IAE OPTIMAL SLIDING MODE CONTROL OF CABLE SUSPENDED LOADS

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Abstract: *In this paper a sliding mode control algorithm for cable suspended loads subject to acceleration and velocity constraints is presented. The algorithm guarantees the reaching phase elimination. Moreover, it ensures fast and monotonic system error convergence to zero without violating the acceleration and velocity constraints. At the initial time* $t = t_0$ *the switching line passes through the representative point of the system in the error state space. Afterwards, the line moves with a constant velocity and a constant angle of inclination to the origin of the space and having reached the origin it stops moving. By this means insensitivity of the system with respect to external disturbance and model uncertainty from the very beginning of the control action is ensured. Parameters of the line are selected in such a way that Integral Absolute Error (IAE) is minimised.*

Keywords: *suspended loads, sliding mode control, variable structure systems, time-varying switching line, reaching phase elimination*

1. INTRODCUTION

The problem of effective carrying cable suspended (or rope hanging) loads has recently become an important research issue. This problem is significant in various applications including lifts, overhead cranes [2, 3, 15, 16, 19], rope hanging robots [8, 9, 10, 11, 12, 14] and helicopters manoeuvring heavy objects [13]. The main difficulty in control of suspended loads is maintaining positive tensions in cables while manipulating the load. In other words, the main problem is caused by the fact that ropes can exert only unidirectional forces on the payload. On the other hand, variable structure systems with sliding modes are known to be robust and effective means of controlling complex, possibly non-linear and time varying plants [1, 4, 5, 6, 7, 17, 18]. Therefore, in this paper we explicitly take into account the constraints caused by the need to keep positive tensions in suspension cables and we design an optimal sliding mode control strategy for the vertical position control of a cable suspended load.

In the paper, we consider a rope suspended mass m which is originally at rest at altitude x_0 and is supposed to change its position to x_d. Clearly, the mass cannot be required to move downwards with any acceleration exceeding gravity constant g and therefore in order to maintain some safety margin we assume that the mass acceleration cannot exceed $a_{\max}$, where $a_{\max}$ is a constant smaller than g. Moreover, for practical reasons we also require that the mass velocity cannot be greater than some pre-specified value $v_{\max}$. Further in the paper, in order to obtain fast system response, we will design the sliding mode controller for the considered mass which minimises Integral Absolute Error (IAE) and ensures that the specified constraints are satisfied.

2. CONTROL STRATEGY

In this paper the following second order system, illustrated in Fig. I, is considered

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{\beta}{m}x_2 - \frac{f\left[\boldsymbol{x}(t)\right]}{m} + \frac{F}{m} \end{aligned} \tag{1}$$

where x_1, x_2 are the load position and velocity, $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)]^T$ is the state vector, t denotes time, m is a suspended mass, F is the input signal (driving force), β is a friction coefficient, $f\left[\boldsymbol{x}(t)\right]/m$ is an unknown and bounded function representing the system uncertainty. Therefore, there exists a constant μ which for every pair $(\boldsymbol{x}, t)$ satisfies the following inequality $\left|f\left[\boldsymbol{x}(t)\right]/m\right| \le \mu$. Further in this paper, it is assumed that the load mass is finite, which implies that there exists a strictly positive constant δ which is the lower bound of $1/m$, i.e. $0 < \delta = \inf\{1/m\}$. System (1) is supposed to reach the demand position x_d, so $\boldsymbol{x}_d = [x_d\ \ 0]^T$. The system error is defined by the following vector

$$\boldsymbol{e}(t) = [e_1(t)\ \ e_2(t)]^T = \boldsymbol{x}(t) - \boldsymbol{x}_d \tag{2}$$

Hence, we have $e_1(t) = x_1(t) - x_d$ and $e_2(t) = x_2(t)$. The initial conditions of the system are $\boldsymbol{x}(t_0) = [x_0, 0]^{\mathrm{T}}$ and consequently, at the initial time $t = t_0$, the system error and the error derivative can be expressed as

$$e_1(t_0) = e_0 = x_0 - x_d,\ e_2(t_0) = 0 \tag{3}$$

In order to effectively control system (1), i.e. to eliminate the reaching phase and to obtain the system insensitivity with respect to the model uncertainty from the very beginning of the system motion, we introduce a time-varying switching line illustrated in Fig. II. The line slope does not change during the control process, which implies that the line moves on the phase plane without rotating. In other words, the line is shifted in the state space with a constant angle of inclination. At the beginning the line moves with a constant velocity in the state space and then it stops at a time instant $t_f > t_0$. Consequently, for any $t \in \langle t_0, t_f \rangle$ the switching line is described by the following equation

$$\begin{aligned} &s(\boldsymbol{e},t) = 0 \quad \text{where} \\ &s(\boldsymbol{e},t) = e_2(t) + ce_1(t) + A + Bt \end{aligned} \tag{4}$$

where c, A and B are constants. The selection of these constants will be considered further in this paper. Since the considered line stops moving at the time t_f, for any $t \ge t_f$ it is fixed and can be described as follows

$$s(\boldsymbol{e},t) = 0 \text{ where } s(\boldsymbol{e},t) = e_2(t) + ce_1(t) \tag{5}$$

In order to ensure the stability of the sliding motion on the line described by equations (4) and (5), parameter c must be strictly positive, i.e. $c > 0$.

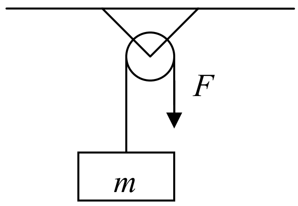

Fig.1. Suspended load.

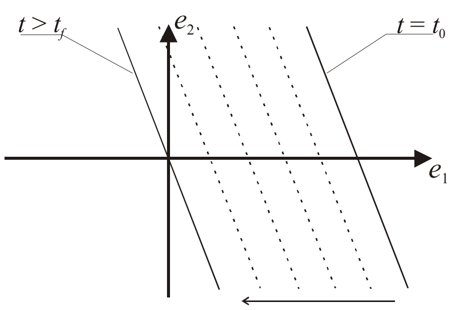

Fig. 2. Time-varying switching line.

In order to eliminate the reaching phase, and consequently to ensure insensitivity of the considered system from the very beginning of its motion, the constants A, B and c should be chosen in such a way that the representative point of the system at the initial time $t = t_0$ belongs to the switching line. For that purpose, the following condition must be satisfied

$$s\left[\boldsymbol{e}(t_0), t_0\right] = e_2(t_0) + ce_1(t_0) + A + Bt_0 = 0 \quad (6)$$

The following control signal ensures the stability of the sliding motion on switching line (4)

$$F = \beta x_2 + m\left\{-ce_2(t) - B - \gamma \operatorname{sgn}\left[s(\boldsymbol{e}, t)\right]\right\} \quad (7)$$

where $\gamma = \eta + \mu$ and η is a strictly positive constant. Consequently, for any time $t \in \langle 0, t_f \rangle$ the system dynamics is described by equation (4) with introduced initial conditions and for $t \geq t_f$ it is governed by equation (5). Therefore, in order to find the system error, we take into account the following equation

$$e_2(t) + ce_1(t) + A + Bt = 0 \quad (8)$$

which determines the considered switching line for any time $t \leq t_f$. Taking into account condition (6) and the assumption that $t_0 = 0$ we obtain

$$A = -ce_0 \quad (9)$$

Solving equation (4) with initial conditions (3), assuming for the sake of clarity that $t_0 = 0$ and furthermore, considering relation (9) we get the system error and its derivative for the time $t \in \langle 0, t_f \rangle$

$$e_1(t) = -\frac{B}{c^2} e^{-ct} + e_0 + \frac{B}{c^2} - \frac{B}{c} t \quad (10)$$

$$e_2(t) = \frac{B}{c} e^{-ct} - \frac{B}{c} = \frac{B}{c}\left(e^{-ct} - 1\right) \quad (11)$$

Notice that for the time $t \geq t_f$ the switching line is fixed and passes through the origin of the error state space. This leads to the condition

$$A + Bt_f = 0 \quad (12)$$

From equations (9) and (12) we obtain

$$t_f = e_0 c / B \quad (13)$$

The fact that parameter c and time t_f are greater than zero implies that constants e_0 and B always have the same signs. Next, we will analyze the behaviour of the system in the second phase of its motion, that is when the switching line does not move. The time invariant switching line is described by relation (5), which is equivalent to equation

$$e_2(t) + ce_1(t) = 0 \quad (14)$$

The initial conditions which are necessary to solve equation (14) can be determined from equations (10) and (11) whose values are evaluated at the time instant t_f

$$e_1(t_f) = \frac{B}{c^2}\left(1 - e^{-c^2 e_0 / B}\right) \quad (15)$$

$$e_2(t_f) = \frac{B}{c}\left(e^{-c^2 e_0 / B} - 1\right) \quad (16)$$

Solving equation (14) with initial conditions (15) and (16), we obtain

$$e_1(t) = \frac{B}{c^2}\left(-1 + e^{e_0 c^2 / B}\right) e^{-ct} \quad (17)$$

$$e_2(t) = \frac{B}{c}\left(1 - e^{e_0 c^2 / B}\right) e^{-ct} \quad (18)$$

These two equations describe the system error for any time $t \geq t_f$. Notice that the error described by equations (10) and (17) does not exhibit any overshoots. In order to facilitate the procedure of determining the switching line parameters A, B and c, we introduce the following constant

$$m = e_0 c^2 / B \quad (19)$$

which is always strictly positive because the signs of B and e_0 are always the same. With the above notation, formulae (17) and (18) can be written more concisely as follows

$$e_1(t) = \frac{B}{c^2}\left(-1 + e^m\right)e^{-ct} \quad (20)$$

$$e_2(t) = \frac{B}{c}\left(1 - e^m\right)e^{-ct} \quad (21)$$

Our considerations presented up to now, as well as equations describing the system error evolution, clearly show that the switching line selection fully determines the system motion and its performance. Therefore, the switching line parameters A, B and c should be carefully chosen according to the user requirements and specifications, so as to obtain the best possible control quality without violating technical and environmental constraints. Therefore, in the next part of this paper several methods of the switching line selection will be proposed. The methods ensure optimal, in the sense of the integral absolute error (IAE) performance of the system without exceeding the maximum admissible values of the acceleration and the system velocity.

3. SWITCHING LINE DESIGN

In this section the switching line parameters are selected. For that purpose we introduce the IAE which will be minimised subject to the acceleration and velocity constraints in order to obtain the optimal switching line parameters. First we present how to select the line when each of these two constraints is satisfied separately, and then we show the method of the switching line design when both constraints are satisfied simultaneously. We begin our analysis with introducing the criterion, i.e. we calculate the IAE

$$J_{\mathrm{IAE}} = \int_{t_0}^{\infty} \left|e_1(t)\right| dt \quad (22)$$

Since the tracking error converges to zero monotonically in the considered system, substituting (10) and (17) into (22) and calculating appropriate integrals, we find criterion (22) to be equivalent to

$$J_{\mathrm{IAE}} = \frac{e_0^2 c}{2|B|} + \frac{|e_0|}{c} \quad (23)$$

On the other hand, calculating c from equation (19) we obtain

$$c = \sqrt{mB/e_0} \quad (24)$$

In order to facilitate the minimisation process, further in the paper we will consider the quality criterion as a function of variables m and B, instead of c and B. Substituting (24) into (23), we obtain the control quality criterion as follows

$$J_{\mathrm{IAE}}(m, B) = \frac{|e_0|^{3/2}}{\sqrt{|B|}}\left(\frac{\sqrt{m}}{2} + \frac{1}{\sqrt{m}}\right) \quad (25)$$

Further in the paper this criterion will be minimised with two constraints.

3.1. Acceleration constraint

First we consider the situation when $a_{\max}$ is the maximum admissible value of the load acceleration. It means that we require that the following inequality holds

$$\left|\dot{x}_2(t)\right| \le a_{\max} \quad (26)$$

where $a_{\max}$ is a constant smaller than g. Let us calculate the greatest value of $\left|\dot{x}_2(t)\right|$. For $t \le t_f$ the system error is given by equation (10). Differentiating this equation two times – or alternatively differentiating equation (11) once – we get

$$\dot{x}_2(t) = -Be^{-ct} \quad (27)$$

The maximum absolute value of this signal, achieved at the initial time $t_0 = 0$ is equal to $\left|\dot{x}_2(0)\right| = |B|$. On the other hand, for $t \ge t_f$ the system error is described by equation (20) whose second derivative right hand side absolute value decreases with time. Consequently, for $t \ge t_f$ the maximum value of $\left|\dot{x}_2(t)\right|$, reached at the time t_f, equals

$$\begin{aligned}&\left|\dot{x}_2\ (t_f\ )\right| = \left|-B\left(1-e^{m}\right)e^{-ct_f}\ \right| = \\ &= \left|-B\left(1-e^{m}\right)e^{-m}\right| = \left|-B\left(e^{-m}-1\right)\right| = \\ &= \left|B\left(1-e^{-m}\right)\right|\end{aligned} \tag{28}$$

As m is greater than zero, the right hand side of the above equation is always smaller than $|B|$. Hence, we conclude that $|B|$ is the extreme value of $\left|\dot{x}_2(t)\right|$, and from relation (26) we obtain the following constraint

$$|B| \le a_{\max} \tag{29}$$

which is the first constraint considered in the paper. Notice that for any given value of m, the minimum of criterion (25) is obtained for the greatest value of $|B|$ satisfying constraint (26). Therefore, the solution of the considered minimisation task can be found as a minimum of a single variable function $J_{\text{IAE}}(m)$. Taking into account constraint (26) we get

$$J_{\text{IAE}}^{a}\left(m\right) = \frac{\left|e_0\right|^{3/2}}{\sqrt{a_{\max}}}\left(\frac{\sqrt{m}}{2}+\frac{1}{\sqrt{m}}\right) \tag{30}$$

Equating the derivative of criterion (30) to zero we obtain the following optimal solution $m_{a\ opt} = 2$. Then for that value of m, we get the optimal value of parameter B

$$B_{a\ opt} = a_{\max}\ \text{sgn}\left(e_0\right) \tag{31}$$

The other parameters of the switching line are given by the following formulae

$$c_{a\ opt} = \sqrt{2\,a_{\max}\,/\left|e_0\right|} \tag{32}$$

$$A_{a\,opt} = -\text{sgn}\left(e_0\right)\sqrt{2\,a_{\max}\,\left|e_0\right|} \tag{33}$$

$$t_{f\ a\ opt} = \sqrt{2\left|e_0\right|/a_{\max}} \tag{34}$$

This concludes the minimisation of IAE subject to acceleration constraint (26).

3.2. Velocity constraint

Next we take into account the velocity constraint expressed by the following inequality

$$\left|x_2\ (t)\right| \le v_{\max} \tag{35}$$

where $v_{\max}$ is the maximum admissible velocity of the considered system. For any time $t \le t_f$ the system velocity is described by equation (11) and for the time $t \ge t_f$ by equation (21). It can be proved that the extreme value of the velocity is achieved at the time instant t_f. This extreme value

$$x_2\ \left(t_f\ \right) = Bc^{-1}\left(e^{-m}-1\right) \tag{36}$$

Thus, in order to satisfy constraint (35), we require that

$$\left|\frac{B}{c}\left(e^{-m}-1\right)\right| \le v_{\max} \tag{37}$$

This inequality will be used further in the minimisation of the IAE performance index. From inequality (37), using relation (24) we find the maximum admissible value of $|B|$

$$|B| \le \left[\frac{v_{\max}\ \sqrt{m}}{\sqrt{\left|e_0\right|}\left(1-e^{-m}\right)}\right]^2 \tag{38}$$

and substituting this value into expression (25) we get

$$J_{\text{IAE}}^{v}\left(m\right) = \frac{e_0^2\left(1-e^{-m}\right)}{v_{\max}}\left(\frac{1}{2}+\frac{1}{m}\right) \tag{39}$$

This function, for any fixed m expresses the minimum value of criterion $J_{\text{IAE}}(m, B)$ which can be achieved when velocity constraint (35) is satisfied. Criterion (39) is a decreasing function of its argument m, and therefore in this case, i.e. when the velocity constraint is taken into account, we get $m_{v\ opt} \to \infty$. Since parameter B can be calculated substituting $m_{v\ opt}$ into

$$B = \left[\frac{v_{\max}\ \sqrt{m}}{\sqrt{\left|e_0\right|}\left(1-e^{-m}\right)}\right]^2\ \text{sgn}\left(e_0\right) \tag{40}$$

then $B_{v\ opt} \to \text{sgn}(e_0)\cdot\infty$. Moreover, $A_{v\ opt} \to -\text{sgn}(e_0)\cdot\infty$, $c_{v\ opt} \to \infty$ and

$$t_{f\,v\,opt} = \frac{|e_0|}{v_{\max}} \tag{41}$$

Even though in the situation considered here both $m_{v\,opt}$ and $B_{v\,opt}$ tend to infinity, the switching line moves along the horizontal axis of the (e_1, e_2) coordinate frame at the finite rate equal to minus $v_{\max}$ sgn(e_0). Notice that in this case, when only the velocity constraint is taken into account, the input signal becomes infinite. Therefore, further in the paper we consider the situation where two constraints are satisfied at the same time, i.e. the velocity and acceleration constraints, which gives more practical application.

3.3 Acceleration and velocity constraint

Now we consider system (1) subject to constraints (26) and (35). It means that the input signal and velocity constraint are required to be satisfied at the same time. In this case, in order to find the optimal switching line parameters the minimum of the following criterion

$$J_{\mathrm{IAE}}^{av}(m) = \max\left[J_{\mathrm{IAE}}^{a}(m), J_{\mathrm{IAE}}^{v}(m)\right] \tag{42}$$

should be determined.

Functions $J_{\mathrm{IAE}}^{a}(m)$ and $J_{\mathrm{IAE}}^{v}(m)$ are illustrated in Fig. III. That situation requires the following two cases to be considered: the first one $J_{\mathrm{IAE}}^{v}(2) \le J_{\mathrm{IAE}}^{a}(2)$ and the other one $J_{\mathrm{IAE}}^{v}(2) > J_{\mathrm{IAE}}^{a}(2)$. In the first case we get that the optimal value of parameter m is $m_{av\,opt} = m_{a\,opt} = 2$ and the optimal value $B_{av\,opt} = B_{a\,opt}$ can be found from (31). The other optimal switching line parameters, can be calculated from relations (32), (33) and (34).

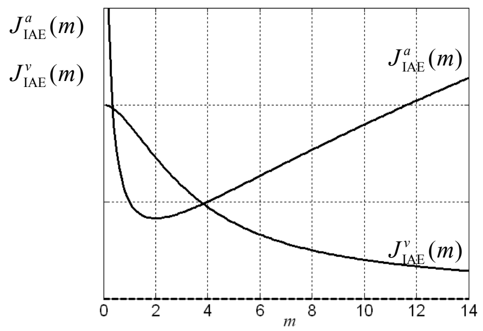

Fig. 3. Criteria $J_{\mathrm{IAE}}^{a}(m)$ and $J_{\mathrm{IAE}}^{v}(m)$.

Analysing the latter case $J_{\mathrm{IAE}}^{v}(2) > J_{\mathrm{IAE}}^{a}(2)$ it can be proved that in order to find $m_{av\,opt}$ the root of function

$$f(m) = J_{\mathrm{IAE}}^{v}(m) - J_{\mathrm{IAE}}^{a}(m) = \\ = |e_0|^{3/2}\left(\frac{1}{2}+\frac{1}{m}\right)\left[\frac{\sqrt{|e_0|}\left(1-e^{-m}\right)}{v_{\max}} - \sqrt{\frac{m}{a_{\max}}}\right] \tag{43}$$

may be numerically calculated in the interval (2, m_γ) where

$$m_\gamma = |e_0|\, a_{\max} / v_{\max}^2 \tag{44}$$

Then the optimal value $B_{av\,opt}$ of parameter B can be found from (31) or (40). The other parameters of the line are determined by (9), (13) and (24).

4. SIMULATION EXAMPLE

In order to verify the proposed method of the switching line design we consider a suspended load m described as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{0.15}{m}x_2 + \frac{1}{m}F - \frac{f(x_1, x_2)}{m} \end{aligned} \tag{45}$$

where m = 1 kg and

$$f(x_1, x_2) = 0.1\,\mathrm{sgn}(x_2) + 0.049\frac{x_2}{|x_2| + 0.1} \tag{46}$$

represents model uncertainty, i.e. unknown friction in the system. Consequently, $\gamma = 0.15$. The initial condition

$$x_0 = 4 \text{ m} \tag{47}$$

The demand position of system (45) is

$$x_d = 2 \text{ m} \tag{48}$$

We require that $a_{\max} = 0.1$ m/s^2 and $v_{\max} = 0.25$ m/s. Then, using the proposed method we obtain that $J^{v}_{\mathrm{IAE}}(2) > J^{a}_{\mathrm{IAE}}(2)$ and the optimal parameter m belongs to the interval $(2, m_\gamma)$, where $m_\gamma = 3.2$. Finding numerically the root of function $f(m)$ given by (43) we obtain $m_{av\,opt} \approx 2.834$. The other optimal switching line parameters are $A_{av\,opt} \approx -0.753$ m/s, $B_{av\,opt} = -0.1$ m/s^2 and $c_{av\,opt} \approx 0.376$ 1/s. The line stops moving at the time instant $t_{f\,av\,opt}$ equal to 7.53 s. Simulation results for the system with this line are shown in Fig. IV – VI. It can be seen from the figures that the load reaches its demand position without oscillations or overshoots. Furthermore, both the velocity and acceleration constraints are satisfied and the system is insensitive from the very beginning of the control process.

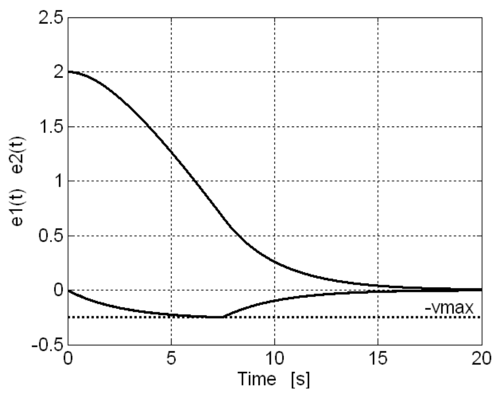

Fig. 4. Tracking error evolution and its derivative.

5. CONCLUSION

In the paper, we proposed a method of sliding mode control for a rope suspended mass. This method employs the time-varying switching line which moves with a constant velocity and a constant angle of inclination to the origin of the error state space. Parameters of this line are selected in such a way that Integral Absolute

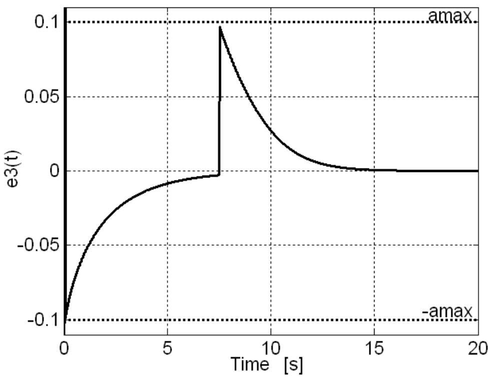

Fig. 5. System acceleration.

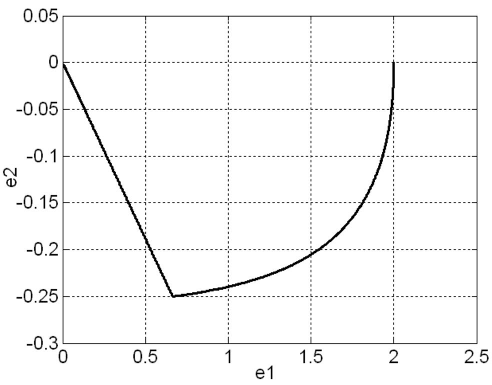

Fig. 6. Phase trajectory.

Error (IAE) is minimised with two constraints (i.e. with the mass acceleration and the mass velocity constraint). Furthermore, the tracking error converges to zero monotonically and the system is insensitive with respect to external disturbance from the very beginning of the control action.

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