Solution for the model adequacy verification in indirect adaptive control

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Abstract: The aim of this paper, is to propose a solution for the model adequacy verification in indirect adaptive control beside the classical known validation tests. The dynamic system that was selected to verify the proposed solution, is represented by a flexible transmission described by a complex four order model. Using the selected model adequacy index, it is studied the influence of the SPAB signal and perturbation amplitude on the model adequacy and control solution, for two different tracking performances selections. The proposed solution, comes to overcome the model validation problems encountered in real-time applications, with the main advantage of improving the model identification and control solutions.

Keywords: model adequacy, indirect adaptive control, closed-loop identification, pole-placement control, tracking performances.

1. INTRODUCTION

The ***indirect adaptive control*** approach, was introduced by Kalman in 1958 in relation to the digital process control (Kalman, 1958; Landau et al., 2011). For the indirect adaptive control approach, the studies were concentrated mainly on: the introduction of unstable zeros for plant models represented in discrete form; the singularities that may appear in the closed-loop system and the presence of the persistence of excitation. In the following sections, a solution will be proposed for the indirect adaptive control, to verify the model adequacy of the closed-loop estimated model. As a plant model, a flexible transmission system was selected. This has a complex **4th** order model identified in open-loop (Landau et al., 2011). In the scientific literature, (Karimi, 1998; Jones, 1995; Walker, 1995; Landau et al., 1995a; M'Saad, 1994; Galdos, 2012; Landau et al., 1995b) a series of studies were realized with the flexible transmission system using different control approaches, as follows: multi-model adaptive control, H_∞ control, convex optimization and others.

Section 2 of the paper, makes a general presentation of the indirect adaptive control approach, with a general control scheme and variants of adaptive control strategies developed for practical needs.

Section 3 makes a theoretical presentation of the proposed model adequacy index, that is used beside the classic validation tests for selecting the most appropriate model of a dynamic system identified in closed-loop.

Section 4 describes the flexible transmission system, for which is tested the proposed solution and the robustness performances of the selected controller with different variants of the performance parameters.

Section 5 puts in discussion the influence of the SPAB signal and perturbation amplitude on the model adequacy and control performances, for the proposed solution from section 4.

2. INDIRECT ADAPTIVE CONTROL

The indirect adaptive control strategy, was introduced in theory and practice in order to select in real-time applications an appropriate control solution for a dynamic system, when the plant model is estimated from the available input-output measurements. The adaptation is called ***indirect***, due to the fact that the controller parameters are adapted in two stages: a **1st** stage, represented by the real-time estimation of the plant parameters and a **2nd** stage, represented by the computation of the controller parameters using the current estimated model. A basic scheme for the indirect adaptive control approach, is shown in Fig. 1. (Landau et al., 2011).

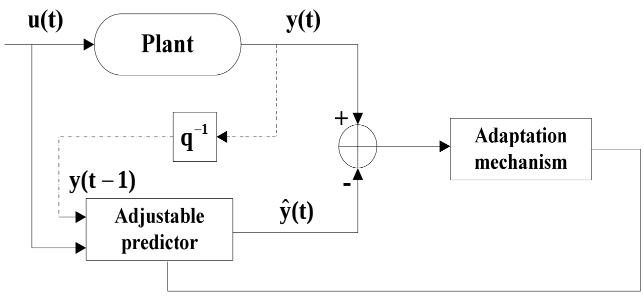

Fig. 1. Indirect adaptive control scheme.

When the plant model has unknown but constant parameters over a large horizon, the following adaptive control strategies are possible: adaptive control with the controller actualization at each sampling interval; adaptive control with controller actualization at N sampling intervals; plant model identification in closed-loop followed by the redesign of the controller. (Landau et al. (2011)) The choice of one of the above control solutions, depends on the dynamic system particularities and also the proposed solution, must take into consideration the available computation power. The most popular control techniques used in practice with this control approach, are: ***the adaptive pole-placement*** (de Larminat, 2007); Landau et al., 2011) and ***adaptive generalized predictive control.*** (M'Saad, 1993; Landau et al., 2011).

3. MODEL ADEQUACY VERIFICATION

After the computation of the regression coefficients, it is necessary to verify the model adequacy of the plant. This means to make an analysis of the regression, in order to appreciate how much the closed-loop estimated model expresses correctly the dependency between the input and output measurements (u, y). Because the standard deviation doesn't put in evidence very clearly the reasons which determined the dispersion degree of the data with respect to the model, the point associated to the center of the experimental data set is computed (corresponding to the average values on the inpus $\bar{u}$ and output $\bar{y}$ signals) and the dispersion of the points y_k and $\hat{y}_k$ is estimated with respect to $\bar{y}$. (Tertisco et al., 1991; Calin et al., 1988).

On the basis of the dispersion degree of the output measurements y_k and $\hat{y}_k$ with respect to the average value, it can be appreciated if the deviation is caused by the inadequacy of the regression model used for the real behavior of the plant, or is caused by the big amplitudes of the measurement errors (additive noise at the output). The dispersion degree of the real-data with respect to the average, can be described by the following formula:

$$S_y = \frac{1}{N}\sum_{k=1}^{N}(y_k - \bar{y})^2 \tag{1}$$

where N represents the measurements horizon. The deviation of the closed-loop estimated model with respect to the average value is:

$$S_{\hat{y}} = \frac{1}{N}\sum_{k=1}^{N}(\hat{y}_k - \bar{y})^2 \tag{2}$$

The dispersion residues between the real-data and the estimated model is:

$$S_R = \frac{1}{N}\sum_{k=1}^{N}(y_k - \hat{y}_k)^2 \tag{3}$$

Between (1), (2) and (3), there is a dependency relation, which can be defined by the coefficient:

$$R^2 = \frac{S_{\hat{y}}}{S_y} \tag{4}$$

which tends to 1, when the estimated model is close to the behaviour of the real plant. (Tertisco et al., 1991; Calin et al., 1988).

4. PLANT DESCRIPTION AND CONTROLLER DESIGN

The flexible transmission system, is made of three pulleys linked by two elastic belts (Fig. 2). One of these pulleys, is constrained to the axis of a DC motor. The motor position, is controlled by a local servo (speed and position feedback). The dynamics of the local position control, is very fast when compared to the mechanical system. (Landau et al., 2011) The control problem, is to obtain the desired position of the **3rd** pulley by modifying the input voltage of the position control that drives the **1st** pulley. The output $y(t)$ of the system, is the axis position of the **3rd** pulley and the command signal $u(t)$, is the reference for the **1st** pulley axis position. The mechanical loads that can be added on the **3rd** pulley, modify the system inertia and consequently also the resonant modes of the mechanical system. (Landau et al., 2011).

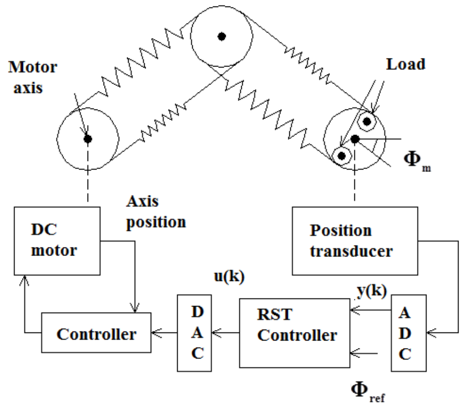

Fig. 2. Control scheme of the flexible transmission system.

For the selection of the tracking performances of the controller, two possible choices were proposed. The **1st** choice, assumes the existence of a reference model:

$$T(q^{-1}) = G \cdot P(q^{-1}) \tag{5}$$

where:

$$G = \begin{cases} \dfrac{1}{B(1)}, B(1) \neq 0 \\ 1, B(1) = 0 \end{cases} \tag{6}$$

is selected in order to have: a unit static gain between the desired trajectory y^* and the output y of the dynamic system; to compensate the regulation dynamics defined by $P(q^{-1})$. (Landau et al. (2011)) The **2nd** choice, corresponds to the situation where the regulation dynamics are the same as the tracking dynamics and the polynomial $T(q^{-1})$ of the tracking performances, is replaced by a gain (Landau et al., 2011):

$$T(q^{-1}) = G = \begin{cases} \dfrac{P(1)}{B(1)}, B(1) \neq 0 \\ 1, B(1) = 0 \end{cases} \quad (7)$$

This assures a unit static gain between the reference and the output signal. In the above equations, $P(q^{-1})$ represents the desired closed-loop poles of the dynamic system, $B(q^{-1})$ represents the numerator of the dynamic plant model and $T(q^{-1})$ represents the selected tracking performances. The initial model of the flexible transmission system, for which is verified the model adequacy of the proposed control solution presented below by simulation, is represented by the open-loop estimated model obtained using the **OEEPM** method (Landau et al., 2011):

$$A(q^{-1}) = 1 - 1.609555\,q^{-1} + 1.87644\,q^{-2} - 1.49879\,q^{-3} + 0.88574\,q^{-4}$$
$$B(q^{-1}) = 0.3053\,q^{-1} + 0.3943\,q^{-2}, d = 2 \quad (8)$$

In the following lines, will be presented the controller performances obtained with four different values of the overshoot M, without pre-specified parts or with pre-specified parts (for a steady-state error). From this will be selected the solution with better controller performances. For an overshoot $M = 0.5\%$, a rising time $t_R = 0.305s$, a natural frequency $\omega_0 = 11.97$ and no pre-specified parts for the controller, the robustness performances represented by the output sensitivity function $S_{yp}\left(e^{-j\omega}\right)$, are represented in Fig. 3.

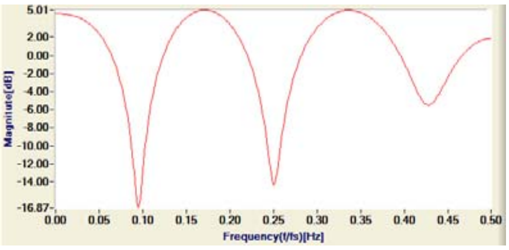

Fig. 3. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

With an overshoot $M = 1\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 11.15$ and no pre-specified parts for the controller performances, the frequency response of the output sensitivity function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 4.

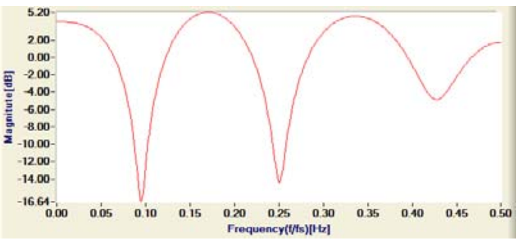

Fig. 4. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

For an overshoot $M = 1.5\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 10.49$ and no pre-specified parts selected for the controller performances, the frequency response of the robustness function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 5.

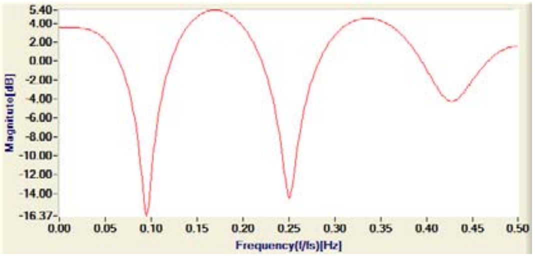

Fig. 5. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

When an overshoot $M = 2\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 9.84$ and no pre-specified parts are proposed for the controller performances, the frequency response of the output sensitivity function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 6.

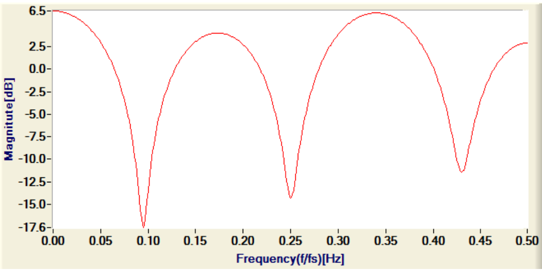

Fig. 6. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

When an overshoot $M = 0.5\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 11.97$ and a set of pre-specified parts (for a steady-state error) are introduced for the controller performances, the frequency response of the output sensitivity function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 7.

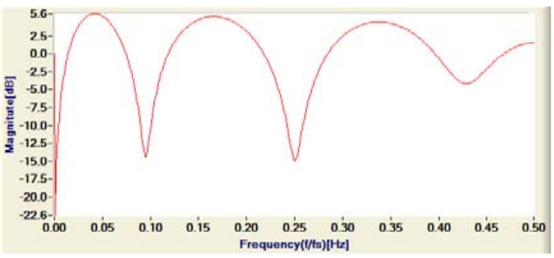

Fig. 7. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

With an overshoot $M = 1\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 11.15$ and a set of pre-specified parts introduced in the controller parameters (for a steady-state error), the frequency response of the output sensitivity function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 8.

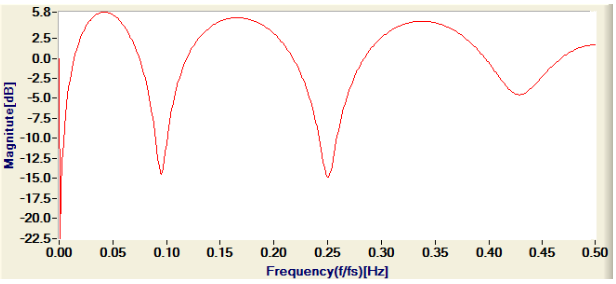

Fig. 8. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

By selecting an overshoot $M = 1.5\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 10.49$ and a set of pre-specified parts (for a steady-state error), the frequency response of the output sensitivity function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 9.

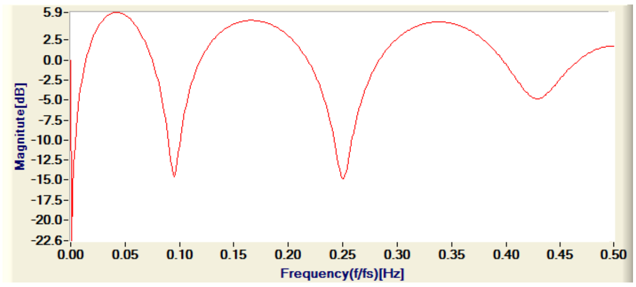

Fig. 9. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

When selecting an overshoot $M = 2\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 9.84$ and a set of pre-specified parts (for a steady-state error) for the controller performances, the frequency response of the robustness function $S_{yp}\left(e^{-j\omega}\right)$, is represented in Fig. 10.

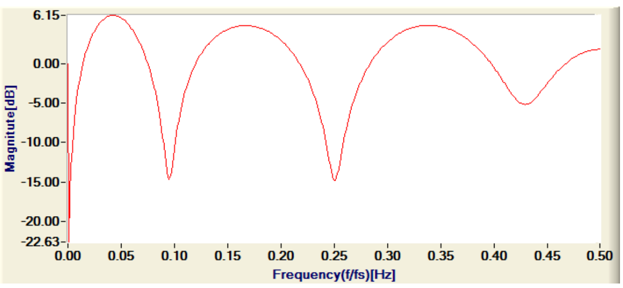

Fig. 10. Frequency representation of the output sensitivity function $S_{yp}(e^{-j\omega})$.

From the above frequency representations of the robustness functions $S_{yp}\left(e^{-j\omega}\right)$, it can be observed that a better control solution is obtained for an overshoot $M = 0.5\%$, a rising time $t_R = 0.305$, a natural frequency $\omega_0 = 11.97$ and a gain margin of $\Delta M = -5.01dB$, as in Fig. 3. The controller parameters obtained using the pole-placement strategy for the **1st** tracking performances selection from equation (5) and (6) with the controller performances imposed above, are the following:

$$\begin{aligned} R(q^{-1}) &= 1.1209q^{-1} - 2.2520q^{-2} + 1.1397q^{-3} \\ S(q^{-1}) &= 1 + 0.9309q^{-1} - 0.2488q^{-2} - 0.5073q^{-3} \\ T(q^{-1}) &= 1.4293 - 0.9700q^{-1} + 0.1846q^{-2} \end{aligned} \tag{9}$$

For the **2nd** tracking performances selection from equation (7) with the above controller performances, the parameters are:

$$\begin{aligned} R(q^{-1}) &= 1.1209q^{-1} - 2.2520q^{-2} + 1.1397q^{-3} \\ S(q^{-1}) &= 1 + 0.9309q^{-1} - 0.2488q^{-2} - 0.5073q^{-3} \\ T(q^{-1}) &= 0.6441 \end{aligned} \tag{10}$$

5. SIMULATION RESULTS AND DISCUSSION USING THE INDIRECT ADAPTIVE CONTROL SOLUTION

A series of simulations are realized in this section using the indirect adaptive control approach for different SPAB signal and perturbation amplitudes with the proposed types of tracking performances from section 4, to verify the closed-loop model adequacy (Landau et al., 2011). For this simulations, the following notations were introduced: A represents the maximum admitted value of the system response; $Y_{\max}$ represents the maximum value of the system response, when no perturbation is present in the closed-loop system. For the closed-loop identification of the flexible transmission system, it was selected a SPAB signal with $N = 8$ cells, a frequency divider $d = 2$, a sampling period $T_e = 0.05$ and a command $U_0 = 20\% A$. For the perturbation model, it was selected a filter of the Gaussian White noise as follows: $C(q^{-1})e(k)$ where $C(q^{-1}) = 1 + 0.7q^{-1} + 0.25q^{-2}$ and $e(k)$ represents the model and Gaussian White noise generated by software. For the closed-loop identification of the flexible transmission system, it was proposed the **G**eneralized-**C**losed-**L**oop-**O**utput-**E**rror algorithm (Landau et al. (2011)) because it permits a better representation and estimation of the plant and perturbation model. Beside the closed-loop validation test used for the estimated model, the formula of the index used to check the closed-loop model adequacy, is the following (Tertisco et al. (1991); Calin et al. (1988)):

$$a\_index = \frac{\sum_{i=1}^{N} (y_i - \tilde{y})^2}{\sum_{i=1}^{N} (\hat{y}_i - \tilde{y})^2} \tag{11}$$

where: $\tilde{y}$ represents the average value of the output signal from the real dynamic system, described by the **4th** order degree model from equation (8); y_i is the output signal of the real plant model from equation (8) at step i of identification; $\hat{y}_i$ is the output of the closed-loop estimated plant model at step i and N is the horizon length selected for identification. It is assumed that a good estimated model by closed-loop identification, has an adequacy index $a\_index \geq 0.7$. Beside the model adequacy index presented above, it is also calculated the medium value of the output signal for each sampling interval. This is realized in order to see the variation of the output signal in rapport with the applied reference signal and also, for the improvement of the control performances. The simulations using the indirect adaptive control solution, are realized for two successive sampling intervals with the selected tracking performances presented in section 4 for different amplitudes of the SPAB signal and perturbation amplitudes. For the **1st** sampling interval, the amplitude of the Gaussian White noise perturbation selected is assumed to be $10\%Y_{max}$ and it represents the simulation of the closed-loop system with the real open-loop model from (8). In the **2nd** sampling interval, the amplitude of the Gaussian White noise perturbation is increased to $15\%Y_{max}$ and it represents the simulation of the closed-loop system after the plant model re-identification and controller updating. For a better understanding of the important points of the system response using the proposed control solution, beside the reference signal drawn with green, the following three suplimentary lines were introduced in the graphical representations: a ***blue*** line, representing the $35\%A$ amplitude of the system response; a ***cyan*** line, representing the $40\%A$ amplitude of the system response; a ***black*** line, representing the $45\%A$ amplitude of the system response. For the **1st** tracking performances selection, five set of simulations were realized for different SPAB signal amplitudes. It was determined that: for a SPAB signal amplitude between $0:10\%U_0$ in the **2nd** sampling interval, the estimated and validated model by closed-loop identification, has an accepted model adequacy index a_index between 0.83 and 0.99. For a higher SPAB signal amplitude, it was determined that the estimated closed-loop model doesn't have a valid model anymore with an accepted adequacy index ($a\_index \geq 0.7$). In the **1st** simulation, for a SPAB signal amplitude $DU = 2.5\%U_0$, the graphical representation using the indirect adaptive control solution, is presented in Fig. 11.

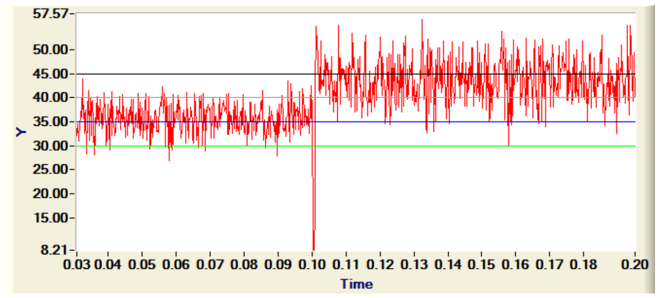

Fig. 11. Indirect adaptive control solution for a $2.5\%U_0$ SPAB signal amplitude.

From the above graphic, it can be observed that the control performances of the **1st** sampling interval are better compared to the **2nd** sampling interval, for a higher perturbation amplitude. The model adequacy index obtained in the **2nd** sampling interval after the closed-loop re-identification of the plant model, is $a\_index = 0.86$. The medium value of the output signal in the **1st** interval is 35.45, while in the 2nd sampling interval is 43.60. For a SPAB signal amplitude $DU = 5\%U_0$, the graphical representation of the simulation result with the indirect adaptive control solution, is presented in Fig. 12.

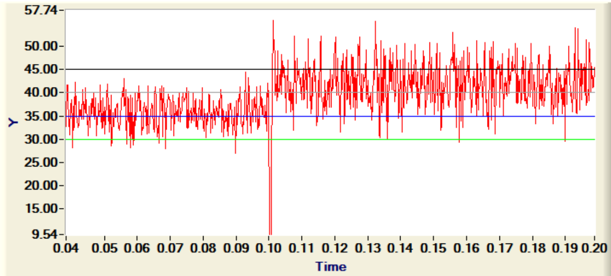

Fig. 12. Indirect adaptive control solution for a $5\%U_0$ SPAB signal amplitude.

From the above graphic, it can be seen that for a smaller perturbation amplitude ($10\%Y_{max}$) of the closed-loop system in the **1st** sampling interval, the control solution has improved performances compared to the **2nd** sampling interval, for a higher perturbation amplitude ($15\%Y_{max}$). The model adequacy index of the estimated and validated model obtained in the **2nd** sampling interval, has the following accepted value $a\_index = 0.92$. This shows a better estimation compared to the previous case considered. The medium value of the output signal in the **1st** sampling interval is 35.60, while the medium value in the **2nd** sampling interval is 41.87. This shows an improvement of the control performances compared to the previous case, for the **2nd** sampling interval considered after re-computing the plant and controller parameters. For a $DU = 6.25\%U_0$ SPAB signal amplitude, the graphical representation of the simulation result using the indirect adaptive control solution, is represented in Fig. 13.

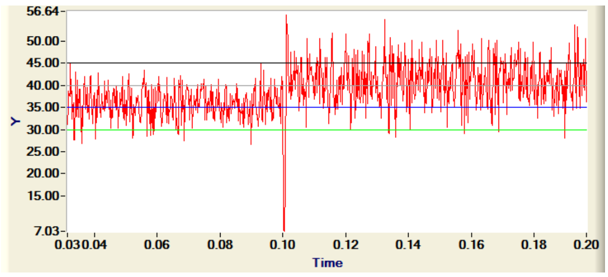

Fig. 13. Indirect adaptive control solution for a $6.25\%U_0$ SPAB signal amplitude.

From the above graphic, it can be observed that the control solution in the **1st** sampling interval obtains improved performances compared to the **2nd** sampling interval, when a higher perturbation amplitude was selected.The adequacy index of the estimated and validated model computed in the **2nd** sampling interval after the closed-loop re-identification of the model, has the following accepted value $a\_index = 0.95$. This shows an improvement of the closed-loop estimated model, compared to the previous cases presented. The medium value of the output signal in the **1st** sampling interval is 35.67, while for the **2nd** sampling interval is 40.94. This result gives an improvement of the control performances in the **2nd** sampling interval, compared to the previous cases considered. For a higher SPAB signal amplitude of $7.5\%U_0$, the graphical representation of the output signal obtained using the indirect adaptive control solution, is presented in Fig. 14.

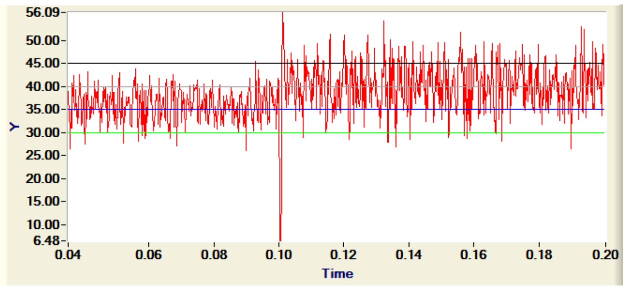

Fig. 14. Indirect adaptive control solution for a $7.5\%U_0$ SPAB signal amplitude.

The above graphic, shows that the control performances of the proposed solution are better in the **1st** sampling interval for a smaller perturbation amplitude, compared to the **2nd** sampling interval for a higher perturbation amplitude. The adequacy index computed for the estimated and validated model in the **2nd** sampling interval, is $a\_index = 0.99$. This shows an improvement of the closed-loop estimated model adequacy, compared to the previous cases. The medium value of the output signal in the **1st** sampling interval is 35.75, while the medium value of the output signal in the **2nd** sampling interval is 40.06. This shows that the control performances, are improved in the **2nd** sampling interval compared to the previous cases considered. For a $DU = 10\%U_0$ SPAB signal amplitude, the graphical representation of the simulation result obtained using the indirect adaptive control solution, is presented in Fig. 15.

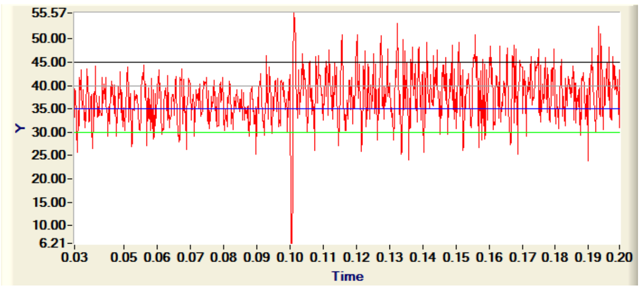

Fig. 15. Indirect adaptive control solution for a $10\%U_0$ SPAB signal amplitude.

The graphical representation from Fig. 15, shows that the control performances in the **1st** sampling interval for a $10\%Y_{\max}$ perturbation amplitude, have better results compared to the **2nd** sampling interval, for a $15\%Y_{\max}$ perturbation amplitude. The model adequacy index obtained in the **2nd** sampling interval after the closed-loop identification of the plant model, is $a\_index = 0.95$. This shows a decrease of the index compared to the previous case and an increase compared to the other cases presented. The medium value of the output signal in the **1st** sampling interval is 35.90 and in the **2nd** sampling interval is 38.60, which shows better control performances in the **2nd** sampling interval compared to the previous cases presented. For the **2nd** tracking performances selection presented in section 4, five set of simulations have been realized with the indirect adaptive control solution. It was determined that a valid estimated model with an accepted adequacy index between 0.77 and 0.98 in the **2nd** sampling interval after the closed-loop re-identification of the plant model, is obtained for a smaller SPAB signal amplitude between $0:8\%U_0$, compared to the previous tracking performances selection. The graphical representation of the simulation result obtained using the indirect adaptive control solution for a $2.5\%U_0$ SPAB signal amplitude, is presented in Fig. 16.

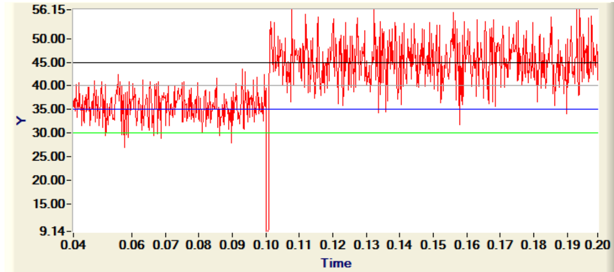

Fig. 16. Indirect adaptive control solution for a $2.5\%U_0$ SPAB signal amplitude.

The above representation, shows that the control performances obtained in the **1st** sampling interval for a $10\%Y_{\max}$ perturbation amplitude, are better compared to the **2nd** sampling interval, for a $15\%Y_{\max}$ perturbation amplitude. The closed-loop estimated and validated model from the **2nd** sampling interval after the closed-loop identification procedure, has an adequacy index $a\_index = 0.77$. This shows that the estimated plant model, has a smaller adequacy to the real-data compared to the same situation for the **1st** tracking performances selection. The medium value of the output signal in the **1st** sampling interval is 35.41, while in the **2nd** sampling interval is 45.10. This shows that the control performances, are smaller compared to the same situation for the **1st** tracking performances selection. For a $DU = 5\%U_0$ SPAB signal amplitude, the graphical representation of the simulation result obtained using the indirect adaptive control solution, is represented in Fig. 17.

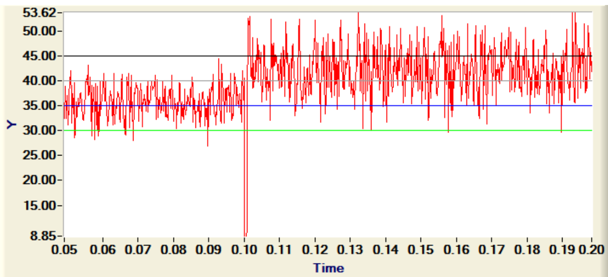

Fig. 17. Indirect adaptive control solution for a $5\%U_0$ SPAB signal amplitude.

From Fig.17, it can be observed that the control solution for a smaller perturbation amplitude ($10\%Y_{\max}$) in the **1**[st] sampling interval, has improved performances compared to the **2**[nd] sampling interval, for a higher perturbation amplitude ($15\%Y_{\max}$). The model adequacy index computed in the **2**[nd] sampling interval after the closed-loop identification is $a\_index = 0.86$, which is smaller compared to the same situation presented for the **1**[st] tracking performances selection and less adequate to the real-data. The medium value of the output signal in the **1**[st] sampling interval is 35.56, while in the **2**[nd] sampling interval is 41.98. This shows an improvement of the control performances in the **2**[nd] sampling interval, compared to the previous case and a decrease of the performances compared to the same situation for the **1**[st] tracking performances selection. For a $DU = 6.25\%U_0$ SPAB signal amplitude, the simulation result obtained using the indirect adaptive control solution, is presented in Fig. 18.

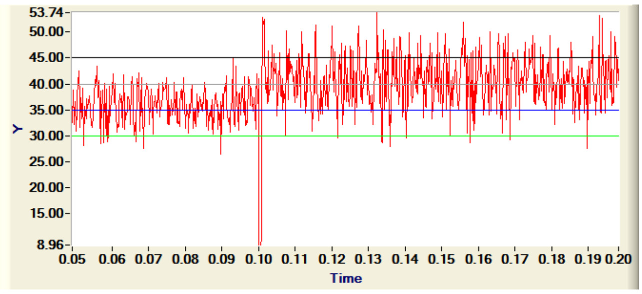

Fig. 18. Indirect adaptive control solution for a $6.25\%U_0$ SPAB signal amplitude.

In the above graphic, the control solution obtains better performances in the **1**[st] sampling interval for a smaller perturbation amplitude ($10\%Y_{\max}$), compared to the **2**[nd] sampling interval for a higher perturbation amplitude ($15\%Y_{\max}$). The model adequacy index of the estimated and validated model computed in the **2**[nd] sampling interval after the closed-loop re-identification procedure, is $a\_index = 0.91$. This shows an improved adequacy to the real-data for the estimated model, compared to the previous cases proposed and a decrease of the adequacy compared to the same situation for the **1**[st] tracking performances selection. The medium value of the output signal in the **1**[st] sampling interval is 35.64, while in the **2**[nd] sampling interval is 40.53. This represents an improvement of the control performances in the **2**[nd] sampling interval, compared to the previous cases considered and also, an improvement compared to the control performances of the same situation with the **1**[st] tracking performances selection. For a $7.5\%U_0$ SPAB signal amplitude, the graphical representation of the simulation result, is presented in Fig.19.

From Fig. 19, the control performances obtained in the **1**[st] sampling interval are higher compared to the **2**[nd] sampling interval for a higher perturbation amplitude. The model adequacy index obtained for the estimated and validated model in the **2**[nd] sampling interval after the plant model re-identification, is $a\_index = 0.96$. This represents an improved adequacy of the model to the real-data compared to the previous cases proposed. The adequacy index in this case, is smaller compared to the same situation for the **1**[st] tracking performances selection. The medium value of the output signal in the **1**[st] sampling interval is 35.71, while the medium value in the **2**[nd] sampling interval is 39.30. This gives an improvement of the control performances in the **2**[nd] sampling interval compared to the previous cases and also compared to the same situation, when the other tracking performances were considered. For a $8\%U_0$ SPAB signal amplitude, the graphical representation of the simulation result obtained using the proposed solution, is presented in Fig. 20.

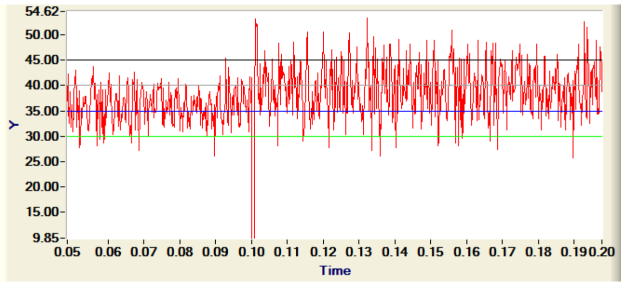

Fig. 19. Indirect adaptive control solution for a $7.5\%U_0$ SPAB signal amplitude.

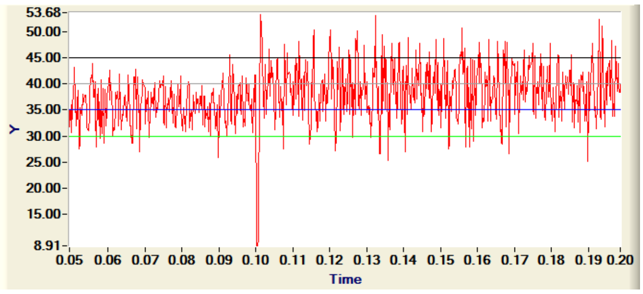

Fig. 20. Indirect adaptive control solution for a $8\%U_0$ SPAB signal amplitude.

The above graphic of the adaptive control solution, shows that the control performances in the **2**[nd] sampling interval decrease in rapport to the **1**[st] sampling interval, for a higher perturbation amplitude of the closed-loop system. The model adequacy index of the estimated plant model in the **2**[nd] sampling interval is $a\_index = 0.98$, which shows an improved adequacy of the model to real-data compared to the previous cases. The medium value of the output signal in the **1**[st] sampling interval is 35.74, while in the **2**[nd] sampling interval is 38.88. This shows that the control performances for the **2**[nd] sampling interval, increase compared to the previous cases considered.

6. CONCLUSIONS

This paper proposed a solution for the model adequacy verification of the indirect adaptive control approach, due to the validation problems that can occur in real-time applications and also, the necessity for improved control solutions. The novelty element of this paper, is represented by the introduction of a model adequacy index (Tertisco et al. (1991); Calin et al. (1988)) between the estimated and real model beside the known validation tests, in order to improve the closed-loop identification and control performances. As a dynamic system for the verification of the proposed solution, a flexible transmission was selected with a complex **4th** order model and as control technique, the pole-placement strategy. For the simulation study with the proposed control solution, two types of tracking performances were selected from the scientific literature (Landau et al. (2011)). The main study, was concentrated on the influence of the SPAB signal and perturbation amplitude on the closed-loop model adequacy and control performances of the dynamic system. It was determined that for the **1st** type of tracking performances selection, the closed-loop estimated model has an accepted model adequacy index a_index between 0.83 and 0.99, for a SPAB signal amplitude in the domain $0-10\%U_0$. For the **2nd** type of tracking performances selection, it was determined that a good estimated and validated model by closed-loop identification, has an accepted model adequacy index a_index between 0.77 and 0.98 with a SPAB signal amplitude in the domain $0-8\%U_0$. From the simulations presented in this paper, it could be observed that the perturbation amplitude has a significant influence on the model adequacy and control performances of the dynamic system. This results can be improved when selecting a higher SPAB signal amplitude. Due to the larger domain of variation for the SPAB signal amplitude, bigger model adequacy index a_index computed for the closed-loop estimated model and improved control performances, the indirect adaptive control solution proposed for the flexible transmission, is recommended to be used with the **1st** tracking performances selection. The model adequacy and control performances of the dynamic system, are restricted by the SPAB signal amplitude (which has a limited domain of variation), but also by the perturbation amplitude as could be seen from the simulations presented in section 5. The model adequacy index presented in this paper, can be used in real-time applications for simple or complex systems, were the dynamic system parameters are time-varying and it is necessary to have an improved accuracy of the identification and control solution, as the system evolves.

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