State Feedback Trajectory Tracking Control of Nonlinear Affine in Control System with Unknown Internal Dynamics and Disturbances

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Abstract: In this paper, a new state adaptive optimal nonlinear control is presented for the realization of the trajectory tracking of nonlinear affine-in control systems with unknown internal dynamics and disturbances. This proposed optimal controller which is developed in four steps is on the basis of a L_2 norm prescribed to minimize the trajectory tracking error between the desired state reference and its state feedback, and a time-delay estimation technique used for the estimation of the nonlinear unknown internal dynamics and disturbances. Because of the application of the time-delay estimation technique, the entire proposed controller has both the adaptive and optimal characteristics. With the systems state feedback, the proposed controller which doesn't require any knowledge of nonlinear internal dynamics and disturbances ensures the state trajectory tracking. Finally, to validate and demonstrate the performance of this proposed method, a satellite numerical example with comparisons to classical Proportional-Integral-Derivative Controller is used to illustrate the proposed controller performance and effectiveness.

Keywords: State Adaptive optimal trajectory tracking control, nonlinear affine in control systems, unknown internal dynamics and disturbances, uncertain dynamic estimation, time-delay estimation technique.

1. INTRODUCTION

The trajectory tracking control of nonlinear systems under state or output feedback have been the topics of considerable interests, and are now established hot fields during last recent years (Dinh et al. (2012); Zargarzadeh et al. (2012)). The design of the proposed trajectory tracking control which can realize one side the stabilization or trajectory tracking of a pre-defined trajectory reference of a nonlinear systems, but also the other side to realize the minimization of a prescribed performance index is difficult, even when the considered nonlinear system dynamics are completely known and without disturbance. And the work is more limited and challenging when the considered nonlinear systems are with unknown internal dynamics and disturbances.

Traditionally, the optimal control of linear systems which is accompanied by quadratic cost functions can be realized by resolving the well-known Riccati Equation (RE). To extend the linear optimal control to nonlinear systems, the State Dependent Riccati Equation (SDRE) was proposed in (Shamma et Cloutier (2003)) under strict constraints and required full knowledge of the controlled system dynamics. For the optimal control of nonlinear systems which is in contrast to linear systems, with partially unknown internal dynamics are not adequately addressed, since the Hamilton-Jacobi-Bellman (HJB) equation which don't have a closed-form solution is required (Zargarzadeh et al. (2012)). These problems becomes more challenging when the nonlinear systems internal dynamics becomes completely uncertain (Dinh et al. (2012); Vrabie et Lewis (2009)), and becomes the study scopes of present paper.

For known dynamic nonlinear systems to solve HJB equations, Riccati Equation (RE), and Algebraic Riccati Equation, are more difficult and complicated with comparison to linear systems to solve their corresponding HJB, RE, or

ARE. And the resolution becomes impractical when their system dynamics present uncertain or unknown dynamics. Recently, online adaptive approximation-based optimal control which refers to an online approximator based Adaptive Critic Designs is proposed in (Vrabie et Lewis (2009)). While in (Zargarzadeh et al. (2012)), an optimal adaptive control is proposed for Multi-Input Multi-Output nonlinear systems in strict feedback form with uncertain internal dynamics but without disturbances. The optimal adaptive feedback scheme is introduced for the affine nonlinear systems to estimate the solution of HJB equation online which becomes the optimal feedback control input for the uncertain closed-loop nonlinear system.

Recently, many various nonlinear systems and methods are discussed in adaptive control world(Tao and Kokotovic (1996)). Nonlinear systems in form of strick feedback in a variety of ways and their stability is studied using the normal Backstepping scheme without any optimality mechanism. The introduction of an inverse optimal control for a strict feedback systems which is composed of an associated cost function based control law is considered in (Li and Krstic (1997)). While in many cases where their systems dynamics are unknown or sometimes with disturbances, the control of unknown strick feedback systems need to use the adaptive neural-network based schemes to approximate their uncertain dynamics (Zhang et al. (2011); Zargarzadeh et al. (2012); Kim et YOO (2014)). And similarly, a direct dynamic programming (DDP) approach is developed for the nearly optimal tracking control of affine nonlinear systems whose internal dynamics are estimated by an online approximator.

Thus in our previous works, a composed adaptive controller which is based on a recursive model free stabilization sub-controller (RMFSSC) and a recursive uncertain dynamic compensation sub-controller (RUDCSC) is proposed for nonlinear systems in (Wang et al. (2013)). The first sub-controller of RMFSSC is a recursive model free controller which is developed and based on the theory of Piecewise-Continuous Systems. These PCS are a particular class of hybrid systems with autonomous switchings and controlled impulses (Koncar and Vasseur (2003); Wang et al. (2010, 2012, 2013)). With the application of this referred PCS theory, piecewise-continuous controllers were firstly developed, enabling sampled trajectory tracking of linear systems with full state or sampled and delayed output (Koncar and Vasseur (2003); Wang et al. (2010, 2012, 2013)), and can be also used in state estimation or prediction (Wang et al. (2012, 2013, 2016, 2015)). Then for the improving trajectory tracking performance, a derived piecewise-continuous controller and recursive model free controller were proposed in (Wang et al. (2012, 2013)). Unfortunately, this referred proposed RMFSSC is defined without any prescribed cost function to be minimized and are not a model-free type trajectory tracking controller.

Therefore in this paper for a general class nonlinear systems with unknown internal dynamics and disturbances, an Adaptive Optimal trajectory tracking Controller (AOC) which is developed based on a $\|\cdot\|$ prescribed to minimize a trajectory tracking error and a time-delay estimation technique based unknown nonlinear systems dynamics estimations is proposed. Reminding that this proposed controller which is characterized of model-free and optimal type has simple structure and can be implemented easily in real-time application systems.

In real-time applications, an ideal case without any disturbances and systems variations don't exist. The presenting disturbance and variation need to be compensated for better trajectory tracking performance. The application of the time delay estimation technique is applied for this objective. And with the proposed AOC the entire method could provide good trajectory tracking performance.

The following of this paper is organized as follows: the problem statement and control objective of state feedback based trajectory tracking of nonlinear affine in control systems with unknown internal dynamics and disturbances is given in Section 2. In Section 3, the four steps design method of AOC controller is introduced. Then in Section 4 based on the proposed approach and compared with PID control, a satellite based numerical example is tested to validate the proposed method performance and robustness. Finally it is followed by some conclusion discussions in Section 5.

2. PROBLEM STATEMENT AND CONTROL OBJECTIVE

In this paper, the following defined non-linear affine in control systems with unknown dynamics and disturbances is considered as follows

$$\dot{x}(t) = f(x(t), t)) + g(x(t), t)u(t) \tag{1}$$

where $x(t) \in \Re^n$ is the system state with an initial condition $x(0) = x_0$, and $u(t) \in \Re^m$ is the system input signal.

The system corresponding output is assumed as in the following form of state feedback $y(t) = x(t)$, equal to the system state which is measurable and accessible.

And more importantly, it is supposed that the above non-linear system internal dynamic and disturbance of $f(x(t), t))$ is assumed to be continuous and unknown, and the controlled continuous functions of $g(x(t), t) \in R^{n\times m}$ is supposed to be known. Noting that the referred $g(x(t), t) \in R^{n\times m}$ can be also defined as unknown without loss of generality.

Thus, the control objective of this paper is to realize a feasible trajectory tracking controller which ensures the following defined trajectory tracking error trends to zero

$$\lim_{t\to\infty} \big(e_x(t) = c_x(t) - x(t)\big) \to 0 \tag{2}$$

where $c_x(t)$ is the desired state reference and supposed to be differentiable. In the case of the desired state reference discontinuous, a smoother filter can be added to remove these discontinuities.

For simplification aim, the notation of $\dot{c}_x(t) = \frac{dc_x(t)}{dt}$ is introduced to represent the derivative of $c_x(t)$.

3. STATE FEEDBACK ADAPTIVE OPTIMAL TRAJECTORY TRACKING CONTROL

For the previous considered system, this section describes the development of the referred state feedback Adaptive

Optimal trajectory tracking Controller (AOC) which is based on the following four steps:

In first step: with a prescribed $\|cdot\|$ norm based derivative state trajectory tracking error cost function, a multi optimal derivative state trajectory tracking controller (ODSTC) is designed;

In second step: to remove the trajectory tracking error of state reference, an adjusted optimal state trajectory tracking controller (AOSTTC) is proposed on the earlier ODSTC;

In third step: to estimate the unknown internal dynamics with disturbances appeared in ODSTC and AOSTTC, a time-delayed based estimation (TDE) techniques is proposed to estimate the nonlinear systems unknown internal dynamics; This TDE technique has a very simple controller structure and can be implemented easily in the real world;

And in final step: with the referred AOSTTC and TDE, the entire AOC controller for the unknown nonlinear dynamic systems with disturbance is developed completely.

3.1 First Step: multi-optimal derivative state trajectory tracking controller Design

The control objective in this subsection is to propose an optimal state derivative trajectory tracking controller which ensures the following trajectory tracking error trends to zero

$$\begin{aligned} e_{\dot{x}}(t) = & \quad \dot{c}_x(t) - \dot{x}(t) \\ & = \dot{c}_x(t) - f(x,t) - g(x,t)u(t) \end{aligned} \tag{3}$$

where $\dot{c}_x = \frac{dc_x}{dt}$ is the desired output derivative reference.

With the following proposed simplification notations

$$b_x = g(x,t) \tag{4}$$

$$z(t) = \dot{c}_x(t) - f(x,t) \tag{5}$$

the minimization of trajectory tracking error which is denoted as

$$\|e_{\dot{x}}^2\| = e_{\dot{x}}^T e_{\dot{x}} \tag{6}$$

is proposed to use.

Then considering the referred defined L_2 error norm and with respect to $u(t)$, one can calculate the following ODSTC sub-controller

$$u(t) = \left(b_x^T b_x\right)^{-1} b_x^T z(t) \tag{7}$$

which can be rewritten as

$$u(t) = K_x(x,t)\left(\dot{c}_x - f(x,t))\right) \tag{8}$$

with

$$b_x = g(x,t) \tag{9}$$

$$K_x(x,t) = \left(b_x^T b_x\right)^{-1} b_x^T \tag{10}$$

$$K_x(x,t)b_x = I \tag{11}$$

State convergence demonstration With multiplication of $K_x(x,t)$ to the left side of (1) and with the above defined ODSTC sub-controller in (8), one has the following equation

$$\begin{aligned} K_x(x,t)\dot{x}(t) = & \quad K_x(x,t)\dot{x}(t) \\ = & \quad K_x(x,t)\big(f(x,t)) + g(x,t)u(t)\big) \\ = & \quad K_x(x,t)f(x,t)) + \underbrace{K_x(x,t)g(x,t)}_{I}u(t) \\ & = K_x(x,t)f(x,t) + K_x(x,t)\dot{c}_x - K_x(x,t)f(x,t) \end{aligned} \tag{12}$$

Then one has

$$K_x(x,t)\left(\dot{c}_x(t) - \dot{x}(t)\right) = 0 \tag{13}$$

The only possible solution for (13) which can minimize $\|e_{\dot{x}}^2\| = e_{\dot{x}}^T e_{\dot{x}}$ is obtained consequently as below

$$\dot{x}(t) \equiv \dot{c}_x(t). \tag{14}$$

3.2 Second step: adjusted optimal state trajectory tracking controller design

Recalling that the pre-defined control objective in this paper is denoted as follow

$$\lim_{t\to\infty} e_x(t) = c_x(t) - x(t) \to 0. \tag{15}$$

While with the developed control in (8), only the state derivative trajectory tracking of $\dot{x}(t) \equiv \dot{c}_x(t)$ is achieved.

Considering the real-time applications, because of their permanent existence of initial state conditions, the above designed controller fails to work sufficiently.

To realize the state feedback trajectory tracking of the nonlinear affine-in control system, which means the state trajectory tracking objective of $c_x(t) = x(t)$, the following new adaptive controller is proposed as follow

$$\begin{aligned} u(t) = K_x(x,t)z(t) - \beta K_x(x,t)\left(c_x - x\right) \\ +\dot{K}_x(x,t)\left(c_x(t) - x(t)\right) \end{aligned} \tag{16}$$

where $\beta \in R^{r\times r}$ is a selected stable square constant matrix and $K_x(x,t)$ is defined as the same in (10). And reminding that the term $\dot{K}_x(x,t)(c_x(t) - x(t))$ is introduced to take into account that the coefficient of $K_x(x,t)$ is dependent explicitly to time with respect to $x(t)$.

Stability demonstration With the application of the proposed new control (16) into the considered nonlinear system(1), one has the following relationship:

$$\begin{aligned} \dot{x}(t) = f(x(t),t) + g(x(t),t) \times \Big(K_x(x,t)(\dot{c}_x - f(x,t)) \\ -\beta K_x(x,t)(c_x - x) + \dot{K}_x(x,t)(c_{xy}(t) - x(t))\Big) \end{aligned} \tag{17}$$

then with multiplication of $K_y(x,t)$ to both sides of the above equation, and with consideration of the above referred defined relationships in (9), (10) and(11), one has

$$K_x(x,t)\left(\dot{c}_x - \dot{x}(t)\right) + \dot{K}_x(x,t)\left(c_x(t) - x(t)\right) = \beta K_x(x,t)\left(c_x(t) - x(t)\right) \quad (18)$$

For simplification aim, with the introduction of notation which is denoted as $E = K_x(x,t)\left(c_x(t) - x(t)\right)$, the above equation can be derived as follow

$$\dot{E} = \beta E \quad (19)$$

which shows that with a selected stable constant matrix β, i.e. with negative proper eigenvalues, one has following convergence

$$\lim_{t\to\infty} E = K_x(x,t)\left(c_x(t) - x(t)\right) \to 0 \quad (20)$$

Thus,the state feedback trajectory tracking for the consider nonlinear system to its desired state output reference is ensured $x(t) = c_x(t)$.

For real-time applications, one can choose $\beta = -\beta_0^2 I_r$ where β_0 and I_r are respectively selected constant and r-dimensional unity matrix. And generally, the bigger value β_0 is, the faster convergence of the state of the controlled system to its desired state reference value, which means that one obtains

$$x(t) = c_x(t). \quad (21)$$

3.3 Third step: time-delay estimation of unknown uncertain system dynamics and disturbances

With the developing quadratic optimal nonlinear control which is defined in (16), one is able to realize the state feedback trajectory tracking with exponential convergence stability in the case where the nonlinear dynamic $f(x,t)$ is known, but for the case considered here in this paper because of the unknown system dynamics and disturbances $f(x,t)$ in $z(t) = \dot{c}_x(t) - f(x,t)$ which is appeared in (16), a practical and robust unknown dynamic and disturbance estimator is needed to propose.

From the derivation of the defined state equation $\dot{x}(t) = f(x,t) + g(x,t)u(t)$, then the unknown nonlinear dynamic and disturbance can be deduced as

$$f(x,t) = \dot{x}(t) - g(x,t)u(t) \quad (22)$$

For real time implementation by application of time delay estimation techniques(Youcef-Toumi and Fuhlbrigge (1989, 1991); Wang et al. (2015a, 2016); Lee et al. (2017)), the following estimate is proposed $\hat{f}(x,t)$ for the system unknown dynamic and disturbance $f(x,t)$ as follow

$$\begin{aligned}\hat{f}(x,t) = \quad & f\big(x, t-\epsilon\big) \\ = & \dot{x}(t) - g(x,t-\epsilon)u(t-\epsilon)\end{aligned} \quad (23)$$

For $\epsilon \to 0$ and under the assumptions that the $f(x,t)$ and $g(x,t)$ are continuous, one obtains the following relationship

$$\hat{f}(\hat{x}, t-\epsilon) \simeq f(x,t) \quad (24)$$

It is important to note that with rapid calculation tools development, the implementation of this assumption of $\epsilon \to 0$ can be realized easily. And normally, the value of ϵ is selected as the sampling period of the simulator.

3.4 Final step: time delay estimation based optimal state trajectory tracking controller

Then with the proposed time delay estimation technique for unknown dynamic $f(x,t)$ in (23), the proposed state feedback adaptive optimal trajectory tracking controller can be defined finally as follows

$$\begin{aligned}u(t) = & K_x(x,t)\Big(\dot{c}_x(t) - c(t)\hat{f}(x,t)\Big) \\ & -\beta K_x\big(x,t\big)\left(c_x(t) - x(t)\right) \\ & +\dot{K}_x\big(x),t\big)\left(c_x(t) - x(t)\right)\end{aligned} \quad (25)$$

Entire closed feedback Stability analysis With the above proposed time delay estimation technique based AOC controller, and replaced it to equation (1), one obtains

$$\begin{aligned}\dot{x}(t) = \quad & f(x(t),t) + g(x(t),t) \times \\ & \Big(K_x(x,t)(\dot{c}_x - \hat{f}(\hat{x},t)) - \beta K_x(x,t)(c_x - x) \\ & +\dot{K}_x(x,t)(c_x(t) - x(t))\Big)\end{aligned} \quad (26)$$

Then by multiplication with $K_x(x,t)$ to the above equation, and with relationships defined in (9), (10), (11), one has

$$\begin{aligned}& K_x(x,t)\left(\dot{c}_x - \dot{x}(t)\right) + \dot{K}_x(x,t)\left(c_x(t) - x(t)\right) \\ & = \beta K_x(x,t)\left(c_x(t) - x(t)\right) + \Delta_f(x,t)\end{aligned} \quad (27)$$

with $\Delta_f(x,t) = K_x(x),t)(\hat{f}(\hat{x},t) - f(x(t),t))$ which is bounded and convergence to zero under $\epsilon \to 0$ (for more detail convergence demonstration, please refer to (Youcef-Toumi and Fuhlbrigge (1989, 1991); Wang et al. (2015a, 2016); Lee et al. (2017)).

If one denotes $E = K_x(x,t)\left(c_x(t) - x(t)\right)$, the above equation can be rewritten as follows

$$\dot{E} = \beta E + \Delta_f \quad (28)$$

which means with the selected stable constant matrix β (with negative proper eigenvalues), one is able to ensure the following relationship

$$\lim_{t\to\infty,\epsilon\to 0} E = K_x(x,t)\left(c_x(t) - x(t)\right) \to 0 \quad (29)$$

and realizing consequently the trajectory tracking of system output to its desired state reference which is denoted as $x(t) = c_x(t)$.

4. APPLICATION TO SATELLITE CONTROL SYSTEMS

To validate the proposed control method, we consider here a satellite which rotates in free space under the influence of gas jets mounted along three mutually orthogonal body-fixed axes as in (Brogan (1985)), and compared to a classical Proportional-Integral-Derivative (PID) Controller whose parameters are tuning by Nichol Zeigler tuning rules and can be defined as follow

$$u(t) = K_p e_x(t) + K_i \int e_x(t)dt + K_d \dot{e}_x(t) \quad (30)$$

where K_p, K_i, and K_d are parameters of classical PID.

4.1 Application system description

In this subsection, the referred satellite rotating is described firstly. Let $\omega = [\omega_x \ \omega_y \ \omega_z]^T$ be the three components of angular velocity denoted with respect to the body-fixed axes and $T = [T_x \ T_y \ T_z]^T$ be the three components of input control torques.

By applying Newton's second law to the satellite rotating body, states that $dH/dt = T$, where H is the angular momentum vector, and the time rate of change d/dt is with respect to a fixed inertial reference. The vector H can be expressed in body coordinates as $H = [J_x\omega_x \ J_y\omega_y \ J_z\omega_z]^T$ where the constants J_i with $i = x, y, z$ are moments of inertia of he body and x, y, z are assumed to be principal axes of inertia.

The inertial rate of change $dH/dt = T$ is related to the apparent rate $\dot{H}$ as seen by an observer moving with the body by

$$dH/dt = [\dot{H}] + \omega \times H \tag{31}$$

Therefore one has the following three input control torques as follows

$$\begin{aligned} T_x &= J_x\dot{\omega}_x + (J_z - J_y)\omega_y\omega_z \\ T_y &= J_y\dot{\omega}_y + (J_x - J_z)\omega_x\omega_z \\ T_z &= J_z\dot{\omega}_z + (J_y - J_x)\omega_y\omega_x) \end{aligned}$$

These referred Euler's dynamical equations which can be rearranged as following nonlinear affine in control system:

$$\dot{\omega} = f(\omega, t) + Gu_T \tag{32}$$

with $\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$, $f(\omega, t) = \begin{bmatrix} \dfrac{J_y - J_z}{J_x}\omega_y\omega_z \\ \dfrac{J_z - J_x}{J_y}\omega_x\omega_z \\ \dfrac{J_x - J_y}{J_z}\omega_x\omega_y \end{bmatrix}$, $u_T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$,

and $G = \begin{bmatrix} \dfrac{1}{J_x} & 0 & 0 \\ 0 & \dfrac{1}{J_y} & 0 \\ 0 & 0 & \dfrac{1}{J_z} \end{bmatrix}$.

It is important to note that $f(\omega, t)$ is considered as an unknown dynamic function and include disturbance in the above controller application, while for the control input matrix G can be considered as a known matrix without loss the generality: because in the case of unknown G, one can introduce a known matrix G_0, then the above equation can be rewritten in the following equivalent form

$$\dot{\omega} = f'(\omega, t) + G_0 u_T \tag{33}$$

with $f'(\omega, t) = f(\omega, t) + G - G_0$. Thus one can always suppose that the matrix G is disposable and known, while $f'(\omega, t)$ is unknown.

Thus, in the general case without loss of generality, one can suppose that in the above equation (32) that $f(\omega, t)$ is unknown including the disturbances, and G can be supposed known.

4.2 Numerical Simulation Results

In this subsection, to demonstrate the proposed AOC controller performance, the above referred satellite nonlinear systems is used to implement and compared it with the most widely applied and effective PID controller.

In the following numerical simulations and corresponding results, the desired state velocities references are chosen as piecewise constants and sinusoidal references which are continuous. Their corresponding numerical results are illustrated respectively in two cases: first case are the figures from Fig.1 to Fig.9 which are under the filtered piecewise constant reference, and second case are figures from Fig.10 to Fig.18 which are under the sinusoidal reference.

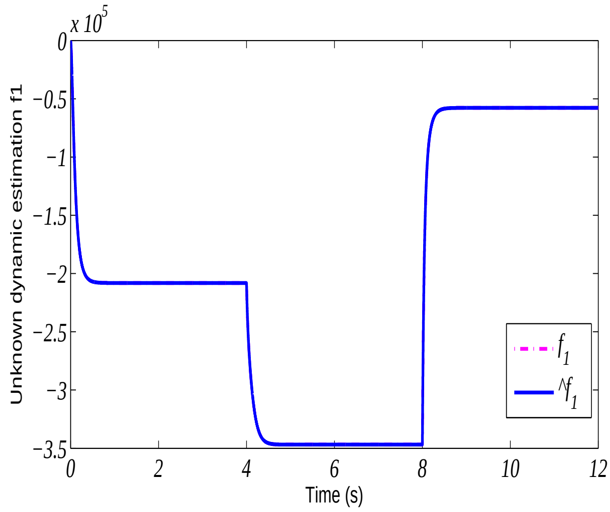

Fig. 1. Estimation of unknown dynamic and disturbance in x-axis under desired state filtered piecewise constant reference

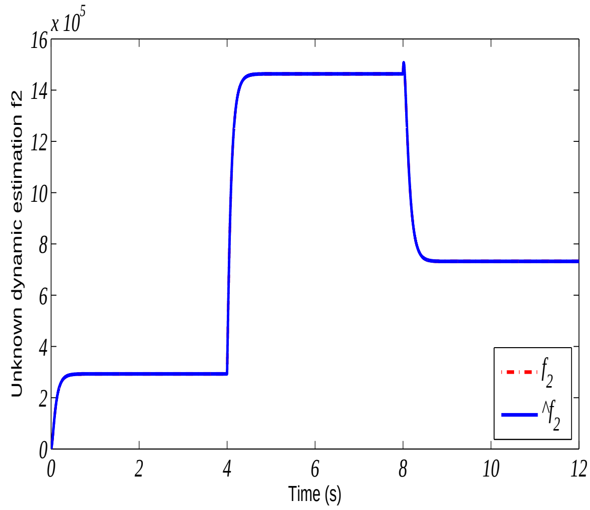

Fig. 2. Estimation of unknown dynamic and disturbance in y-axis under desired state filtered piecewise constant reference

Noting that the implemented filtered piecewise constant reference and the sinusoidal references are the most widely used and re-presentive references in application world.

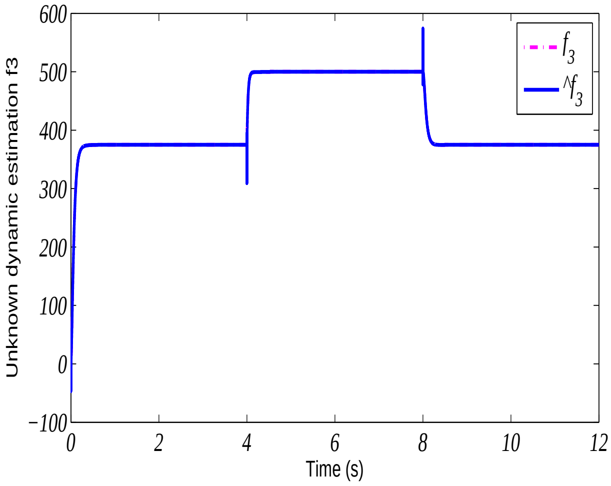

Fig. 3. Estimation of unknown dynamic and disturbance in z-axis under desired state filtered piecewise constant reference

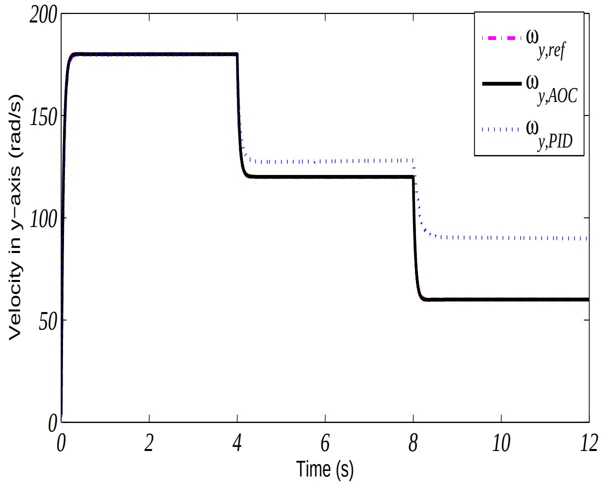

Fig. 5. Velocity tracking in y-axis under desired state filtered piecewise constant reference

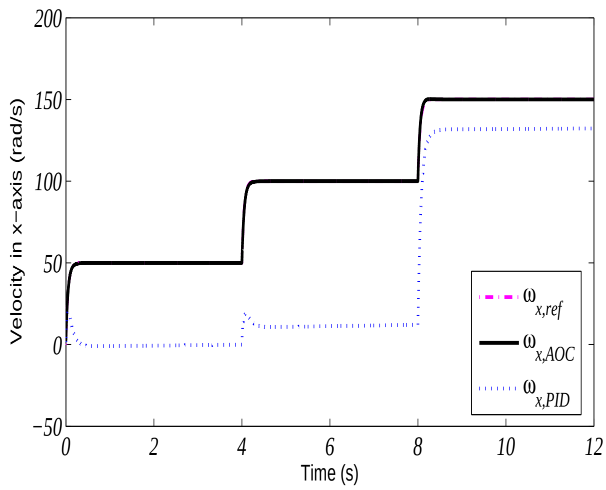

Fig. 4. Velocity tracking in x-axis under desired state filtered piecewise constant reference

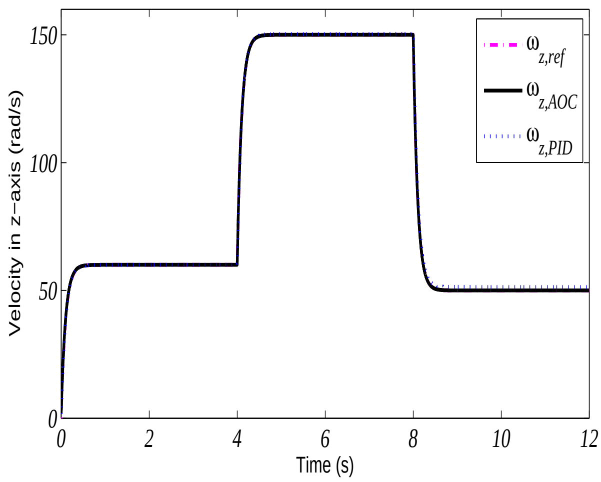

Fig. 6. Velocity tracking in z-axis under desired state filtered piecewise constant reference

For the considered two different references from figures of Fig.1 to Fig.3 and from figures of Fig.10 to Fig.12, it can be concluded easily that the proposed time-delayed estimation technique can estimate very well the unknown internal dynamic and disturbance of the considered satellite controlled systems.

Moreover, the proposed entire AOC controller compared with the classical PID controller, the entire proposed AOC approach which can be tuned easily, ensures better trajectory tracking performances in term of rising times and static state errors under the above implemented two references (illustrated in figures from Fig.4 to Fig.6 and figures from Fig.13 to Fig.15). More importantly, the tuning parameters of the proposed AOC approach can be configured much more easily than PID.

Then concerning their control inputs of the AOC, which are illustrated respectively in figures from Fig.7 to Fig.9 under filtered piecewise constant reference and figures from Fig.16 to Fig.18 under sinusoidal reference, are both s-

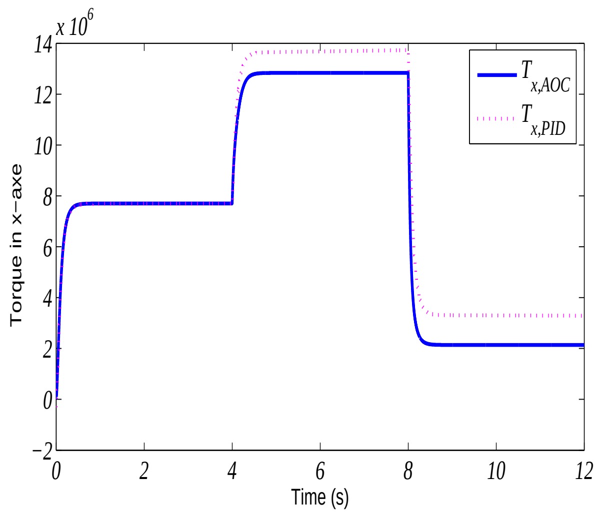

Fig. 7. Torque control input in x-axis under desired state filtered piecewise constant reference

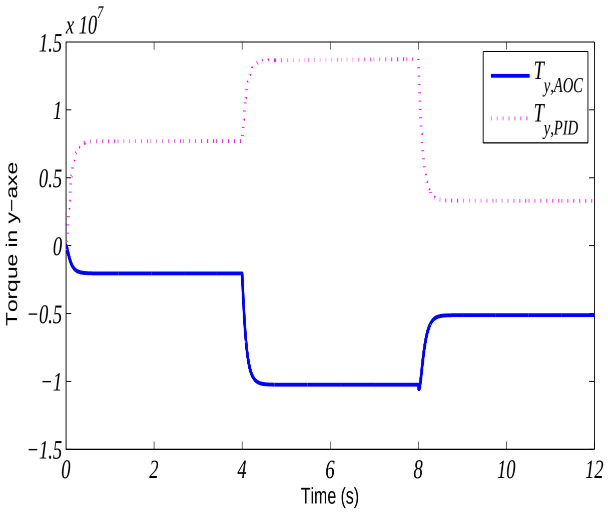

Fig. 8. Torque control input in y-axis under desired state filtered piecewise constant reference

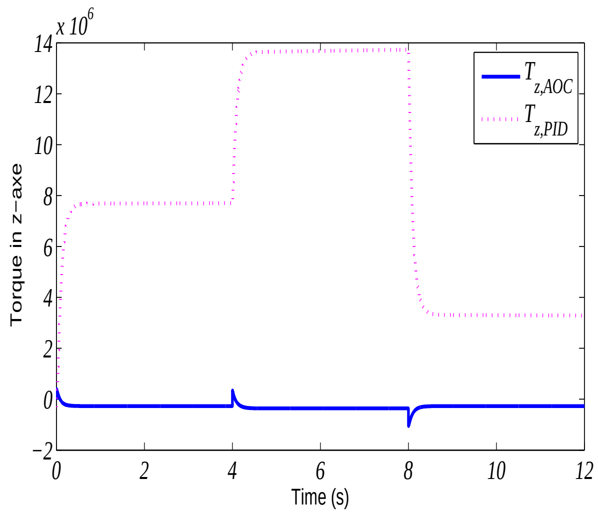

Fig. 9. Torque control input in z-axis under desired state filtered piecewise constant reference

mooth and practical for real-time implementations. And more importantly to note that the obtained input amplitudes obtained by the proposed AOC control are relatively smaller than the classical PID control inputs. From this side of view, one can see that the proposed AOC method consumes less energy and are more economical.

5. CONCLUSION REMARKS

In this paper for a nonlinear affine-in control systems with unknown internal dynamics and disturbance, a new state feedback adaptive optimal trajectory tracking controller has been proposed and analysed completely of its corresponding closed-loop stability. The effectiveness and robustness of the proposed AOC approach have been demonstrated and compared with a classical PID controller via a nonlinear satellite rotating motion system.

This develop AOC controller which is based on a $\|\cdot\|$ prescribed norm to minimize the state trajectory tracking error and the time-delay estimation technique based nonlinear systems unknown perturbed dynamics estimations

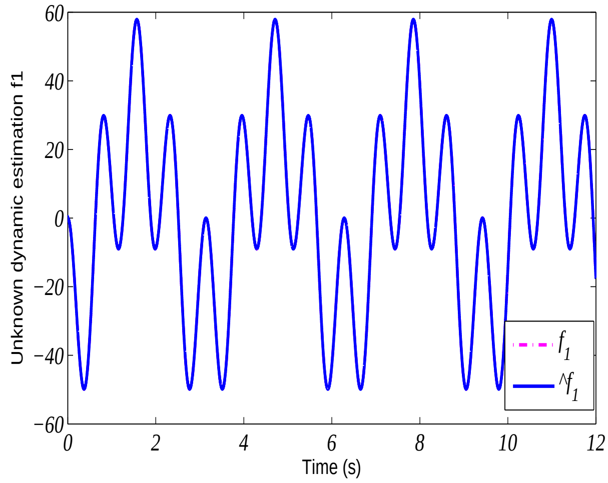

Fig. 10. Estimation of unknown dynamic and disturbance in x-axis under desired state sinusoidal reference

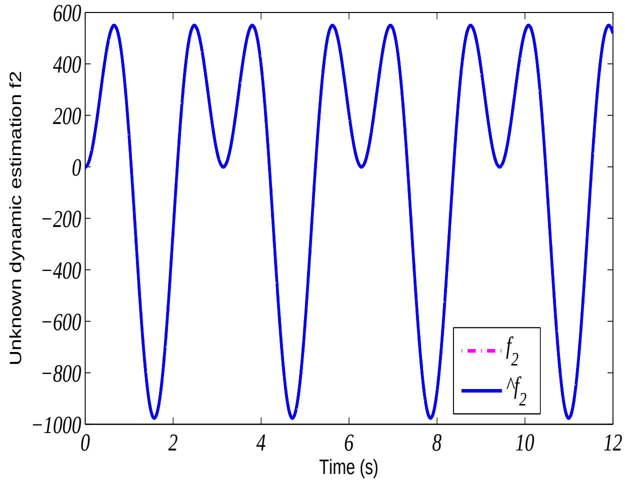

Fig. 11. Estimation of unknown dynamic and disturbance in y-axis under desired state sinusoidal reference

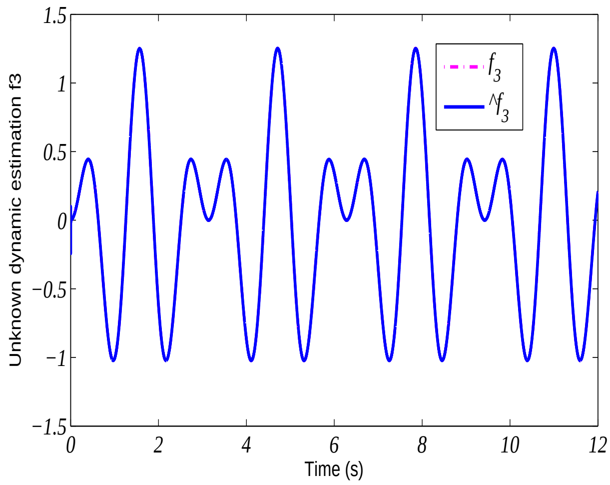

Fig. 12. Estimation of unknown dynamic and disturbance in z-axis under desired state sinusoidal reference

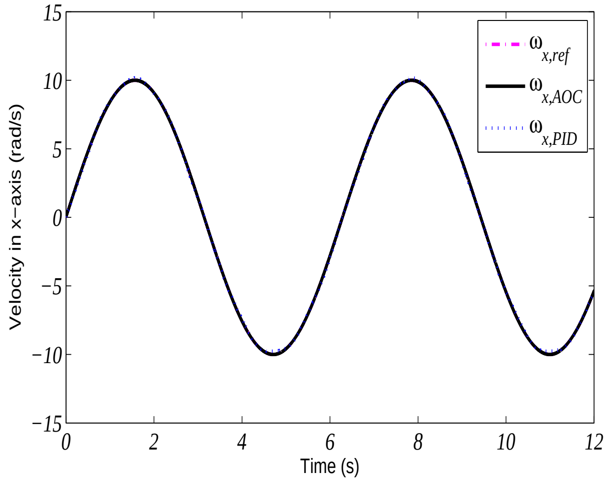

Fig. 13. Velocity tracking in x-axis under desired state sinusoidal reference

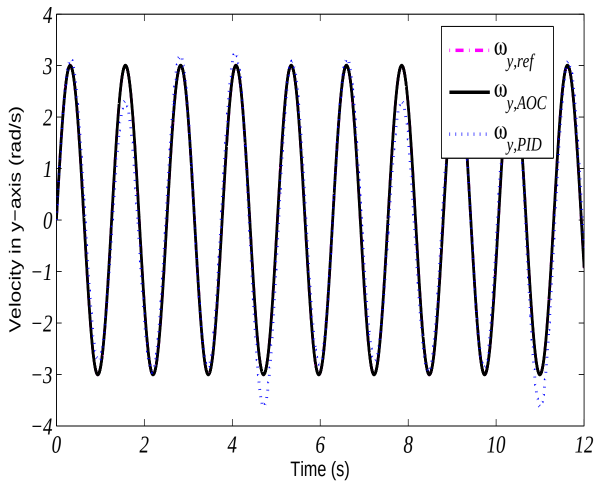

Fig. 14. Velocity tracking in y-axis under desired state sinusoidal reference

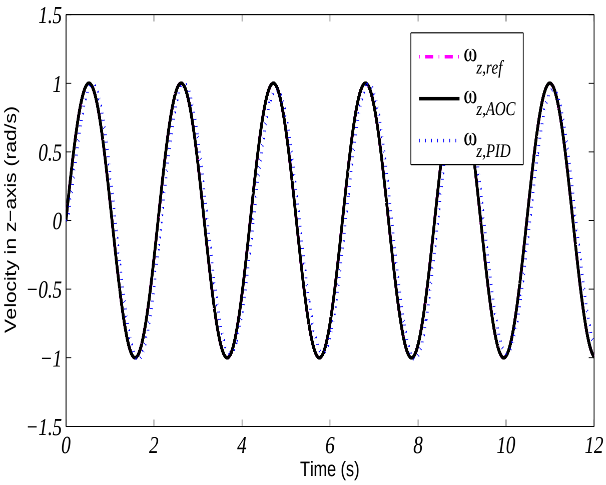

Fig. 15. Velocity tracking in z-axis under desired state sinusoidal reference

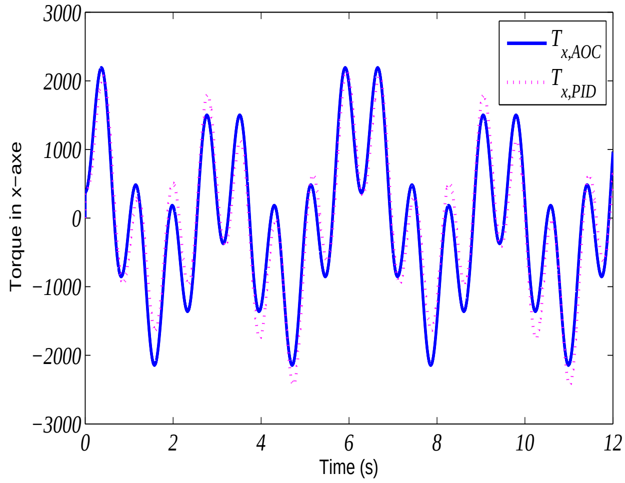

Fig. 16. Torque control input in x-axis under desired state sinusoidal reference

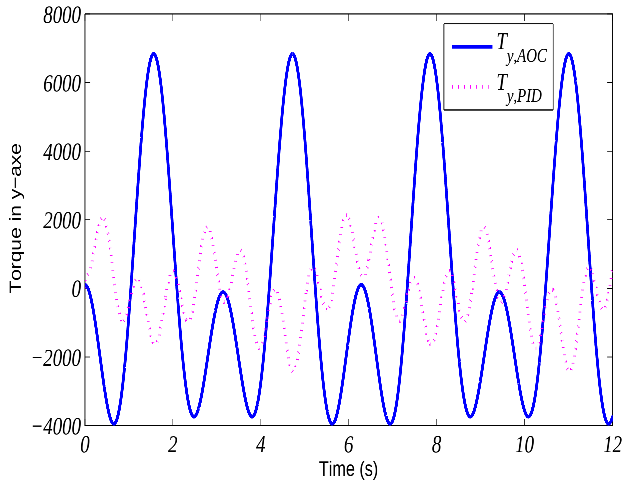

Fig. 17. Torque control input in y-axis under desired state sinusoidal reference

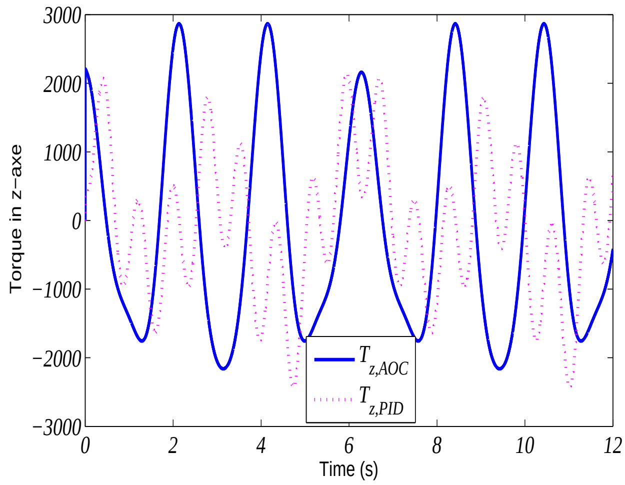

Fig. 18. Torque control input in z-axis under desired state sinusoidal reference

has adaptive and optimal characteristics. With the systems state feedback, the proposed AOC controller dose not require any knowledge of the controlled nonlinear internal dynamics and disturbances, and can ensure the desired state trajectory tracking performance of the controlled nonlinear MIMO systems under two difference references (i.e., the filtered piecewise constant reference and the sinusoidal references).

It is important to note that one side the proposed AOC method has smaller torque inputs by comparing with the PID controller which implies that the proposed AOC method is more economical and consumes less energy and more environment amiable, and the other side for real-time implementation with the rapid development of numerical calculators, although the time-delay estimation technique performance is related to the realization capacity of the time delay value of ϵ to zero, its influence to the stability of the entire controlled systems remains limited. And for high precision application cases, the proposed AOC method can be improved further by the combination of the estimation techniques of the time-delay estimation induced error.

ACKNOWLEDGMENTS

This work was partially supported by the International Science & Technology Cooperation Program of China (2015DFA01710), by the National Natural Science Foundation of China (61304077, 61203115), by the Natural Science Foundation of Jiangsu Province (BK20170094), by the Chinese Ministry of Education Project of Humanities and Social Sciences (13YJCZH171), by the Fundamental Research Funds for the Central Universities (30920130111014), and by the Zijin Intelligent Program of Nanjing University of Science and Technology (2013_ZJ_0105), the authors would like to express theirs deep appreciations.

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