Lyapunov-Based Nonlinear Disturbance Observer for n-Link Flexible Joint Robot Manipulators

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Abstract: In this paper, the nonlinear disturbance observer (NDO) is presented for serial flexible joint robot manipulators (FJM). To this end, a planar robot manipulator with n flexible joints is considered. After deriving the general form of dynamic equations for serial n-link FJM, a nonlinear disturbance observer is proposed on the basis of the system dynamic equations. The main challenge here is to obtain the acceptable range of the observer gain which guarantees the stability condition. So by choosing the proper Lyapunov candidate, stability analysis of the proposed observer is performed by using the Lyapunov's direct method. Since the proposed NDO makes the system robust against internal and external disturbances, no accurate dynamic model is required to achieve the high precision motion control. The effectiveness of the proposed observer in the regulation problem is investigated by numerical simulations for a two-link robot manipulator. To this end, an optimal LQR controller is designed to stabilize the system besides the optimal state observer in order to estimate the angular velocity of the links and motors. Simulation results show the ability of the proposed method to properly estimate and compensate different disturbances.

*Keywords: N*onlinear disturbance observer, Lyapunov's direct method, flexible joint manipulators, disturbances, LQR controller, state observer.

1. INTRODUCTION

Joint flexibility in many robot manipulator systems, introduced by the elements of the drive systems, such as shafts, belts, gears, and chains, cannot be neglected. The effect of flexibility on robot performance will be considerable, particularly, when the manipulator is used to perform a high-speed motion and carry a heavy load. However, the flexibility of the joint in manipulators causes difficulty in modelling the manipulator's dynamics and becomes a potential source of uncertainty that can degrade the performance of the manipulator and in some cases can even destabilize the system (Talebi et al., 2002). Therefore, the joint flexibility must be considered in control design in order to achieve the desired performance. On the other hand, flexible joint manipulators (FJMs) are subject to different types of disturbances that adversely affect their performance such as repeatability and positioning accuracy. There are many sources that exert disturbances to the system. Unmodeled dynamics due to the joints flexibility and uncertainties due to unknown parameters or parameter variations are known as internal disturbance sources. In contrast, external forces on the end-effector, friction in joints, and torque ripple of the actuators are known as external disturbance sources. In fact, disturbances affect the performance of the system significantly. So, in order to reduce or estimate the disturbance effects, a large number of methods have been designed during the last three decades.

The sliding mode control (Shendge et al., 2011; Moldoveanu et al., 2005), Kalman filter (Park et al., 2013), adaptive control (Chien et al., 2007), H-infinity controller (Taghirad et al., 2001), nonlinear optimal control approach (Korayem et al., 2015), active disturbance rejection (Kordaz et al., 2012), robust input shaping technique (Alici et al., 2006), adaptive sliding control (Huang et al., 2004), linear feed-forward torque using the principle of work and energy (Salmasi et al., 2009), Fuzzy logic control (Zirkohi et al., 2013), neural network (Talebi et al., 2002), adaptive neural network sliding mode controller (Sefriti et al., 2012) and decentralized direct adaptive fuzzy control (Fateh et al., 2013) are among the disturbance rejection techniques proposed in the literature for flexible joint manipulators and robot manipulators. An alternative to these methods that has received much attention in recent years is the use of disturbance observers proposed first time by Ohnishi et al., 1996. The idea behind the disturbance observer is to lump all the external and internal unknown torques/forces acting on the manipulator into a single disturbance term, then estimate this unknown term using the disturbance observer. The output of the disturbance observer can be used as a command to compensate the disturbances.

The disturbance observers have many applications in robotics such as, decoupling dynamics of the joints in order to design a simple controller for each DOF (Zhongyi et al., 2008), estimating and compensating the friction in order to improve the manipulator tracking performance (Bona et al., 2005), using in time-delayed bilateral teleportations in order to improve the transparency of telerobotic systems (Mohammadi et al., 2011), employing in sensorless force control systems in order to estimate the contact forces (Shimada et al., 2010), using in shadow robot systems (Katsura at al., 2010) and fault detection systems (Sneider et al., 1996; Mohammadi et al., 2013).

The methods used to design the disturbance observers for robot manipulators are classified to linear and nonlinear methods. But most of the existing literature uses the linearized models or linear system techniques (Kim et al., 2003; Yun et al., 2014). In order to overcome the limitations of linear disturbance observer (Nikoobin et al., 2009; Yang at al., 2006) for the highly nonlinear and coupled dynamics of robotic manipulators, some nonlinear disturbance observers (NDO) have been developed for the flexible joint manipulator. Many works done in this area are based on the state observer design for the augmented system in which disturbances are taken as states (Qin et al., 2012). Variable structure disturbance observers (Lee et al., 2007) and high gain observers (Morales et al., 2001) are designed based on this method.

Another approach for designing the NDO for robotic manipulators is proposed by (Chen et al., 2000), in which the stability of the proposed observer is verified by using Lyapunov's theorem for a two links robot manipulator. Later, Nikoobin et al. generalized Chen's solution to n-link planar manipulators (Nikoobin et al., 2009). After that, Mohammadi et al. (2013), extended NDO proposed by (Nikoobin et al., 2009) for a general robot manipulator without restrictions on the types of the joints and the manipulators configuration. In all the previous works dealing with NDO for robot manipulators (Mohammadi et al., 2013; Nikoobin et al., 2009; Chen et al., 2000), the flexibility in the joints have not been considered yet. The FJM is an underactuated system in which the number of actuators is lower than the degrees of freedom. So deriving the NDO formulation and verifying the stability for the FJM is different with the rigid manipulator. The proposed NDO in comparison with the NDO methods based on the state observer such as variable structure and high gain observer methods which estimate the whole applied disturbances as a single signal, can estimate the applied disturbance on each link and motor separately. So the presented method can be employed in sensorless force control systems in order to estimate the contact forces, or in the fault detection system.

The other challenge in the flexible joint manipulator is the measurement reduction. Reduction in the number of feedback quantities has been always a goal in control design, especially for industrial robots. The works done in this field can be classified into two methods, avoiding the rate measurement by changing the control strategy and using the state observer in order to reduce the number of measurements reviewed completely in (Ozgoli et al., 2006). As a common method the velocity of motors and links are estimated using the different type of observers such as LQG/LTR techniques (Lahdhiri at al., 1999), robust dynamic feedback tracking controller (Chang at al., 2011) and neural network observer (Abdollahi at al., 2006).

In this paper, a nonlinear disturbance observer is proposed for a general n-link planar robot manipulator with flexible joints. Here, on the base of the Spong model of FJM which has no zero dynamics (Luca, 2000), a proper Lyapunov function is chosen and the sufficient condition for stabilizing the system is proved. The paper is organized as follow: A Dynamic model of an n-link planar flexible joint manipulator is presented in Section 2. The proposed NDO is introduced in section 3 and the stability analysis of the proposed observer based on the Lyapunov's direct method is presented in Section 4. By designing the LQR state feedback controller besides the optimal state observer to estimate the velocities, the effectiveness of the proposed method to eliminate the different disturbances on the system is shown in Section 5. Lastly, Section 6 includes the concluding remarks.

2. DYNAMIC MODEL OF FJM

In this paper, a nonlinear disturbance observer is presented for a general n-link planar robot manipulator with flexible joints. Here, on the base of the Spong model of FJM which has no zero dynamics (Luca, 2000), a proper Lyapunov function is chosen and the sufficient condition for stabilizing the system is proved. The paper is organized as follow: A Dynamic model of an n-link planar flexible joint manipulator is presented in Section 2. The proposed NDO is introduced in section 3 and the stability analysis of the proposed observer based on the Lyapunov's direct method is presented in Section 4. By designing the LQR state feedback controller besides the optimal state observer to estimate the velocities, the effectiveness of the proposed method to eliminate the different disturbances on the system is shown in Section 5. Lastly, Section 6 includes the concluding remarks.

The structure of a multiple flexible joints manipulator consisting of n flexible revolute joints and n rigid links is shown in Fig. 1. The links are cascaded in serial fashion and are actuated by individual motors and gearboxes. A payload of mass m_p is connected to the distal link. Let q_{Li}, $i = 1, 2, \dots, n$, denote the generalized coordinate of the ith link, q_{ai}, $i = 1, 2, \dots, n$, denote the generalized coordinate of the ith actuator and u_i, $i = 1, 2, \dots, n$, denote the applied torque of the ith actuator of the robot.

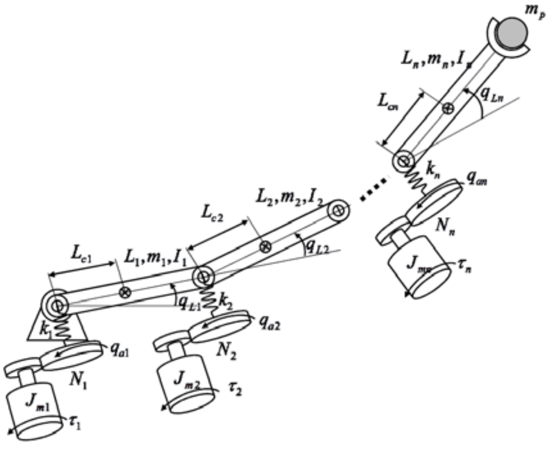

Fig. 1. Serial n-link planar flexible joint manipulator.

The flexible joint is simplified as a linear torsional spring with the spring constant k_i, $i = 1, 2, \dots, n$. Due to the elastic coupling between actuators and links, the n-link flexible joint robot has $2n$ degrees of freedom.

Let $q_L = \begin{bmatrix} q_{L1} & q_{L2} & \dots & q_{Ln} \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^{n\times 1}$ and $q_a = \begin{bmatrix} q_{a1} & q_{a2} & \dots & q_{an} \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^{n\times 1}$ be a set of generalized coordinates.

Based on the dynamic model of the flexible joint robot developed by (Spong, 1987), the dynamic equation of the FJM can be described as

$$
\begin{aligned}
&M_{act}\left(q_L\right)\ddot{q}_L + b_{act}\left(q_L,\dot{q}_L\right) + h_{act}\left(q_L\right) \\
&+K_{act}\left(q_L - q_a\right) = f_{dis.1} \\
&J_{act}\ddot{q}_a + K_{act}\left(q_a - q_L\right) = u + f_{dis.2}
\end{aligned} \tag{1}
$$

in which

$$
\begin{aligned}
J_{act} &= N_{act}^2 J_{mact} \\
u &= N_{act}\tau
\end{aligned} \tag{2}
$$

where $M_{act}(q_L) \in \mathrm{R}^{n\times n}$ is positive definite symmetric inertia matrix, $b\left(q_L,\dot{q}_L\right) \in \mathrm{R}^{n\times 1}$ is the vector of Coriolis and centrifugal forces, $h_{act}(q_L) \in \mathrm{R}^{n\times 1}$ is the vector of gravitational forces, $K_{act} = \mathrm{diag}(k_1, k_2, \dots, k_n) \in \mathrm{R}^{n\times n}$ is the joint torsional stiffness matrix, $J_{mact} = \mathrm{diag}(J_{m1}, J_{m2}, \dots, J_{mn}) \in \mathrm{R}^{n\times n}$ is the motor inertia matrix, $N_{act} = \mathrm{diag}(N_1, N_2, \dots, N_n) \in \mathrm{R}^{n\times n}$ is the gear ratio matrix, $\tau = \begin{bmatrix} \tau_1 & \tau_2 & \dots & \tau_n \end{bmatrix}^{\mathrm{T}} \in \mathrm{R}^{n\times 1}$ is the vector of motor torque, and $f_{dis} \in \mathrm{R}^{n\times 1}$ is the disturbance vector which contains the torque due to the unknown load, friction force, external force, torque ripple and unmodeled dynamics. The subscript "1" denotes the disturbance on the links, subscript "2" denotes the disturbance on the actuators and the subscript "*act*" denotes the actual value of parameters.

Property 1: $M\left(q_L\right)$ is symmetric positive definite, and bounded below and above, i.e., $\exists \alpha \ge \beta > 0$, such that $\beta I_n \le M\left(q_L\right) \le \alpha I_n, \forall q_L \in \mathrm{R}^n$, where I_n is the $n\times n$ identity matrix.

Property 2: The dynamic model of the flexible joint manipulator given in (1) is input-state linearizable and has no zero dynamics, so the closed loop system is internally stable (Luca, 2000).

Assumption 1: By defining the $q = \begin{bmatrix} q_L^{\mathrm{T}} & q_a^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, the angular velocity vector $\dot{q}$ lies in the known bounded set $\Omega_{\dot{q}}$, it means $\dot{q} \in \Omega_{\dot{q}} \overset{\Delta}{=} \left\{ \dot{\mathrm{q}} : \left\| \dot{\mathrm{q}} \right\| \le \dot{\mathrm{q}}_{\max} \right\}$.

Assumption 2: Joint deflections are small, so that flexibility effects are limited to the domain of linear elasticity. The actuators' rotors are modelled as uniform bodies having their own center of mass on the rotation axis. Each motor is located on the robot arm in a position preceding the driven link (Siciliano at al., 2008).

In order to simplifying the computation in the next sections, inertia matrix $M\left(q_L\right)$ can be written as follow

$$
M\left(q_L\right) = B\bar{M}\left(q_L\right)A \tag{3}
$$

in which

$$
\bar{M} = \frac{1}{2}\begin{bmatrix}
M_1 & & & & & \\
X_1^1 C_2 & M_2 & & & \text{sym.} & \\
X_2^1 C_{2,3} & X_1^2 C_3 & M_3 & & & \\
X_3^1 C_{2,4} & X_2^2 C_{3,4} & X_1^3 C_4 & & & \\
X_4^1 C_{2,5} & X_3^2 C_{3,5} & X_2^3 C_{4,5} & \ddots & & \\
\vdots & \vdots & \vdots & & & \\
X_{n-1}^1 C_{2,n} & X_{n-2}^2 C_{3,n} & X_{n-3}^3 C_{4,n} & \cdots & M_n
\end{bmatrix}_{n\times n}, \tag{4}
$$

and

$$
A = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
1 & 1 & \ddots & \vdots \\
\vdots & \vdots & \ddots & 0 \\
1 & 1 & \cdots & 1
\end{bmatrix}_{n\times n}, B = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
0 & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 1 \\
0 & \cdots & 0 & 1
\end{bmatrix}_{n\times n}, \tag{5}
$$

where $C_i = \cos\left(q_{Li}\right), C_{i,j} = \cos\left(q_{Li} + q_{Li+1} + \ldots + q_{Lj}\right)$ and $M_{\mathrm{i}}, Y_{i,j}, X_i^j, i = 1\cdots\mathrm{n}$ are constant parameters, which depend on the masses of the links, payload and the lengths of the links. $\bar{M}\left(q_L\right)$ is also a symmetric matrix,

3. NONLINEAR DISTURBANCE OBSERVER

Assuming all states are available in output, the structure of a control system with disturbance observer is shown in Fig. 2, which indicates the principle of disturbance observer design.

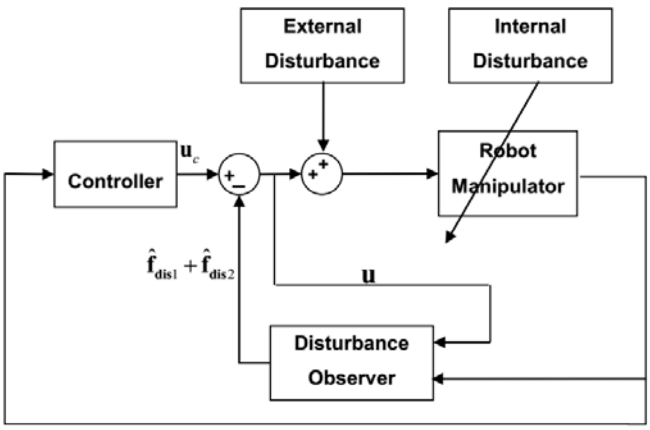

Fig. 2. The structure of disturbance observer.

By defining the nominal value of parameters, one can write (1) as

$$M(q_L)\ddot{q}_L + b(q_L,\dot{q}_L) + h(q_L) + K(q_L - q_a) = f_{dis1}$$
$$J\ddot{q}_a + K(q_a - q_L) = u + f_{dis2} \tag{6}$$

where M, b, h, K and J are the nominal values of M_{act}, b_{act}, h_{act}, K_{act} and J_{act} respectively.

Consequently, using (1) and (6), the disturbance vector can be written as

$$\begin{bmatrix} f_{dis.1} \\ f_{dis.2} \end{bmatrix} = \begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{q}_L \\ \ddot{q}_a \end{bmatrix} + \begin{bmatrix} K(q_L - q_a) \\ K(q_a - q_L) \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} + \begin{bmatrix} h \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ u \end{bmatrix}. \tag{7}$$

So, by defining the

$$f_{dis} = \begin{bmatrix} f_{dis.1} \\ f_{dis.2} \end{bmatrix},\ q = \begin{bmatrix} q_L \\ q_a \end{bmatrix}, \tag{8}$$

disturbance observer can be proposed as

$$\dot{\hat{f}}_{dis} = L(q,\dot{q})\left(\left(\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{q}_L \\ \ddot{q}_a \end{bmatrix} + \begin{bmatrix} K(q_L - q_a) \\ K(q_a - q_L) \end{bmatrix} + \begin{bmatrix} b+h \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ u \end{bmatrix}\right) - \hat{f}_{dis}\right), \tag{9}$$

where $L(q,\dot{q}) \in \mathrm{R}^{2n\times 2n}$, and $\hat{f}_{dis}$ is the estimated of real disturbance f_{dis}. Since, there is no prior information about the derivative of the disturbance, it is reasonable to suppose that the disturbance in (7) varies slowly (Mohammadi et al., 2013; Chen et al., 2000),

$$\dot{f}_{dis} = 0. \tag{10}$$

which implies that the disturbance varies slowly compared with the observer dynamics. But in Section 5, it will be shown that the proposed observer estimates also fast time varying disturbances. The observer error is defined as a difference between actual disturbance and estimated disturbance

$$e = f_{dis} - \hat{f}_{dis}. \tag{11}$$

By differentiation from observer error, dynamic of error can be written as

$$\dot{e} = \dot{f}_{dis} - \dot{\hat{f}}_{dis} = L\left(\hat{f}_{dis} - f_{dis}\right), \tag{12}$$

which it can be expressed as below

$$\dot{e} + Le = 0. \tag{13}$$

L must be chosen in such a way that the dynamic of error be asymptotically stable.

As it can be seen from (9), acceleration signals $\ddot{q}_L$ and $\ddot{q}_a$ are required to realize the disturbance observer. Since acceleration measurement is a hard task in many robotic applications, the problem is circumvented by defining an auxiliary variable $y = \hat{f}_{dis} - p(\dot{q})$. Differentiating the auxiliary variable with respect to time gives

$$\dot{\hat{f}}_{dis} = \dot{\psi} + \frac{\partial p}{\partial \dot{q}}\ddot{q}. \tag{14}$$

So, substituting for $\dot{\hat{f}}_{dis}$ from (14) into (9) gives

$$\dot{\psi} + \frac{\partial p}{\partial \dot{q}}\ddot{q} = -L(q,\dot{q})\left(\psi + p(\dot{q})\right) + L(q,\dot{q}) \times$$
$$\left(\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{q}_L \\ \ddot{q}_a \end{bmatrix} + \begin{bmatrix} b+h \\ 0 \end{bmatrix} + \begin{bmatrix} K(q_L - q_a) \\ K(q_a - q_L) \end{bmatrix} - \begin{bmatrix} 0 \\ u \end{bmatrix}\right). \tag{15}$$

By defining

$$\frac{\partial p}{\partial \dot{q}} = L(q,\dot{q})\begin{bmatrix} M & 0 \\ 0 & J \end{bmatrix}. \tag{16}$$

Equation (14) can be reduced to

$$\dot{\psi} = -L\psi + L\left(\begin{bmatrix} K(q_L - q_a) \\ K(q_a - q_L) \end{bmatrix} + \begin{bmatrix} b+h \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ u \end{bmatrix} - p(\dot{q})\right), \tag{17}$$

in which there is no acceleration signal, and estimated disturbance can be obtained by

$$\hat{\mathrm{f}}_{\mathrm{dis}} = \psi + \mathrm{p}(\dot{\mathrm{q}}). \tag{18}$$

Finally, as it can be shown in Fig. 2, one can write the control law as follow

$$u = u_c - \hat{f}_{dis1} - \hat{f}_{dis2}, \tag{19}$$

in which u is the final control signal, u_c is the controller signal, $\hat{f}_{dis1}$ and $\hat{f}_{dis2}$ are the estimated disturbance signals.

4. STABILITY ANALYSIS

The proposed NDO in (17) has two design parameters L and P. However, they are not independent and related to each other by (16), so one of them must be chosen in order to guarantee the asymptotical stability of the proposed NDO. In the following theorem a value for $p(\dot{q})$ is proposed to guarantee the stability of disturbance observer.

Theorem 1: If $p(\dot{q}) \in R^{2n\times 1}$ be defined as below

$$p(\dot{q}) = c\begin{bmatrix} \dot{q}_{L1} \\ \dot{q}_{L1} + \dot{q}_{L2} \\ \vdots \\ \dot{q}_{L1} + \dot{q}_{L2} + \ldots + \dot{q}_{Ln} \\ \dot{q}_{a1} \\ \dot{q}_{a2} \\ \vdots \\ \dot{q}_{an} \end{bmatrix}, \tag{20}$$

then asymptotical stability of NDO given by (17) is guaranteed by proper choice of parameter c.

Proof: Since derivative of $p(\dot{q})$ with respect to $\dot{q}$ can be expressed as below

$$\frac{\partial p(\dot{q})}{\partial \dot{q}} = c\begin{bmatrix} A_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} \end{bmatrix}, \tag{21}$$

in which $A_{n\times n}$ is defined in (5). So by substituting (20) into (16) one can write

$$L(q,\dot{q}) = c\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} M^{-1} & 0 \\ 0 & J^{-1} \end{bmatrix}. \tag{22}$$

Using (3), the inertia matrix can be written as follow

$$\begin{bmatrix} M(q_L) & 0 \\ 0 & J \end{bmatrix} = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \bar{M}(q_L) & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}, \tag{23}$$

so substituting (23) into (22) yields

$$L(q,\dot{q}) = c\begin{bmatrix} \bar{M}^{-1} & 0 \\ 0 & J^{-1} \end{bmatrix}\begin{bmatrix} B^{-1} & 0 \\ 0 & I \end{bmatrix}. \tag{24}$$

Since $\bar{M}(q_L)$ and J is positive definite for all q, a Lyapunov candidate can be proposed as

$$V(e,q) = e^T\begin{bmatrix} \bar{M} & 0 \\ 0 & J \end{bmatrix}e. \tag{25}$$

The time derivative of the Lyapunov function is

$$\frac{\mathrm{d}V(e,q)}{\mathrm{d}t} = \frac{\partial V(e,q)}{\partial e}\dot{e} + \frac{\partial V(e,q)}{\partial q}\dot{q}. \tag{26}$$

By using (13) and (24), the first term can be determined as

$$\begin{aligned} \frac{\partial V}{\partial e}\dot{e} &= 2e^T\begin{bmatrix} \bar{M} & 0 \\ 0 & J \end{bmatrix}\left(-c\begin{bmatrix} \bar{M}^{-1} & 0 \\ 0 & J^{-1} \end{bmatrix}\begin{bmatrix} B^{-1} & 0 \\ 0 & I \end{bmatrix}e\right) \\ &= -e^T\begin{bmatrix} 2cB^{-1} & 0 \\ 0 & 2cI \end{bmatrix}e = -e^T\begin{bmatrix} 2c\bar{B}^{-1} & 0 \\ 0 & 2cI \end{bmatrix}e, \end{aligned} \tag{27}$$

in which

$$2c\bar{B}^{-1} = \begin{bmatrix} 2c & -c & 0 & \cdots & 0 \\ -c & 2c & -c & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & -c & 2c & -c \\ 0 & \cdots & 0 & -c & 2c \end{bmatrix}, \tag{28}$$

The second term in right-hand side of (26) can be written as

$$\frac{\partial V(e,q)}{\partial q}\dot{q} = -\frac{1}{2}e^T\begin{bmatrix} D_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} \end{bmatrix}e \tag{29}$$

in which

$$D = \begin{bmatrix} 0 & & & & \text{sym.} \\ X_1^1\dot{q}_{L2}S_2 & 0 & & & \\ X_2^1\dot{q}_{L2,3}S_{2,3} & X_1^2\dot{q}_{L3}S_3 & \ddots & & \\ \vdots & \vdots & & 0 & \\ X_{n-1}^1\dot{q}_{L2,n}S_{2,n} & X_{n-2}^2\dot{q}_{L3,n}S_{3,n} & \cdots & X_1^{n-1}\dot{q}_{Ln}S_n & 0 \end{bmatrix}$$

where $\dot{q}_{Li,j} = \dot{q}_{Li} + \dot{q}_{Li+1} + \cdots + \dot{q}_{Lj}$, $S_i = \sin(q_{Li})$ and $S_{i,j} = \sin(q_{Li} + q_{Li+1} + \ldots + q_{Lj})$. By substituting (27) and (29) into (26), the time derivative of the Lyapunov candidate becomes

$$\begin{aligned} \frac{\mathrm{d}V(e,q)}{\mathrm{d}t} &= -\frac{1}{2}e^T Pe = -\frac{1}{2}e^T\begin{bmatrix} 4c\bar{B}^{-1} & 0 \\ 0 & 4cI \end{bmatrix}e - \\ &\frac{1}{2}e^T\begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix}e = -\frac{1}{2}e^T\begin{bmatrix} 4c\bar{B}^{-1}+D & 0 \\ 0 & 4cI \end{bmatrix}e \end{aligned} \tag{30}$$

in which

$$4c\bar{B}^{-1} + D =$$

$$\begin{bmatrix} 4c & & & & \text{sym.} \\ -2c + X_1^1\dot{q}_{L2}S_2 & 4c & & & \\ X_2^1\dot{q}_{L2,3}S_{2,3} & -2c + X_1^2\dot{q}_{L3}S_3 & \ddots & & \\ \vdots & \vdots & & 4c & \\ X_{n-1}^1\dot{q}_{L2,n}S_{2,n} & X_{n-2}^2\dot{q}_{L3,n}S_{3,n} & \cdots & -2c + X_1^{n-1}\dot{q}_{Ln}S_n & 4c \end{bmatrix}$$

Now, it must be proved that P in (30) is positive definite matrix. P will be the positive definite matrix, if determinants of all principal minors of $4c\bar{B}^{-1}+D$ and $4cI$ be positive. It is clear that, $\forall c > 0$, $4cI$ is a positive definite matrix. The minors of $4c\bar{B}^{-1}+D$ can be derived as follow

$$\begin{aligned} &Minor_1 = 4c \\ &Minor_2 = (6c - X_1^1\dot{q}_{L2}S_2)(2c + X_1^1\dot{q}_{L2}S_2) \\ &Minor_3 = 32c^3 + c^2\left(16X_1^1\dot{q}_{L2}S_2 + 8X_2^1\dot{q}_{L2,3}S_{2,3} + 16X_1^2\dot{q}_{L3}S_3\right) \\ &+c\begin{pmatrix} -4(X_1^1\dot{q}_{L2}S_2)^2 - 4X_1^1\dot{q}_{L2}S_2X_2^1\dot{q}_{L2,3}S_{2,3} - (X_2^1\dot{q}_{L2,3}S_{2,3})^2 - \\ 4X_2^1\dot{q}_{L2,3}S_{2,3}X_1^2\dot{q}_{L3}S_3 - 4(X_1^2\dot{q}_{L3}S_3)^2 \end{pmatrix} \\ &+2X_1^1\dot{q}_{L2}S_2X_2^1\dot{q}_{L2,3}S_{2,3}X_1^2\dot{q}_{L3}S_3 = \\ &(3+1)(2c)^3 + \text{lower order of } c \\ &\vdots \\ &Minor_i = (i+1)(2c)^i + \text{lower order of } c \end{aligned} \tag{31}$$

It can be seen that the determinant of each minor is the polynomial of degree i, and its greatest element is $(i+1)(2c)^i$. If c is chosen big enough, the determinant of all minors will be positive and the stability of disturbance observer is guarantied. The first and second minors must be positive, so the following condition must be satisfied

$$c > 0.5\left|X_1^1\dot{q}_{L2\max}\right|, \tag{32}$$

where $\dot{q}_{2\max}$ denotes the maximum velocity of the second link. Now, in (31) by taking $a_1 = X_1^1\dot{q}_2 S_2$, $a_2 = X_1^2\dot{q}_3 S_3$ and $a_3 = X_2^1\dot{q}_{2,3} S_{2,3}$, the third minor becomes

$$\begin{aligned} Minor_3 &= 32c^3 + c^2\left(16a_1 + 16a_2 + 8a_3\right) + \\ & c(-4a_1^2 - 4a_2^2 - 4a_1a_3 - a_3^2 - 4a_2a_3) + 2a_1a_2a_3 \end{aligned} . \tag{33}$$

The worst condition that causes the determinant to be negative is occurred when $a_1 < a_2 < a_3 < 0$. So, replacing a_2 and a_3 in (33) with a_1 yields

$$Minor_3 > 32c^3 + c^2\left(40a_1\right) + c(-17a_1^2) + 2a_1^3 . \tag{34}$$

If $c = 3|a_1| = -3a_1$ then $Minor_3 > -451a_1^3 > 0$. In the other word, the following condition for c must be satisfied

$$c > 3 \times \max\left\{\left|X_1^1\dot{q}_{L\max 2}\right|, \left|X_2^1\dot{q}_{L\max 2,3}\right|, \left|X_1^2\dot{q}_{L\max 3}\right|\right\} . \tag{35}$$

By repeating the above procedure, it can be shown that for n-link robot manipulator by taking the parameter c as

$$c > \left(\frac{n\times(n-1)}{2}\right) \times \max\left\{\begin{matrix} \left|X_1^1\dot{q}_{L\max 2}\right|, \ldots, \left|X_{n-1}^1\dot{q}_{L\max 2,n}\right|, \\ \left|X_1^2\dot{q}_{L\max 3}\right|, \ldots, \left|X_{n-2}^2\dot{q}_{L\max 3,n}\right|, \\ \ldots, \left|X_{n-(n-1)}^{n-1}\dot{q}_{L\max n}\right| \end{matrix}\right\} \tag{36}$$

matrix P is positive definite, and the observer (17) and (18) is globally asymptotically stable. According to Property 2, the dynamic model of the FJM has no zero dynamics and it is internally stable, so the proposed NDO by compensating the estimated disturbances can improve the control response performance.

5. SIMULATION STUDY

In this section, the proposed NDO is tested for the two-link flexible joint robot manipulator. Here, the effectiveness of the presented observer is verified by numerical simulations. In the performed simulations external disturbances on the links and motors are taken into account. The state feedback controller on the base of the linear quadratic regulator (LQR) is designed to stabilize the system and an optimal state observer is designed to estimate the angular velocities. The controllers are designed for the nominal model of manipulators without considering the disturbances.

5.1. Deriving the Dynamic equations

According to the general form of the dynamic equation (1), the dynamic equation of two-link planar flexible joint robot manipulator can be written as follow (Korayem et al., 2008).

$$\begin{aligned} &\begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_{L1} \\ \ddot{q}_{L2} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \\ &\begin{bmatrix} k_1\left(q_{L1} - q_{a1}\right) \\ k_2\left(q_{L2} - q_{a2}\right) \end{bmatrix} = \begin{bmatrix} f_{dis.11} \\ f_{dis.12} \end{bmatrix} \\ &\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{q}_{a1} \\ \ddot{q}_{a2} \end{bmatrix} + \begin{bmatrix} k_1\left(q_{a1} - q_{L1}\right) \\ k_2\left(q_{a2} - q_{L2}\right) \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} f_{dis.21} \\ f_{dis.22} \end{bmatrix}, \end{aligned} \tag{37}$$

in which

$$\begin{aligned} &M_{11} = I_1 + I_2 + m_1L_{c1}^2 + m_2L_1^2 + m_2L_{c2}^2 + m_pL_1^2 + m_pL_2^2 + \\ &2m_2L_1L_{c2}C_2 + 2m_pL_1L_2C_2 \\ &M_{12} = I_2 + m_2L_{c2}^2 + m_pL_2^2 + m_2L_1L_{c2}C_2 + m_pL_1L_2C_2 \\ &M_{22} = I_2 + m_2L_{c2}^2 + m_pL_2^2 \\ &b_1 = -2m_2L_1L_{c2}\dot{q}_{L1}\dot{q}_{L2}S_2 - m_2L_1L_{c2}\dot{q}_{L2}^2S_2 - \\ &2m_pL_1L_2\dot{q}_{L1}\dot{q}_{L2}S_2 - m_pL_1L_2\dot{q}_{L2}^2S_2 \\ &b_2 = m_2L_1L_{c2}\dot{q}_{L1}^2S_2 + m_pL_1L_2\dot{q}_{L1}^2S_2 \\ &h_1 = m_1L_{c1}gC_1 + m_2L_1gC_1 + m_pL_1gC_1 + \\ &m_2L_{c2}gC_{1,2} + m_pL_2gC_{1,2}, h_2 = m_2L_{c2}gC_{1,2} + m_pL_2gC_{1,2} \\ &J_1 = N_1^2J_{m1}, J_2 = N_2^2J_{m2} \end{aligned} \tag{38}$$

where L_i, L_{ci}, I_i and m_i denote the length, mass center, moment of inertia and mass of link i, i=1,2 respectively. The parameters used in the simulation study are given in Table 1.

Table 1. Parameters of the two-link FJM.

Parameters	Values	Unit
Payload mass	m_p=0	Kg
Mass of links	m_1=m_2=2	Kg
Length of links	L_1=L_2=2	m
Center of mass	L_{c1} =L_{c2}=1	m
Inertai of links	I_1=I_2=0.6667	Kg.m^2
Inertia of motors	J_{m1}=J_{m2}=0.1	Kg.m^2
Spring constants	k_1=k_2=5000	N.m/rad
Gear ratio	N_1=N_2=3	
Gravity acceleration	g=0	m/s^2

5.2. Disturbance observer design

From (20) and (22), P and L become

$$p\left(\dot{q}\right) = c\begin{bmatrix} \dot{q}_{L1} \\ \dot{q}_{L1} + \dot{q}_{L2} \\ \dot{q}_{a1} \\ \dot{q}_{a2} \end{bmatrix}, L = c\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} M^{-1} & 0 \\ 0 & J^{-1} \end{bmatrix}. \tag{39}$$

Consequently, the nonlinear disturbance observer can be obtained by substituting P and L into (17) and (18). The parameter c can be chosen based on (36) and by assuming that the maximum joint velocity of each link does not exceed 1 rad/s, so one can write

$$c > 0.5\left|X_1^1\dot{q}_{2\max}\right| \tag{40}$$

According to (40), the observer would be globally asymptotically stable, if parameter c is bigger than 4. Since the bigger values of c accelerate the convergence of the observer error, so the parameter c is chosen as 10 to have a reasonable convergence rate. In the following, the details of designing the state feedback controller on the base of the linear quadratic regulator (LQR) are presented.

5.3. State feedback controller and state observer

The state feedback controller is a more common approach in the control of manipulator systems. In order to design this controller, a linear state-space model of the manipulator is obtained by linearizing the equations of motion of the system about the operating point. The dynamic equations (37) can be put in the nonlinear state space form as follow

$$\dot{x} = f(x) + g(x)u, \tag{41}$$

where the state vector x is defined as $x = \begin{bmatrix} q_L^{\mathrm{T}} & q_a^{\mathrm{T}} & \dot{q}_L^{\mathrm{T}} & \dot{q}_a^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x_1 & L & x_8 \end{bmatrix}^{\mathrm{T}}$, the control input vector as $u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^{\mathrm{T}}$, and the functions $f(.)$ and $g(.)$ are given by

$$\begin{gathered} f_1 = x_5, f_2 = x_6, f_3 = x_7, f_4 = x_8, \\ \begin{bmatrix} f_5 \\ f_6 \end{bmatrix} = \begin{bmatrix} M_{11}(x_2) & M_{12}(x_2) \\ M_{12}(x_2) & M_{22} \end{bmatrix}^{-1} \\ \left(-\begin{bmatrix} b_1(x_2, x_5, x_6) \\ b_2(x_2, x_5) \end{bmatrix} - \begin{bmatrix} h_1(x_1, x_2) \\ h_2(x_1, x_2) \end{bmatrix} + \begin{bmatrix} k_1(x_3 - x_1) \\ k_2(x_4 - x_2) \end{bmatrix} \right) \\ \begin{bmatrix} f_7 \\ f_8 \end{bmatrix} = \begin{bmatrix} J_1^{-1} k_1 (x_1 - x_3) \\ J_2^{-1} k_2 (x_2 - x_4) \end{bmatrix} \\ g_1 = g_2 = g_3 = g_4 = g_5 = g_6 = 0, \\ g_7 = J_1^{-1} u_1, g_8 = J_2^{-1} u_2 \end{gathered} \tag{42}$$

This obtained nonlinear model, can be used for developing the linear model. The linearized model about the operating point (x_0, u_0) is obtained by applying the Taylor series expansion to the set of equations (41) and (42) for a two-link FJM which yields the following system

$$\dot{x} = Ax + Bu, \tag{43}$$

where matrices **A** and **B** are given by

$$A = \left.\frac{\partial f}{\partial x}\right|_{(x_0, u_0)}, B = \left.\frac{\partial g}{\partial u}\right|_{(x_0, u_0)}. \tag{44}$$

So one can obtain the matrices A and B as follow

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & 0 & 0 \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & 0 & 0 \\ J_1^{-1} k_1 & 0 & -J_1^{-1} k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2^{-1} k_2 & 0 & -J_2^{-1} k_2 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{45}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & J_1^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_2^{-1} \end{bmatrix}^T$$

where $S_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{(\mathbf{x_0}, \mathbf{u_0})}$ for i=5,6 and j=1,…6. An appropriate way for designing the state feedback controller is linear quadratic regulator (LQR) technique. According to LQR method, for the LTI system in (43), the following feedback control law

$$u_c = -Kx, \tag{45}$$

minimize the quadratic cost function

$$J = \int_0^\infty (x^{\mathrm{T}} Q x + u_c^{\mathrm{T}} R u_c) dt \tag{46}$$

where Q is the symmetric positive semi definite matrix and R is the symmetric positive definite matrix. The matrices Q and R are called the state and control penalty matrices, respectively. Here the lqr command in MATLAB software is used to calculate the controller gain matrix *K*. In order to evaluate the matrices *A* and *B*, the manipulator parameters are substituted from Table 1 into Eq. (45). So by choosing the matrices R and Q as follow

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}, Q = 20 \times \mathrm{diag}(1,1,1,1,1,1,1,1). \tag{47}$$

The controller gain matrix *K* is obtained as

$$K = \begin{bmatrix} -16.7 & 45.6 & 22.8 & -47.2 & 5.4 & 2.2 & 5.6 & -2.5 \\ 5.8 & -6.9 & -5.1 & 9.6 & 2.6 & 0.7 & -0.5 & 2.9 \end{bmatrix} \tag{48}$$

The matrices *R* and *Q* are adjusted in such a way that the rise time of the system response is about 6 seconds. From the practical point of view, often all the states are not measured, so it is required to estimate the necessary unmeasured states. Here, it is assumed that only the position variables are available. So using the optimal state observer as follow

$$\dot{\hat{x}} = (A - BK - G^T C)\hat{x} + G^{\mathrm{T}} y, \tag{49}$$

the velocity of links and motors can be estimated. In (49), $\hat{x} = [\hat{q}^{\mathrm{T}} \ \ \hat{\dot{q}}^{\mathrm{T}}]^{\mathrm{T}}$ is the vector of estimated states, K is the controller gain matrix, $\mathbf{y}$ is the measured variables defined as $y = \begin{bmatrix} q_L^{\mathrm{T}} & q_a^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x_1 & \cdots & x_4 \end{bmatrix}^{\mathrm{T}}$ and **G** is the observer gain. In the presence of state observer, the control law (46) is rewritten as follow

$$u_c = -K\tilde{x}, \tag{50}$$

where $\tilde{\mathbf{x}} = [q^{\mathrm{T}} \ \ \hat{\dot{q}}^{\mathrm{T}}]^{\mathrm{T}}$. By choosing the *C* and weighting matrices *W* and *V* as follow

$$\begin{gathered} C = \begin{bmatrix} I_{4\times4} & 0_{4\times4} \end{bmatrix}, \\ V = (0.0001)^2 diag(1,1,1,1), W = 50\mathrm{diag}(0,0,0,0,1,1,1,1) \end{gathered} \tag{51}$$

The observer gain matrix *G* using the lqr(A^{T}, C^{T},*W*,*V*) in MATLAB can be obtained. The values of weighting matrices

W and V, depends on the amount of disturbance and sensor noise exerted to the system. By increasing the W, the disturbance are estimated more accurately, but the system sensitivity to the sensor noise is increased. In the performed simulations, noise on the measured signal is not considered and the W is adjusted in such a way that the disturbance signals are estimated with reasonable accuracy.

The control structure, including the state feedback controller, state observer and the nonlinear disturbance observer, is shown in Fig. 3, where the effect of disturbances is compensated for by the outputs of the NDO.

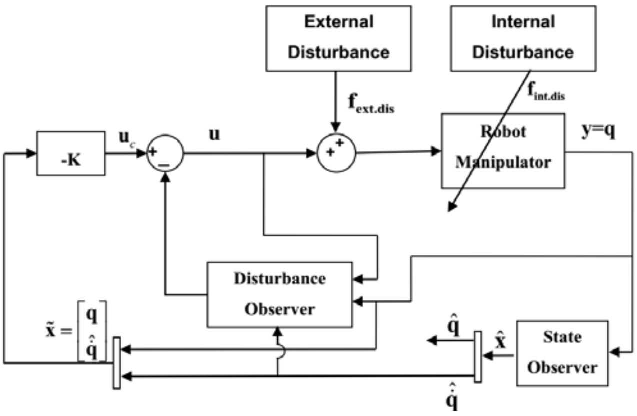

Fig. 3. State feedback controller with the NDO and state observer.

This control structure is implemented for the two-link FJM using the simulation toolbox of MATLAB. In this simulation, the external disturbances are applied to the links and motors as the different functions and the internal disturbances are not considered. The initial value of the state vector and estimated state vector are chosen to be

$$\mathbf{x}(0) = \hat{\mathbf{x}}(0) = [1.5\ 1.5\ 1.5\ 1.5\ 0\ 0\ 0\ 0].$$

The other robot parameters used in these simulations are listed in the Table 1. All simulations are performed for two cases, controller without disturbance observer and controller with disturbance observer. The applied disturbance functions to the links and motors, and their estimated functions are shown in Fig. 4 to Fig. 7. As it can be seen, the NDO can estimate the applied disturbance acceptably, even for rapid time-varying signals. The angular positions of the links with NDO (solid line) and without NDO (dotted line) are demonstrated in Fig. 8 and Fig. 9. The steady state errors of the links with NDO and without NDO are less than 0.02 and 0.27 Rad, respectively. The angular positions of motors are very similar to the angular position of links, so they are not shown here. The angular velocities of the links and motors and their estimated values are shown in Fig. 10 to Fig. 13. As it can be seen, in the presence of NDO, the regulating performance is improved and the effect of disturbance on the system outputs is reduced significantly, also the state observer estimates the velocities accurately. In Fig. 14 and Fig. 15, the applied control torque in motors with and without NDO is shown. It can be seen that adding the NDO to the system does not exert the large control effort to the system and there is no big difference in two cases. Consequently, it can be seen from the simulation results, adding the NDO to the control loop improve the regulating performance and increase the stability of the system against the applied disturbance.

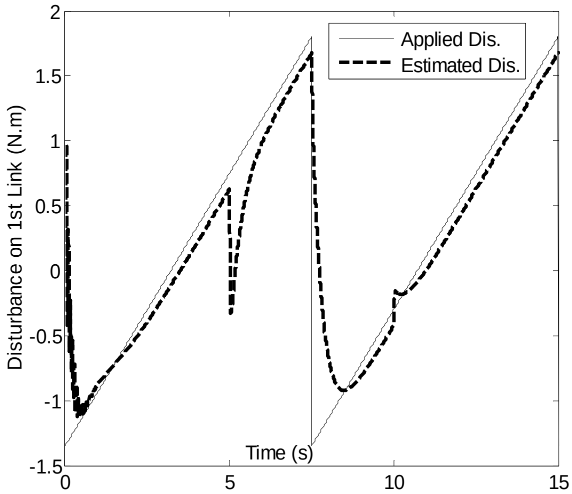

Fig. 4. Applied and estimated disturbance on the first link.

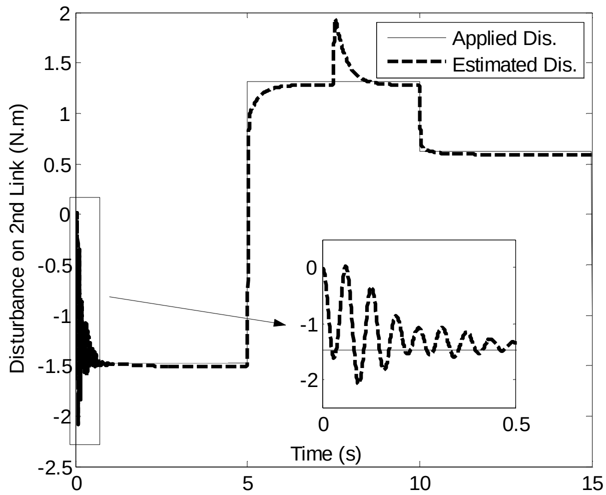

Fig. 5. Applied and estimated disturbance on the second link.

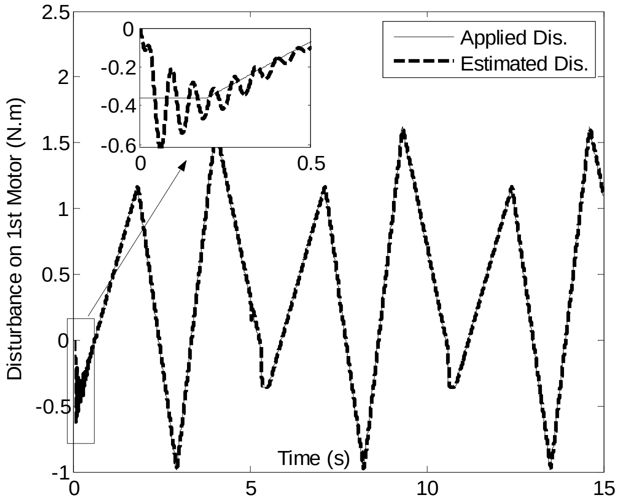

Fig. 6. Applied and estimated disturbance on the first motor.

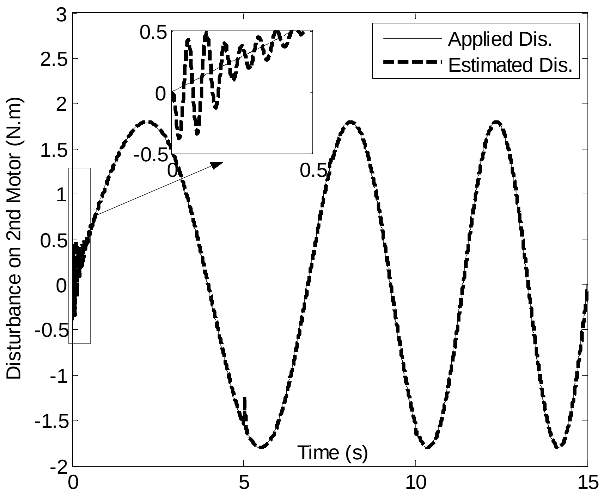

Fig. 7. Applied and estimated disturbance on the second motor.

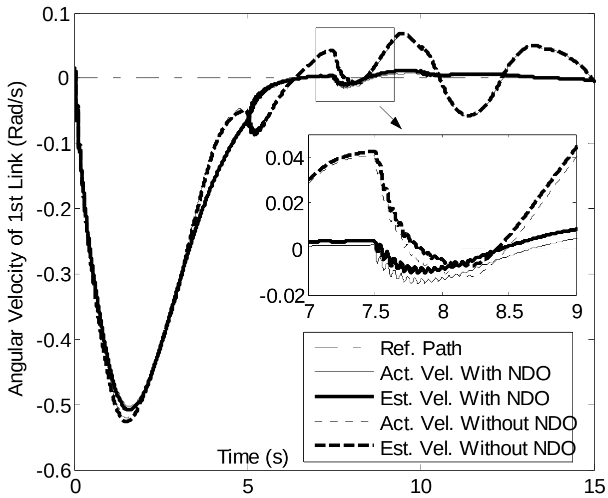

Fig. 10. Angular velocity of the first link.

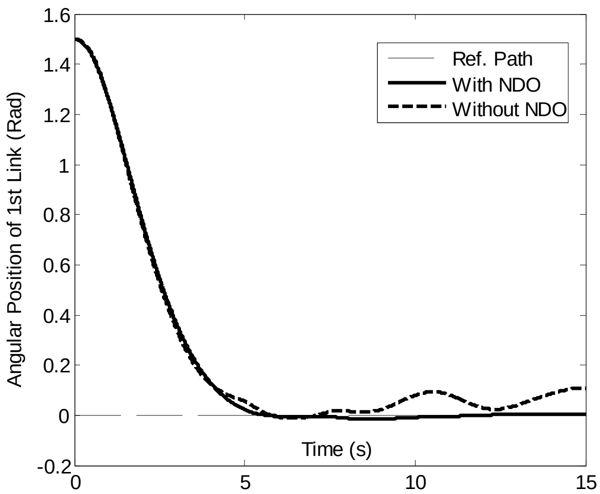

Fig. 8. Angular position of the first link.

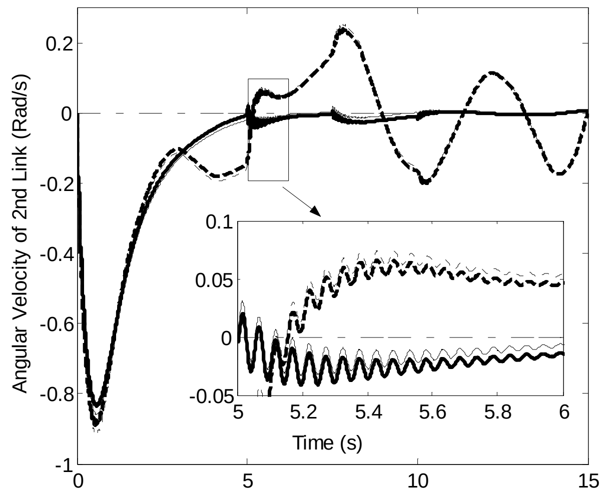

Fig. 11. Angular velocity of the second link.

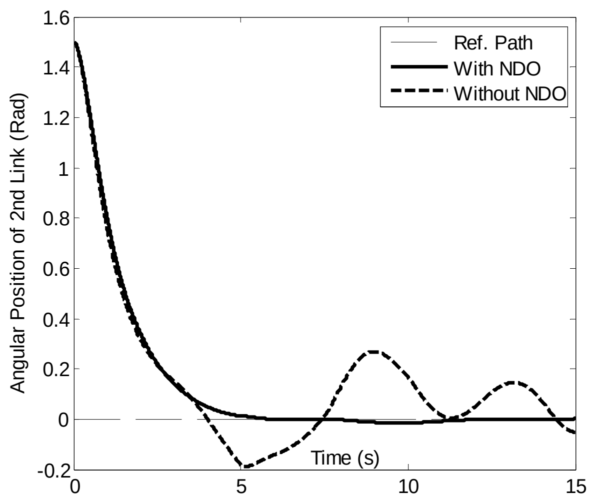

Fig. 9. Angular position of the second link.

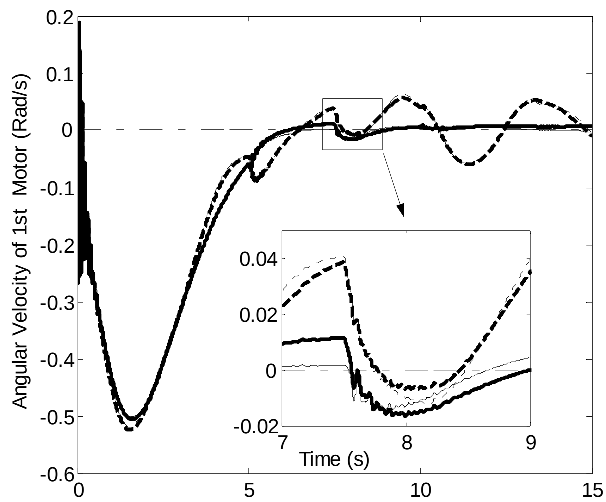

Fig. 12. Angular velocity of the first motor.

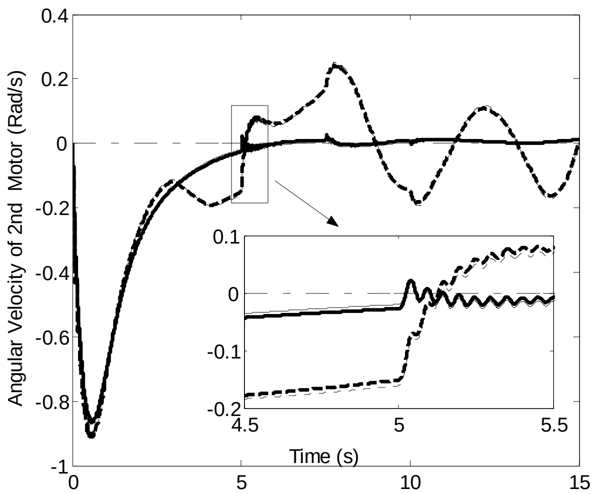

Fig. 13. Angular velocity of the second motor.

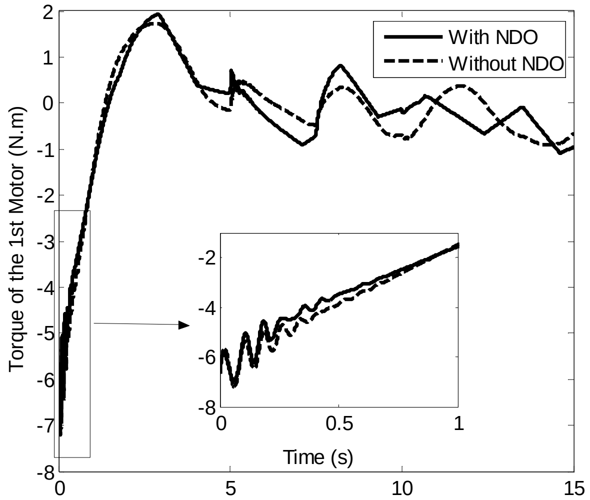

Fig. 14. Control input of the first motor.

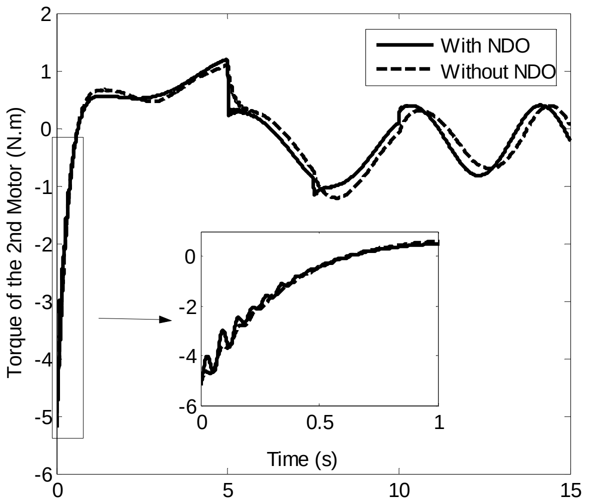

Fig. 15. Control input of the second motor.

6. CONCLUSION

In this paper, a nonlinear disturbance observer for the serial n-link planar FJM is presented. The FJM is an underactuated system; that is, the number of actuators is lower than the degrees of freedom. But according to the Spong model of the FJM, it is proved that its dynamic model has no zero dynamics and the closed loop system is internally stable. On the base of this property, the NDO law is proposed for FJM and by choosing the proper Lyapunov candidate, the stability analysis is performed by using the Lyapunov's direct method. Designing the proposed disturbance observer is so simple so that it is not required to have the disturbance model, and only one design parameter must be tuned to attain the desirable performance. By compensating the estimated disturbance via the NDO, the control response performance is considerably improved in the regulation control. Applicability of the observer is tested for a two-link FJM. To this end beside the proposed NDO, the LQR state feedback controller and optimal state observer are also designed to stabilize the system. Simulation results show the effectiveness of the method to estimate and compensate the applied disturbances accurately.

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