POSITIONING SYSTEM BASED ON ELECTROMAGNETIC LEVITATION USING FUZZY LEARNING CONTROL

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Abstract: This paper describes the design and implementation of a positioning system based on electromagnetic levitation which uses a learning Fuzzy-PD control strategy. Electromagnetic levitation exhibits nonlinear force-current-distance characteristics, and if controllers are to be designed by using linear analysis, the air gap is restricted to a small region around the chosen operating point. In this work, an attempt has been made to improve the operating range and the trajectory tracking capabilities of an electromagnetic suspension system by adding a learning/adaptive mechanism to a direct Fuzzy-PD controller, developed earlier to stabilize the system around a single operating point.

Keywords: Electromagnetic Levitation, Fuzzy Learning Control

1. INTRODUCTION

Most of the design techniques for control systems are based on good understanding of the controlled plant. However, there are cases when the controlled plant is too complex or the physical phenomenon that drive it cannot be completely understood. So, in the presence of structural or parametric uncertainties classical control strategies can no longer satisfy all the required performances. This doesn't mean that the knowledge used to design classical controllers is without importance. It can be incorporated in the design of modern controllers (fuzzy, adaptive etc.) that provide better performances.

The capacity of fuzzy systems to deal with nonlinear systems affected by uncertainties is a characteristic worth considering in the design of controllers. Their easily adaptable structure makes them a good choice for controlling nonlinear systems in adaptive control schemes. [7,8]

The study of electromagnetic suspension systems presents a great potential due to a large number of applications they were used in. The applications include: frictionless bearings, vibration damping, suspension of wind tunnel models, high speed passenger trains etc. These systems, also known as Maglev systems, can be divided into two types: (1) *repulsion type* which are naturally stable, and therefore easier to control, and (2) *attraction type* which are inherently unstable.[2,10]

In this paper an educational Maglev system is considered. The control objective is to keep a ferro-magnetic object (steel ball) suspended in midair by controlling the current through an electromagnet. The basic principle is to use the current to manipulate the electromagnetic force which can counteract the weight of the steel ball and keep it suspended in the air. By measuring the location of the object using a non-contact sensor, and adjusting the current, the levitated object can be positioned very accurately.[2,9]

This paper examines the implementation of Learning Fuzzy-PD control for an attraction type electromagnetic levitation device with one degree of freedom. This plant is characterized by a nonlinear dynamic that is open-loop unstable and, as a result, feedback fuzzy control based on conventional PD control will be used to stabilize it and learning control will be applied to reduce the stationary error and to improve the trajectory tracking capabilities of the system.

2. DYNAMICAL MODEL OF THE PLANT

The dynamic of the magnetic levitation system is described by the following nonlinear equation:

$$m\frac{d^2x(t)}{dt^2} = mg + f_e(x, i, t) \tag{1}$$

where $f_e(x,i,t)$ is the electromagnetic force that counteracts the weight of the ball, x is the distance between the electromagnet and the steel ball, m is the mass of the ball and g is the gravitational constant.

The electromagnetic force produced by current i(t), which acts on the steel ball, is found using the magnetic energy equation as follows:

$$W(i, x, t) = \frac{L(x)i^{2}(t)}{2}$$
(2)

$$f_e(x,i,t) = \frac{\partial W(i,x,t)}{\partial x} = \frac{i^2(t)dL(x)}{2dx} = -C\left(\frac{i(t)}{x(t)}\right)^2 \qquad (3)$$

C is a nonlinear electromagnetic parameter which depends on the incremental inductivity caused by the steel ball and the levitation distance (X_0) .

Figure 1 shows the principle of the electromagnetic levitation.



Fig. 1. Electromagnetic levitation

A linear model for the system was developed to help design the direct fuzzy-PD controller. Equation (1) was linearized around an equilibrium point (I_0 , X_0), where I_0 is the current of the electromagnetic coil when the ball is at X_0 [9].

Based on the linear model, the following transfer function resulted for the plant:

$$H(s) = \frac{-k_1}{ms^2 - k_2}$$
(4)

where k_1 and k_2 are two constants depending on parameters C, I_0 and X_0 .

The position of the steel ball is determined using a position sensor with:

$$K_{sensor} = 3333[V/m] \tag{5}$$

and a current amplifier is used as execution element:

$$K_{amp} = 0.1[A/V] \tag{6}$$

The plant parameters were determined experimentally based on measurements on the process and are given in Table 1.

Parameter	Value
т	0.0101 kg
g	9.81 m/s^2
X_0	0.0076 m
I_0	0.421 A
С	$3.2084 \cdot 10^{-5} \text{ Nm}^2/\text{A}^2$
k_{I}	0.4677
k_2	25.9087

Table 1: Plant parameters

3. DIRECT FUZZY CONTROL

This section discusses the design of the direct fuzzy controller developed from the more conventional phase-lead (PD + filter) controller designed for the linear plant in equation (4).

A fuzzy controller is nothing more than a nonlinear controller, having one or more inputs and outputs. The shape of the nonlinear characteristic can be modeled into various shapes by adequately choosing the different parameters in the structure of the fuzzy controller [8].

By dynamically processing the inputs / outputs, the fuzzy controller is capable of generating all the known conventional controllers, including the PID.

The design method used to obtain the direct fuzzy controller in this paper, makes use of an already implemented PID-type controller for the plant and it follows the next steps:

- tuning the PID-type controller;
- replacing it with an equivalent linear fuzzy controller;
- making the fuzzy controller nonlinear.[5]

3.1 Phase lead control

Transfer function (4) shows the system has a stable pole, while the other one is unstable. The control system needs to be designed in such a way as to stabilize the working point of the levitation system. The simplest way to stabilize the system is to use a phase-lead controller to cancel the unstable pole. This is a PD-controller + a filter of the following form:

$$H_C(s) = K_P \frac{1 + T_D s}{1 + \alpha T_D s}$$
⁽⁷⁾

where $H_C(s)$ is the controller's transfer function, K_P is the proportional gain, T_D is the differential time constant and α is a filter constant. The following values were determined for these parameters: $K_P = -1.2$, $T_D = 0.02$ and $\alpha = 0.1$.

Since fuzzy systems are soft computing algorithms and require digital implementation, the continuous controller (7) was digitized with a sampling rate T = 0.0002s. The regression model of the digitized controller is presented next:

$$u(k) = q_0 u(k-1) + q_1 e(k) + q_2 e(k-1)$$
 (8)

Parameters q_i took the following values: $q_0 = 0.9048$, $q_1 = -12$, $q_2 = 11.89$.

To indicate the PD component, equation (8) was brought to the following form:

$$u(k) = q_0 u(k-1) + \Delta u(k) \tag{9}$$

$$\Delta u(k) = K'_{P} \left(e(k) + T'_{D} \frac{e(k) - e(k-1)}{T} \right)$$
(10)

The proportional gain of the digital PD component is:

$$K_{P}' = q_1 + q_2 = -0.11 \tag{11}$$

and the differential time constant: [2]

$$T_D = -\frac{q_2 T}{q_1 + q_2} = 0.0216 \tag{12}$$

3.2 Fuzzy-PD control

It is possible to create an equivalent linear fuzzy system, by adequately modeling the system's components, which will replace the PD component (10) in the final Fuzzy-PD controller.

In order to achieve this, the following design choices are recommended:

- the use of symmetric triangular membership functions, overlapped at 50% for the input variables;
- the use of algebraic product for the *and* connective;
- the use of singleton membership functions for the output variable;
- the use of the centre of gravity (COG) defuzzification method [5].

According to (10) it comes very natural to chose the error e(k) and change-in-error c(k) = (e(k) - e(k-1)) / T as input variables, and the variation in command $\Delta u(k)$ as output variable.

Figure 2 shows the general structure of the fuzzy system which replaces the PD component of the controller in equation (10).



Fig. 2. The Fuzzy-PD system

The elements of the system were designed according to the specification mentioned earlier. For each of the input variables five membership functions were used (meaning $25 = 5^2$ rules in the rule base). These were symmetric, 50% overlapping triangular shaped membership functions. For the output variable, five singletons were used.

The gains g_e , g_c and g_u of the fuzzy system will be determined as follows:

$$\Delta u(k) = g_u \cdot f(g_e \cdot e(k), g_c \cdot c(k)) \tag{13}$$

Usually a fuzzy system makes a nonlinear mapping of the input space, according to equation (13), where f represents a nonlinear function.

Using the following linear approximation for
$$f$$
:
 $f = g_e \cdot e(k) + g_c \cdot c(k)$ (14)

equation (13) becomes:

$$\Delta u(k) = g_u \cdot \left(g_e \cdot e(k) + g_c \cdot c(k)\right) \tag{15}$$

$$\Delta u(k) = g_e \cdot g_u \left(e(k) + \frac{g_c}{g_e} c(k) \right)$$
(16)

which is very similar in form to equation (10).

By identifying the terms in equations (16) and (10), the gains of the fuzzy system can be determined based on the following relations: [5]

$$g_e \cdot g_u = K_P \tag{17}$$

$$\frac{g_c}{g_e} = T_D' \tag{18}$$

The fuzzy controller implements a rule base made of a set of IF-THEN type rules. These rules were determined heuristically based on the knowledge of the plant

Table 2: Rule base for the Fuzzy-PD system [1,3]

var. in com	change in error (c)					
(∆u)	NB	NS	Z	PS	PB	
	NB	PB	PB	PB	PS	Z
	NS	PB	PB	PS	Z	NS
error	Z	PB	PS	Z	NS	NB
(0)	PS	PS	Z	NS	NB	NB
	PB	Z	NS	NB	NB	NB

After choosing the gains and with the other components designed according to the specifications mentioned earlier, the resulting fuzzy system acts the same way as the PD component of the initial controller.

The final step in the design procedure refers to gradually transforming the linear fuzzy system determined earlier into a nonlinear one.

The final structure for the fuzzy PD component was obtained after having made the following transformations:

- the universe of discourse for the input variables was modified according to effective measurements of the process;
- the shape of the membership functions for the output variables was changed, and the universe of discourse was modified;
- Mamdani inference was adopted.

The domain of variation for the variables of the Fuzzy-PD system is given next:

$$E = g_e \cdot e = [-100, 100];$$

$$C = g_c \cdot c = [-100, 100]$$

$$U = \Delta u = [-40, 40] \cdot g_u$$
(19)



Fig. 3. Membership functions of the Fuzzy-PD system variables

The final structure of the direct Fuzzy-PD controller is given by equation (9), where $\Delta u(k)$ is the output of the nonlinear fuzzy system and is determined according to equation (13).

4. LEARNING FUZZY PD CONTROL

The success with the direct fuzzy controller, applied to the maglev plant, presented in the previous section is used in the design of a learning Fuzzy-PD controller. This approach is based on the Fuzzy Model Reference Learning Control structure which was first introduced in [6]. The learning algorithm is based on the online adaptive tuning of the centers of the output membership functions of the fuzzy-PD system in the controller presented earlier.

The control structure consists of four main parts: the process (plant, sensor, execution element), the reference model, the direct Fuzzy-PD controller, and the learning mechanism (inverse fuzzy model, rule base modifier).

Figure 4 shows the general Learning Fuzzy-PD Control structure as applied for the positioning system based on electromagnetic levitation.



Fig. 4. Learning Fuzzy-PD control structure

The learning mechanism tries to adjust the controller parameters so that the closed loop system (expressed through r(kT) and y(kT)) behaves as the reference model (expressed by r(kT) and $y_m(kT)$). This way, two loops are used to control the plant: the control loop (lower) in which the controller acts by modifying the command (u(kT)) so that the output y(kT) follows the reference r(kT) and the adaptation loop (upper) which makes the output of the plant y(kT) follow the output of the reference model $y_m(kT)$ by adjusting the fuzzy controller's parameters [4].

The *reference model* is chosen to generate the desired trajectory, y_m , for the plant output y to follow. Generally, this model can take any form. It can be a continuous or discrete system, variant or non-variant, linear or nonlinear etc. In the case of the Maglev control system, we considered the output of the reference model identical to the reference.

$$y_m(kT) = r(kT) \tag{20}$$

This allowed simplified computations, expressing at the same time the desired trajectory.

An additional fuzzy system is developed called *"fuzzy inverse model"* which adjusts the centers

of the output membership functions of the Fuzzy-PD system developed earlier, used to control the process. This fuzzy system acts like a second controller, which updates the rule base of the Fuzzy-PD controller by acting upon the output variable (its membership functions centers).

It uses the adaptation error $y_e(k)$ and the variation of the adaptation error $y_c(k)$ as inputs:

$$y_{e}(k) = y_{m}(k) - y(k)$$

$$y_{c}(k) = \frac{y_{e}(k) - y_{e}(k-1)}{T}$$
 (21)

The output of the inverse fuzzy model is an adaptation factor p(kT) which is used by the rule base modifier to adjust the centers of the output membership functions of the Fuzzy-PD system in the controller. The adaptation is stopped when p(kT) gets very small and the changes made to the rule base are no longer significant.

The *fuzzy inverse model* has a similar structure to that of the fuzzy system designed in paragraph 3 (the same rule base, membership functions, inference engine, fuzzification and defuzzification interfaces). The only notable

difference between the two fuzzy systems is the normalizing gains values g_{ye} , g_{yc} , g_p .

These values were determined based on measurements of the system controlled by the direct fuzzy controller.

The determined input domains are expressed by the following relation:

$$Y_{e} = g_{y_{e}} \cdot y_{e} = [-0.5, 0.5]$$

$$Y_{c} = g_{y_{c}} \cdot y_{c} = [-100, 100]$$
(22)

and the resulting shape for the input variables of the fuzzy inverse model is given in Figure 5.



Fig. 5. Membership functions for the input variables of the fuzzy inverse model.

The output gain g_p of the fuzzy inverse model controls the speed of the adaptation (see Section 5). Therefore, the normalized interval will be considered next:

$$P = [-1.0, 1.0] \cdot g_p \tag{23}$$



Fig. 6. Normalized domain and membership functions for the output variable of the fuzzy inverse model

For better learning control, the initial structure of the direct fuzzy-PD controller, presented in the previous Section, was modified to allow the use of a larger number of output membership functions, a separate one for each input combination. This leads to $5^2=25$ different output membership functions which will be considered over-lapped at first, thus if the system works with no adaptation the same structure as in Section 3.2 can be assumed.

Table 3 presents the linguistic values corresponding to the consequences of the fuzzy rules expressed using the centers ($\mu^{\Delta u} = 1$) of the output membership functions of the Fuzzy-PD system.

 Table 3: Rule table for the fuzzy-PD system

var. in command		change in error (c)					
(Au)		NB(1)	$NS_{(2)}$	Z ₍₃₎	$PS_{(4)}$	PB(5)	
error (e)	$NB_{(1)}$	+26.67	+26.67	+26.67	+13.33	0.00	
	$NS_{(2)}$	+26.67	+26.67	+13.33	0.00	-13.33	
	Z(3)	+26.67	+13.33	0.00	-13.33	-26.67	
	$PS_{(4)}$	+13.33	0.00	-13.33	-26.67	-26.67	
	$PB_{(5)}$	0.00	-13.33	-26.67	-26.67	-26.67	

At first these centers are overlapped (there are only 5 different values for the centers, see Table 3), but once the learning process starts their values will change (see Table 4). Using a larger number of membership functions for the output means that, a larger memory is available to store information.

The rule base modifier adjusts the centers of the output membership functions in two stages:

1. The active set of rules for the fuzzy controller at time (k-1)T is first determined:

$$\mu_i^e(e(k-1)) > 0, i = \overline{1,5}$$

$$\mu_i^c(c(k-1)) > 0, j = \overline{1,5}$$
(24)

The pair (i, j) will determine the activated rule (see Table 3).

2. The centers of the output membership functions, which were found in the active set of rules determined earlier, are adjusted. The centers of these membership functions (b_l) at time kT will have the following value:

$$b_l(k) = b_l(k-1) + p(k)$$
(25)

By l we denoted the consequences (element ij in Table 3) of the rules introduced by the pair (i, j).

The centers of the output membership functions, which are not found in the active set of rules (i, j), will not be updated [6, 7].

Due to practical consideration, in order to reduce the oscillations determined by fast changes of the rule base, the centers of the output membership functions were modified at predetermined time intervals: kT+nT. For the duration of the n=50 samples the rule base of the fuzzy-PD system remains constant. Consequently, equation (25) changes to:

$$b_l(k+n) = b_l(k-1) + p(k), \quad n = 0, 50$$
 (26)

We can easily notice that only local changes are made to the controller's rule base, since the inverse model updates only the output centers of the rules which apply at that time instant and does not change the outcome of the other rules. The larger number of output membership functions means a better capacity to map different working conditions. That is, the controller will remember the adjustments it made in the past for a wider range of specific conditions. This represents an advantage for this method since time consuming re-learning is avoided. At the same time this is one of the characteristics that differences learning control from the more conventional adaptive control.

5. IMPLEMENTATION AND EXPERIMENTAL RESULTS

This section discusses the implementation of the magnetic levitation system and presents some of the experimental results for this control problem. Figure 7 shows the structure for the implemented electromagnetic levitation Learning Fuzzy-PD control system.



Fig. 7. The implementation of the positioning system based on magnetic levitation controlled by a Learning Fuzzy-PD controller

The electromagnetic levitation control system consists of several important parts:

- the electromagnet coil having inductance L = 0.3699H and resistance $R = 14.01\Omega$;
- the position sensor: an infra red LED and a phototransistor;
- the current amplifier;
- a 4th order Butterworth antialias low-pass filter;
- assorted operational amplifiers, resistors, capacitors, potentiometers for sensor calibration, summing, inverting, gain adjustment etc.;
- power supplies
- data acquisition board: 16-bit A/D conversion with up to 20 kHz sampling rate, D/A conversion, digital input/output, analog input/output and counter/timer;
- a software-based control algorithm implemented on a Personal Computer (PC)

Since an accurate positioning system was desired, great attention was given to the quality

of the signal coming from the process. Data acquisition required filtering of the input signals for several reasons like: noise reduction, antialias effect removal, assuring the bandwidth of the sampled signal in the desired domain.

For this reason, an analog 4th order Butterworth low-pass filter was designed, having a cutoff frequency ($f_c=1.6kHz$) chosen according to: $f_s \in [3f_c, 5f_c]$ (27)

where $f_s = 1/T$ represents the sampling frequency of the signal.

An additional digital FIR filter was designed and implemented in the control software to complete the signal conditioning path from the process to the learning algorithm.

Several other components of the learning algorithm were implemented in the control software like the initial digital phase-lead controller (see Section 3.1), the direct fuzzy-

PD controller (see Section 3.2), the learning mechanism and a reference generator.

The configuration of the experimental device allows switching from one control structure to another, making it easy, this way, to perform a comparative study of the system's response for three different situations: discrete phase-lead control, direct fuzzy-PD control and learning fuzzy-PD control.

Next some experimental results are presented.

In figure 8 a reference corresponding to the initial linearization point (I_0, X_0) is considered. This means the steel ball levitates at a distance equal to X_0 from the electromagnet.

Two different values for the output gain g_p of the fuzzy inverse model were considered to show the way this parameter influences the adaptation speed. At first the system is controlled only by the direct fuzzy-PD controller and then once the adaptation is started, the learning mechanism makes changes in the structure of the fuzzy-PD system reducing the stationary error.



A comparative response between three control strategies is shown in figure 9. For the three cases, the set-point of the closed loop system corresponded to the initial linearization point (I_0, X_0) .Discrete phase-lead control, direct Fuzzy-PD control and Learning Fuzzy-PD control were considered.

The changes made to the initial fuzzy-PD linear controller (which behaved identical to the phase lead system) show just a little

improvement in the response. However, the nonlinear transformation did not reduce sufficiently enough the stationary error, specific to PD control strategies. A dynamical change in the structure of the fuzzy-PD controller, introduced by the learning system, shows the reduction of the stationary error and a more accurate tracking of the reference.



The tracking capabilities of the system are shown in figures 10 and 11, where a square and a sine trajectory are considered for exemplification. The figures show the system's ability to position very accurately the levitated steel ball along the required trajectories, with very fast convergence.



Fig. 11. The tracking of a sine trajectory $(g_p = 1/300)$

lime[s]

Since the learning algorithm doesn't modify in any way the input of the fuzzy-PD system, by considering the same input domain, we show in the following figures the way the control surface of the fuzzy-PD system is modified after a learning process. The initial control surface of the system is shown in figure 12 and the variation of the fuzzy-PD control surface is presented in figure 13. We mention that the resulting control surface is not unique and it depends on working conditions, desired trajectory, reference model etc.



Fig. 12. Control surface of the initial Fuzzy-PD system



Fig. 13. Variation of the Fuzzy-PD control surface

As figure 12 shows, only local changes are made to the controller, since the difference between the initial and final control surfaces is zero for a great part. An updated rule table obtained by learning also confirms these observations.

able	4:	Modiff	ed rule	lable	aner	learning	

var. in command		Change in error (c)					
(44)		NB(1)	$NS_{(2)}$	Z ₍₃₎	PS ₍₄₎	PB(5)	
	$NB_{(1)}$	+26.67	+26.67	+26.67	+13.33	0.00	
Error	$NS_{(2)}$	+26.67	+27.07	+13.61	-0.15	-13.33	
	Z ₍₃₎	+26.67	+13.85	+0.02	-13.84	-26.67	
(6)	$PS_{(4)}$	+13.33	+0.12	-13.60	-27.02	-26.67	
	PB(5)	0.00	-13.33	-26.67	-26.67	-26.67	
Initial rules Modified rules							

6. CONCLUSION

Conventional PD controllers have poor performances when dealing with nonlinear

plants outside the domain for which they have been designed. However, they can provide a framework for developing fuzzy systems and help solve some of the problems that designers of fuzzy systems are faced with, like: setting the controller gains or choosing other fuzzy parameters.

A Fuzzy-PD control strategy that evolved from heuristic knowledge and classical control methods was presented in this paper.

Since both classical and Fuzzy-PD control laws couldn't make the tracking error sufficiently small enough and the desired performances were not satisfied, a learning algorithm was incorporated into the Fuzzy-PD control system.

The new approach improved the tracking accuracy for the closed loop nonlinear electromagnetic levitation process. It achieved this with very fast convergence speed, it extended the working conditions outside the initial linear domain, it presented robustness against model uncertainties and allowed very precise positioning of the levitated object.

The electromagnetic levitation system used here can serve for academic instruction in the field of Control Engineering, due to the many problems it deals with like: system modeling and simulation, linearization, controller design, filter design, data acquisition etc., as well as a test bed for the design of various control algorithms.

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