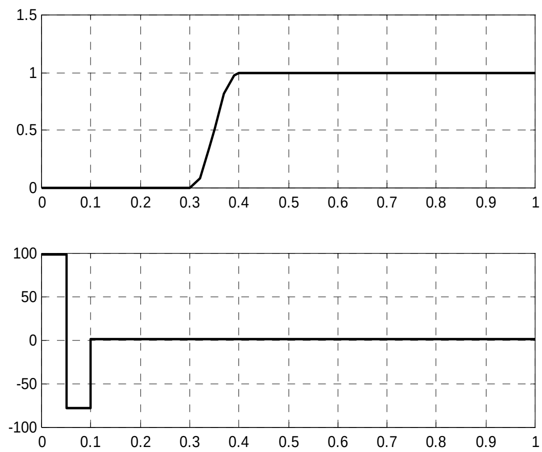

Fig.8. Dead-beat response for bigger dead-time

The controller represents the equivalent controller from the IMC structure in delta domain. Applying unit step as reference signal, it can be seen that there are no significant changes in the system responses, and in both cases the control signal has a quite high value (Fig.9).

In order to show the advantage of delta representation as far as sensitivity to model-plant mismatch is concerned, let the following experiment be performed: the coefficients of the established model will be given with three digits accuracy. This way, both delta- and *z*-domain controller will become:

$$C(\gamma)=\frac{0.247\gamma^2+1.05\gamma+1}{(0.05\gamma+1)^2}=\frac{(0.694\gamma+1)(0.355\gamma+1)}{(0.05\gamma+1)^2} \quad (23)$$

and
$$C(z)=\frac{z^2-1.787z+0.797}{0.0101z^2} \quad (24)$$

Ideally there is no plant-model mismatch and also no disturbances, so the control is open-loop. In case if there are still no disturbances, but there is plant-model mismatch, that is the plant model is given with three digits accuracy as mentioned above, let the open loop behaviours be compared in Fig. 9 (line: delta representation, dot-line: *z*-domain representation).

Comparing the outputs it can be noticed that the *z*-domain representation is more sensitive to parameter mismatch, reflected in the static error.

If closed-loop control is performed using the above presented IMC structrue, the static error disappeares due to the integrator introduced by the structure itself (see Fig. 11). Comparing the results, it can be seen that the pure *z*-domain structure (dot-line) is more sensitive to parameter mismatch than the hybrid structure (line).

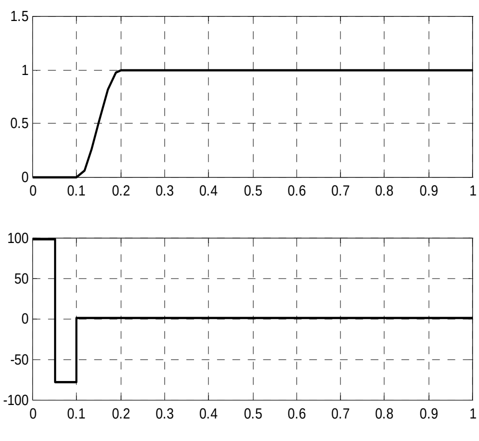

Fig.9. Dead-beat responses for both z-domain and hybrid representation

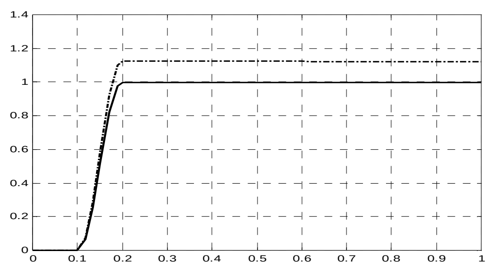

Fig.10. Open loop behaviour for z-domain and hybrid structures

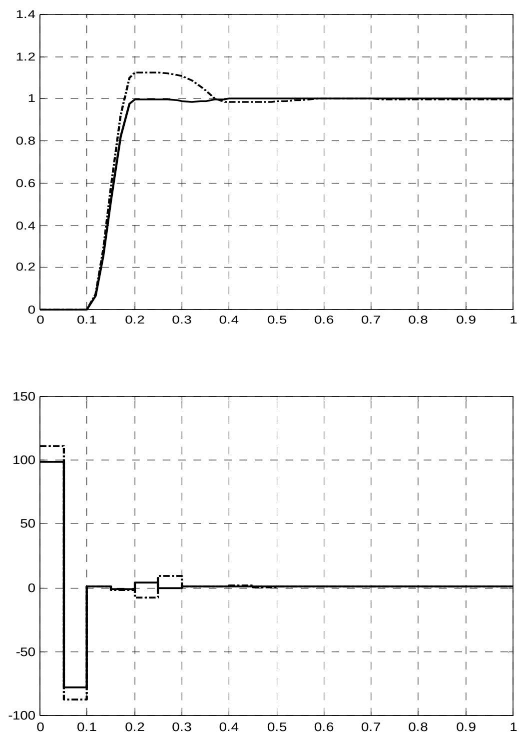

Fig.11. IMC structure in for both *z*-domain and hybrid implementation in case of three digits accuracy of the model

2.2. *IMC-based Smith predictor for plants with dead-time*

In this case first the controller *C* is designed and in accordance to this the Smith-predictor is calculated, followed by the verification of the system behaviour. The basic idea in designing a Smith-predictor is reflected in Fig.12, where the dead-time is virtually excluded from the closed loop, and the controller that is designed accordingly will be finally converted to the Smith predictor (design exemplified in *z*-domain) [9]:

$$C_{SM}(z)=\frac{C_{PID}(z)}{1+C_{PID}(z)\cdot P(z)(1-z^{-d})} \quad (25)$$

Transposed to the hybrid *z*-delta domain IMC structure representation, the difference to the DB control scheme is depicted in Fig.12. In this case the controller t.f. $C_{SM}(\gamma)$ has the expression:

$$C_{SM}(\gamma) = \frac{C_{PID}(\gamma)}{1 + C_{PID}(\gamma) \cdot B(\gamma)/A(\gamma)} \qquad (26)$$

where, $C_{PID}(\gamma)$ denoting the t.f. of PID controller designed according to Fig.12. The dead-time is represented in *z*-domain and it can be easily adapted to the dead-time of the plant.

To exemplify the procedure, let the same transfer function be analysed, which is described with equation (18) and has its delta transfer function (19). The PID controller is designed to the plant model without dead-time, using pole cancellation and imposing a phase margin of $\varphi_m \approx 60°$.

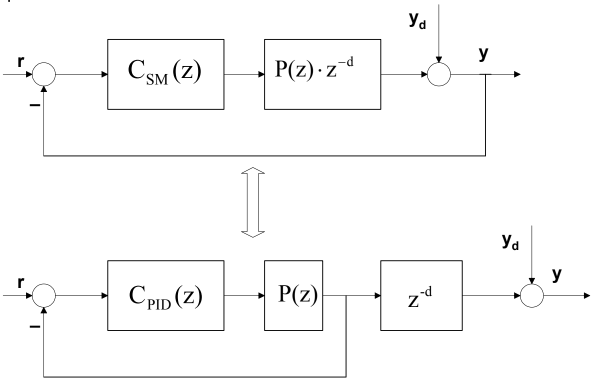

Fig. 12. Theoretical transformation of dead-time compensation scheme

The delta t.f. of a PID controller is:

$$C_{PID}(\gamma) = \frac{K_C}{T_I\gamma} \cdot \frac{(T_I\gamma+1)(T_D\gamma+1)}{T_f\gamma+1} \quad \text{with} \quad k_C = \frac{K_C}{T_I} \qquad (27)$$

where if applying the pole cancellation technique then $T_I = T_1$ and $T_D = T_2$ and k_C can be calculated imposing the desired phase margin. T_1 and T_2 are the biggest two time constants of the plant. T_f will be chosen in order to make the system quicker, but keeping in view the limitations in the control signal (in this case $T_f=T_2/4$ was chosen). The numerical values of the PID controller will be:

$$C_{PID}(\gamma) = \frac{1.6237\gamma^2 + 6.9007\gamma + 6.5662}{\gamma(0.0889\gamma + 1)} \qquad (28)$$

The controller from the IMC structure (see Fig. 13) will be in this case:

$$C(\gamma) = \frac{18.2623\gamma^2 + 77.6169\gamma + 73.8546}{\gamma^2 + 13.1635\gamma + 73.8546} \qquad (29)$$

In case there is perfect match between plant and the model, the control is actually open loop, its response to unit step input being shown in Fig. 14.

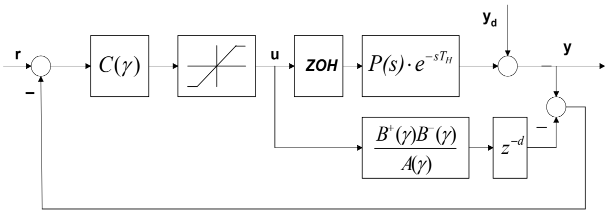

Fig.13. Hybrid *z*-delta domain Smith-predictor using IMC structre

Remark: For this application two other adequate design techniques can be used: once the use in delta domain of the well-known Modulus Optimum method [5] and other, the use in delta domain of the extension to the Extended Symmetrical Optimum method [11].

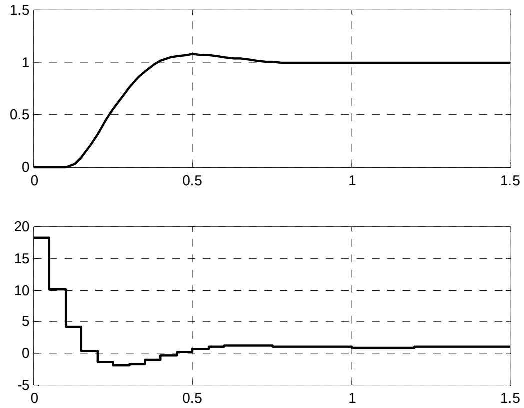

Fig.14. Dead-time compensation using PID controller

In this case the overshoot is quite little, and the control signal does not have such a high value as at DB control (of course, settling time is much bigger).

If there are limitations in the control signal, for example $u_{max}=10$, $u_{min}=-10$, the behaviour of the system is reflected in Fig.15.

In case of Smith predictor algorithm also a detailed or an illustrative sensitivity analysis can be performed. If there is no mismatch and the model parameters are given with a very high accuracy, the behaviours in *z* and delta domain are mainly the same. If the model parameters are given with three digits accuracy (see (30) and (31)) and the controllers are calculated accordingly, they result in form of (32) and (33), respectively.

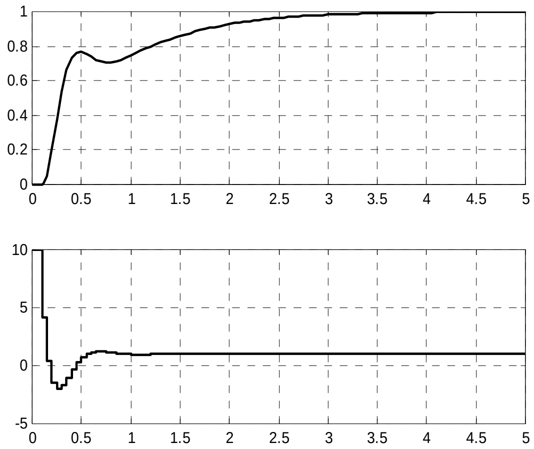

Fig.15. Dead-time compensation using PID controller and limiting the control signal

$$P'(\gamma) = \frac{0.105\gamma + 4.04}{\gamma^2 + 4.25\gamma + 4.04} \qquad (30)$$

and $$P'(z) = \frac{0.0053z + 0.0049}{z^2 - 1.79z + 0.798} \qquad (31)$$

The controllers t.f.s are:

$$H'_C(\gamma) = \frac{18.3273\gamma^2 + 77.8909\gamma + 74.0422}{\gamma^2 + 13.1830\gamma + 74.0422} \qquad (32)$$

and $$H'_C(z) = \frac{18.4214z^2 - 32.9743z + 14.7003}{z^2 - 1.3379z + 0.5241}. \qquad (33)$$

Simulating the behaviour of the two systems first in open loop, it can be noticed that there is a big difference between them (Fig.16): the *z*-domain representation (dot-line) is worse than the hybrid *z*-delta domain (solid line). As expected, the closed loop behaviour eliminates the steady-state error (Fig. 17).

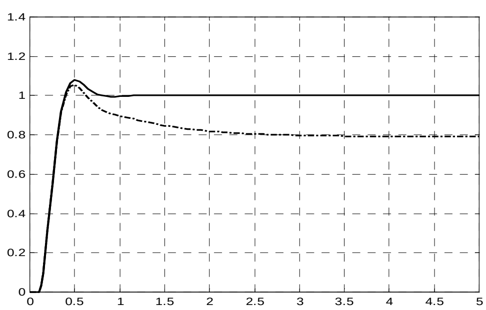

Fig.16. Open loop behaviour in case of plant-model mismatch

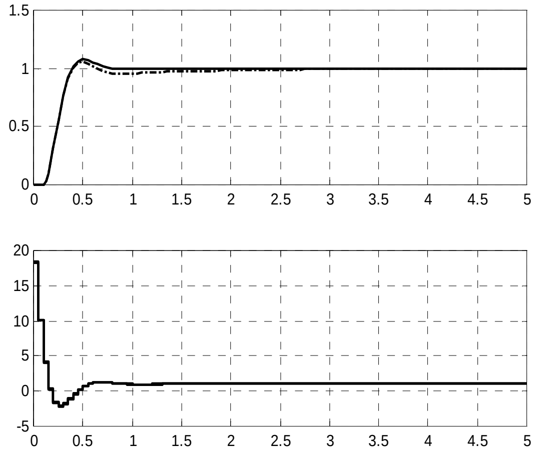

Fig.17. Closed loop behaviour in case of plant-model mismatch

3. CONCLUSIONS

In case of sampled data, the shift operator representation is very sensitive to parameter changes for fast sampling. In this case the use of delta models is advantageous. When controlling plants with long dead-time, the delta representation of dead-time deteriorates its superiority. The paper introduces a hybrid handling of control algorithms using IMC structure, handling the dead-time in *z*-domain and the model dynamics in delta domain. The hybrid algorithms are given for dead-beat control and for Smith-predictor control. Simulations showing the better performance of the hybrid form both for dead-beat and Smith predictor controls for parameter mismatch are performed. The effect of control signal limitation is also shown.

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