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PROPAGATION, DELAYS AND STABILIZATION I

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Abstract: *The paper starts from the well established idea of the association of Functional Differential Equations with some structures of hyperbolic Partial Differential Equations defining propagation. Here an application is considered – stabilization of the overhead crane involving distributed elastic phenomena. Conservativeness of the system requires stabilization and non-uniform elasticity will introduce distributed delays.*

Keywords: *Systems with propagation, Time lag, Stabilization.*

1. INTRODUCTION. PROBLEM STATEMENT

It is now a well established fact that a strong connection exists between the so-called *systems with propagation* and various classes of Functional Differential Equations of neutral type.

By systems with propagation we understand a class of systems with distributed parameters in one dimension, whose models are given by initial / boundary value problems for hyperbolic Partial Differential Equations (PDE) in two dimensions – time plus one space dimension (the "propagation dimension") ; such systems contain long transmission lines for physical signals – usually electric signals but also water, steam or gas flow/pressure. In various references we had selected and analyzed such systems. The strong motivation was the one-to-one correspondence between the two mathematical objects which at its turn offered the possibility of solving various problems for both systems by solving them for the system of FDE (assumed to be simpler). The problems we deal with arose from control: stability and absolute stability of feedback systems, forced oscillations, numerical procedures and control synthesis. From time to time we used to point out this approach – a straightforward application of the d'Alembert method and signal new and open problems [1], [2], [3], [4], [5], [6].

In this paper we consider a problem of the same type but with specific features. The *control of an overhead crane* appears to be *benchmark* in control, even if considered from other different viewpoints. For instance, its nonlinear model with lumped parameters, i.e. described by Ordinary Differential Equations (ODEs) [7] was used as an application of recent nonlinear control techniques [8], [9].

A more recent model [10] takes into account the distributed elasticity of the crane's pendulum rod. If this elasticity is uniform then the associated system of FDE is standard i.e. with lumped time delays, thus allowing a “neutral” FDE approach [11]. The resulting system is conservative, hence does not have asymptotic stability. Otherwise, the associated FDE are by far more complicated.

There exist however, various simplifications of the basic model. These simplifications will be discussed in the context of model parameters - their relative “smallness”. In this way the more complicated models may be viewed as perturbations of the simpler ones. Starting from these considerations, the paper is organized as follows. First, the general basic model is presented and the basic parameters are displayed. Further, some simplified models are discussed, both lumped and distributed parameters. An important element is their inherent (without any control action) stability. The cases requiring feedback stabilization are discussed and some feedback structures are considered. Finally robustness is discussed with reference to an energy-like Liapunov functional which may also act as a c.l.f. (Control Liapunov Functional).

2. MODELS, EQUATIONS AND PARAMETERS

Following [10] we consider the control model of the overhead crane incorporating the elasticity effects due to the mass distribution along the length of the crane’s rod (fig.1)

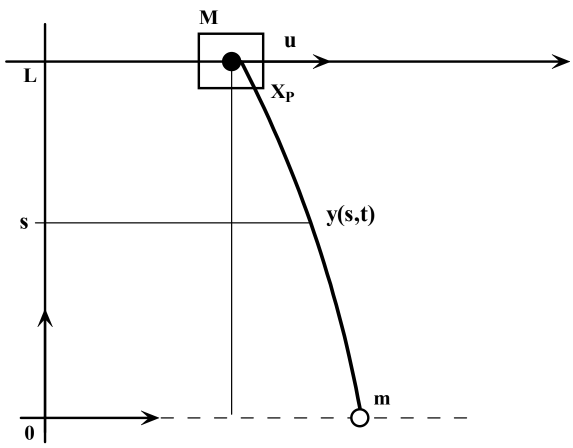

Fig.1. Overhead crane modeling mechanics

$$\begin{aligned}
&y_{tt} - \frac{\partial}{\partial s}(a(s)y_s) = 0, t > 0, 0 \le s \le L \\
&y_{tt}(0,t) = g\ y_s(0,t), \quad y(L,t) = X_p \\
&\ddot{X}_P = K(a(s)y_s)\ (L,t) + u(t)
\end{aligned} \tag{2.1}$$

where most of the notations are obvious from fig.1 but we have to explain the two aggregate parameters a(s) and K. We have:

$$a(s) = g(s + \frac{m}{\rho}) \quad , K = \frac{m + \rho L}{Ma(L)} = \frac{\rho}{Mg}$$

where g=9.81 ms^{-2} is the gravity acceleration, and ρ is the mass density (per length) of the crane’s rod.

If the normalized rod length σ=s/L is introduced, we may find after some elementary (but targeted manipulation) the following model:

$$\begin{aligned}
&\frac{\rho L}{m} \cdot \frac{L}{g} y_{tt} - \frac{\partial}{\partial \sigma}((1 + \frac{\rho L}{m}\sigma)y_\sigma) = 0, \ t > 0, \ 0 \le \sigma \le 1 \\
&\frac{L}{g} y_{tt}(0,t) = y_\sigma(0,t), \ y(1,t) = X_p \\
&\frac{L}{g}\ddot{X}_p = \frac{m}{M}(1 + \frac{\rho L}{m})y_\sigma(1,t) + \frac{L}{g}u(t)
\end{aligned} \tag{2.2}$$

where we have now two parameters which are adimensional as mass ratios: the ratio of the total rod mass versus load mass (ρ L)/m and the ratio of the load mass versus platform mass m/M; the third parameter L/g accounts for some equivalent pendulum of the rod and is the square of a time constant.

With this, the partial differential equation of (2.2) is adimensional, as well as the boundary condition at σ = 0, while the boundary condition at σ=1 has the dimension of a length.

A. A standard assumption is that the rod mass is negligible with respect to the load mass ρL/m << 1 – see [8], [9], where this assumption is mentioned as such. Taking this aggregate as 0 in (2.2), the distributed dynamics is lost and the model becomes:

$$\begin{aligned}
&y_{\sigma\sigma} = 0, \quad t > 0, \quad 0 \le \sigma \le 1 \\
&\frac{L}{g} y_{tt}(0,t) = y_\sigma(0,t)\,, \quad y(1,t) = X_p \\
&\frac{L}{g}\ddot{X}_P = \frac{m}{M} y_\sigma(1,t) + \frac{L}{g}u(t)
\end{aligned} \tag{2.3}$$

From the first equation we deduce:

$$y(\sigma,t)=\varphi_0(t)+\varphi_1(t)\sigma \tag{2.4}$$

where $\varphi_0 : R_+ \to R$ and $\varphi_1 : R_+ \to R$ will be determined from the boundary conditions. It is not difficult to see that φ_0 and X_P are solutions of the following system of ordinary differential equations, while φ_1 is some system output:

$$\begin{aligned}
&\frac{L}{g}\ddot{\varphi}_0+\varphi_0=X_p\\
&\frac{L}{g}\ddot{X}_p=\frac{m}{M}(X_p-\varphi_o)+\frac{L}{g}u(t)\\
&\varphi_1=X_p-\varphi_0
\end{aligned} \tag{2.5}$$

More interesting from the control point of view is to have φ_0 and φ_1 as state variables and X_p as measurable output; this is done by subtracting the first equation from the second:

$$\begin{aligned}
&\frac{L}{g}\ddot{\varphi}_0=\varphi_1\\
&\frac{L}{g}\ddot{\varphi}_1=-(1-\frac{m}{M})\varphi_1+\frac{L}{g}u(t)\\
&X_p=\varphi_0+\varphi_1
\end{aligned} \tag{2.6}$$

This is a standard linear system whose characteristic equation is:

$$\frac{L}{g}s^2(\frac{L}{g}s^2+1-\frac{m}{M})=0 \tag{2.7}$$

and has a double zero root and a pair of purely imaginary roots. This whole configuration is well known for the standard model of lumped parameter overhead crane hence our model reduction is consistent with previous knowledge. The stabilization problem is also standard and we do not insist on it.

B. The assumption on the boundary condition at σ=0 made in [10] reads as follows: the acceleration of the load mass is negligible with respect to the gravitational acceleration g. If we consider the boundary condition we see that this is a "predictive" assumption since it requires the validity of the property for all **t** which cannot be guaranteed *apriori*. More reasonable would be to take L/g ≈ 0 but this would destroy the entire system dynamics – distributed and on its boundaries. If we admit nevertheless the physical assumption of [10], a new model would be obtained; we may discuss this model separately.

To conclude this section we note the following: a difference is made between "legitimate" model reduction and "illegitimate" one; the first is obtained by setting a parameter to 0 everywhere it appears – as showed on the case of ρL/m – while the second one is obtained by applying some physical assumption – as showed previously – and thus obtaining a new model.

C. We shall return to the model (2.2) and transform it by introducing the forward and backward waves, i.e. the Riemann invariants and integrate them along the characteristics. Consider first the new distributed variables:

$$v(\sigma,t):=y_t(\sigma,t)\,,\quad w(\sigma,t):=(1+\frac{\rho L}{m}\sigma)y_\sigma(\sigma,t) \tag{2.8}$$

and write down the transformed equations (2.2):

$$\begin{aligned}
&\gamma_0T^2v_t-w_\sigma=0\,,\quad w_t-(1+\gamma_0\sigma)v_\sigma=0,\quad t>0,\ 0\le\sigma<1\\
&T^2v_t(0,t)=w(0,t)\,,\ v(1,t)=\dot{X}_p\\
&T^2\ddot{X}_p=\delta_0 w(1,t)+T^2u(t)
\end{aligned} \tag{2.9}$$

where we denoted $\gamma_0=(\rho L)/m$ and $\delta_0=m/M$ the two mass ratios and $T=\sqrt{L/g}$ the time constant of the pendulum; it is clear that X_p appears as a cyclic variable. We define the Riemann invariants from the formulae:

$$\begin{aligned}
&v(\sigma,t)=u^+(\sigma,t)+u^-(\sigma,t)\\
&w(\sigma,t)=c(\sigma)(u^-(\sigma,t)-u^+(\sigma,t))
\end{aligned} \tag{2.10}$$

where u^+ and u^- are the backward and forward waves respectively; c(σ) is chosen from the equality:

$$c(\sigma)=T\sqrt{\gamma_0(1+\gamma_0\sigma)}$$

which will ensure decoupling of the derivatives:

$$u_t^+ + \frac{1}{T\sqrt{\gamma_0}}\sqrt{(1+\gamma_0\sigma)}u_\sigma^+ = \frac{1}{4T\sqrt{\gamma_0}}\frac{\gamma_0}{\sqrt{(1+\gamma_0\sigma)}}(u^- - u^+)$$
$$u_t^- - \frac{1}{T\sqrt{\gamma_0}}\sqrt{(1+\gamma_0\sigma)}u_\sigma^- = \frac{1}{4T\sqrt{\gamma_0}}\frac{\gamma_0}{\sqrt{(1+\gamma_0\sigma)}}(u^- - u^+) \quad (2.11)$$

The boundary condition will become, accordingly:

$$T(u_t^+(0,t) + u_t^-(0,t)) = \sqrt{\gamma_0}(u^-(0,t) - u^+(0,t))$$
$$u^+(1,t) + u^-(1,t) = \dot{X}_p \quad (2.12)$$
$$T\ddot{X}_p = \delta_0\sqrt{\gamma_0(1+\gamma_0)}(u^-(1,t) - u^+(1,t)) + Tu(t)$$

We turn now to the system of the characteristics. There are two characteristic families defined by:

$$\frac{dt}{d\sigma} = \frac{\pm T\sqrt{\gamma_0}}{\sqrt{1+\gamma_0\sigma}} \quad (2.13)$$

the first family being the family of the increasing characteristics, the second – of the decreasing ones, as below:

$$\tau^+(\lambda;\sigma,t) = t + \frac{T}{\sqrt{\gamma_0}}\int_0^\lambda \frac{\gamma_0 d\eta}{\sqrt{1+\gamma_0\eta}} = t + \frac{2T}{\sqrt{\gamma_0}}(\sqrt{1+\gamma_0\lambda} - \sqrt{1+\gamma_0\sigma})$$
$$\tau^-(\lambda;\sigma,t) = t - \frac{T}{\sqrt{\gamma_0}}\int_0^\lambda \frac{\gamma_0 d\eta}{\sqrt{1+\gamma_0\eta}} = t + \frac{2T}{\gamma_0}(\sqrt{1+\gamma_0\sigma} - \sqrt{1+\gamma_0\lambda})$$

The propagation time along the characteristic is given by the following equalities:

$$T^+ = \tau^+(1;0,t) - t = \frac{2T}{\sqrt{\gamma_0}}(\sqrt{1+\gamma_0} - 1) = \frac{2T\sqrt{\gamma_0}}{1+\sqrt{1+\gamma_0}}$$
$$T^- = \tau^-(0;1,t) - t = \frac{2T\sqrt{\gamma_0}}{1+\sqrt{1+\gamma_0}} = T^+$$

We shall present now briefly the integration along the characteristics. The equation of the forward wave $u^+(\sigma,t)$ is considered along an increasing characteristic, while the backward one is considered along a decreasing one:

$$\Phi^+(\lambda) = u^+(\lambda;\tau^+(\lambda;\sigma,t))$$
$$\Phi^-(\lambda) = u^-(\lambda;\tau^-(\lambda;\sigma,t))$$

Differentiation with respect to λ will give, after straightforward manipulation, the following equations:

$$\frac{d\Phi^+}{d\lambda} + \frac{\gamma_0}{4(1+\gamma_0\lambda)}\Phi^+ = \frac{\gamma_0}{4(1+\gamma_0\lambda)}u^-(\lambda,\tau^+(\lambda;\sigma,t))$$
$$\frac{d\Phi^-}{d\lambda} + \frac{\gamma_0}{4(1+\gamma_0\lambda)}\Phi^- = \frac{\gamma_0}{4(1+\gamma_0\lambda)}u^+(\lambda,\tau^-(\lambda;\sigma,t)) \quad (2.14)$$

These equations have to be integrated; formally it is easy to do it, but their forcing terms which represent mixtures of waves and their characteristics will lead to some functional equations that are difficult to connect to the boundary conditions. The reason is the presence of the term $1+\gamma_0\sigma$ in (2.9), i.e. the non-uniform elastic effects.

In order to simplify the equations *we shall change the model* by replacing the term with our equivalent constant one; the replacement should be acceptable for sufficiently small γ_0. For the replacement we shall choose the following condition: the propagation time along the characteristic straight lines thus obtained must remain the same. Therefore, by taking the modified families of characteristics:

$$\frac{dt}{d\sigma} = \pm\frac{T\sqrt{\gamma_0}}{\sqrt{1+\gamma_1}}$$

the propagation time will be:

$$T_d = T^+ = T^- = \frac{T\sqrt{\gamma_0}}{\sqrt{1+\gamma_1}} = \frac{2T\sqrt{\gamma_0}}{1+\sqrt{1+\gamma_0}}$$

and this will give:

$$0 < \gamma_1 = \sqrt{1+\gamma_0} + \frac{\gamma_0}{2} - 1 < \gamma_0 \quad (2.15)$$

This modification will concern model (2.9) which will be replaced by the following one:

$$\gamma_0 T^2 v_t - w_\sigma = 0,\quad w_t - (1+\gamma_1)v_\sigma = 0,\quad t>0,\ 0\le\sigma\le 1$$
$$T^2 v_t(0,t) = w(0,t),\quad v(1,t) = \dot{X}_p \quad (2.16)$$
$$T^2\ddot{X}_p = \delta_0 w(1,t) + T^2 u(t)$$

The remaining part of the paper will be concerned with this model.

3. THE MODIFIED MODEL AND ITS INHERENT STABILITY

A. We introduce first the Riemann invariants using (2.10) but with constant c, which will be taken as $c(\sigma) = T\sqrt{\gamma_0(1+\gamma_1)}$ as follows:

$$\begin{aligned} & u_t^+ + \frac{1}{T\sqrt{\gamma_0}}\sqrt{1+\gamma_1}u_\sigma^+ = 0 \\ & u_t^- - \frac{1}{T\sqrt{\gamma_0}}\sqrt{1+\gamma_1}u_\sigma^- = 0 \end{aligned} \tag{3.1}$$

$$\begin{aligned} & T(u_t^+(0,t)+u_t^-(0,t)) = \sqrt{\gamma_0(1+\gamma_1)}(u^-(0,t)-u^+(0,t)) \\ & u^+(1,t)+u^-(1,t) = \dot{X}_P \\ & T\ddot{X}_P = \delta_0\sqrt{\gamma_0(1+\gamma_1)}(u^-(1,t)-u^+(1,t))+Tu(t) \end{aligned} \tag{3.2}$$

If we use the propagation time T_d (3.1) and (3.2) may be written as:

$$\begin{aligned} & u_t^+ + \frac{1}{T_d}u_\sigma^+, u_t^- - \frac{1}{T_d}u_\sigma^- = 0,\ \ 0 \le \sigma \le 1,\ \ t>0 \\ & T_d(u_t^+(0,t)+u_t^-(0,t)) = \gamma_0(u^-(0,t)-u^+(0,t)) \\ & u^+(1,t)+u^-(1,t) = \dot{X}_P,\ \ T_d\ddot{X}_P = \gamma_0\delta_0(u^-(1,t)-u^+(1,t))+T_d u(t) \end{aligned} \tag{3.3}$$

The characteristic equations are now:

$$\tau^+(\lambda;\sigma,t) = t+T_d(\lambda-\sigma),\ \tau^-(\lambda;\sigma,t) = t-T_d(\lambda-\sigma) \tag{3.4}$$

and we shall have:

$$\Phi^+(\lambda) = u^+(\lambda, t+T_d(\lambda-\sigma)),\ \Phi^-(\lambda) = u^-(\lambda, t-T_d(\lambda-\sigma)) \tag{3.5}$$

Since u^+ and u^- are solutions of (3.3) it follows that $d\Phi^+/d\lambda = d\Phi^-/d\lambda = 0$. From here we obtain, if σ=1, i.e. on the characteristic that crosses (1,t):

$$u^+(1,t) = u^+(0,t-T_d)$$

Also, on the characteristic that crosses (0,t) and decreases:

$$u^-(0,t) = u^-(1,t-T_d)$$

Denoting:

$$y^+(t) = u^+(0,t), \qquad y^-(t) = u^-(1,t) \tag{3.6}$$

we substitute in the boundary conditions of (3.3) to find:

$$\begin{aligned} & T_d\frac{d}{dt}(y^+(t)+y^-(t-T_d)) = -\gamma_0(y^+(t)-y^-(t-T_d)) \\ & y^-(t)+y^+(t-T_d) = \dot{X}_P \\ & T_d\ddot{X}_P = \gamma_0\delta_0(y^-(t)-y^+(t-T_d))+T_d u(t) \end{aligned} \tag{3.7}$$

This is a system of functional differential equations of neutral type, that may be given various forms.

Eliminating X_P we obtain:

$$\begin{aligned} & T_d\frac{d}{dt}(y^+(t)+y^-(t-T_d)) = -\gamma_0(y^+(t)-y^-(t-T_d)) \\ & T_d\frac{d}{dt}(y^-(t)+y^+(t-T_d)) = \gamma_0\delta_0(y^-(t)-y^+(t-T_d))+T_d u(t) \\ & \dot{X}_P = y^-(t)+y^+(t-T_d) \end{aligned} \tag{3.8}$$

B. The first two equations define a genuine (standard) system of neutral functional differential equations as described e.g. in [12], while the third one may be viewed as the system's output.

The inherent stability of the system (3.7) is given by its characteristic equation below:

$$\begin{aligned} & \det\begin{pmatrix} T_d\lambda+\gamma_0 & (T_d\lambda-\gamma_0)e^{-\lambda T_d} \\ (T_d\lambda+\gamma_0\delta_0)e^{-\lambda T_d} & T_d\lambda-\gamma_0\delta_0 \end{pmatrix} = \\ & = (T_d\lambda+\gamma_0)(T_d\lambda-\gamma_0\delta_0)-(T_d\lambda-\gamma_0)(T_d\lambda+\gamma_0\delta_0)e^{-2\lambda T_d} = 0 \end{aligned} \tag{3.9}$$

This quasi-polynomial obviously has a zero root since:

$$p(0) = -\gamma_0^2\delta_0 + \gamma_0^2\delta_0 = 0$$

If we take into account that the output $\dot{X}_P$ is a cyclic variable, we rediscover a fact known for the lumped parameter case – see (2.7) – a double zero pole of the controlled configuration with *u*(*t*) as input and X_P as measurable control output.

We may want to continue the analogies and check the existence of purely imaginary roots of (3.8).

Consider $p(j\omega)$, write down its real and imaginary parts, and equate them to 0; without any loss of roots, the possible purely imaginary root of (3.8) are of the form $\pm jx_K / T_d$ where x_K are the positive roots of:

$$tg\ x = \frac{\gamma_0 \left(1-\delta_0\right) x}{\delta_0\gamma_0 + x^2} \tag{3.10}$$

This equations is well studied - see [13]: it has real roots of the form $\nu\pi + \delta_\nu$ where $\{\delta_\nu\}_\nu$ is a positive bounded sequence approaching 0 for $\nu \to \infty$. We have thus discovered an infinity of purely imaginary roots; this infinity of oscillating modes is well known in the theory of elastic rods: mathematically its presence can be explained by the fact that the difference operator of (3.7) has its roots on the unit circle.

Another simple analysis of (3.8) will show the existence of complex and real roots with arbitrary negative real parts – what is only natural for such equations whose solutions define a strongly continuous semi-group (see [12]), but also complex and real roots located within the disk $|z| < \gamma_0 \sqrt{\delta_0} T_d$, i.e. with bounded positive real part; also according to the general theory of such equations [12], the number of such roots is at most finite.

We deduce that the model (3.7) is inherently unstable since it has many roots on jR and possibly in the right half plane C^+. While the lumped parameter model had only a pair of roots on jR, the elasticity of the rod adds an infinity of such roots plus a strong instability. This points to the feedback stabilization of the system.

C. It is interesting to discuss the same problem in the context where the assumption of [10] is adopted, and the model is modified once more: the boundary condition at σ=0 in (2.9) becomes $w(0,t) = 0$. Consequently (3.6) becomes:

$$\begin{aligned} & y^+(t) - y^-(t-T_d) = 0 \\ & y^-(t) + y^+(t-T_d) = \dot{X}_p \\ & T_d \ddot{X}_p\, \delta_0\gamma_0(y^-(t) - y^+(t-T_d)) + T_d u(t) \end{aligned} \tag{3.11}$$

We may eliminate both y^+ and $\dot{X}_P$ which become thus output variables, to obtain the dynamics described by a first order neutral equation.

$$T_d \frac{d}{dt}(y^-(t) + y^-(t-2T_d)) = \delta_0\gamma_0(y^-(t) - y^-(t-2T_d)) + T_d u(t) \tag{3.12}$$

Its characteristic equation is:

$$\Pi(\lambda) \equiv (T_d\lambda - \delta_0\gamma_0) + (T_d\lambda + \delta_0\gamma_0)e^{-2\lambda T_d} = 0 \tag{3.13}$$

which also has λ=0 as simple root. Its possible purely imaginary roots may be obtained from:

$$\delta_0\gamma_0 \sin\omega T_d - \omega T_d \cos\omega T_d = 0 \tag{3.14}$$

as $\pm jx_K / T_d$ where x_K are the positive real roots of:

$$tg\, x = x / \delta_0\gamma_0 \tag{3.15}$$

Concerning other roots, it follows that only roots with negative real parts are possible. It thus appears that stabilizing this last model is easier than stabilizing the previous one.

4. CONCLUSIONS

The variety of models that may be associated to the overhead crane confirm its importance as a benchmark problem of control theory. In all cases modeling imposes feedback stabilization that might have various degrees of difficulty. Stabilization will be discussed in a companion paper. Among possible approaches, the C.L.F. (Control Liapunov Functional) appears to be the best, since all advantages of the energy integral of hyperbolic partial differential equations may be used.

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