Robust Fuzzy Output Regulator Design for Nonlinear Systems without Virtual Desired Variable Calculation

Kuang-Yow Lian * **Chien-Hung Liu** ** **Chian-Song Chiu** **

* *Department of Electrical Engineering , National Taipei University of Technology, Taipei 10608, Taiwan*
** *Department of Electrical Engineering, Chung-Yuan Christian University, Chung Li 32023, Taiwan,*
(e-mail: acs.chiu@gmail.com)

Abstract: This paper proposes a robust T-S fuzzy output regulator for affine nonlinear systems in the presence of parametric uncertainties and external disturbance. First, we introduce the fuzzy output regulator by involving an integral error state and PDC compensation, where a set of virtual desired variables (VDVs) is solved for error coordinate transformation. The benefit of the VDV-based regulator is with systematic design. However, since the VDV-based fuzzy regulator is unavailable when the system is subject to uncertainty and external disturbance, the controller is further reduced to the non-VDV fuzzy output regulator. The VDV calculation is removed in a more simplified manner, while the exponential output regulation is assured. To reject uncertainty and external disturbance, the robust H_∞ theorem is derived with linear matrix inequality (LMI) stability condition. Finally, numerical simulation of the DC-DC buck converter is given to show the benefits of the non-VDV fuzzy output regulator.

Keywords: T-S Fuzzy regulator, virtual-desired-variable(VDV), linear matrix inequality (LMI), robustness.

1. INTRODUCTION

Fuzzy logic systems have been successfully applied to many control problems of complex or poorly modeled systems over the past decades, e.g., Marcu (2011), Cuibus and Letia (2012), Wallam and Abbasi (2014), Tasar *et al.* (2015), Khelchandra *et al.* (2014), Ameur *et al.* (2013). Recently, the T-S fuzzy model approach has been developed for stability analysis and stabilization of nonlinear systems (Ma and Fei, 2015; Jabri *et al.*, 2012; Chuang *et al.*, 2011; Sadeghi and Vafamand, 2014) because of its ability to accurately approximate complex nonlinear systems by a set of linear subsystems with associated membership functions (Takagi and Sugeno, 1985; Wang *et al.*, 1996). The stability can be rigorously proven by Lyapunov theory and the controller can be designed with the framework of the parallel distributed compensation (PDC)(Tanaka and Wang, 2001). In addition, the control gains also can be quickly obtained via powerfully solving linear matrix inequalities (LMIs)(Boyd *et al.*, 1994). For example, many fuzzy control methods incorporated with LMI-based strategies have been developed for stabilization (Tanaka *et al.*, 1998; Chen *et al.*, 2000; Tuan *et al.*, 2001; Liu and Zhang, 2003).

Besides stabilization problem, regulation is also an important task for the control systems. The output regulation problem studies the stabilization of dynamic systems with the system output asymptotically rejecting unwanted disturbances and tracking prescribed trajectories (Huang, 2004). For example, the regulation theory (Ma and Sun, 2000) based method is extended to solve the output regulation control problem of T-S fuzzy systems, e.g., Yuan and Li (2007), Meda-Campaña *et al.* (2012). These papers have to solve some regulation equations for exact output regulation and are failed when considering system uncertainty/disturbance or bias terms, cf. (Byrnes and Isidori, 2000). This means that the regulation theory based approaches are difficultly applied on more general uncertain nonlinear systems. On the other hand, some alternative methods which calculate equilibrium point or virtual desired variable (VDV) for T-S fuzzy output regulation of nonlinear systems have been developed in (Chiu *et al.*, 2004), and (Lian *et al.*, 2005), respectively. In (Chiu *et al.*, 2004), the operational point reference approach is provided to take coordinate transformation and to construct the regulation error system. However, this method may lead to a nontrivial control task when solving the equilibrium points of complex system is infeasible. Afterward, a novel VDV based method is occurred to overcome drawbacks of the above method. In (Lian *et al.*, 2005), a set of virtual desired variables (VDVs) is introduced to synthesize the control law for the speed control of induction motors. After the output tracking system is transformed to a stabilization problem via the VDV concept, the original controller becomes as two-parts of control law including PDC and VDV calculation. Although the VDV concept scheme can avoid searching equilibrium point to design fuzzy regulator, the design procedure still has computational complexity to solve the VDVs for high-order systems. Meanwhile, the VDV based fuzzy regulator is unavailable to control the systems with uncertainty and external disturbance. In other words, its practical applications are limited due to the computational-intensive characteristic and unavailable uncertainty for VDVs. This motivated us to further improve the above regulator with a simpler form and robust performance.

This paper will present an easy implement condition to design output regulator for nonlinear affine systems with uncertainty and external disturbance. First, we augment the nonlinear affine system with an added integral state of the output regulation error. Second, we introduce a set of VDVs to construct the output regulation error system and represent it in a T-S fuzzy model. Then, the output regulation problem is converted to the stabilization problem in a straight-forward design manner. Third, we further remove the VDVs to simplify the controller as a non-VDV fuzzy regulator. Meanwhile, the robustness of the resulted regulation error system to uncertainty and external disturbance is enhanced by the non-VDV fuzzy regulator. As a result, the exponential output regulation and H_∞ performance are assured under LMI conditions, i.e., the H_∞ disturbance attenuation is obtained based on the proposed gain design. In contrast to the work (Lian *et al.*, 2005), this paper comes with such merits: (i) it is not necessary to determine VDVs; (ii) an error coordinate transformation is not required; (iii) controller gains guarantee the stability of the nonlinear system in the presence of system uncertainty and external disturbance; (iv) the output regulation stability is transformed to the feasible LMI problem; (v) even if the desired output signal is changing, the piecewise regulation performance can be achieved without redesigning the controller. In addition, to demonstrate the effectiveness of the robust output regulator, numerical simulation for a DC-DC buck converter with uncertainty and external disturbance is given.

The rest of this paper is organized as follows. A VDV-based T-S fuzzy regulator design for nonlinear affine systems is introduced in Section 2. In Section 3, a non-VDV T-S fuzzy regulator synthesis is proposed by removing VDV and redesigning control gain. In Section 4, robustness design is presented. Numerical simulation is carried out for a DC-DC buck converter to verify the proposed two regulators and comparison results are included in Section 5. Finally, some conclusions are given in Section 6.

2. VDV-BASED REGULATOR

First of all, a VDV-based design scheme is introduced for the T-S fuzzy output regulation control. Consider a kind of affine nonlinear system described by the following dynamic equation:

$$\dot{x}_p(t) = f(x_p(t)) + g(x_p(t))u(t) + \xi \tag{1}$$
$$y(t) = h(x_p(t)) \tag{2}$$

where $x_p \in R^n$, $u \in R^q$, $y \in R^m$ $(q \leq m)$ are the state, the input, and the output respectively; $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are nonlinear function vectors with appropriate dimensions; ξ is a bias term. Through this paper, the nonlinear dynamic functions $f(\cdot)$ and $h(\cdot)$ are assumed to be expressed in the forms: $f(x_p(t)) = A(x_p(t))x_p(t)$ and $h(x_p(t)) = C(x_p(t))x_p(t)$, where the matrices $A(x_p(t))$, $C(x_p(t))$ and $g(x_p(t))$ are well defined in the discussed region Ω. For this system, the control objective is to drive the output $y(t)$ to a desired constant value y_d. To achieve zero steady-state regulation error, the integral compensation is used to cope with the bias term and uncertainty. A new state variable is added to account the integration of output regulation error i.e.,

$$\dot{x}_e(t) = y_d - y(t) \tag{3}$$

where x_e is the error integration. For multi-output systems, $x_e(t)$ denotes a stack of m integrators. By combining (1), (2) and (3), the overall augmented dynamics becomes

$$\begin{aligned} \dot{x}_p(t) &= A(x_p(t))x_p(t) + g(x_p(t))u(t) + \xi \\ \dot{x}_e(t) &= y_d - C(x_p(t))x_p(t) \end{aligned} \tag{4}$$

By letting the augmented state $x = \left[x_p^T \; x_e^T\right]^T \in R^{n+m}$, the above dynamics is rewritten in the state-space form:

$$\dot{x}(t) = \overline{A}(x_p(t))x(t) + \overline{B}(x_p(t))u(t) + \overline{\xi}$$

where $\overline{A}(x_p(t))$, $\overline{B}(x_p(t))$, and $\overline{\xi}$ are denoted as follows:

$$\overline{A}(x_p(t)) = \begin{bmatrix} A(x_p(t)) & 0 \\ -C(x_p(t)) & 0 \end{bmatrix}, \overline{B}(x_p(t)) = \begin{bmatrix} g(x_p(t)) \\ 0 \end{bmatrix},$$
$$\overline{\xi} = \begin{bmatrix} \xi \\ y_d \end{bmatrix}$$

If the output regulation control is achieved, then there is an operational point $x_d = [x_{pd}^T \; x_{ed}^T]^T$ for $y(t) = y_d$, where x_{pd}, x_{ed} are the operational point for the state x_p, and the integral state x_e, respectively. Since we only need to control the output, the operational point x_d has freedom to be designed later. As a result, x_d is called the virtual desired variable (VDV) for the system state. Let us define the error states $\tilde{x}_p(t) = x_p(t) - x_{pd}$, $\tilde{x}_e(t) = x_e(t) - x_{ed}$, $e(t) = x(t) - x_d$. The regulation error system is obtained below:

$$\begin{aligned} \dot{e}(t) &= \overline{A}(x_p(t))e(t) + \overline{B}(x_p(t))u(t) + \overline{\xi} + \overline{A}(x_p(t))x_d \\ &= \overline{A}(x_p(t))e(t) + \overline{B}(x_p(t))\tau(t) + \psi(t) \end{aligned} \tag{5}$$

where the terms $\overline{B}(x_p(t))\tau(t)$ and $\psi(t)$ satisfy

$$\psi(t) = \overline{B}(x_p(t))u(t) + \overline{\xi} + \overline{A}(x_p(t))x_d - \overline{B}(x_p(t))\tau(t) \tag{6}$$

If letting $\psi(t) = 0$, the term $\tau(t)$ becomes a virtual control input in (5) to be designed later. Next, according to the T-S fuzzy modeling method Lian et al., (2001), and letting $\psi(t) = 0$, the nonlinear regulation error system (5) is further expressed in terms of T-S fuzzy rules as follows:

Plant Rule i :
IF $z_1(t)$ is F_{1i} and $\cdots$ and $z_s(t)$ is F_{si}
THEN $\dot{e}(t) = \overline{A}_i e(t) + \overline{B}_i \tau(t), \quad i = 1, 2, ..., r$ (7)

where $z_1(t) \sim z_s(t)$ are premise variables which consist of proper state variables of the system; $F_{ji} (j = 1, 2, \ldots, s)$ are the fuzzy sets; r is the number of fuzzy rules; $\overline{A}_i$ and $\overline{B}_i$ are system matrices with appropriate dimensions. Using singleton fuzzifier, product inference, and weighted defuzzifier, the fuzzy system is inferred as the dynamic equation:

$$\dot{e}(t)=\sum_{i=1}^{r}\mu_i(z(t))\overline{A}_i e(t)+\sum_{i=1}^{r}\mu_i(z(t))\overline{B}_i\tau(t) \qquad (8)$$

where $z(t) = [z_1(t)\ z_2(t)\ \cdots\ z_s(t)]^T$ and $\mu_i(z(t)) = \frac{\prod_{j=1}^{s} F_{ji}(z_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{s} F_{ji}(z_j(t))} \geq 0$ for all t in the regarded discussion region Ω; and $\sum_{i=1}^{r}\mu_i(z(t)) = 1$. In the fuzzy model (8), the membership function $F_{ji}(z_j(t))$ and the subsystem matrices $\overline{A}_i$, $\overline{B}_i$ will be properly chosen, such that $\sum_{i=1}^{r}\mu_i(z(t))\overline{A}_i = \overline{A}(x_p(t))$ and $\sum_{i=1}^{r}\mu_i(z(t))\overline{B}_i = \overline{B}(x_p(t))$. At this step, if the error state $e(t)$ is stabilized to zero, then the state $x(t)$ is driven to the VDV x_d for the regulation objective. The output regulation control is transformed to a stabilization problem by designing the virtual control input $\tau(t)$, while the desired variables x_d and the control input $u(t)$ are conformed to (6). Based on the IF-THEN fuzzy rules in (7), the T-S fuzzy stabilization law is constructed as follows:

Controller Rule i :
IF$z_1(t)$ is F_{1i} and… and $z_s(t)$ is F_{si}
THEN $\tau(t) = -K_i e(t),\ i = 1, 2, \ldots, r$ (9)

where the $K_i's$ are the feedback gains. By substituting the inferred output of (9) into the regulation error system (8), the closed-loop system is thus obtained below:

$$\dot{e}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))G_{ij}e(t) \qquad (10)$$

where $G_{ij} = \overline{A}_i - \overline{B}_i K_j$. Afterward, according to Lyapunov stability method, the gain design can be performed by solving the following LMIs

$$\begin{bmatrix} X\overline{A}_i^T + \overline{A}_i X - M_j^T\overline{B}_i^T - \overline{B}_i M_j & XD \\ DX & -X \end{bmatrix} < 0, \qquad (11)$$

for $i, j = 1, 2, \ldots, r$, where $M_j = K_j X$ (i.e., $K_j = M_j X^{-1}$); D is a diagonal positive-definite matrix chosen by designer for adjusting the control performance. In other word, if the above LMIs have a feasible solution, then the error system (10) is asymptotically stable. This means that the output regulation objective is achieved.

Next, the remaining design for the output regulation is to determine x_d and $u(k)$. From the constraint $\psi(t) = 0$ in (6), we partition the matrices as follows:

$$\overline{B}(x_p(t))=\begin{bmatrix} B_u(x_p(t)) \\ ------ \\ 0_{(n+m-q)\times q}\end{bmatrix},\ \overline{A}(x_p(t))=\begin{bmatrix} A_u(x_p(t)) \\ ------ \\ A_d(x_p(t))\end{bmatrix},$$
$$\overline{\xi}=\begin{bmatrix}\xi_u \\ -- \\ \xi_d\end{bmatrix}$$

where $B_u(x_p) \in R^{q\times q}$ is the nonzero term of $g(x_p) \in R^{n\times q}$; $A_u(x_p) \in R^{q\times(n+m)}$; $A_d(x_p) \in R^{(n+m-q)\times(n+m)}$; $\xi_u \in R^{q\times 1}$; and $\xi_d \in R^{(n+m-q)\times 1}$. Then, the condition $\psi(t) = 0$ in (6) can be rewritten below:

$$\begin{bmatrix} B_u(x_p(t))(u(t)-\tau(t)) \\ ----------- \\ 0_{(n+m-q)\times 1}\end{bmatrix}=\begin{bmatrix} -A_u(x_p(t))x_d-\xi_u \\ ---------- \\ -A_d(x_p(t))x_d-\xi_d\end{bmatrix} \qquad (12)$$

From the upper part of (12), the practical T-S fuzzy regulator is formed as follows:

$$u(t) = -\sum_{i=1}^{r}\mu_i(z(t))K_i e(t) - B_u^{-1}(x_p(t))\{A_u(x_p(t))x_d + \xi_u\} \qquad (13)$$

where the virtual control input (9) has been applied; x_d is solved from the lower part of (12), i.e.,

$$0 = A_d(x_p(t))x_d + \xi_d \qquad (14)$$

Since the number of the above constraint is lower than the dimension of x_d, there exist q redundant freedoms for solutions of x_d, i.e. x_{ed} can be unlimited under the output regulation objective $y_d = C(x_{pd})x_{pd}$. From the above, the structure of the VDV-based integral T-S fuzzy regulation control is shown in Fig. 1.

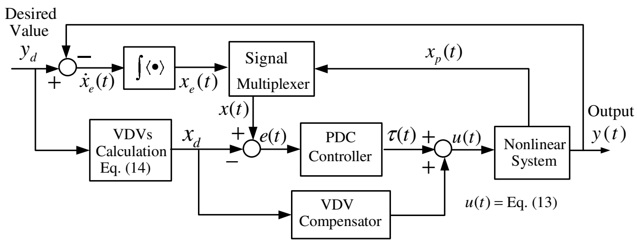

Fig. 1. Diagram of the VDV-based T-S fuzzy output regulation control.

Remark1 : From the aforemention, the VDV-based control is a systematic scheme for output regulation. But (13) is unavailable as well as x_d cannot be solved from (14) when there exists uncertainty or disturbance. To overcome the disadvantage, we will design a new T-S fuzzy output regulator by removing VDV design.

3. NON-VDV T-S FUZZY REGULATOR DESIGN

From the above VDV based regulator design, the controller is complex due to solving the virtual desired variables which are dependent on the dynamic function $A_d(x_p)$. To reduce the complexity of the regulator, the controller redesign is proposed in this section.

Step1. Remove the VDV

To avoid solving VDVs and calculating the complex control law, the constraint (6) is firstly modified. By applying the definition $e = x - x_d$, the term $\psi(t)$ in (6) with the virtual control input (9) is rewritten in the form:

$$\begin{aligned}\psi(t) &= \overline{B}(x_p(t))(u(t) + \sum_{i=1}^{r}\mu_i(z(t))K_ie(t)) + \overline{\xi} \\ &\quad + \overline{A}(x_p(t))x_d \\ &= \overline{B}(x_p(t))(u(t) + \sum_{i=1}^{r}\mu_i(z(t))K_ix(t)) \\ &\quad + \widehat{\psi}(x_d) + \Delta\psi(t) \qquad (15)\end{aligned}$$

where

$$\begin{aligned}\widehat{\psi}(x_d) &= -\overline{B}(x_{pd})\sum_{i=1}^{r}\mu_i(z_d)K_ix_d + \overline{\xi} + \overline{A}(x_{pd})x_d \\ \Delta\psi(t) &= -[\overline{B}(x_p(t)\sum_{i=1}^{r}\mu_i(z(t))K_ix_d \\ &\quad - \overline{B}(x_{pd})\sum_{i=1}^{r}\mu_i(z_d)K_ix_d] \\ &\quad + [\overline{A}(x_p(t)) - \overline{A}(x_{pd})]x_d \qquad (16)\end{aligned}$$

and z_d is the premise variable composed of x_d w.r.t. z. From the above equation, we simplify the fuzzy output regulator to

$$u(t) = -\sum_{i=1}^{r}\mu_i(z(t))K_ix(t), \qquad (17)$$

while the controller constraint is changed to

$$\widehat{\psi}(x_d) = 0 \qquad (18)$$

As a result, this implies $\psi(t) = \Delta\psi(t)$ in (5) under (17) and (18), i.e., the term $\Delta\psi(t)$ becomes an error to the closed-loop system. The dynamics of the closed-loop system by applying the designed virtual control input (9) becomes

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z(t))\mu_j(z(t))G_{ij}e(t) + \Delta\psi(t) \qquad (19)$$

On the other hand, by taking the same partition as (12) on the new controller constraint $\widehat{\psi}(x_d) = 0$, the following two equations are obtained below:

$$B_u(x_{pd})\sum_{i=1}^{r}\mu_i(z_d)[K_{pi}x_{pd} + K_{ei}x_{ed}] = A_u(x_{pd})x_d + \xi_u \qquad (20)$$

$$A_d(x_{pd})x_d + \xi_d = 0 \qquad (21)$$

From the above, x_{pd} can be first solved from (21), then x_{ed} is found from (20). The number of the constraint equals to the dimension of x_d, so that there exist a feasible solution x_d satisfying the constraint $\widehat{\psi}(x_d) = 0$. Since the real control law (17) is not dependent on the VDV x_d, the VDV x_d is not required to solve for the controller, i.e. the VDV x_d is removed in the controller implementation. As a result, the configuration of the non-VDV T-S fuzzy output regulation control is shown in Fig. 2.

Step2. Design controller gains

To assure the stability, let us consider the closed-loop error system (19) again. Without loss of generality, the error term $\Delta\psi(t)$ has a benefited property for Lipschitz dynamic functions $\overline{A}(x_p(t))$, $\overline{B}(x_p(t))$ and $\mu_i(z(t))$. The Lipschitz condition implies the term $\Delta\psi(t)$ can be expressed in

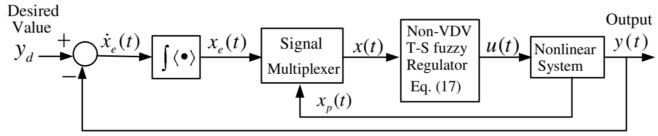

Fig. 2. Diagram of the non-VDV T-S fuzzy output regulation control.

terms of the error $e(t)$ by the mean value theorem. It yields that there is a constant matrix ΔE such that

$$\Delta\psi(t)^T\Delta\psi(t) \le e(t)^T\Delta E^T\Delta Ee(t) \qquad (22)$$

Afterward, the gain design is given in the following theorem.

Theorem 1. By using the simplified fuzzy output regulator (17), the closed-loop system (19) is exponentially stable and assures the output regulation objective if there exist a symmetric positive matrix $X > 0$ and control gain K_j satisfying the following LMIs

$$\begin{bmatrix} \begin{pmatrix} X\overline{A}_i^T + \overline{A}_iX - M_j^T\overline{B}_i^T - \overline{B}_iM_j \\ +X\Delta E^T + \Delta EX \end{pmatrix} & XD \\ DX & -X \end{bmatrix} < 0 \qquad (23)$$

where $M_j = K_jX$; $D = D^T > 0$; and $i, j = 1, 2, \ldots, r$.

Proof. The result can be proven via the Lyapunov's stability method. By choosing the Lyapunov function candidate $V(e(t)) = e(t)^TPe(t)$ with $P = P^T > 0$ and taking the time derivative of $V(e(t))$ along the error dynamics (19), we have

$$\begin{aligned}\dot{V}(e(t)) &= \left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x_p(t))\mu_j(x_p(t))G_{ij}e(t) + \Delta\psi(t)\right\}^T \\ &\quad \times Pe(t) + e(t)^TP \\ &\quad \times \left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x_p(k))\mu_j(x_p(t))G_{ij}e(t) + \Delta\psi(t)\right\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x_p(t))\mu_j(x_p(t)) \\ &\quad \times e(t)^T\{G_{ij}^TP + PG_{ij}\}e(t) \\ &\quad + \Delta\psi(t)^TPe(t) + e(t)^TP\Delta\psi(t) \qquad (24)\end{aligned}$$

Due to the Property (22), $\dot{V}(e(t))$ further satisfies the following inequality:

$$\begin{aligned}\dot{V}(e(t)) &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x_p(t))\mu_j(x_p(t))e(t)^T\{G_{ij}^TP + PG_{ij} \\ &\quad + \Delta E^TP + P\Delta E + DPD\}e(t) \\ &\quad - e(t)^TDPDe(t) \qquad (25)\end{aligned}$$

If the following inequality is satisfied, then $\dot{V}(e(t)) < -e(t)^TDPDe(t)$, i.e.,

$$G_{ij}^TP + PG_{ij} + P\Delta E + \Delta E^TP + DPD < 0 \qquad (26)$$

Due to

$$\dot{V}(e(t)) < -e(t)^TDPDe(t) \le -\sigma V(e(t))$$

it results in

$$V(e(t)) \leq V(0)e^{-\sigma t} \tag{27}$$

with $\sigma = \lambda_{min}(DPD)/\lambda_{max}(P)$, where $\lambda_{min}(M)$, $\lambda_{max}(M)$ denote the minimal and maximal eigenvalue of a matrix M, respectively. Since $V(e(t)) > 0$, $\dot{V}(e(t)) < -e(t)^T DPDe(t) < 0$, and $\|e(t)\|^2 \leq V(0)e^{-\sigma t}/\lambda_{min}(P)$, the error $e(t)$ exponentially converges to zero as $t \to \infty$. Therefore, the simplified regulator assures the output regula-tion control objective. Furthermore, the stability condition can be transformed to LMI conditions. After pre-multiplying and post-multiplying (26) by $X = P^{-1}$, it leads to

$$X\overline{A}_i^T + \overline{A}_i X - M_j^T \overline{B}_i^T - \overline{B}_i M_j + X\Delta E^T + \Delta E X + XDX^{-1}DX < 0 \tag{28}$$

where $M_j = K_j X$. Then the LMIs (23) are obtained by applying Schur's complement technique on (28).

Remark2 : Based on the simplified fuzzy output regulator (i.e, the non-VDV fuzzy output regulator), the output regula-tion does not require solving the equilibrium points and VDVs as well as performing coordinate transforma-tion. Even if there exists uncertainty on $\overline{A}(x_p)$, the controller (17) can be realized. The control performance can be adjusted by the LMI design.

4. ROBUSTNESS DESIGN

Furthermore, both system uncertainty and disturbances are considered in this section. From taking observation on (19), the uncertain closed-loop system can be assumed in the following form:

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(x_p(t))\mu_j(x_p(t))(G_{ij} + \Delta G_{ij})e(t) + \Delta\psi(t) + J\omega(t) \tag{29}$$

where $\omega(t)$ denotes an external disturbance; J is a known matrix; $\Delta G_{ij} = \Delta\overline{A}_i - \Delta\overline{B}_i K_j$ denotes the system uncertainty; $\Delta\overline{A}_i$, and $\Delta\overline{B}_i$ are unknown time-varying parametric uncertainties, which hold the norm-bounded condition:

$$[\Delta\overline{A}_i \ \Delta\overline{B}_i] = U_i\Phi_i(t)[E_{1i} \ E_{2i}] \tag{30}$$

where U_i, E_{1i}, and E_{2i} are known real constant matrices; and $\Phi_i(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $\Phi_i(t)^T\Phi_i(t) \leq I$ for all t, in which I is an identity matrix with appropriate dimension. According to (30), the uncertain term ΔG_{ij} can be expressed in the form:

$$\begin{aligned} \Delta G_{ij} &= \Delta\overline{A}_i - \Delta\overline{B}_i K_j \\ &= U_i\Phi_i(t)(E_{1i} - E_{2i}K_j) \\ &\equiv U_i\Phi_i(t)E_{ij} \end{aligned}$$

i.e., $E_{ij} = E_{1i} - E_{2i}K_j$. This implies that the closed-loop system (29) along with the simplified fuzzy output regulator (17) arrived with the following form:

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(x_p(t))\mu_j(x_p(t))\{G_{ij} + U_i\Phi_i(t)E_{ij}\}e(t) + \Delta\psi(t) + J\omega(t) \tag{31}$$

For the disturbance, the control objective becomes to achieve the H_∞ criterion

$$\int_0^{t_f} e(t)^T D^T PDe(t)dt < e(0)^T Pe(0) - e(t_f)^T Pe(t_f) + \frac{1}{\rho^2}\int_0^{t_f} \omega(t)^T\omega(t)dt \tag{32}$$

where $\tilde{x}(0)$ is the initial error; P is a symmetric positive-definite matrix; and ρ is an attenuation factor. The effect of the disturbance will be attenuated below a desired level $(1/\rho)$. If no disturbances exists, the control law (17) will stabilize $e(t)$ to zero exponentially, i.e., the output signal $y(t)$ is regulated to the prescribed value y_d. To this end, the robust gain design is given in the following theorem.

Theorem 2. Considering the closed-loop system (31) endowed with the simplified fuzzy control law (17), the H_∞ regulation performance (32) is guaranteed for a desired attenuated level $\frac{1}{\rho}$ of disturbance if there exist a symmetric positive definite matrix $X > 0$ and $M_j = K_j X$ satisfying the LMIs:

$$\begin{bmatrix} \begin{pmatrix} X\overline{A}_i^T + \overline{A}_i X \\ -M_j^T\overline{B}_i^T - \overline{B}_i M_j \\ +X\Delta E^T + \Delta E X \end{pmatrix} & * & * & * & * \\ \epsilon U_i^T & -\epsilon I & * & * & * \\ E_{1i}X - E_{2i}M_j & 0 & -\epsilon I & * & * \\ DX & 0 & 0 & -X & * \\ J^T & 0 & 0 & 0 & -\frac{1}{\rho^2}I \end{bmatrix} < 0 \tag{33}$$

for $i, j = 1, 2, ..., r$.

Proof. Choosing the Lyapunov function candidate $V(e(t)) = e(t)^T Pe(t)$ with $P = P^T > 0$ and taking the time derivative of $V(e(t))$ along the error dynamics (31) yields

$$\begin{aligned} \dot{V}(e(t)) = \{&\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(x_p(t))\mu_j(x_p(t))[G_{ij}e(t) \\ &+ U_i\Phi_i(t)E_{ij}]e(t) + \Delta\psi(t) + J\omega(t)\}^T Pe(t) \\ &+ e(t)^T P\{\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(x_p(t))\mu_j(x_p(t)) \\ &\times [G_{ij}e(t) + U_i\Phi_i(t)E_{ij}]e(t) + \Delta\psi(t) + J\omega(t)\} \end{aligned} \tag{34}$$

According to the property in (22), $\dot{V}(e(t))$ satisfies

$$\begin{aligned} \dot{V}(e(t)) \leq &\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(x_p(t))\mu_j(x_p(t))e(t)^T\{\overline{A}_i^T P + P\overline{A}_i \\ &- K_j^T\overline{B}_i^T P - P\overline{B}_i K_j + E_{ij}^T\Phi_i(t)^T U_i^T P \\ &+ PU_i\Phi_i(t)E_{ij} + \Delta E^T P + P\Delta E\}e(t) \\ &+ \omega(t)^T J^T Pe(t) + e(t)^T PJ\omega(t) \end{aligned} \tag{35}$$

Due to $\Phi_i(t)^T\Phi_i(t) \le I$, we have

$$\dot{V}(e(t)) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(x_p(t))\mu_j(x_p(t))e(t)^T\{\overline{A}_i^T P + P\overline{A}_i - K_j^T\overline{B}_i^T P - P\overline{B}_i K_j + \epsilon P U_i U_i^T P + \frac{1}{\epsilon}E_{ij}^T E_{ij} + \Delta E^T P + P\Delta E + DPD + \rho^2 PJJ^T P\}e(t) - e(t)^T DPDe(t) + \frac{1}{\rho^2}\omega(t)^T\omega(t) \quad (36)$$

for some positive constant $\epsilon > 0$ and $DPD > 0$. If the matrix inequality

$$\overline{A}_i^T P + P\overline{A}_i - K_j^T\overline{B}_i^T P - P\overline{B}_i K_j + \epsilon P U_i U_i^T P + \frac{1}{\epsilon}E_{ij}^T E_{ij} + \Delta E^T P + P\Delta E + DPD + \rho^2 PJJ^T P < 0 \quad (37)$$

is satisfied, then

$$\dot{V}(e(t)) < -e(t)^T DPDe(t) + \frac{1}{\rho^2}\omega(t)^T\omega(t) \quad (38)$$

Therefore, by integrating both sides of the inequality (38), the H_∞ criterion (32) is assured under the condition (37).

Next, the robust gain design is performed in the following. After pre-multiplying and post-multiplying (37) by X^{-1} and denoting $M_j = K_j X$, the inequality (37) is equivalent to

$$X\overline{A}_i^T + \overline{A}_i X - M_j^T\overline{B}_i^T - \overline{B}_i M_j + \epsilon U_i U_i^T + \frac{1}{\epsilon}(E_{1i}X - E_{2i}M_j)^T(E_{1i}X - E_{2i}M_j) + X\Delta E^T + \Delta E X + XDX^{-1}DX + \rho^2 JJ^T < 0 \quad (39)$$

Applying Schur complement to (39), the LMI stability condition (33) is obtained. Therefore, if there exists a feasible solution satisfying the LMIs (33), the non-VDV fuzzy output regulator (17) can drive the system to the desired operational point with $y(t) = y_d$ even if the system uncertainty and disturbance are considered. The effect of disturbance is attenuated to $\frac{1}{\rho}$ level.

5. NUMERICAL SIMULATIONS

In this section, the VDV and non-VDV based T-S fuzzy output regulators will be applied on a DC-DC buck converter to verify the theoretical validity. Moreover, some comparisons are performed to demonstrate differences between the two proposed regulation control methods and to show the benefits of the non-VDV based T-S fuzzy regulator. Consider the DC-DC buck converter (Sun and Grotstollen, 1992) with the equivalent circuit illustrated in Fig. 3. The DC-DC buck converter is described by the dynamics:

$$\dot{I}_L(t) = -\frac{I_L(t)}{L}\left(R_L + \frac{RR_C}{R+R_C}\right) - \frac{V_c(t)R}{L(R+R_C)} + \frac{u(t)(V_{in}+V_D-R_M I_L(t))}{L} - \frac{V_D}{L}$$
$$\dot{V}_c(t) = \frac{I_L(t)R}{C(R+R_C)} - \frac{V_c(t)}{C(R+R_C)}$$
$$V_o(t) = \frac{RR_C}{R+R_C}I_L(t) + \frac{R}{R+R_C}V_c(t) \quad (40)$$

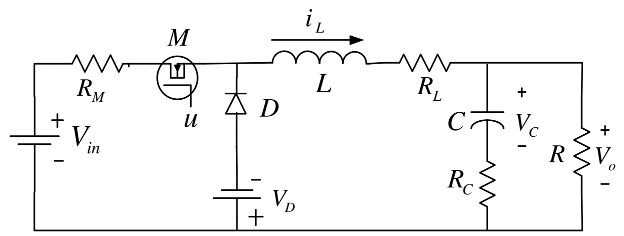

Fig. 3. The equivalent circuit of a DC-DC buck converter

where $I_L(t)$ is the inductor current; $V_c(t)$ is the capacitor voltage; $V_o(t)$ is actual output voltage; V_{in} is a voltage source; V_D is the forward voltage of the diode; $u(t)$ is a duty ratio of the PWM signal to control the power MOSFET; R is the load resistance; L is the inductance; C is the capacitance; R_C is the capacitor resistance; R_L is the inductor resistance; and R_M is the MOSFET resistance.

In general, voltage regulation is the most common objective for the DC-DC converter. To this end, we define an error state $x_e(t)$ by the following dynamic equation:

$$\dot{x}_e(t) = V_d - V_o(t)$$

where V_d is the desired voltage. Let $x(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T = [I_L(t)\ V_c(t)\ x_e(t)]^T$, the overall system dynamics is written in the form:

$$\dot{x}(t) = \begin{bmatrix} -\frac{1}{L}\left(R_L + \frac{RR_C}{R+R_C}\right) & -\frac{R}{L(R+R_C)} & 0 \\ \frac{R}{C(R+R_C)} & -\frac{1}{C(R+R_C)} & 0 \\ -\frac{RR_C}{R+R_C} & -\frac{R}{R+R_C} & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{L}(V_{in}+V_D-R_M x_1(t)) \\ 0 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} -\frac{V_D}{L} \\ 0 \\ V_d \end{bmatrix}$$
$$\equiv \overline{A}(x_p(t))x(t) + \overline{B}(x_p(t))u(t) + \overline{\xi} \quad (41)$$

where $x_1(t)$ is well-defined in the discussion region Ω; and the system matrices $\overline{A}(x_p(t)), \overline{B}(x_p(t))$, $\overline{\xi}$ are thus defined by (41). From the state-space model form, the state $x_1(t)$ is taken as the premise variable $z(t)$. According to the fuzzy modeling method (Lian *et al.*, 2001), the membership function of $z(t)$ is chosen such that $\overline{A}(x_p(t)) = \sum_{i=1}^{r}\mu_i(z(t))\overline{A}_i$, and $\overline{B}(x_p(t)) = \sum_{i=1}^{r}\mu_i(z(t))\overline{B}_i$. Then, (41) can be exactly represented by the T-S fuzzy model as follows:

Plant Rule i :
IF $x_1(t)$ is F_i
THEN $\dot{x}(t) = \overline{A}_i x(t) + \overline{B}_i u(t) + \overline{\xi},\ i = 1, 2 \quad (42)$

where the fuzzy set are $F_1 = \{about\ \alpha\ mA\}$ and $F_2 = \{about\ \beta\ mA\}$ which corresponding membership functions are denoted below:

$$F_1(z(t)) = \frac{x_1(t)-\beta}{\alpha-\beta}, \quad F_2(z(t)) = \frac{\alpha - x_1(t)}{\alpha-\beta}$$

where $\alpha \equiv max_{x_1\in\Omega}x_1(t)$, $\beta \equiv min_{x_1\in\Omega}x_1(t)$, $x_1(t) = F_1\alpha + F_2\beta$, $F_1 + F_2 = 1$. The subsystem matrices are given by:

$$\overline{A}_1 = \overline{A}_2 = \begin{bmatrix} -\frac{1}{L}\left(R_L + \frac{RR_C}{R+R_C}\right) & -\frac{R}{L(R+R_C)} & 0 \\ \frac{R}{C(R+R_C)} & -\frac{1}{C(R+R_C)} & 0 \\ -\frac{RR_C}{R+R_C} & -\frac{R}{R+R_C} & 0 \end{bmatrix},$$

$$\overline{B}_1 = \begin{bmatrix} \frac{1}{L}(V_{in} + V_D - R_M\alpha) \\ 0 \\ 0 \end{bmatrix},$$

$$\overline{B}_2 = \begin{bmatrix} \frac{1}{L}(V_{in} + V_D - R_M\beta) \\ 0 \\ 0 \end{bmatrix}, \ \overline{\xi} = \begin{bmatrix} -\frac{V_D}{L} \\ 0 \\ V_d \end{bmatrix}.$$

On the other hand, from (6), the constraint of the VDVs is

$$\begin{bmatrix} \frac{1}{L}(V_{in} + V_D - R_M x_1(t)) \\ 0 \\ 0 \end{bmatrix} (\tau(t) - u(t)) = \begin{bmatrix} -\frac{1}{L}\left(R_L + \frac{RR_C}{R+R_C}\right) & -\frac{R}{L(R+R_C)} & 0 \\ \frac{R}{C(R+R_C)} & -\frac{1}{C(R+R_C)} & 0 \\ -\frac{RR_C}{R+R_C} & -\frac{R}{R+R_C} & 0 \end{bmatrix} \begin{bmatrix} x_{1d} \\ x_{2d} \\ x_{3d} \end{bmatrix} + \begin{bmatrix} -\frac{V_D}{L} \\ 0 \\ V_d \end{bmatrix} \tag{43}$$

where $x_d = [x_{1d} \;\; x_{2d} \;\; x_{3d}]^T$. Applying the partition as (12) on the above equations yields the following conditions for determining x_d:

$$\frac{R}{C(R+R_C)}x_{1d} - \frac{1}{C(R+R_C)}x_{2d} = 0$$
$$\frac{RR_C}{R+R_C}x_{1d} + \frac{R}{R+R_C}x_{2d} = V_d \tag{44}$$

This implies that $x_{1d} = \frac{V_d}{R}$, $x_{2d} = V_d$, and x_{3d} is not limited. Here we set $x_{3d} = 0$ for simplification. Meanwhile, we can easily obtain the control input:

$$u(t) = -\sum_{i=1}^{2} \mu_i(z(t))K_i e(t) + \frac{1}{V_{in} + V_D - R_M x_1(t)} \times \left\{ \left(R_L + \frac{RR_C}{R+R_C}\right) x_{1d} + \frac{R}{R+R_C}x_{2d} + V_D \right\} \tag{45}$$

Obviously, if there exist parametric uncertainties, the solutions of x_{1d}, x_{2d}, u are unavailable. This means that the VDV-based fuzzy output regulator fails to control uncertain systems. To show the benefits of the proposed non-VDV fuzzy regulator, the uncertain buck converter is considered. The system parameters are set to $V_{in} = 30V$; $V_D = 0.82V$; $R = 10\Omega$; $L = 3mH$; $C = 470\mu F$; $R_C = 0.162\Omega$; $R_L = 0.0485\Omega$; $R_M = 0.27\Omega$, while all system parameters are uncertain but bounded within 1% of their nominal values. In other words, the norm-bounded condition of uncertainties is regarded with

$$U_1 = U_2 = I_{3\times 3},$$
$$E_{11} = 0.01\overline{A}_1, \ E_{12} = 0.01\overline{A}_2,$$
$$E_{21} = 0.01\overline{B}_1, \ E_{22} = 0.01\overline{B}_2.$$

Moreover, the external disturbance is assumed as $\omega(t) = 2\sin(200\pi t)$ with $J = [0 \ 1 \ 0]^T$. For the T-S fuzzy model (42), the fuzzy parameters are chosen as $\alpha = 5$ and $\beta = 0.01$.

First, the fixed regulation is considered to drive the buck converter to generate the desired output $y_d = 12V$. For the non-VDV fuzzy output regulator, the control parameters are set with $D = diag\{10, 8, 5\}$, $\Delta E = diag\{48, 48, 20\}$, $\rho = 60$, $\epsilon = 2^{-4}$. After solving the LMIs (33) in Theorem 2, the feasible controller gains are obtained as follows:

$$K_1 = [\,0.5605 \quad 0.2127 \quad -31.2302\,],$$
$$K_2 = [\,0.5605 \quad 0.2127 \quad -31.2302\,].$$

By letting the initial states of the buck converter to be $x(0) = [3 \;\; 5 \;\; 0.1]^T$. The output regulation results are obtained in Fig. 4 and Fig. 5. The output quickly tends to the desired value without solving the VDVs and performing coordinate transformation. Next, a piecewise constant signal (V_d is changed from 12V → 24V → 12V) is also taken as the desired output. Based on the above controller setting, the control responses are shown in Fig. 6 and Fig. 7. The output also quickly tracks the desired piecewise constant signal without VDV calculation and controller redesign.

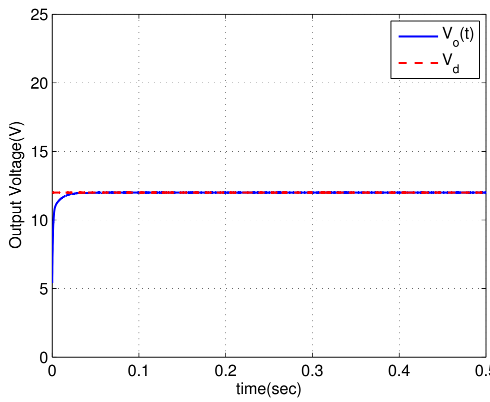

Fig. 4. Result of the fixed regulation of buck converter.

To demonstrate the benefit of the non-VDV fuzzy regulator, the VDV-based fuzzy controller is also applied on the buck converter. The VDVs are solved from the nominal system by omitting uncertainty and disturbance. The controller gains of the VDV-based fuzzy regulator is solved according to LMIs (11) and we obtain

$$K_1 = [\,-0.0097 \ 0.0832 \ -2.3333\,],$$
$$K_2 = [\,-0.0097 \ 0.0832 \ -2.3333\,].$$

For the piecewise constant output regulation as Fig. 6, the VDVs of (44) should be solved simultaneously according to the varying desired command $y_d(t)$. However, the VDVs cannot be exactly obtained due to considering uncertainty.

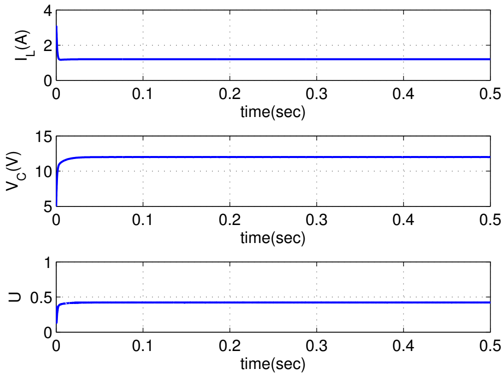

Fig. 5. State responses of the buck converter for the fixed regulation.

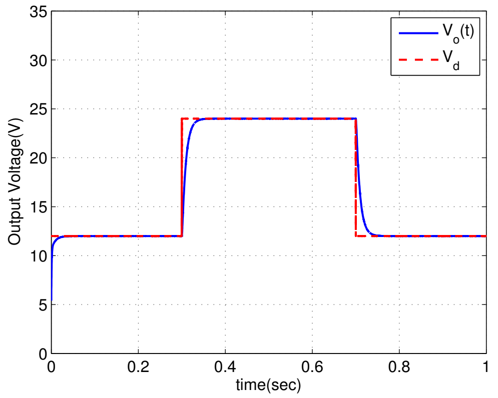

Fig. 6. The piecewise constant regulation of buck converter by using non-VDV T-S fuzzy regulator.

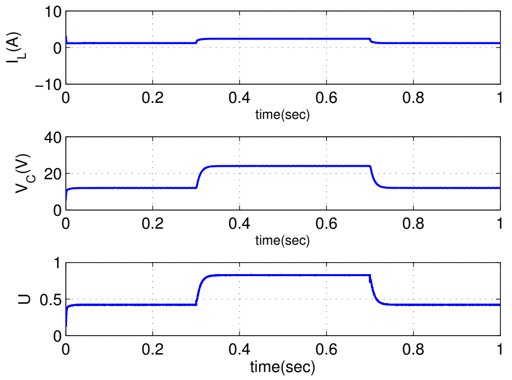

Fig. 7. State responses of the piecewise constant regulation of buck converter by using non-VDV T-S fuzzy regulator.

As a result, the VDV-based fuzzy regulator is difficultly realized. Under the same initial condition, the results of the VDV-based fuzzy regulation control for the piecewise constant regulation are obtained in Fig. 8 and Fig. 9.

Compared with Figs. 6 and 7, the VDV-based fuzzy regulator lead to large oscillation in the transient response. In addition, since the input source voltage V_{in} is usually not fixed from AC voltage rectification, the variation

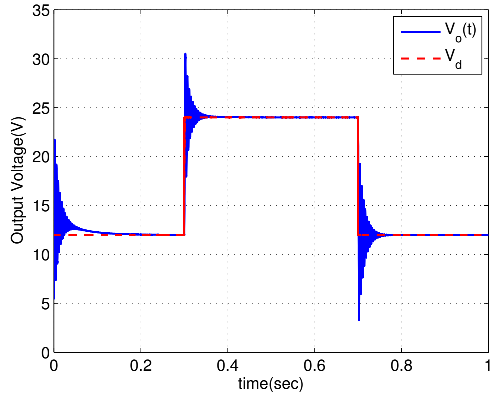

Fig. 8. The piecewise constant regulation of buck converter by using VDV-based T-S fuzzy regulator.

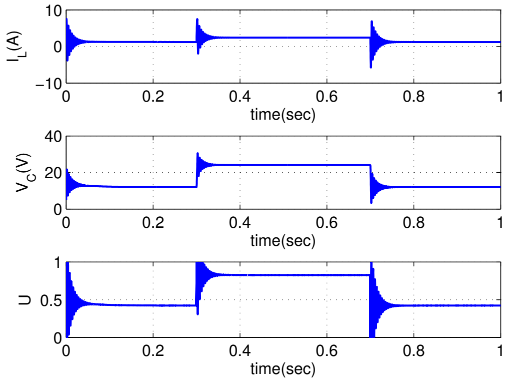

Fig. 9. State responses of the piecewise constant regulation of buck converter by using VDV-based T-S fuzzy regulator.

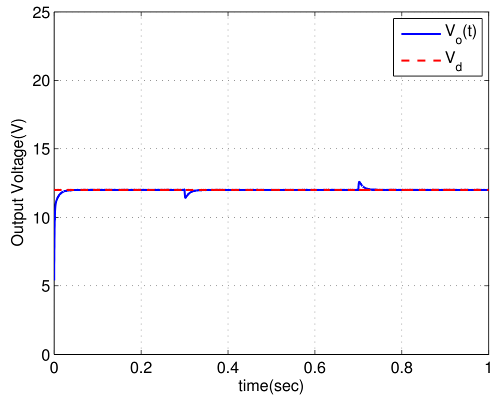

Fig. 10. The fixed regulation of buck converter when V_{in} is changed from 30V $\rightarrow$ 20V $\rightarrow$ 30V for $R = 10\Omega$ by using non-VDV T-S fuzzy regulator.

of the input voltage is taken as an uncertainty. Also, the output load changes in various applications of a buck converter. To show and observe the benefits of the proposed control method, we further consider piecewise varying input voltage source V_{in} and output load R which are worst cases with suddenly changing input voltage (due to suddenly changed AC line voltage) and load (due to

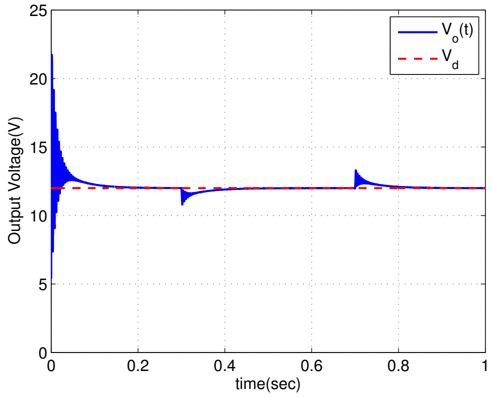

Fig. 11. The fixed regulation of buck converter when V_{in} is changed from 30V $\rightarrow$ 20V $\rightarrow$ 30V for $R = 10\Omega$ by using VDV-based T-S fuzzy regulator.

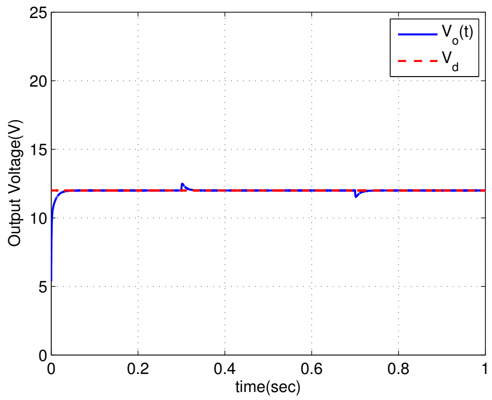

Fig. 12. The fixed regulation of buck converter when R is changed from 8 Ω $\rightarrow$ 10 Ω $\rightarrow$ 8 Ω for V_{in}= $30V$ by using non-VDV T-S fuzzy regulator.

users' applications). Comparisons of non-VDV and VDV based fuzzy output regulators for coping with piecewise varying input voltage and output load are made and shown in Figs. 10 - 13. Obviously, the non-VDV based fuzzy regulator has better control performance than the VDV based fuzzy regulator. The effect of the uncertainty and disturbance is attenuated from the proposed gain design.

Furthermore, to test the robustness during the starting up procedure of regulation, a worst case with V_{in} changing from 20V $\longrightarrow$ 30V $\longrightarrow$ 20V respectively at 0.01 and 0.06 sec is applied to the DC-DC buck converter before the regulation is settled by the T-S fuzzy output regulators. For the reference command $y_d = 12V$ and the load $R = 10\Omega$, the results by using the non-VDV and VDV fuzzy output regulators are shown in Figs. 14 and 15, respectively. In comparison, the VDV-based fuzzy regulator results in large oscillation and steady-state error due to lacking an exact solution of the VDVs for uncertain model with disturbance. In contrast, the non-VDV fuzzy regulator has better robust performance with short settling time, small overshot, and fast recovery. As a result, the proposed non-VDV fuzzy output regulator is better and simpler than the VDV-based fuzzy regulator.

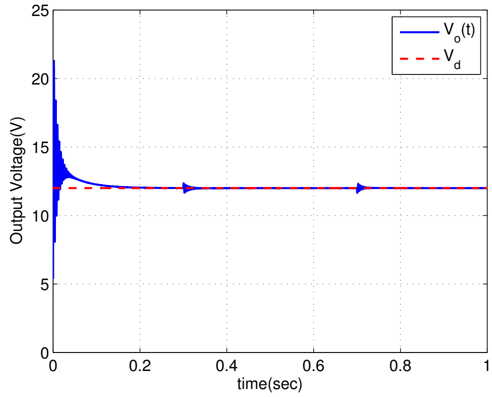

Fig. 13. The fixed regulation of buck converter when R is changed from 8 Ω $\rightarrow$ 10 Ω $\rightarrow$ 8 Ω for V_{in}= $30V$ by using VDV-based T-S fuzzy regulator.

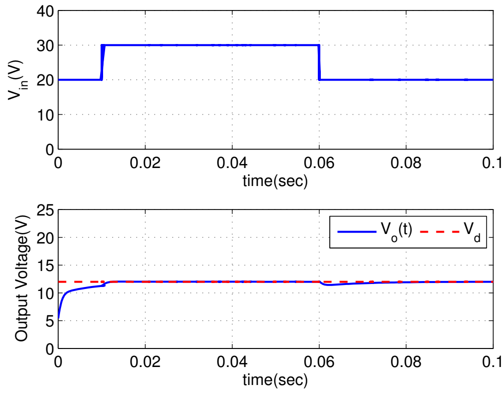

Fig. 14. The fixed regulation of buck converter when V_{in} is changed from 20V $\rightarrow$ 30V $\rightarrow$ 20V respectively at 0.01 and 0.06 sec by using non-VDV T-S fuzzy regulator.

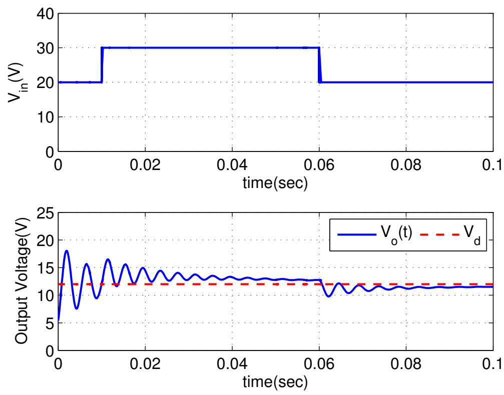

Fig. 15. The fixed regulation of buck converter when V_{in} is changed from 20V $\rightarrow$ 30V $\rightarrow$ 20V respectively at 0.01 and 0.06 sec by using VDV-based T-S fuzzy regulator.

6. CONCLUSION

In this paper, the concept of robust integral fuzzy output regulator design with and without solving VDVs has been discussed. In comparison, the non-VDV regulator removes the main drawbacks of the VDV-based fuzzy regulator —

the error coordinate transformation and VDV calculation so that only the system states and the output regulation objective are required. Furthermore, the uncertainty and disturbance are allowed by the proposed non-VDV fuzzy regulator, but the traditional VDV-based fuzzy regulator cannot. The robustness can be enhanced by proper LMI gain design. From the simulation results, H_∞ performances have been shown by using the non-VDV fuzzy regulator. In contrast, the VDV-based fuzzy regulator has lower robustness.

ACKNOWLEDGEMENTS

This work was supported by Ministry of Science and Technology, R.O.C., under grants MOST 102-2221-E-033-053 and MOST 103-2221-E-027-074.

REFERENCES

Ameur, A., Mokhtari, B., Essounbouli, N., and Nollet, F., (2013). Modified direct torque control for permanent magnet synchronous motor drive based on fuzzy logic torque ripple reduction and stator resistance estimator, *Journal of Control Engineering and Applied Informatics*, 15(3), 45–52.

Boyd, S., Ghaoui, L.El, Feron, E., and Balakrishnan, V. *Linear Matrix Inequalities in System and Control Theory.* Philadelphia, PA: SIAM, 1994.

Byrnes, C.I., and Isidori, A. (2000). Output regulation for nonlinear systems: an overview, *Int. J. of Robust and Nonlinear Control*, 10(5), 323–337.

Chen, B.S., Tseng, C.S., and Uang, H.J. (2000). Mixed H_2/H_∞ fuzzy output feedback control design for nonlinear dynamic systems: An LMI approach, *IEEE Trans. Fuzzy Syst.*, 8(3), 249–265.

Chiu, C.S., Chiang, T.S., and Liu, P. (2004). Output regula-tion control via fuzzy operational point reference approach, *2004 IEEE International Conference on Fuzzy Systems(FUZZ 2004)*, 1613–1617.

Chuang, H.F., Wang, W.J., Sun, Y.J., and Chen, Y.J. (2011). T-S fuzzy model based H_∞ finite-time synchronization design for chaotic systems, *Int. J. Fuzzy Syst.*, 13(4), 358–368.

Cuibus, O. and Letia, T. (2012). Cooperative control achieved by generic genetic fuzzy logic, *Journal of Control Engineering and Applied Informatics*, 14(3), 43–53.

Huang, J. *Nonlinear Output Regulation: Theory and Applications.* Philadelphia, PA: SIAM, 2004.

Jabri, D., Guelton, K., Manamanni, N., Jaadari, A., and Chinh, C.D., (2012). Robust stabilization of nonlinear systems based on a switched fuzzy control law, *Journal of Control Engineering and Applied Informatics*, 14(2), 40–49.

Khelchandra, T., Huang, J., and Debnath, S. (2014). Path planning of mobile robot with neuro-genetic-fuzzy technique in static environment, *Int. J. Hybrid Intell. Syst.*, 11(2), 71–80.

Lian, K.Y., Chiang, T.S., Chiu, C.S., and Liu, P. (2001). Synthesis of fuzzy model-based designs to synchronization and secure communications for chaotic systems, *IEEE Trans. SMC Part B, Cybernetics,*, 31(1), 66–83.

Lian, K.Y., Hung C.Y., and Chiu, C.S. (2005). Induction motor control with friction compensation: an approach of virtual-desired-variable synthesis, *IEEE Trans. Power Electronics*, 20(5), 1066–1074.

Liu, X., and Zhang, Q. (2003). New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI, *Automatica*, 39(9), 1571–1582.

Ma, K., and Fei, J. (2015). Model reference adaptive fuzzy control of a shunt active power filter, *Journal of Intelligent and Fuzzy Systems*, 28, 485–494.

Ma, X.J., and Sun, Z.Q. (2000). Output tracking and regulation on nonlinear system based on Takagi-Sugeno fuzzy model,*IEEE Trans. SMC Part B, Cybernetics*, 30(1), 47–59.

Marcu, E. (2011). Fuzzy logic approach in real-time UAV control, *Journal of Control Engineering and Applied Informatics*, 13(1), 12–17.

Meda-Campaña, J.A., Gõmez-Mancilla, J.C., and Castillo-Toledo, B. (2012). Exact output regulation for nonlinear systems dscribed by Takagi-Sugeno fuzzy models, *IEEE Trans. Fuzzy Syst.*, 20(2), 235-247.

Sadeghi, M.S., and Vafamand, N. (2014). More relaxed stability conditions for fuzzy TS control systems by optimal determination of membership function information, *Journal of Control Engineering and Applied Informatics*, 16(2), 67–77.

Sun, J., and Grotstollen, H. (1992). Averaged modelling of switching power converters: reformulation and theoretical basis, *Power Electronics Specialists Conference, 1992. PESC '92 Record.,* 2, 1165–1172.

Takagi, T., and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Syst., Man and Cybernetics*, 15(1), 116–132.

Tanaka, K., and Wang, H.O. *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach.* John Wiley & Sons, Canada. 2001.

Tanaka, K., Ikeda, T., and Wang, H.O. (1998). A unified approach to controlling chaos via an LMI-based fuzzy control system design, *Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on* , 45(10), 1021–1040.

Tasar, B., Giiler, H., and Özdemir M., (2015). The investigation of fuzzy logic-PI based load frequency control of Keban HEPP, *Journal of Control Engineering and Applied Informatics*, 17(4), 71–80.

Tuan, H.D., Apkarian, P., Narikiyo, T., and Yamamoto, Y. (2001). Parameterized linear matrix inequality techniques in fuzzy control system design, *IEEE Trans. Fuzzy Syst.*, 9(2), 324–332.

Wallam, F., and Abbasi, A.R. (2014). Evaluating the transient handling capability of a fuzzy logic controller for a pressurized heavy water reactor, *Journal of Control Engineering and Applied Informatics*, 16(2), 40–48.

Wang, H.O., Tanaka, K., and Griffin, M. (1996). An approach to fuzzy control of nonlinear systems: Stability and design issues, *IEEE Trans. Fuzzy Syst.*, 4(1), 14–23.

Yuan, L.Y., and Li, S.Y., Missle guidande law design using robust nonlinear output regulation and T-S model, *Industrial Electronics and Applications, 2007. ICIEA 2007. 2nd IEEE Conference on,* 1037–1042.