Metaheuristic strategy for the hierarchical predictive control of large scale energy networks

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Abstract: The short term optimization and control of energy networks is of great interest for Energy Industries because of the technical, economical and environmental benefits which could be gained from an appropriate management. However, models of such complicated systems are strongly non linear due to the energy propagation modeling and suffer from important uncertainties. Furthermore, some of the control variables are binary ones (on/off status of production units) and some of them are real ones (amount of energy to be produced). In this article, the goal is to encompass the whole technological string "production - distribution - consumption" by defining a suitable hierarchical predictive control strategy. In a first step, a global predictive control law is defined to compute the global amount of energy to be produced by each production site. In the second stage, this energy is dispatched between the production units of each site by a local predictive law. Due to the complexity of the system, an exact solution of the on line optimization problems to be solved in the predictive control strategy is untractable, and metaheuristic optimization methods are used. The global law is computed by a Particle Swarm Optimization (PSO) method whereas the local law is computed by ant colony and genetic algorithm. Numerical results exhibit more than satisfactory results and prove the viability of the approach.

Keywords: Predictive control, large scale systems, particle swarm optimization, ant colony, energy networks, hierarchical strategy.

1. INTRODUCTION

The short term optimal scheduling and control of power systems has emerged as a crucial point. Indeed, energy markets have become more and more competitive. Producers and network managers have to drive their power systems, which are more and more complicated, to fulfill consumers' power demands with the lowest global costs. Producers are also made to be aware of environmental issues by environmental laws. They are compelled to reduce their rate of polluting emissions. Thus, technical, economical and environmental constraints have to be simultaneously dealt with. The optimization problem stated from this multi field area can hardly be solved as it is a mixed non linear programming problem, made of numerous variables. The optimal control of district heating networks, for which propagation delays cannot be neglected and mechanical and thermal losses have nonlinear expressions, picks up all these harsh difficulties.

In most studies, consumers' demands are considered as given and perfectly known data; see Benonysson et al. (1995), Zhao et al. (1998) and Ravn and Ryggard (1994). In this case the minimization of operational costs is an ideal reference trajectory for the district heating system, but is not robust against load prediction errors. In Palsson (1993) and Nielsen and Madsen (2002), a closed loop control strategy is depicted for district heating networks with one thermal power production point and leads to very coherent behaviours: as heat losses in the distribution network increase with the supply temperature, the optimal strategy is to keep this temperature as low as possible while satisfying consumers' demands. However, in the general case, this control approach may fail as supply temperature is difficult to compute. This difficulty occurs for multi supply point networks, time varying operational costs or networks with heat storage tanks. For these cases, it may be economically interesting to locally produce more power than required to reduce global costs. The supply temperature may be temporary higher than supposed to achieve better global efficiencies. Furthermore, the combinatorial complexity is never taken into account by considering each production site as a single global production unit, instead of considering each individual unit in the system.

In this paper a new approach is presented, based on predictive control principle depicted in Clarke et al. (1987) and Maciejowski (2002). This approach aims to be quite versatile and could be apply to various kinds of district networks. It aims also to be robust against load prediction errors and model uncertainties. The idea is to compute a suitable hierarchical predictive control stratgey. In a first step (section 2), a global predictive control law is defined to compute the global amount of energy to be produced by each production site. In the second stage (section 3), this energy is dispatched between the production units of each site by a local predictive law. Due to the complexity of the system, an exact solution of the on line optimization

problems to be solved in the predictive control strategy is untracatble, and metaheuristic optimization methods are used. The global law is computed by a Particle Swarm Optimization (PSO) method whereas the local law is computed by ant colony and genetic algorithm. Numerical results exhibit more than satisfactory results and prove the viability of the approach.

2. GLOBAL PARTICLE SWARM OPTIMIZATION BASED PREDICTIVE CONTROL LAW

2.1 District heating networks modeling

A district heating network is depicted in figure 1. It is a part of a more general district heating network which has been reported in Sandou et al. (2004). It is made of two main subnetworks which are interconnected with the help of two valves. The main components which have to be modeled are the producers, the energy supply network made of pipes, pumps and nodes, and consumers.

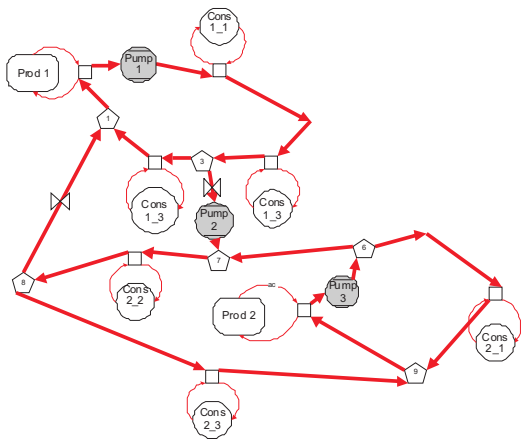

Fig. 1. District heating network benchmark

Producers A production site s is globally modeled by a characteristic, identified from technical data. At this stage, the different production units of the site are not individually considered. For hour n, production costs can be derived from produced thermal power Q_n^s:

$$c_{prod}^s(Q_n^s, Q_{n-1}^s) = a_2^s(Q_n^s)^2 + a_1^s Q_n^s + a_0 + \lambda(Q_n^s - Q_{n-1}^s)^2 \quad (1)$$

where λ is a weighting factor penalizing the control increments, and globally models the dynamics of production units. Coefficients a_i^s are technical data which are usually identified from experiments by a least square method.

The thermal power given to the primary or distribution network is related to network temperatures by:

$$Q_n^s = c_p \cdot m \cdot (T_s - T_r) \quad (2)$$

where $m[\text{kg.s}^{-1}]$ is the mass flow in the energy supply network, $T_s(\text{K})$ the supply temperature, $T_r(\text{K})$ the return temperature in primary network and $c_p[\text{J.kg}^{-1}.\text{K}^{-1}]$ the specific heat of water.

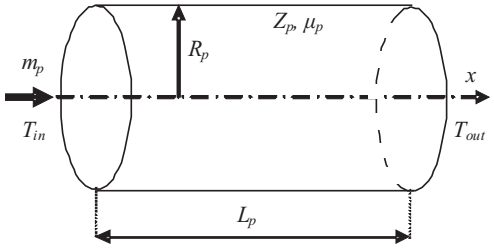

Fig. 2. Notations for pipes modeling

Energy supply network The energy supply network is concerned with pipes, valves, nodes and pumps. Notations for pipe modeling are given in figure 2.

Mechanical losses in pipes can be expressed by:

$$H_{out} = H_{in} - Z_p \cdot m_p^2 \quad (3)$$

with $m_p[\text{kg.s}^{-1}]$ the mass flow in pipe, H_{in} (resp. H_{out}) [m] the pressure at the beginning (resp. the end) of the pipe, and $Z_p(\text{m.kg}^{-2}.\text{s}^2)$ the friction coefficient. For a valve, this coefficient becomes Z_p/d, where d is the opening degree of the valve (from 0 for a closed valve to 1 for an open one) which is a control input of the system.

The thermal energy propagation in pipes can then be modeled by a partial differential equation:

$$\frac{\partial T}{\partial t}(x,t) + \frac{m_p(t)}{\pi\rho R_p^2}\frac{\partial T}{\partial x}(x,t) + \frac{2\mu_p}{c_p\rho R_p}(T(x,t) - T_0) = 0 \quad (4)$$

To counterbalance mechanical losses in pipes, pumps are installed in the network leading to an increase in pressure:

$$\Delta H = b_2(m\frac{\omega_0}{\omega})^2 + b_1 m\frac{\omega_0}{\omega} + b_0 \quad (5)$$

with $m[\text{kg.s}^{-1}]$ is the mass flow through the pump, $\omega[\text{rad.s}^{-1}]$ its rotation speed and ω_0 its nominal rotation speed. Coefficients b_i are obtained from technical data.

Finally, nodes in the network are easily modeled using mass flow balance equations and energy balance equations.

Consumers Secondary networks of consumers are connected to the primary network by way of heat exchangers. Notations are those of figure 3.

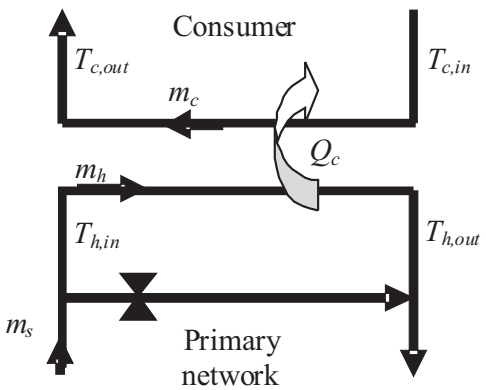

Fig. 3. Notations for consumers modeling

The following equation is the classical equation for a counter flow heat exchanger with $S[\mathrm{m}^2]$ the surface of the heat exchanger, and $e[\mathrm{W.K}^{-1}.\mathrm{m}^{-2}]$ its efficiency:

$$Q_c = eS\frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln(T_{h,in} - T_{c,out}) - \ln(T_{h,out} - T_{c,in})} \quad (6)$$

Assuming no thermal energy loss between primary and secondary networks, the thermal power given by the primary network can be also expressed by:

$$Q_c = c_p m_h (T_{h,in} - T_{h,out}) \quad (7)$$

$$Q_c = c_p m_c (T_{c,out} - T_{c,in}) \quad (8)$$

Assuming that m_c and $T_{c,out}$ are given, and that mass flow m_h is determined by the opening degree of the valve, then $T_{c,in}$, Q_c and $T_{h,out}$ can be computed from $T_{h,in}$. Q_c is an increasing function of m_h: the maximal thermal power which can be given to a consumer is obtained for $m_h = m_s$. Consequently, the given power is finally expressed by:

$$Q_c = \min(Q_{dem}, Q\max) \quad (9)$$

where Q_{dem} is the heat demand of the consumer, and Q_{max} is the maximum power that can be given by the primary network. Q_{max} is computed by solving the system made of 6, 7 and 8, in the particular case $m_h = m_s$.

2.2 Receding horizon based control of district heating networks

Open loop and optimization Consider a district heating network, with S production sites, V valves and C consumers. For simplicity, rotation speeds of pumps are supposed to be constant. The open loop control law of the whole system can be computed from the solution of the optimization problem:

$$\begin{array}{c} \min \\ \{Q_n^s, d_n^v\}, \\ n \in \{1,\dots,N\} \\ s \in \{1,\dots,S\}, \\ v \in \{1,\dots,V\} \end{array} \quad \sum_{n=1}^{S}\sum_{k=1}^{K} c_{prod}^s(Q_n^s, Q_{n-1}^s) \quad (10)$$

subject to physical constraints of (2), (3), (4), (5), (9)

where, Q_n^s is the thermal power produced by site s during time interval n and d_n^v is the opening degree of valve v during time interval n. N is the total number of considered time intervals.

Constraints are also the satisfaction of technical constraints (pressures and mass flows in the energy supply network) and the fulfilling of consumers demands $Q_{dem,n}^c, c \in \{1,\dots,C\}$. To compute these constraints, one has to simulate the whole network. From the modeling details presented in the previous section, this implies the numerical solution of non linear algebraic systems of equations for the mass flow and pressure computation and the simulation of systems of partial differential equations for the thermal energy propagation part.

Finally, the solution of this problem is hard to be solved with a classical deterministic method. A PSO method is then chosen as a solution algorithm.

Closed loop control The open loop computed by the solution of (10) cannot be directly applied to the real system. Indeed, consumers demands $Q_{dem,n}^c$ are not known in advance, but only predicted. To get a robust behavior of the system, one has to control the system in a closed loop framework. The real control inputs are the supply temperatures of producers. These values are bounded due to physical limitations of steam boilers. Further, consumers take power from the energy supply network if temperatures are sufficiently high (if not, the consumer demand is not fulfilled, but the behavior of the energy supply network remains correct). An important remark is that whatever the control strategy is employed, due to these physical limitations, all temperatures in the network remain in the acceptable range. In conclusion there is no instability danger for the control law, and the receding horizon strategy can be applied, even if a stochastic optimization problem is used without global optimality guarantee.

2.3 Classical PSO algorithm

Particle swarm Optimization (PSO) was firstly introduced by Kennedy and Eberhart Kennedy and Eberhart (1995). This optimization method is inspired by the social behavior of bird flocking or fish schooling. Consider the following optimization problem:

$$\min_{x\in\chi} f(x) \quad (11)$$

P particles are moving in the search space. Each of them has its own velocity, and is able to remember where it has found its best performance. Each particle has some "friends". The following notations are used:

- x_k^p (resp. v_k^p): position (resp. velocity) of particle p at iteration k;
- $b_k^p = \mathrm{argmin}(f(x_{k-1}^p), f(b_{k-1}^p))$: best position found by particle p until iteration k;
- $V(x_k^p) \subset \{1,\dots,P\}$: set of "friend particles" of particle p at iteration k;
- $g_k^p = \mathrm{argmin}(f(x_k^j), j \in V(x_k^p))$: best position found by the friend particles of particle p until iteration k.

The particles move in the search space according to the transition rule:

$$\begin{array}{l} v_{k+1}^p = w \times v_k^p + c_1 \otimes (b_k^p - x_k^p) + c_2 \otimes (g_k^p - x_k^p) \\ x_{k+1}^p = x_k^p + v_{k+1}^p \end{array} \quad (12)$$

where w is the inertia factor, $\otimes$ denotes the element wise multiplication of vectors and c_1 (resp. c_2) is a random vector whose length is the number of optimization variables, and whose components are in the range $[0, \bar{c_1}]$ (resp. $[0, \bar{c_2}]$).

The construction of the transition rule 12 is represented in figure 4.

The choice of parameters is very important to ensure the satisfying convergence of the algorithm. Important results have been reported on the topic; see for instance Shi and Eberhart (1998), Eberhart ans Shi (2000). Is beyond the scope of the present study to present the exhaustive description of tuning strategies (the Automatic Control community being less enthusiastic about metaheuristics details). Standard values, which are given in Kennedy and

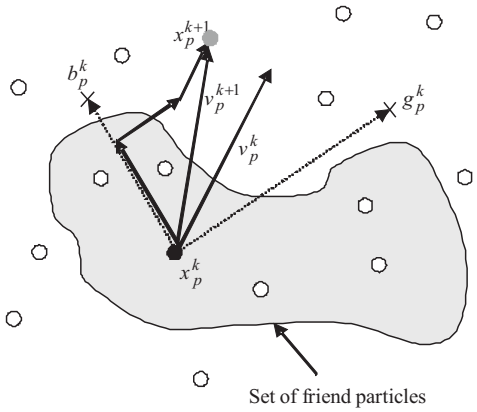

Fig. 4. Geometric representation of the transition rule

Clerc (2006) will be used: swarm size $P = 10 + \sqrt{n}$, where n is the number of optimization variables, $w = \frac{1}{2 \cdot \ln(2)}$, $\bar{c_1} = \bar{c_2} = 0.5 + \ln(2)$

Several topologies exist for the design of the set of *friend* particles. For a comprehensive study of this topic, see Kennedy (1999). In particular, if these sets do not depend on k, neighborhoods are said to be "social". This choice is the simplest for the implementation of the algorithm and so a social neighborhood will be used in this paper.

2.4 Numerical results

The receding horizon based control law has been applied for the control of the district heating benchmark depicted in figure 1. Tests have been performed for a total time horizon of 24 or 48 hours, with a sampling time of one hour. The prediction horizon for the optimization problem is $N = 12$ hours. Thus, as the benchmark represented in figure 1 has 2 producers and 2 valves, the optimization problem is made of $12 \times (2+2) = 48$ optimization variables, which are the global production of both production sites and the opening degrees of valves. The solution of the optimization problem is performed in 120 seconds on a Pentium IV, 2.5 GHz with Matlab 2007, for 50 iterations of the PSO algorithm.

Robustness of the closed loop structure To validate the control law, a worst case experiment has been performed. It is assumed that all consumer demands are always underestimated by a factor of 10%. This represents a worst case experiment as in the real world, load error predictions can partially compensate each other. Tests of the proposed approach have shown that consumers demands are always fulfilled, by using the receding horizon control structure.

Economical benefit of the receding horizon strategy In the district heating network (figure 1), producer 1 is a cogeneration site. Cogeneration refers to the simultaneous production of electric and thermal powers, leading to high global efficiencies. Briefly speaking, the main goal of the producer is to satisfy the thermal power demand. But he has the opportunity to use the exhaust fumes to produce and to sold electric power. Finally, for the thermal power point of view, the higher the price of sold electricity, the lower the thermal power production costs. The simulation has been performed for different electricity prices, and corresponding total productions over the whole horizon (24 or 48 hours) are given in figure 5.

	Producer 1		**Producer 2**	
Electricity price	**24h**	**48h**	**24h**	**48h**
$40E/MWh$	535	947	537	1016
$0E/MWh$	541	963	492	950

Fig. 5. Numerical Results: total production (in MWh)of producers for different configurations

The price $40E/MWh$ corresponds approximately to the price in France from November 1^{st} to March 31^{st}, whereas the null price corresponds to the price from April 1^{st} to October 31^{st}.

Results show that the higher the price, the higher the production of the cogeneration site. The control law uses the interconnection valves to make the extra amount of power to pass from subnetwork 1 to subnetwork 2. Although obvious, the possibility is not used in classical district heating networks: controls laws only use local information, and the interconnections are often viewed as safety means, and are rarely used. The receding horizon law is able to take into account the whole technological string "production - distribution - consumption" and the whole system through the solution of the optimization problem 10. The solution of this problem is made tractable by the use of a stochastic approximated optimization method.

Note that in the future, the price of sold electricity may depend on the electricity market. In such a situation, production costs would be predicted, and the closed loop structure is also a good trend to get a robust behavior against cost uncertainties.

3. LOCAL ANT COLONY AND GENETIC ALGORITHM BASED PREDICTIVE CONTROL LAW

The global predictive control law compute "optimal" global values for the amounts of energy Q_n^s to be produced by each production site. However, this global value has to be dispatched between the different production units of the site. This problem refers to the so called *Unit Commitment* problem. The aim of the section is to develop a predictive control law for each decentralised production site.

3.1 Open loop control and Unit Commitment

Unit Commitment is a classical large scale mixed integer problem in power systems, which aims to compute the optimal scheduling of several production units while satisfying consumer demand and technical constraints:

$$\min_{\{u_n^k, Q_n^k\}} \sum_{n=m}^{m+N-1} \left(\sum_{k=1}^{K} \begin{pmatrix} c_{prod}^k(Q_n^k, u_n^k) \\ +c_{on/off}^k(u_n^k, u_{n-1}^k) \end{pmatrix} \right). \quad (13)$$

N is the length of time horizon, K the number of production units, u_n^k (resp. Q_n^k) the on/off status (resp. produced power) of production unit k during time interval n. Production costs and start up and shut down costs are defined by:

$$\begin{cases} c^k_{prod}(Q^k_n, u^k_n) = \alpha^k_1 Q^k_n + \alpha^k_0 u^k_n \\ c^k_{on/off}(u^k_n, u^k_{n-1}) = \begin{pmatrix} c^k_{on} u^k_n (1-u^k_{n-1}) \\ +c^k_{off} u^k_{n-1}(1-u^k_n) \end{pmatrix} \end{cases}. \quad (14)$$

Integer variables are on/off status of production units (u^k_n), and real variables are produced powers (Q^k_n). Coefficient α^k_i, c^k_{on} and c^k_{off} are technical data. The usual restriction comes from:

- capacity constraints

$$\begin{array}{c} Q^k_{\min} u^k_n \leq Q^k_n \leq Q^k_{\max} u^k_n, \\ \forall n \in \{m, \ldots, m+N-1\}, \forall k \in \{1, \ldots, K\}, \end{array} \quad (15)$$

- distribution network demand satisfaction (remember that those values are computed by the PSO global predictive law of section 2)

$$\sum_{k=1}^{K} Q^k_n \geq \hat{Q}^{dem}_n, \forall n \in \{m, \ldots, m+N-1\}, \quad (16)$$

- time up and time down constraints

$$\begin{cases} \left(u^k_{n-1} = 0, u^k_n = 1\right) \\ \Rightarrow \left(u^k_{n+1} = 1, \cdots, u^k_{n+T^k_{up}-1} = 1\right) \\ \left(u^k_{n-1} = 1, u^k_n = 0\right) \\ \Rightarrow \left(u^k_{n+1} = 0, \cdots, u^k_{n+T^k_{down}-1} = 0\right) \end{cases}, \quad (17)$$

- ramp constraints

$$\begin{array}{c} \left|Q^k_n - Q^k_{n-1}\right| \leq \Delta Q^k, \\ \forall n \in \{m, \ldots, m+N-1\}, \forall k \in \{1, \ldots, K\}. \end{array} \quad (18)$$

Discrete dynamics on the system are expressed through logical equations (17) and continuous dynamics are stated as power increment limitations (18).

3.2 Closed loop control

For optimisation, the distribution network demand (computed by the PSO global law) is supposed to be perfectly known over the whole time horizon: the computation of the optimal scheduling is a reference trajectory for integer and real control inputs. However, prediction errors of the consumer load may lead to prediction errors on the distribution network demand and so to a possible deficient behavior. Thus, a closed loop control is also required for the production sites: the problem refers to the control of a hybrid system. As in section 2 for the computation of the distribution network demand, a convenient way to extend optimisation results in a closed loop framework is the receding horizon. The idea is still to compute the optimal scheduling on time interval $[m, m+N-1]$, considering predicted distribution network demand $\hat{Q}^{dem}_n$ (in fact, Q^s_n computed by the global control law). The first values of integer scheduling (on/off status of production units) are applied to the system. Simultaneously, real variables (amounts of produced energy) are slightly updated so as to fulfill, if possible, the real demand at time m, Q^{dem}_m. Production unit local regulations are assumed to be perfect: if production unit k has produced Q^k_{m-1} during time interval $m-1$, it can produce, if still switched on, whatever power in the range $[\max(Q^k_{m-1} - \Delta Q^k, Q^k_{\min}), \min(Q^k_{m-1} + \Delta Q^k, Q^k_{\max}]$ during time interval m. The scheduling algorithm will now be detailed.

3.3 Optimization procedure

Optimization methods analysis and proposition of a well suited algorithm Numerous methods have been applied to solve Unit Commitment and related problems such as facility location. They are listed for instance in Sen and Kothari (1998) and are here briefly depicted. Exact solution methods (exhaustive enumeration, Branch and Bound Chen and Wang (1993), dynamic programming Ouyang and Shahidehpour (1991)) have been tested. These methods suffer from combinatorial complexity: an efficient approximated method is required. Deterministic approximated methods can be used (priority lists in Senjyu et al. (2004)). But, due to numerous constraints, they are often strongly suboptimal. Constraints are explicitly considered using Lagrangian relaxation Zhai and Guan, (2002). Coupling constraints are relaxed, and the problem is divided into several optimisation problems (one per production unit). However, no guarantee can be given on the actual optimality. Further, an iterative procedure has to be performed: solution of the optimisation problems and updating of Lagrange multipliers. The update can be made with genetic algorithms Chen et al. (2000) or subgradient methods Dotzauer et al. (1999). For large scale cases, metaheuristics are interesting methods: simulated annealing in Yin Wa Wong (1998), tabu search in Rajan and Mohan (2004) and genetic algorithms in Swarup and Yamashiro (2002). No guarantee can be given on the actual optimality of the solution, but an often suitable solution with low computation times can be found. One of the problems of these methods is the handling of constraints. The algorithm "moves" randomly in the search space, and so, there is no guarantee that the final solution is in the feasible set. This is particularly the case for Unit Commitment, as the feasible set is much smaller than the search space.

Considering these arguments, ant colony appears to be an efficient way to solve this kind of problems, as it is able to find near optimal solutions with an explicit handling of all constraints. Indeed, ant colony is a constructive stochastic algorithm and solutions are explicitly built as feasible ones. From this initial population of "medium quality solutions" quickly computed by ant colony, a feasibility criterion is defined. Genetic algorithm is then used to intensively explore the search space, with an implicit management of problem constraints. Indeed, due to a positive feedback in the ant colony algorithm formulation, it may converge to a local minimum. An intensive exploration of the search space is thus required to circumvent this issue, and genetic algorithm, which is an efficient stochastic algorithm for unconstrained problems is used. Note that the algorithm supposes that real variables are quickly computed: the developed cooperative algorithm is also hybridized with an exact solution algorithm for real variables. Finally, this method allows simultaneously using the interesting properties of ant colony (explicitly handling of constraints) and of genetic algorithm (deep exploration of the search space, and so high quality of the solution). The general synopsis of the method is depicted on figure 6.

Ant colony optimization for the Unit Commitment Ant colony optimization was firstly introduced by Marco Dorigo Dorigo et al. (1996) and Dorigi and Gambardella

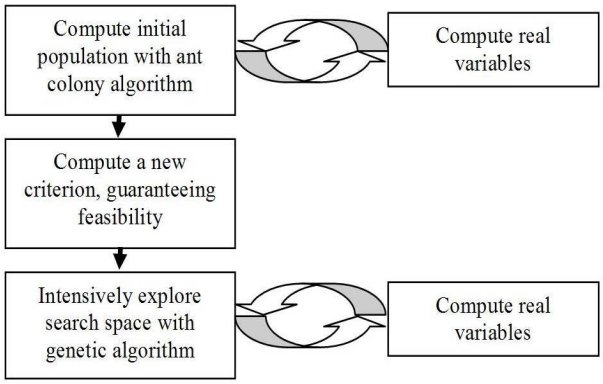

Fig. 6. Cooperative algorithm synopsis.

(1997). It is based on the way ants are looking for food (see figure 7).

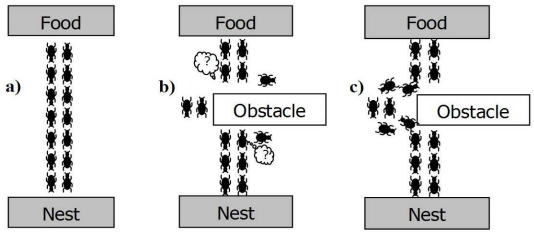

Fig. 7. Ants looking for food.

Suppose, that ants have managed to find food. Each particular ant does not know where to go. It only chooses its path depending on the pheromone trail which has been laid on the ground by previous ants. If the path of pheromone is broken because of an obstacle, first ants randomly choose their path. But, the ants which have chosen the shortest path will arrive first: the trail of pheromone in the shortest path is increasing faster than in the longest path. The positive feedback structure makes all ants finally choose the shortest path at the end of the experience.

The Unit Commitment problem can be formulated as a graph exploration problem as shown in figure 8, as in previous work Sandou et al. (2004).

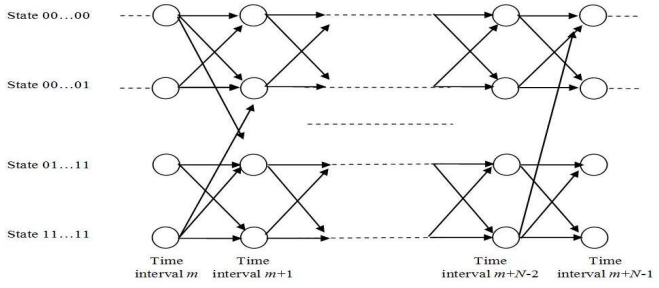

Fig. 8. Graph structure for Unit Commitment

The nodes of the graph represent all the possible states of production system, for all time intervals: $(u_n^1, \ldots, u_n^K)$. The aim is to go from one of the possible states at time m, to one of the possible states at time $m+N-1$, while satisfying all the constraints and minimizing global costs defined in equation (13). For each edge $(u_n^1, \ldots, u_n^K) \rightarrow (u_{n+1}^1, \ldots, u_{n+1}^K)$ of the graph, start-up and shut-down costs are added. Production costs are also associated to nodes.

During iteration t of the algorithm, F ants walk on this graph. If an ant f has reached state $i = (u_n^1, \ldots, u_n^K)$, the probability that it chooses the next state $j = (u_{n+1}^1, \ldots, u_{n+1}^K)$ is defined by the probabilistic law:

$$p_i^{(f)}(j) = \frac{\eta_{ij}^{\alpha} \tau_{ij}(t)^{\beta}}{\sum_{m \in J_f(i)} \eta_{im}^{\alpha} \tau_{im}(t)^{\beta}}. \quad (19)$$

- $\tau_{ij}(t)$ is the pheromone trail on edge $i = (u_n^1, \ldots, u_n^K) \rightarrow j = (u_{n+1}^1, \ldots, u_{n+1}^K)$ during iteration t. Its value depends on the results of previous ants.
- η_{ij} is the attractiveness. It refers to the " local choice ". When next node has to be chosen, the best local candidate is the node for which the gap between the maximum produced power and the predicted demand is the smallest. It is not sure that this is the best "global" choice, as the security margin is quite low. For more details, see previous work Sandou et al. (2004).
- α and β are weighting factors.
- $J_f(i)$ is the feasible set. This feasible set contains a priori all 2^K states. But, those states which do not satisfy time up and time down constraints, and those states which do not satisfy consumers' demands, are to be removed. Note that, even if produced powers are not known yet, it is possible to check the possibility of consumer's demand satisfaction with the equation:

$$\sum_{k=1}^{K} \left(\left(Q_{\min}^k (1 - u_n^k) + \left(\min \begin{pmatrix} Q_{\max}^k, \\ Q_n^k + \Delta Q^k \end{pmatrix} \right) u_n^k \right) u_{n+1}^k \right) \geq \hat{Q}_{n+1}^{dem}. \quad (20)$$

$J_f(i)$ sets are recursively constructed for each ant, and lead to the guarantee of the feasibility of solutions. Indeed, these sets have to be recursively constructed, as it is neccesary to know the number od time interval each units have been shitched on or off for the satisfaction of time up and time down constraints.

After the ant has completed its path, it is possible to evaluate the solution by solving the real optimisation problem defined in equation (13), with fixed binary variables. Due to the positive feedback of the algorithm the past mistakes have to be forgotten to avoid premature convergence. This is done by the pheromone evaporation. The pheromone trail is updated:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t) \quad (21)$$

ρ is the evaporation coefficient. This coefficient is viewed as an analogy with natural evaporation. $\Delta\tau_{ij}$ is the updating coefficient, depending on the results of ants in iteration t. An elitism algorithm is used: only the best ant is allowed to lay some pheromone on each edge it has used. The ant evaluation supposes the computation of real variables, which will now be depicted.

Computation of real variables Binary variables are computed by ant colony. For each feasible sequence u_n^k the corresponding real variables Q_n^k are computed as the solution of:

$$\begin{aligned}
&\arg\min_{\{Q_n^k\}} \sum_{n=m}^{m+N-1} \left(\sum_{k=1}^{K} \begin{pmatrix} c_{prod}^k \left(Q_n^k, u_n^k\right) \\ +c_{on/off}^k \left(u_n^k, u_{n-1}^k\right) \end{pmatrix} \right) \\
&= \arg\min_{\{Q_n^k\}} \left(\sum_{n=m}^{m+N-1} \sum_{k=1}^{K} \left(c_{prod}^k \left(Q_n^k, u_n^k\right)\right) \right) \\
&= \arg\min_{\{Q_n^k\}} \left(\sum_{n=m}^{m+N-1} \sum_{k=1}^{K} \alpha_1^k Q_n^k u_n^k + \alpha_0^k u_n^k \right) \\
&= \arg\min_{\{Q_n^k\}} \left(\sum_{n=m}^{m+N-1} \sum_{k=1}^{K} \alpha_1^k Q_n^k u_n^k \right)
\end{aligned} \tag{22}$$

As there are no temporally coupling constraints anymore (they have been guaranteed by the constructive ant colony algorithm), the problem can be divided into N successive optimisation problems:

$$\min_{\{Q_n^k, k=1,\ldots,K\}} \left(\sum_{k=1}^{K} \alpha_1^k Q_n^k u_n^k \right) \text{ subject to}$$
$$\begin{cases} \sum_{k=1}^{K} Q_n^k \geq \hat{Q}_n^{dem} \\ Q_{\min}^k u_n^k \leq Q_n^k \\ Q_n^k \leq \underbrace{\begin{pmatrix} Q_{\min}^k (1 - u_{n-1}^k) + \\ \left(\min(Q_{\max}^k, Q_{n-1}^k + \Delta Q^k)\right) u_{n-1}^k \end{pmatrix}}_{=Q_{\max}^k(n)} u_n^k \end{cases} \tag{23}$$

Without loss of generality, consider that $\alpha_1^1 \leq \alpha_1^2 \leq \ldots \leq \alpha_1^K$. Then, the optimal solution of problem (22) is to produce as much as possible with low-cost units, while satisfying capacity constraints. Then, the following recursive algorithm is performed:

$$\begin{cases} Q_n^1 = \min\left(\max\left(\begin{matrix} \hat{Q}_n^{dem} \\ -\sum_{i=2}^{K} Q_{\min}^i u_n^i \end{matrix}, Q_{\min}^1 \right), Q_{\max}^1(n) \right) u_n^1 \\ \vdots \\ Q_n^k = \min\left(\max\left(\begin{matrix} \hat{Q}_n^{dem} \\ -\sum_{i=1}^{k-1} Q_n^i - \\ \sum_{i=k+1}^{K} Q_{\min}^i u_n^i \end{matrix}, Q_{\min}^k \right), Q_{\max}^k(n) \right) u_n^k \\ \vdots \\ Q_n^K = \min\left(\max\left(\begin{matrix} \hat{Q}_n^{dem} \\ -\sum_{i=1}^{K-1} Q_n^i \end{matrix}, Q_{\min}^K \right), Q_{\max}^K(n) \right) u_n^K \end{cases} \tag{24}$$
$$Q_{\max}^k(n+1) = Q_{\min}^k(1 - u_n^k) + \left(\min(Q_{\max}^k, Q_n^k + \Delta Q^k)\right) u_n^k$$

Feasibility criterion To compute a feasibility criterion one has just to know a feasible solution. If the cost of this feasible known solution is c^f, the feasibility criterion can be:

$$\min_{\substack{\left\{u_n^k, Q_n^k\right\} \\ n=1,\cdots,N \\ k=1,\ldots,K}} \left(\begin{matrix} \sum_{n=1}^{N} \sum_{k=1}^{K} \begin{pmatrix} c_{prod}^k \left(Q_n^k, u_n^k\right) \\ +c_{on/off}^k \left(u_n^k, u_{n-1}^k\right) \end{pmatrix} \\ + \begin{pmatrix} (1+\varepsilon) c^f \\ +h\left(\left\{u_n^k, Q_n^k\right\}\right) \end{pmatrix} B\left(\left\{u_n^k, Q_n^k\right\}\right) \end{matrix} \right) \tag{25}$$

where one can distinguish:

- ϵ is a small positive real,
- $h\left(\left\{u_n^k, Q_n^k\right\}\right)$ is a penalty function for non feasible solutions $\left\{u_n^k, Q_n^k\right\}$,
- $B\left(\left\{u_n^k, Q_n^k\right\}\right)$ is a boolean function with value 1 for non feasible solutions and 0 for feasible ones.

With this criterion, all infeasible solutions will have a higher cost than the cost of the feasible known solution. For feasible solutions, the penalty is null, and the initial cost function is just considered. Finally, any unconstrained optimisation algorithm can be used to solve the problem, the constraints being implicitly taken into account. In this study, the known feasible solution is the best solution found by ant colony optimization.

Knowledge based genetic algorithm A knowledge based genetic algorithm, similar to the one developed in current work Sandou et al. (2008) is used. The general flow chart of a genetic algorithm is called up in figure 9.

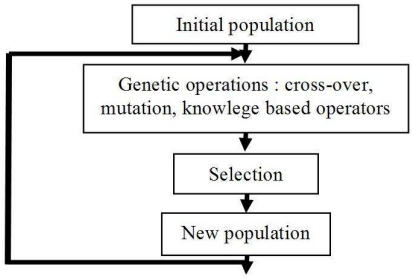

Fig. 9. Flow chart of a genetic algorithm

The main idea is to make a population of potential solutions evolves to create new solutions by using stochastic (or "genetic") operators. In this cooperative method, the initial population is made of all feasible solutions computed by ant colony optimization. Classical operators are crossing-over operator and mutation operator. For the crossing over operation, two potential solutions are randomly chosen in the population. They randomly merge their variables (or "genes") to create two new solutions. The mutation operation is the random selection of a potential solution and of one of its genes. This gene is changed to another. The aim of this operator is to keep the population genetic diversity.

The selection operator is an operator which aims to choose a new population from parents and children. This operation is made using the roulette wheel selection. After having computed the fitness value of each individual in the population, the probability of selection is proportional to the quality of individuals.

As already explained, the "optimal" scheduling of the production site is computed from values computed by the global PSO law $\hat{Q}_n^{dem} = Q_n^s$ on time interval $[m, m+N-1]$. The first values of integer scheduling u_m^k are applied to the system. Real variables are updated at time m, when real value Q_m^{dem} is known, using the equation (24) considering Q_m^{dem} instead of $\hat{Q}_n^{dem}$. These values are given to the scheduling procedure at time m+1, for instance for the satisfaction of ramp constraints.

Unit	Q_{min} (MW)	Q_{max} (MW)	ΔQ (MW)	α_0	α_1	c_{on}	c_{off}
1	10	40	10	25	2.6	10	2
2	10	40	10	25	7.9	10	2
3	10	40	10	25	13.1	10	2
4	10	40	10	25	18.3	10	2

Table 1. Characteristics of the benchmark example

Case study To test the algorithm, a "four unit" academic case is considered for producers 1 and 2. The characteristics of this case are given in table 1. A worst case is considered: the distibution network demand, which is computed by the global PSO predictive law is considered to be always underestimated. For the aim of testing the local law, the prediction error is a random value in the range $[-5\%, 0\%]$. The time horizon is $N = 24$ hours. The distribution network has a daily oscillation because the consumers' demands has also its global shape. Thus, the dynamic of the system is about 24 hours and the time horizon has to be greater: a high value has to be given to N. The simulation is performed on a 4 day total horizon. Results, obtained with Matlab 6.5 on a PIV 2GHz, are given on figure 10.

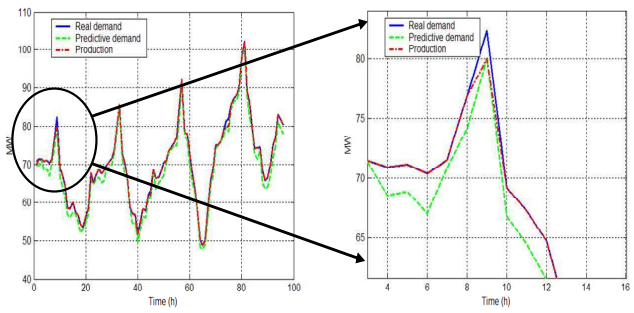

Fig. 10. Simulation results

Results show that the production is very close to the real demand, except for some peaks which have been underestimated. The optimization of the 96 binary variable problem is performed in just 25 seconds with the developed ant colony/genetic algorithm method. Due to the computation of successive economical near optimal solutions and real time slight updates, global costs are very close to global optimal costs. More precisely, a comparison has been performed with classical MILP solver, with a "Branch and Bound" method. It shows that the developed metaheuristic optimisation algorithm can compute a sub-optimal solution with a mean slight increase of 0.4% compared with the optimal solution. Furthermore, the MILP solver is very sensitive to the problem characteristics such as start up and shut down costs, leading to varying computation times going from 10 seconds to 10 hours. Finally, the algorithm has been tested for 10-unit case, leading to satisfying results with 10 minutes computation times, and for which MILP solver is untractable.

4. CONCLUSIONS

In this paper, metaheuristic optimization methods have been used to define a hierarchical predictive law for the control of district heating networks. The idea is to compute a global law, aiming at defining global values of energy to be produced by each production site. A local law computes the classical Unit Commitment for each production site. District heating networks are concerned with numerous binary variables, partial differential equations and nonlinear algebraic equations. Due to the complexity of the corresponding optimization problems which have to be solved, the use of classical deterministic solvers is untractable. In this paper a Particle Swarm Optimization method is used for the global law and a cooperative method (ant colony and genetic algorithm) is used for the local law. Numerical results prove the viability of the approach.

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