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Delta Domain Predictive Control For Fast, Unstable And Non-Minimum Phase Systems

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Abstract: The design of the advanced control strategies is usually based on a discrete time model of the plant. The usual approach has been to use the shift operator q, but this gives numerical difficulties with the small sampling periods. Many algorithms are better-conditioned numerically using δ operator implementation than that ones based on shift operator implementation. This study concerns the predictive control strategy in a state space form because it is attractive during implementation and is particularly easy to extend to the multivariable case. In view of design parameters, the suggested method is very simple, and opposing to the conventional discrete time GPC, the algorithm proposed exhibits at high sampling rates lower computational complexity and better numerical conditioning for unstable and non-minimum phase systems.

Keywords: fast systems, unstable systems, δ operator, predictive control.

1. INTRODUCTION

The predictive control is a control methodology widely used in the process industry where system dynamics are sufficiently slow to permit its implementation. In opposition, the applications of predictive control for fast unstable dynamic systems are rather limited. The main reason is referred to the computational consideration and to the accuracy of predictors, which can be quite poor due to the accumulation of numerical errors.

The interest for the δ operator in the academic community has been renewed in the last years and therefore has been widely investigated in problems in the areas of signal processing [1-2], systems modeling [3-7] and control [8-15].

The delta operator has the ability to improve on the numerical ill-conditioning problems found when using shift operator q [9]. An advantage of modeling linear systems in δ domain consists in the fact that it provides an exact discrete-time representation of the system [7]; the identified model has structural similarity to the continuous-time differential equation describing system dynamics. It has been concluded that the utilization of the δ operator has the advantage to overcome the problem of numerical similarity especially in conditions of fast sampling. It has been demonstrated that the model description in δ domain directly results in the improvement of the numerical properties of structure detection. Anywhere where digital control is to be applied, it is desirable to develop a discrete-time model for analysis purposes.

Generalized predictive control [16-18] is the most popular controller among of all predictive control formulations. At high sampling rates, the conventional GPC suffers from the large number of samples that must be taken into account at each sampling instant. The complexity involved in developing such a controller leads to the necessity that Generalized Predictive Control algorithm should be formulated in the δ-domain. As the δ-operator is simply a linear transformation of the shift operator, it is possible to find an exact δ-domain transformation of the q domain GPC.

A wide range of GPC algorithms was reported in literature, but during the last few years, some research paid attention to δ-domain GPC to emphasize the close connection between discrete time and continuous time theory. The suggestion of connecting the GPC with the advantages offered by a δ parameterization has been discussed in [16-18] using an emulator in a state-space approach. The δ-domain emulator based GPC has been further investigated in connection to discrete-time GPC in [15] using a diophantine formulation. It is emphasized throughout the emulator research that as the sample time tends to zero, the approach of mapping between δ and q-domain taken in [14] is liable to numerical problems. The significant relationship in fast sampling is the ratio between the dominant time constant of the system and the sample time. To highlight the value of the algorithm using delta operator, the control of a non-minimum phase is considered. Although this class of systems is difficult to control, some aspects must take into account to be able to provide adequate control.

This study is based on [14] and brings new framework for study predictive control scheme for complex systems in delta domain.

The paper investigates the GPC algorithm derived from the delta state-space model. This framework has the advantage of working within a discrete time δ domain, without the requirement of mapping the system model to the q domain. The investigation emphasizes the outstanding capacity of the algorithm to control a fast system with a very small sample period, knowing that the predictive control techniques need high value for sample time.

The study is organized as follows: the second part of the paper develops the proposed predictive control scheme based on the delta operator. In the Section 3 it is examined the performances of algorithm considering a simplified dynamic model for the Single Machine Infinite Bus subsystem. The simulation studies are performed for different sampling periods and different values for tuning parameters. Finally, the Section 5 concludes the paper.

2. PREDICTIVE CONTROL ALGORITHM FOR FAST SAMPLED SYSTEMS IN Δ DOMAIN

The discrete δ operator is defined for the different values of the sampling period as [9]:

$$\delta = \begin{cases} \frac{d}{dt}(.), \text{ for T=0} \\ \frac{q-1}{T}, \text{ for T} \neq 0 \end{cases} \tag{1}$$

where T is the sampling period and q is the usual forward-shift operator. It is to be noticed that for $T = 0$, the continuous derivative of a function is obtained:

$$\lim_{T\to 0} \delta x(t) = \frac{d}{dt}x(t). \tag{2}$$

The δ operator is known in the numerical analysis and it is quoted as the Euler's approximation of derivative.

Although there is a linear transformation between the two discrete domains, the two operators have distinct conceptual roles:

$$\begin{aligned} q^j &= (1+\delta T_s)^j = \sum_{n=0}^{j} C_j^n (T_s\delta)^j \\ C_j^n &= \frac{j!}{n!(j-n)!}. \end{aligned} \tag{3}$$

The generalized predictive control algorithm proposed in this paper is based on the state-space approach. The δ discrete state space model can be written:

$$\begin{aligned} \delta x_k &= \mathbf{A}_\delta x_k + \mathbf{B}_\delta u_k \\ y_k &= \mathbf{C}_\delta x_k + \mathbf{D}_\delta u_k \end{aligned} \tag{4}$$

where $x_k \in \mathbb{R}^n$ is the state vector at moment k, $u_k \in \mathbb{R}^m$ is the input vector, $y_k \in \mathbb{R}^p$ is the output vector, $A_\delta \in \mathbb{R}^{nxn}$ is the state transition matrix, $B_\delta \in \mathbb{R}^{nxm}$ is the input matrix and $C_\delta \in \mathbb{R}^{pxn}$ is the measurement matrix. In [9], Middleton and Goodwin propose an artless procedure in order to obtain the δ discrete model based on the continuous model. Therefore the relation between the matrices of the system is:

$$\begin{aligned} A_\delta &= \frac{e^{AT_s} - I}{T_s} = \Omega A_c \\ B_\delta &= \Omega B_c \\ C_\delta &= C_c \\ D_\delta &= D_c \end{aligned} \tag{5}$$

with

$$\Omega = \frac{1}{T_s}\int_0^{T_s} e^{A_c\tau}d\tau = \frac{1}{T_s}(e^{A_cT_s} - I)A_c^{-1} = I + \frac{A_cT_s}{2!} + \frac{A_c^2T_s^2}{3!} + ...$$

The δ derivative states of the system of j order are obtained from [12]:

$$\delta^j x_k = \mathbf{A}_\delta{}^j x_k + \sum_{i=0}^{j-1} \mathbf{A}_\delta{}^{j-i-1}\mathbf{B}_\delta \delta^i u_k, \tag{6}$$

with $j = \overline{0, N_y}$, N_y being the prediction horizon.

The standard state-space predictions can be utilized to predict higher order δ- outputs:

$$\delta^j y_k = C_\delta \delta^j x_k = \mathbf{C_\delta A_\delta}{}^j x_k + \sum_{i=0}^{\min\{j,Ny\}-1} \mathbf{C_\delta} A_\delta{}^{j-i-1}\mathbf{B_\delta}\delta^i u_k. \tag{7}$$

The expression of predictors of order $0...N_y$, in the matrix form is:

$$\begin{aligned} \delta^j y_k &= \boldsymbol{\beta}\mathbf{x_k} + \mathbf{G}\mathbf{u_\delta} = \mathbf{f} + \mathbf{G}\mathbf{u_\delta}, \\ &\Leftrightarrow \hat{\mathbf{y}}_\delta = \mathbf{f} + \mathbf{G}\mathbf{u_\delta} \end{aligned} \tag{8}$$

with the extended Toeplitz matrix G which contains the Markov parameters in the δ domain [13]:

$$\mathbf{G} = \begin{bmatrix} \mathbf{0} & .... & ... & ...... & 0 \\ \mathbf{C_\delta B_\delta} & .... & ... & ...... & 0 \\ \mathbf{C_\delta A_\delta B_\delta} & \mathbf{C_\delta B_\delta} & ... & ... \; ...... & 0 \\ \vdots & & & & \\ \mathbf{C_\delta A_\delta{}^{N_y-1} B_\delta} & \mathbf{C_\delta A_\delta{}^{N_y-2} B_\delta} & \mathbf{...} & \mathbf{C_\delta B_\delta} & \end{bmatrix}, \tag{9}$$

and $\boldsymbol{f}$ is the free response:

$$\mathbf{f} = \begin{bmatrix} \mathbf{C_\delta A_\delta{}^0} \\ \mathbf{C_\delta A_\delta{}^1} \\ \mathbf{.} \\ \mathbf{C_\delta A_\delta{}^{Ny}} \end{bmatrix} \mathbf{x_k} = \boldsymbol{\beta}\mathbf{x_k} \quad . \tag{10}$$

The expression (8) predicts the change in output $\delta^j y_k$ for $j = \overline{0, N_y}$.

The control sequence $\mathbf{u}_\delta$, as well as the vector containing the predicted outputs $\hat{\mathbf{y}}_\delta$ are defined as:

$$\begin{aligned}
\mathbf{u}_\delta &= \left[u_k \quad \delta u_k \quad \delta^2 u_k ......\delta^{N_y-1} u_k\right]^T \\
\hat{\mathbf{y}}_\delta &= \left[y_k \quad \delta y_k \quad \delta^2 y_k ......\delta^{N_y} y_k\right]^T
\end{aligned} \tag{11}$$

The optimal control sequence in the δ domain is derived from the minimization of a cost function. Due to a very small sampling period, the controller design can be performed in different strategies. Seeing that the delta model is very close to the continuous time model, one advantageous approach is to design the controller in continuous time and to implement it in the delta domain. Another possible approach is to design the controller based on the δ model, which must be obtained in advance. In this paper, the second approach is suggested. Therefore, a mapping must be defined from future changes in output to the future output vector, *i.e.* $\delta^j y_k \to y_{k+j}$ for $j = \overline{0, N_y}$. The control law is described in terms of δ operator by mapping the terms associated to classical domain, using the binomial transformation [4]:

$$\begin{aligned}
q^j &= (1+\delta T_s)^j = \sum_{n=0}^{j} C_j^n (T_s\delta)^j \frac{n!}{r!(n-r)!} \\
C_j^n &= \frac{j!}{n!(j-n)!} \xrightarrow{notation} \binom{n}{j}.
\end{aligned} \tag{12}$$

The link between the future outputs in the two discrete forms is described in the matrix form:

$$\begin{bmatrix} q^0 y_k \\ q^1 y_k \\ \vdots \\ q^j y_k \\ \vdots \\ q^{N_y} y_k \end{bmatrix} = \begin{bmatrix} T_s^0 & 0 & 0 & \dots & 0 \\ T_s^0 & 1\mathrm{T}_s^1 & 0 & \dots & 0 \\ T_s^0 & 2\mathrm{T}_s^1 & \mathrm{T}_s^2 & \dots & \vdots \\ \vdots & \vdots & \vdots & \dots & \\ T_s^0 & \frac{j!}{(j-1)!} T_s^1 & \dots & \dots & \frac{j!}{(j-i)!i!} T_s^i \\ \binom{N_y-1}{0} T_s^0 & \binom{N_y-1}{1} T_s^1 & \binom{N_y-1}{2} T_s^2 & \dots & \binom{N_y-1}{N_y-1} T_s^{N_y-1} \end{bmatrix} \begin{bmatrix} \delta^0 y_k \\ \delta^1 y_k \\ \vdots \\ \delta^j y_k \\ \vdots \\ \delta^{N_y} y_k \end{bmatrix}$$

$$\hat{\mathbf{y}}_q = \mathbf{K}_y \cdot \hat{\mathbf{y}}_\delta \tag{13}$$

The corresponding expression for control description is:

$$q^j u_k = \sum_{i=0}^{j} C_j^i T_s{}^i \delta^i u_{\delta,} \quad j = \overline{0, N_y - 1} \tag{14}$$

or in the matrix form:

$$\mathbf{u}_q = \mathbf{K}_u \cdot \mathbf{u}_\delta . \tag{15}$$

The GPC strategy in the δ domain follows the idea from the classical q-domain [13-15]. In the computational context, it should be noted that the matrices $\mathbf{K}_y$, $\mathbf{K}_u$ could be computed off-line, because the elements are dependent only on the output horizon N_y and the sample time, which are known a priori.

The characteristic design control parameters for SS δ GPC algorithm are set in the same way as in the case of the classical algorithm. In the discrete time implementation of the GPC algorithm, the control horizon N_u is a key parameter whose signification is reflected in the condition:

$$u_{k+j} = 0, \quad \mathrm{j}=\overline{N_u + 1 : N_y - 1} \;\; . \tag{16}$$

Expression (16) is the simplest case applied where the system incorporates an integrator.

The sequence of future control increments is preferred instead of absolute values of the control action in order to avoid the steady state error for systems without integrator:

$$\Delta u_{k+i} = 0, \; i = \overline{N_u + 1, N_y - 1} \tag{17}$$

It is well known that only for the weighting factor $\lambda = 0$ or an integrator incorporated in the system is able to resolve the control problem. The GPC algorithm performed in the continuous time domain and the derived emulator GPC formulation suppose the following conditions, which intentionally do not have a correspondence to the discrete time GPC representation:

$$\delta^j u_k = 0, \qquad \mathrm{j}=\overline{N_u \;\; + 1 : N_y - 1} \tag{18}$$

The control increments are included in the cost function, using the Q matrix:

$$\begin{bmatrix} \Delta u_{k+j} \\ \Delta u_{k+j+1} \\ \vdots \\ \Delta u_{k+Nu-1} \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 1 & 0\dots & \\ 0 & -1 & 1 & 0\dots \\ \vdots & & & \\ 0\dots & & -1 & 1 \end{bmatrix}}_{Q} \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+N_u} \end{bmatrix} \tag{19}$$

Usually, the restrictions, which act on the control vector, are not encouraged in the case of very small sampling period [14].

In delta domain, the control vector $\mathbf{u}_\delta$ can be partitioned into:

$$\begin{aligned}
\mathbf{u}_\delta &= \begin{bmatrix} \mathbf{u}_\delta^1 \\ \mathbf{u}_\delta^2 \end{bmatrix} \\
\mathbf{u}_\delta^1 &= [\delta^0 u \quad \delta^1 u \quad ...\delta^{N_u} u]^T \\
\mathbf{u}_\delta^2 &= [\delta^{N_u+1} u \;\; \delta^{N_u+2} u \;\; ...\delta^{N_y-1} u]^T .
\end{aligned} \tag{20}$$

Once the control horizon is reached as is pointed out in (17), further adjustment of the control action is not possible and so the action must remain constant up to the output horizon. Therefore, the control action in the δ domain beyond the control horizon must be defined so that there is no change in the absolute control action [14]:

$$u_{k+i} = u_{k+N_u}, \; i = \overline{N_u + 1, N_y - 1} \tag{21}$$

which leads to:

$$\begin{bmatrix} u_{k+N_u+1} \\ u_{k+N_u+2} \\ \vdots \\ u_{k+N_y-1} \end{bmatrix} = \begin{bmatrix} u_{k+N_u} \\ u_{k+N_u} \\ \vdots \\ u_{k+N_u} \end{bmatrix} = \vec{\mathbf{\Gamma}} \cdot \mathbf{u}_\delta^1 \tag{22}$$

with:

$$\vec{\Gamma} = \left[\begin{pmatrix} N_u \\ 0 \end{pmatrix} T_s^0 \quad \begin{pmatrix} N_u \\ 1 \end{pmatrix} T_s^1 \quad \cdots \quad \begin{pmatrix} N_u \\ N_u \end{pmatrix} T_s^{N_u} \right] \tag{23}$$

From the relations (19) and (22), the following expression can be written:

$$\mathbf{\Gamma}_u \mathbf{u}_\delta^1 + \mathbf{\Gamma}_y \mathbf{u}_\delta^2 = \vec{\mathbf{\Gamma}} \mathbf{u}_\delta^1 \tag{24}$$

where $\mathbf{\Gamma}_u$ and $\mathbf{\Gamma}_y$ are submatrices from the matrix $\mathbf{K}_u$ with the number of columns equals the dimension of the vectors $\mathbf{u}_\delta^1$ and $\mathbf{u}_\delta^2$ respectively.

From (24), the corresponding vector $\mathbf{u}_\delta^2$ is defined as:

$$\mathbf{u}_\delta^2 = \mathbf{\Gamma}_y^{-1} (\vec{\mathbf{\Gamma}} - \mathbf{\Gamma}_u) \mathbf{u}_\delta^1 \tag{25}$$

The cost function is expressed as:

$$J = \sum_{i=N_1}^{N_y} \left[\hat{y}_{k+i} - r_{k+i} \right]^2 + \sum_{i=1}^{N_u} \left[\Delta u_{k+i-1} \right]^2 . \tag{26}$$

The cost function can be re-written in terms of δ domain:

$$J_\delta = \mathbf{K}_y \sum_{j=N_1}^{Ny} (\delta^j \hat{y}_k - \delta^j w_k)^2 + \lambda \mathbf{Q} \mathbf{K}_u \sum_{j=1}^{Nu} (\delta^j u_k)^2 . \tag{27}$$

The reference vector is given in the form:

$$\mathbf{w}_\delta = \left[\delta^0 w \quad \delta^1 w \quad \delta^2 \; w ...... \; \delta^{N_y} w \right]^T . \tag{28}$$

Expanding the expression (7) and taking into account (25) the predictor is obtained as:

$$\begin{aligned} \hat{\mathbf{y}}_{\boldsymbol{\delta}} &= \mathbf{f} + (\mathbf{G}_u \mathbf{u}_\delta^1 + \mathbf{G}_y \mathbf{u}_\delta^2) \\ &= f + \left[\mathbf{G}_u + \mathbf{G}_y \mathbf{\Gamma}_y^{-1} (\vec{\mathbf{\Gamma}} - \mathbf{\Gamma}_u) \right] \mathbf{u}_\delta^1 \\ &= f + \overset{\approx}{\mathbf{G}} \mathbf{u}_\delta^1 \end{aligned} \tag{29}$$

with the matrices $\mathbf{G}_u \in \mathbb{R}^{n(N_y+1) \times m(N_u+1)}$ and $\mathbf{G}_y \in \mathbb{R}^{n(N_y+1) \times m(N_y - N_u - 1)}$ partitioned from G matrix.

Now (27) can be expressed as:

$$J_\delta = u_\delta^{1T} \mathbf{H} u_\delta^1 + u_\delta^{1T} \mathbf{L} + \mathbf{L}^T u_\delta^1 + \mathbf{f_0} \tag{30}$$

where

$$\begin{aligned} \mathbf{H} &= \overset{\approx}{G}^T \mathbf{K}_y{}^T \mathbf{K}_y \overset{\approx}{G} + \lambda \mathbf{K}_u{}^T \mathbf{Q}^T \mathbf{Q} \mathbf{K}_u \\ \mathbf{L} &= \overset{\approx}{G}^T \mathbf{K}_y{}^T \mathbf{K}_y (\mathbf{f} - \mathbf{w}) \\ f_0 &= (\mathbf{f} - \mathbf{w})^T \mathbf{K}_y{}^T \mathbf{K}_y (f - w) \end{aligned} \tag{31}$$

and $\mathbf{H}$ being a symmetric matrix and f_0 incorporates the free response.

Differentiating J_δ with respect to $\mathbf{u}_\delta^1$ and setting it to zero gives the optimal solution:

$$u_\delta^1 = -(\mathbf{H}^T)^{-1} \mathbf{L} \tag{32}$$

The control signal applied is the first control move:

$$u_\delta^* = e^T u_\delta^1 \tag{33}$$

where $e = [1, 0, ... \quad 0]^T$.

3. CASE STUDY

This section illustrates the performances of the SS-δ-GPC algorithm via simulations applied on a Single Machine to Infinite Bus system (SMIB) [20]. The characteristics of the SS-δ-GPC are investigated in terms of the ability to function effectively at fast and slow sample rates, the performance when varying the control and output horizon.

In most practical situations, the disturbances as load changes, are sufficiently small such that the system can be linearized around the operating point. Small disturbances in practical power systems usually result in oscillations of rotor angles of some generators to others. Instability arises if oscillation magnitude constantly increases in time up to the separation of the system. In large actual power systems, this kind of unstable separation is eliminated due to the large amount of controllers. Another kind of instability refers to voltage instability, which is a consequence of loss of equilibrium between load demand and load supply. Steady state operation of the synchronous generator is guaranteed by the action provided by several control devices. The main feedback controllers are the turbine governor, which determines the mechanical torque input to the machine, the Automatic Voltage Regulator (AVR) whose aim is to control the output voltage and the Power System Stabilizer (PSS) which provides a damping action to rotor oscillations.

This study demonstrates that the δ-GPC- AVR can retain the system stability and keep stable the system operation.

The Lyapunov theory has been for a long time an important tool in linear as well as nonlinear control. Its use has been limited by the difficulties to find a Lyapunov function for a given system. If one can be found, the system is known to be stable, but the task of finding such a function has often left to the experience of the designer [21].

Moreover, the application of digital techniques for controlling this type of fast system has expanded in recent years with the introduction of low cost digital controller hardware. Therefore, the need for other strategies must be considered

The advanced control strategy proposed in this paper requires a δ discrete time model. Although the common way of describing discrete-time model is to use the forward shift operator, this has some disadvantages at fast sampling, loss of information may occur because of finite word length and arithmetical operations.

In order to demonstrate a better transient response in the case of instability region, the continuous-time domain matrices of the SMIB are considered [19-20]:

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 \\ 6.00\cdot10^5 & -6.25\cdot10^{-1} & 5.04\cdot10^6 \\ 2.98\cdot10^1 & 0 & -2.80\cdot10^{-1} \end{bmatrix}$$
$$\mathbf{B}_c = \begin{bmatrix} 0 \\ 0 \\ 1.44\cdot10^{-1} \end{bmatrix} \quad (34)$$
$$\mathbf{C}_c = \begin{bmatrix} -1.13\cdot10^3 & 0 & 2.79 \end{bmatrix}$$

The excitation control input is referred as input signal and the output, the alternator output voltage [20].

The associated transfer function is:

$$G(s) = \frac{0.40s^2 + 0.25s - 8.27\cdot10^8}{s^3 + 0.90s^2 - 6\cdot10^5 s - 1.50\cdot10^8} \quad (35)$$

The relative degree of the system is one and the poles and zeros are:

$$p_1 = 8.78\cdot10^2, p_2 = -5.86\cdot10^2, \; p_3 = -2.93\cdot10^2$$
$$z_1 = -4.51\cdot10^4, \; z_2 = 4.51\cdot10^4 \quad (36)$$

The selection of the sampling period is normally based on Shannon's reconstruction theorem. It has been proved that sampling at a rate less than ten times the bandwidth involves a loss of information regarding inter-sample behavior. Accordingly, sampling rates up to 50 times the closed loop bandwidth are sometimes chosen in fast, high precision digital control system as is illustrated in the power system application [11].

The system bandwidth is found from the 3 db points to be $\omega_0 = 590 rad/\sec$, suggesting a sampling rate in the range of $939.01 \le f \le 4695.07$. Therefore a suggestion for the choice of sampling period is given by the range: $2.12\cdot10^{-4} \le T \le 1.06\cdot10^{-3}$.

The most convenient feature of the δ operator is that the poles and zeros approach those of the continuous time representation as T tends to zero.

Control design is performed in the δ domain, where the continuous time model was mapped accordingly to the δ domain for the sample time $T = 4\cdot10^{-4}$.

$$G(\delta) = \frac{-21.75\delta^2 - 3.33\cdot10^5\delta - 8.33\cdot10^8}{\delta^3 - 253.4\delta^2 - 6.96\cdot10^5\delta - 1.51\cdot10^8} \quad (37)$$

In the case of δ operator model, the poles and zeros of the discrete transfer function are:

$$p_1 = 1.05\cdot10^3, p_2 = -0.52\cdot10^3, \; p_3 = -0.27\cdot10^3$$
$$z_1 = -1.21\cdot10^4, \; z_2 = -0.31\cdot10^4 \quad (38)$$

Fig. 1 shows that the system is controllable by the SS-δ-GPC algorithm for varying values of prediction horizon.

It is apparent that the output horizon has the dominant effect on the system response. This demonstrates that control action becomes less active as the output horizon increases for fixed control weighting.

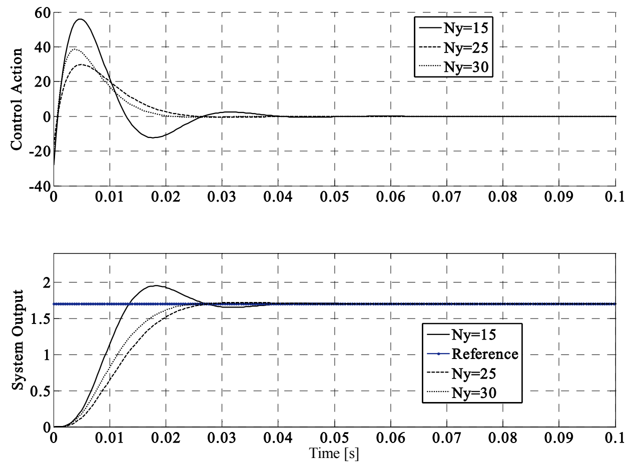

Fig. 1. Influence of varying N_y =15,25,30 for fixed N_u =2 and λ=0.004

In the Fig. 2, the simulation results demonstrate the ability of the algorithms to perform good performances even for very small sample periods. An advantage of working in the δ domain is that numerical properties are typically improved at fast sample rates.

The sample time of the discretized model was set to $T = 0.0004s, 0.0006s$ and 0.001s. For each sample time the control action and the system output were set $N_y = 20, N_u = 2$.

4. CONCLUSION

The objective of this paper was to formulate a state space generalized predictive control for fast, unstable, and non-minimum phase systems by mapping the discrete GPC into SS-δ-GPC. A δ discrete time representation of a Single Machine Infinite Bus system has been used in order to be of

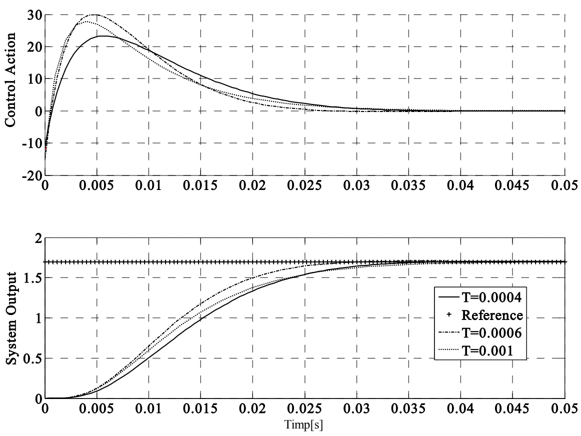

Fig. 2. Performance for varying values of sample times for fixed N_y =20, N_u =2 and λ=0.004.

interest in control design. The study proposes a new control strategy. The generalized predictive control scheme has been applied to the delta domain. It has been chosen the state space form because it is attractive during implementation and is particularly easy to extend to the multivariable case. In view of design parameters, the suggested method is very simple, and opposing to the conventional discrete time GPC, the algorithm proposed, exhibit at high sampling rates lower computational complexity and better numerical conditioning. The limitation of algorithm arises when is used a very large prediction horizon combined with fast sampling. However, the prediction horizon must be chosen according to the system's dynamic. The results obtained in the simulations illustrate the efficiency of the algorithm. Moreover, the strategy can cope with those processes, wherefore the classical control schemes fail.

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