Robust Model Predictive Controller for Uncertain Systems Modelled by Orthonormal Laguerre Functions

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Abstract: This paper aims to synthesize a new method for robust predictive control of a class of linear discrete-time uncertain systems represented by Laguerre orthonormal functions. This controller adopts a worst-case strategy solved by a min max optimization problem taking into account the constraints relative to parameter uncertainties of the resulting Laguerre model and to measurement signals. In order to ensure a significant reduction of the Laguerre model coefficients, a new algorithm is developed to select the optimal Laguerre model. The unknown but bounded error approach is used to update the parameter domain selected as an ellipsoid. The effectiveness of the robust controller is shown by an example of simulation.

Keywords: Predictive control, robust control, uncertain dynamic system, min-max, Laguerre base, ellipsoid.

1. INTRODUCTION

Model Predictive Control (MPC) has become one of the most popular strategy for controlling discrete-time uncertain systems with a large frame of applications (Lazar et al., 2008; Nisha and Pillai, 2014; Fesharaki and Talebi, 2014; Sun et al., 2017; Gouta et al., 2016; Yamashita et al., 2016; Kutasi et al., 2017). The design of MPC uses a mathematical model to predict the behavior of the process by minimizing an objective function subjects to constraints over a finite-time horizon. Only the first of control inputs is applied to the system and after retrieving the next process output, the optimization problem is solved again for the next control.

There has been interest in developing methods for systems approximation and discrete-time model predictive control using Laguerre orthonormal functions (Oliveira et al., 2000; Wang, 2004; Zhang et al., 2006; Asad and Hasan, 2012; Hidayat and Medvedev, 2012; Bouzrara et al., 2012; Kannan et al., 2013; Samuel et al., 2014; Mbarek et al., 2017; El Anes et al., 2018). However, the use of such Laguerre model structure implicitly involves expanding the true discrete-time transfer function of the system around poles near the dominating slow mode (Wahlberg, 1991; Wahlberg and Mäkilä, 1996). In this way, the Laguerre model requires a very large number of terms in the series expansion to represent systems with various representative modes, and this may lead to poor accuracy in the estimated model as well as the control strategy. In reality, the choice of the optimal Laguerre pole is primordial to permit the truncating order of the Laguerre network filters as a minimal order without too alter the quality of approximation (Oliveira e Silva, 1994; Belt and Den Brinker, 1995; Fu and Dumont, 1993).

To situate the dynamics of the system and to permit an optimal and adequate choice of the Laguerre pole, we propose to develop a reduced complexity Laguerre model in order to synthesize a new robust predictive control strategy for SISO linear uncertain systems. The proposed control design uses a worst-case strategy to solve a min-max optimization problem, subjects to constraints on the measurement signals and the parameter uncertainty of the resulting Laguerre model. By introducing linear matrix inequalities (LMIs), the min-max optimization problem can be reformulated as a linear programming problem with a reduced number of constraints (Kothare et al., 1996; Boyd et al., 1998; Apkarian and Tuan, 2000; Jia et al., 2005).

On the other hand, when no statistical information on the noise is available at the exception is bounded and of known boundary, the set of Fourier coefficients is updated using the unknown but bounded error method (UBBE). With this hypothesis, the uncertainty domain of parameters is usually considered under a convex polytope delimited by its vertices. This set is compatible with the measures, the model structure and the error bounds. Because of the complexity of the convex polytope, a simpler geometrical shape such as an ellipsoid is used to approach it (Fogel and Huang, 1982; Favier and Arruda, 1996). The need of such approximation is that the number of inequalities describing the polytope increases the amount of data to be processed which leads to complex computational (Dabbenea et al., 2003).

The main contribution of the paper are threefold. First, a reduced complexity model based on Laguerre functions is developed. The key advantage of this model structure is it requires no prior knowledge about the system modes during the identification process and hence as few parameters as possible have to be estimated. Second, the derived controller ensures the robustness of the closed loop system against parameter uncertainties. Finally, it synthesizes a robust predictive controller handling a class of linear systems having several modes. The main features of the proposed control approach using a reduced Laguerre model is that it is not sensitive to the choice of the sampling interval, it doesn't requires a prior knowledge of the system delay and it operates on a small number of parameters. Furthermore, the criterion to be minimized is convex on the uncertainty domain of model coefficients.

The structure of this paper is as follows. Section 2 presents the Laguerre model structure for a class of linear dynamical systems. Section 3 describes the method developed for estimating the optimal Laguerre pole and the concept of the UBBE approaches used to update the set of Fourier coefficients. The main results of the robust predictive control are included in Section 4, in which the output predictor is expressed and the optimization problem is reformulated and solved by introducing LMIs techniques. Section 5 ends with a simulation example and finally, concluding remarks and perspectives are, made in section 6.

2. LAGUERRE MODEL STRUCTURE

This paper considers a linear discrete-time-invariant SISO (Single-Input Single-Output) system described by the following standard input-output model:

$$y(k) = G(q)u(k) + e(k) \tag{1}$$

where $u(k) \in \mathbb{R}$ is the system input, $y(k) \in \mathbb{R}$ is the output and $e(k)$ is the measurement noise assumed to have finite variance (a zero-mean stationary sequence).

The scalar transfer function $G(q)$ describes the (assumed stable) unknown system dynamics, q is the forward shift operator defined as $qx(k) = x(k+1)$.

In (Wahlberg, 1991; Wahlberg and Hannan, 1993), the discrete orthonormal Laguerre functions are given by:

$$L_n(z,\xi) = \frac{z\sqrt{1-\xi^2}}{z-\xi}\left(\frac{1-\xi z}{z-\xi}\right)^n, \quad n = 0,1,\cdots \tag{2}$$

where $|\xi| \prec 1$ is called the (real valued) Laguerre pole and n is the order of Laguerre functions.

The following recurrent relations are defined from (2):

$$L_0(z,\xi) = \frac{z\sqrt{1-\xi^2}}{z-\xi} \tag{3}$$

$$L_n(z,\xi) = \left(\frac{1-\xi z}{z-\xi}\right)L_{n-1}(z,\xi) \tag{4}$$

The model given in (1) is parameterized by representing the transfer function $G(q)$ as truncated series expansion in the Laguerre orthonormal basis.

Then, the Laguerre model output is written as follows:

$$\hat{y}(k) = \sum_{n=0}^{N-1} g_n x_n(k) \tag{5}$$

$$x_n(k) = Z^{-1}\left\{L_n(z,\xi)\right\}u(k) \tag{6}$$

where $\{x_n(k)\}_{n=0,\cdots,N-1}$ are the Laguerre filter outputs, N is the truncating order, $\{g_n\}_{n=0,\cdots,N-1}$ are the Fourier coefficients and Z^{-1} is the inverse Z-transform.

Using the Z-transform of relations given in (5)-(6) and assume that initial conditions are equal zero's, the discrete-time Laguerre filter network is given by Fig. 1.

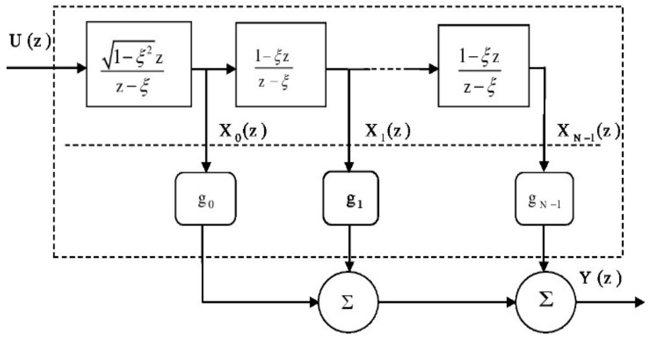

Fig. 1. Discrete-time Laguerre filter network.

According to the filter network of Fig. 1, the following recurrent equations are established:

$$\begin{cases} x_0(k+1) = \xi x_0(k) + \sqrt{1-\xi^2}u(k) \\ x_1(k+1) = \left(1-\xi^2\right)x_0(k) + \xi x_1(k) - \xi\sqrt{1-\xi^2}u(k) \\ \quad\vdots \\ x_{N-1}(k+1) = (-\xi)^{N-2}\left(1-\xi^2\right)x_0(k) + \cdots + \xi x_{N-1}(k) + \\ \qquad\qquad + (-\xi)^{N-1}\sqrt{1-\xi^2}u(k) \end{cases} \tag{7}$$

Let $x(k) = \left[x_0(k)\, x_1(k) \cdots x_{N-1}(k)\right]^T$ be the state vector of dimension N.

From (5)-(7), the Laguerre model representation is given by:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ \hat{y}(k) = \theta^T x(k) \end{cases} \tag{8}$$

where:
$A = \left(a_{i,j}\right) \in \mathbb{R}^{N\times N}$ is a triangular state matrix, $B = \left[b_1\, b_2 \cdots b_N\right]^T \in \mathbb{R}^N$ is the control gain vector and $\theta = \left[g_0\, g_1 \cdots g_{N-1}\right]^T$ is the parameter vector, where $a_{i,j}$ and b_j are respectively given by:

$$a_{i,j} = \begin{cases} \xi & if \quad i = j \\ \left(1-\xi^2\right)\left(-\xi\right)^{i-j-1} & if \quad i \geq j+1 \\ 0 & if \quad i < j \end{cases} \tag{9}$$

$$b_j = \left(-\xi\right)^{j-1}\sqrt{1-\xi^2}, \quad j = 1,2,\cdots,N \tag{10}$$

3. MODEL PARAMETER IDENTIFICATION

In this section, we focus on determining the optimal Laguerre pole and the Fourier coefficients of the resulting model given in (8)-(10).

3.1 Estimation of the Laguerre pole

Consider the following criterion defined over the horizon M of data as:

$$J(\xi) = \sum_{k=1}^{M} \left(y(k) - \sum_{n=0}^{N-1} g_n x_n(k) \right)^2 \tag{11}$$

Adopting the vectorized notation:

$$\underline{x} = \left[x(1)\, x(2) \cdots x(M)\right]^T, \quad \underline{y} = \left[y(1)\, y(2) \cdots y(M)\right]^T$$

The criterion given in (11) can be written in matrix form as:

$$J(\xi) = \left(\underline{y} - \underline{x}\,\theta\right)^T \left(\underline{y} - \underline{x}\,\theta\right) \tag{12}$$

Using the Gauss-Newton algorithm, the Laguerre pole at iteration $(i+1)$ is given by:

$$\xi^{(i+1)} = \xi^{(i)} - \eta \left(\frac{\partial^2 J(\xi)}{\partial^2 \xi} \right)^{-1}_{\xi=\xi^{(i)}} \left(\frac{\partial J(\xi)}{\partial \xi} \right)_{\xi=\xi^{(i)}} \tag{13}$$

where η is the Newton adaptive step.

From (12), the Gradient $\frac{\partial J(\xi)}{\partial \xi}$ and the approximate Hessian $\frac{\partial^2 J(\xi)}{\partial^2 \xi}$ are given by:

$$\frac{\partial J(\xi)}{\partial \xi} = -2\theta^T W \left(\underline{y} - \underline{x}\,\theta\right) \tag{14}$$

$$\frac{\partial^2 J(\xi)}{\partial^2 \xi} \approx 2\theta^T W W^T \theta \tag{15}$$

with:
W is an $(N \times M)$ dimensional matrix given by:

$$W = \left[w(1)\, w(2) \cdots w(M)\right]^T$$

where $w(k) = \left[w_0(k)\, w_1(k) \cdots w_{N-1}(k)\right]^T \in \mathbb{R}^N$ is the vector of filter sensitivities given by:

$$w_\ell(k) = \frac{\partial x_\ell(k)}{\partial \xi}, \quad \ell = 0, 1, \cdots, N-1 \tag{16}$$

However, computing the gradient given in (14) and the approximate Hessian given in (15) requires determination of the filter sensitivities regrouped in the matrix W.
Differentiating the state equation (8) with respect to ξ yields:

$$w(k+1) = A w(k) + C x(k) + D u(k) \tag{17}$$

with:

$C = \left(c_{i,j}\right)$ is an $N \times N$ dimensional matrix and $D = \left[d_1\, d_2 \cdots d_N\right]^T$ is a vector of dimension N.

The coefficients $c_{i,j} = \left(\frac{\partial a_{i,j}}{\delta \xi} \right)$ and $d_j = \frac{\partial b_j}{\partial \xi}$ $i, j = 1, 2, \cdots, N$ are given by:

$$c_{i,j} = \begin{cases} 1 & \text{if } i = j \\ \zeta_{i,j} & \text{if } i \geq j+1 \\ 0 & \text{if } i < j \end{cases} \tag{18}$$

$$\zeta_{i,j} = 2(-\xi)^{i-j-2} \left[1 - \left(1-\xi^2\right)(i-j-1)\right] \tag{19}$$

$$d_j = (-\xi)^{j-2} \left[\frac{1}{\sqrt{1-\xi^2}} - (j-1)\sqrt{1-\xi^2} \right] \tag{20}$$

The most obvious scheme for estimating the parameter vector θ is using least squares techniques (Lujung, 1987). It is well known that in this case the least squares estimate has closed form solution, which is given by:

$$\hat{\theta} = \left(\underline{x}^T \underline{x}\right)^{-1} \underline{x}^T \underline{y} \tag{21}$$

The optimal Laguerre pole can be estimated by iterating the following steps:

- determine the filter sensitivities regrouped in the matrix W by applying (17);
- determine the vector of Fourier coefficients by applying (21);
- determine the Gradient and the approximate Hessian from (14)-(15);
- determine the optimal Laguerre pole from (13).

3.2 Estimation of the Fourier coefficients

The model output for the SISO system given in (1) can be rewritten as follows:

$$y(k) = \theta^T x(k) + e(k) \tag{22}$$

where $x(k)$ is the state vector, θ is the parameter vector and $e(k) = y(k) - \hat{y}(k)$ is the model error.

Assumption1: The model error is assumed to be unknown but bounded and of known boundary $\gamma(k) \in \mathbb{R}$, such that:

$$|e(k)| \leq \gamma(k) \tag{23}$$

According to (22) and (23) one gets:

$$y(k) - \gamma(k) \leq \theta^T x(k) \leq y(k) + \gamma(k) \tag{24}$$

The double inequalities given in (24) generates at each time instant k two hyperplanes H_{k1} and H_{k2} in the parametric space of the vector θ and normal to the state vector $x(k)$:

$$H_{k1} = \left\{ \theta \middle/ \theta^T x(k) = y(k) + \gamma(k) \right\} \tag{25}$$

$$H_{k2} = \left\{ \theta \middle/ \theta^T x(k) = y(k) - \gamma(k) \right\} \tag{26}$$

It follows from (25)-(26) that each hyperplane $H_{kj}\ (j = 1,2)$ generates two closed half-spaces $H_{kj}^{+}\ (j = 1,2)$ and $H_{kj}^{-}\ (j = 1,2)$. Therefore, the parameter vector θ satisfying the double inequalities (24) belongs to the domain defined by the intersection of the positive closed half-spaces H_{k1}^{+} and H_{k2}^{+} generated by the hyperplanes H_{k1} and H_{k2} respectively.

$$H_k^{+} = H_{k1}^{+} \cap H_{k2}^{+} \tag{27}$$

The membership of the vector θ, S_M, obtained following to the acquisition of M measurements, must thus satisfy:

$$S_M = \bigcap_{k=1}^{M} H_{k1}^{+} \cap H_{k2}^{+} \tag{28}$$

In this way the UBBE approach consists in determining at each time instant k, the smallest set of parameters $S_M \subset \mathbb{R}^N$ consisting with measurements, the model structure and the error bounds.

Remark 1: Notice that the complexity of the geometrical form S_M, increases with the number of measurements and the number of parameters to be estimated. To overcome this complexity, this exact set S_M is approached by an ellipsoid.

4. ROBUST PREDICTIVE CONTROL DESIGN

4.1 Step-ahead predictor

The incremental form of (8) yields:

$$\delta x(k+1) = A\,\delta x(k) + B\,\delta u(k) \tag{29}$$

$$\hat{y}(k) = \hat{y}(k-1) + \theta^T \delta x(k) \tag{30}$$

where $\delta x(k)$ and $\delta u(k)$ are the state increment vector and the control increment signal, defined by:

$$\delta x(k) = x(k) - x(k-1) \tag{31}$$

$$\delta u(k) = u(k) - u(k-1) \tag{32}$$

When the error on the Laguerre model is unknown but bounded, the Fourier coefficients are defined by uncertainty intervals.

Equation (30) can be then rewritten as:

$$\hat{y}(k) = \hat{y}(k-1) + \theta^T(\varepsilon)\delta x(k) \tag{33}$$

where $\varepsilon \in \Omega$ is the vector of parameter uncertainties and Ω is the parameter uncertainty domain.

By (33), the i-step ahead predictor is given as:

$$\begin{aligned}\hat{y}(k+i/k) = \hat{y}(k+i-1/k) + \\ + \theta^T(\varepsilon)\delta x(k+i),\ \forall\ i \ge 1\end{aligned} \tag{34}$$

By successive substitution, equation (29) yields:

$$\delta x(k+i) = A^i \delta x(k) + \sum_{j=1}^{i} A^{i-j} B\,\delta u(k+j-1) \tag{35}$$

Thus, by successive substitution of (35) into (34) one gets:

$$\begin{aligned}\hat{y}(k+i/k) = \hat{y}(k) + \theta^T(\varepsilon)\left(K_i - I_N\right)\delta x(k) + \\ + \theta^T(\varepsilon)\sum_{j=1}^{i} K_{i-j} B\,\delta u(k+j-1)\end{aligned} \tag{36}$$

where I_N is the identity matrix of dimension N and K_i is an $(N \times N)$ dimensional matrix defined by:

$$K_i = \begin{cases} \sum_{j=0}^{i} A^j & for\ \ i \ge 0 \\ 0 & for\ \ i < 0 \end{cases} \tag{37}$$

According to (36), the predictor can be written as a sum of two components as follows:

$$\hat{y}(k+i/k) = \hat{y}_l(k+i/k) + \hat{y}_f(k+i/k) \tag{38}$$

with $\hat{y}_l(k+i/k)$ is the free part and $\hat{y}_f(k+i/k)$ is the forced part given successively as:

$$\hat{y}_l(k+i/k) = \hat{y}(k) + \theta^T(\varepsilon)\left[K_i - I_N\right]\delta x(k) \tag{39}$$

$$\hat{y}_f(k+i/k) = \theta^T(\varepsilon)\sum_{j=1}^{i} K_{i-j} B\,\delta u(k+j-1) \tag{40}$$

Remark 2: From (39) and (40), it is noted that $\hat{y}_l(k+i/k)$ and $\hat{y}_f(k+i/k)$ are affine functions of the uncertainty vector ε. Hence, the predictor $\hat{y}(k+i/k)$ is also affine function of ε.

Let $\hat{Y}(k,\varepsilon) \in \mathbb{R}^{N_p}$ be the vector of predicted values of the output given by:

$$\hat{Y}(k,\varepsilon) = \left[\hat{y}(k+1)\ \cdots \hat{y}(k+N_p)\right]^T$$

where N_p is the prediction horizon of the output.

Equation (38), can be rewritten in matrix form as:

$$\hat{Y}(k,\varepsilon) = \hat{Y}_l(k,\varepsilon) + \hat{Y}_f(k,\varepsilon) \tag{41}$$

where $\hat{Y}_l(k,\varepsilon) = \left[\hat{y}_l(k+1)\ \cdots \hat{y}_l(k+N_p)\right]^T \in \mathbb{R}^{N_p}$ can be computed using (39) and $\hat{Y}_f(k,\varepsilon) \in \mathbb{R}^{N_p}$ is expressed as:

$$\hat{Y}_f(k,\varepsilon) = H(\varepsilon)\delta U(k) \tag{42}$$

with:

$\delta U(k) = \left[\delta u(k)\,\delta u(k+1)\cdots\delta u(k+N_u-1)\right]^T \in \mathbb{R}^{N_u}$ is the control increment vector, $N_u\ (N_u \prec N_p)$ is the prediction horizon of the control and $\delta u(k+i)=0$ for $i \geq N_u$.

$H(\varepsilon)$ is an $(N_p \times N_u)$ dimensional matrix representing the system response coefficients:

$$H(\varepsilon)=\begin{bmatrix} h_1(\varepsilon) & 0 & \cdots & 0 \\ h_2(\varepsilon) & h_1(\varepsilon) & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_u}(\varepsilon) & \cdots & \cdots & h_1(\varepsilon) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_p}(\varepsilon) & \cdots & \cdots & h_{N_p-N_u+1}(\varepsilon) \end{bmatrix}$$

where $h_i(\varepsilon),\quad i=1,2,\cdots,N_p$ are affine functions of ε given by:

$$h_i(\varepsilon)=\theta^T(\varepsilon)K_{i-1}B=\sum_{j=0}^{i-1}\theta^T(\varepsilon)A^jB \tag{43}$$

4.2 The set of constraints

The constraints are resulting from the uncertainty on the Laguerre model coefficients and bounds on control signals and its increments over the horizon N_u, given as follow:

$$u_{\min} \leq u(k+i) \leq u_{\max} \qquad \forall\ i \in [0, N_u-1] \tag{44}$$

$$\delta u_{\min} \leq \delta u(k+i) \leq \delta u_{\max} \quad \forall\ i \in [0, N_u-1] \tag{45}$$

where $u_{\min}$, $u_{\max}$, $\delta u_{\min}$ and $\delta u_{\max}$ are bounds on control signals and control increment signals.

Let be define the following vectors of dimensions N_u:

$$U_{Max}=[u_{\max}\cdots u_{\max}]^T\ ,\ U_{Min}=[u_{\min}\cdots u_{\min}]^T$$

$$\delta U_{Max}=[\delta u_{\max}\cdots\delta u_{\max}]^T\ ,\ \delta U_{Min}=[\delta u_{\min}\cdots\delta u_{\min}]^T$$

By (44)-(45), the set $\delta\Psi$ of the constraints on control signals and control increment signals is defined as:

$$\delta\Psi=\{\delta U\ /\ \Gamma\delta U \leq V\} \tag{46}$$

with:

Γ is an $4N_u \times N_u$ dimensional matrix and V is a vector of dimension $4N_u$, given by:

$$\Gamma=\begin{bmatrix} I_{N_u} \\ -I_{N_u} \\ \Delta \\ \Delta \end{bmatrix},\quad V=\begin{bmatrix} \delta U_{Max} \\ -\delta U_{Min} \\ U_{Max}-\varphi \\ -U_{Min}+\varphi \end{bmatrix}$$

where I_{N_u} is the identity matrix of dimension N_u, Δ is an $N_u \times N_u$ dimensional matrix and φ is a vector of dimension N_u, given by:

$$\Delta=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & 1 & 1 \end{bmatrix},\quad \varphi(k-1)=\begin{bmatrix} u(k-1) \\ u(k-1) \\ \vdots \\ u(k-1) \end{bmatrix}$$

4.3 Convex optimization problem and solution

The robust predictive control based on a worst-case strategy consists in solving a min-max problem given by:

$$\min_{\delta U \in \delta\Psi}\ \max_{\varepsilon\in\Omega} J_2(\delta U,\varepsilon) \tag{47}$$

where J_2 is the quadratic criterion to be minimized given by:

$$J_2(\delta U,\varepsilon)=\sum_{i=1}^{N_p}\left(\hat{y}(k+i/k,\varepsilon)-r(k+i)\right)^2+ \\ +\sum_{i=0}^{N_u-1}\lambda_i\,\delta u^2(k+i) \tag{48}$$

where $r(k+i)$ represents the reference signal defined over the prediction horizon N_p, $\lambda_i \geq 0$ is a weighting factor generally considered constant and equals to λ.

In matrix form, one has

$$J_2(\delta U,\varepsilon)=\left\|\hat{Y}(k,\varepsilon)-R(k)\right\|^2+\left\|\Sigma^{1/2}\delta U(k)\right\|^2 \tag{49}$$

where $R(k)=\left[r(k+1)\cdots r(k+N_p)\right]^T$ is the reference vector of dimension N_p and $\Sigma=diag\left(\lambda_0,\lambda_2,\cdots,\lambda_{N_u-1}\right)$ is a weighting diagonal matrix of dimension N_u.

From (49), one gets:

$$J_2(\delta U,\varepsilon)\ =\left(\hat{Y}(k,\varepsilon)-R(k)\right)^T\left(\hat{Y}(k,\varepsilon)-R(k)\right)+ \\ +\delta U^T(k)\Delta\delta U(k) \tag{50}$$

Using (41), the relation given in (50) yields:

$$J_2(\delta U,\varepsilon)=\delta U^T(k)\Phi(\varepsilon)\delta U(k)+2\rho^T(\varepsilon)\delta U(k)+\beta(\varepsilon) \tag{51}$$

where $\Phi(\varepsilon)$ is an $N_u \times N_u$ dimensional positive-definite matrix, $\rho(\varepsilon)$ is a vector of dimension N_u and $\beta(\varepsilon)$ is a scalar defined respectively as follow:

$$\Phi(\varepsilon)=H^T(\varepsilon)H(\varepsilon)+\Sigma \tag{52}$$

$$\rho(\varepsilon)=H^T(\varepsilon)\left[\hat{Y}_l(k,\varepsilon)-R(k)\right] \tag{53}$$

$$\beta(\varepsilon)=\left[\hat{Y}_l(k,\varepsilon)-R(k)\right]^T\left[\hat{Y}_l(k,\varepsilon)-R(k)\right] \tag{54}$$

Since the criterion $J_2(\delta U, \varepsilon)$ is convex over the set Ω, the maximization problem over this set is reduced to the maximization over its vertices.

Therefore, the optimization problem given in (47) becomes:

$$\min_{\delta U \in \delta\Psi} \max_{\varepsilon \in S} J_2(\delta U, \varepsilon) \quad (55)$$

where S is the set of vertices of the orthotope containing the ellipsoid.

Let μ a positive scalar.

The problem given in (55) can then be formulated as:

$$\min_{\mu, \delta U} \mu \quad (56)$$

Subject to:

$$\mu \geq J_2(\delta U, \varepsilon), \quad \forall\, \varepsilon \in S \quad (57)$$

$$\Gamma \delta U \leq V \quad (58)$$

By (51) and (57), one gets:

$$J_2(\delta U, \varepsilon) = \delta U^T \Phi(\varepsilon) \delta U + 2\rho^T(\varepsilon)\delta U + \beta(\varepsilon) - \mu < 0 \quad (59)$$

Since the matrix $\Phi(\varepsilon)$ is definite-positive, one has:

$$-\Phi^{-1}(\varepsilon) = -\left[H^T(\varepsilon) H(\varepsilon) + \Sigma\right]^{-1} < 0 \quad (60)$$

by (59)-(60) and applying the Schur's lemma, the following LMI inequality:

$$\begin{bmatrix} 2\rho^T(\varepsilon_j)\delta U + \beta(\varepsilon_j) - \mu & \delta U^T \\ \delta U & -\Phi^{-1}(\varepsilon_j) \end{bmatrix} < 0 \quad (61)$$
$$\left(\forall\, j = 1, \cdots, 2^N\right)$$

is equivalent to:

$$\begin{cases} -\Phi^{-1}(\varepsilon_j) < 0 \\ \delta U^T \Phi(\varepsilon_j) \delta U + 2\rho^T(\varepsilon_j)\delta U + \beta(\varepsilon_j) - \mu < 0 \end{cases} \quad (62)$$
$$\left(\forall\, j = 1, \cdots, 2^N\right)$$

where ε_j, $\left(j = 1, \cdots, 2^N\right)$ represents the uncertainties vector associated to the jth vertex of S.

By (61)-(62), the optimization problem given in (56)-(58) yields:

$$\min_X Q^T X \quad (63)$$

Subject to:

$$\left(LX\right)^T \Phi(\varepsilon_j) LX + 2\underline{\rho}^T(\varepsilon_j)\delta U + \beta(\varepsilon_j) - Q^T X < 0 \quad (64)$$
$$\left(\forall\, j = 1, \cdots, 2^N\right)$$

$$EX - F < 0 \quad (65)$$

with:

$$X = \begin{bmatrix} \delta U \\ \mu \end{bmatrix} \in \mathbb{R}^{N_u + 1}, \; Q = \begin{bmatrix} \underbrace{0 \cdots 0}_{N_u} 1 \end{bmatrix}^T \in \mathbb{R}^{N_u + 1}$$

$$L = \begin{bmatrix} I_{N_u} & 0_{N_u} \end{bmatrix}, \; \underline{\rho}^T(\varepsilon_j) = \begin{bmatrix} \rho^T(\varepsilon_j) & 0 \end{bmatrix}, \; j = 1, \cdots, 2^N$$

$$E = \begin{bmatrix} I_{N_u} & \underline{0}_{N_u} \\ -I_{N_u} & \underline{0}_{N_u} \\ \Delta & \underline{0}_{N_u} \\ -\Delta & \underline{0}_{N_u} \end{bmatrix}, \; F = V$$

where $\underline{0}_{N_u}$ is a vector of dimension N_u containing zero's.

Remark 3: Noting that the number of constraints $2^N + 4N_u$ allowed in the problem (56)-(58) is reduced to $2^N + 1$ constraints allowed in the convex problem given in (63)-(65).

5. SIMULATION EXAMPLE

To illustrate the utility of the robust predictive control based on the Laguerre model structure, consider a highly oscillating system described by the following discrete transfer function (Malti et al., 1998):

$$G(z) = \frac{3.8z^4 - 3.71z^3 + 0.3995z^2 - 0.2439z + 0.254}{z^5 - 0.85z^4 - 0.555z^3 + 0.616z^2 - 0.0733z - 0.0612} \quad (66)$$

with $n_a = 5$ and $n_b = 4$ are the orders of the denominator and the numerator of the transfer function $G(z)$.

The performances of the Laguerre model is evaluated by the following criteria:

The signal noise to ratio (SNR):

$$SNR = \frac{\sum_{k=1}^{M} \left(y(k) - y_m\right)^2}{\sum_{k=1}^{M} \left(e(k) - e_m\right)^2}$$

where y_m and e_m are the mean values of y and e respectively.

The normalized mean squared error (NMSE):

$$NMSE = \frac{\sum_{k=1}^{M} \left(y(k) - \hat{y}(k)\right)^2}{\sum_{k=1}^{M} \left[y(k)\right]^2}$$

The parametric reduction of rate (PRR):

$$PRR = 1 - \frac{N}{n_a + n_b}$$

The Gaussian sequence used as input to generate the output signal is plotted in Fig. 2 on a window of $M = 1000$ iterations.

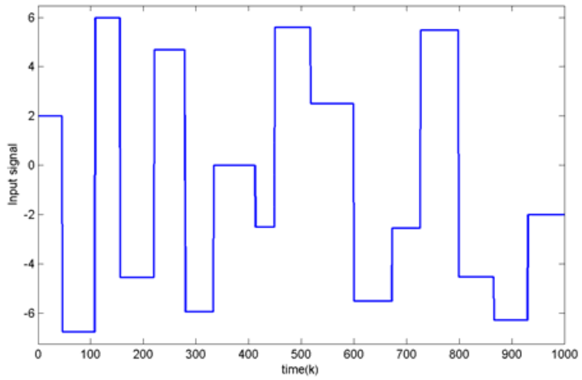

Fig. 2. Sequence of Gaussian input $u(k)$.

The tuning parameters and bounds on control signals and control increment signals used in this simulation are:

$N_p = 4$, $N_u = 2$, $\lambda = 10$, $-27 \le u \le 27$, $-7 \le \delta u \le 7$

Tab. 1 summarizes the set of vertices of the orthotope containing the ellipsoid updated using the UBBE approach for SNR=10.

Table 1. List of vertices.

s_1	s_2	s_3	s_4	s_5	s_6
2.9373	0.2438	2.9147	0.7464	3.0068	0.8242
-1.090	0.4128	-1.0397	1.1540	-1.2225	1.3157
0.7457	0.2170	0.7236	0.5281	0.8067	0.6536

Using the computational algorithm in subsection 3.1, the optimal Laguerre pole is $\xi = 0.7208$. By applying the Monte Carlo method, Fig. 3 shows the result of the estimated pole for 100 realizations of noise (SNR=10) when a stationary zero mean Gaussian white noise corrupts the output signal.

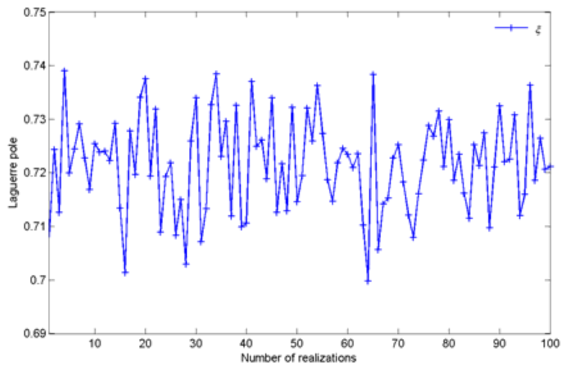

Fig. 3. Monte Carlo simulation of the Laguerre pole for 100 realizations of noise.

Figure 4 plots the Laguerre model output and the process output. The representation of the true system by orthonormal Laguerre functions leads to the following resulting Laguerre model given by:

$$A = \begin{pmatrix} 0.7208 & 0 & 0 \\ 0.4805 & 0.7208 & 0 \\ -0.3463 & 0.4805 & 0.7208 \end{pmatrix}, \quad B = \begin{pmatrix} 0.6932 \\ -0.4996 \\ 0.3601 \end{pmatrix}$$
$$\theta = \begin{pmatrix} 2.9546 \\ -1.1307 \\ 0.7735 \end{pmatrix} \tag{67}$$

It can be observed from Fig. 4 that the approximation of the oscillating system (66) by the Laguerre model (67) is guaranteed ($NMSE = 3.9 \times 10^{-3}$ and $PRR = 67\%$).

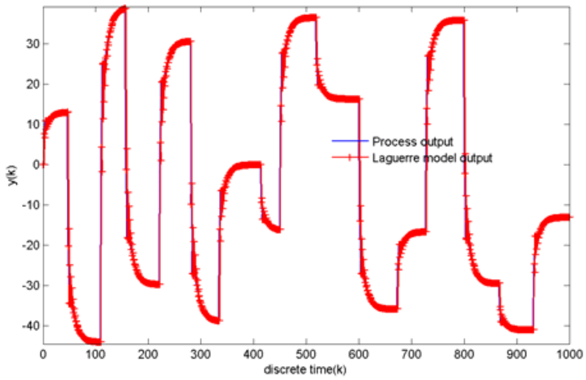

Fig. 4. Validation of the Laguerre model.

Under the robust predictive controller using the Laguerre model, the simulation results are depicted in Figs. 5-6. From Fig. 5, it is clear that reference signal can be tracked with good dynamical performance. This is predictable since a tracking criterion is optimized. However, the tuning parameters N_p, N_u and λ determine the convergence rate of the system output.

In order to verify the robustness of the controller, we plot in Figs. 7-8 the results for different error bounds. It can be clearly seen from Figs. 7-8 that under the proposed controller, the predicted outputs can be rapidly tracked. Thus, the robustness of the closed loop system is ensured.

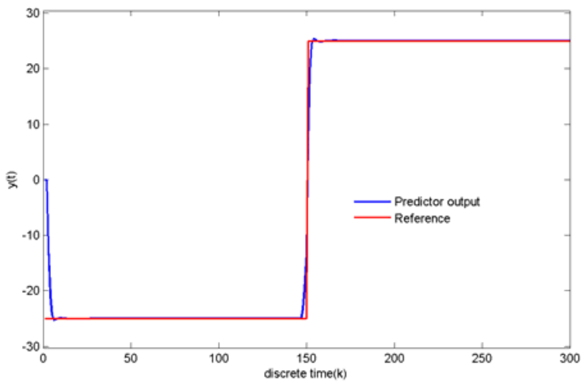

Fig. 5. Evolution of the predicted under the robust controller.

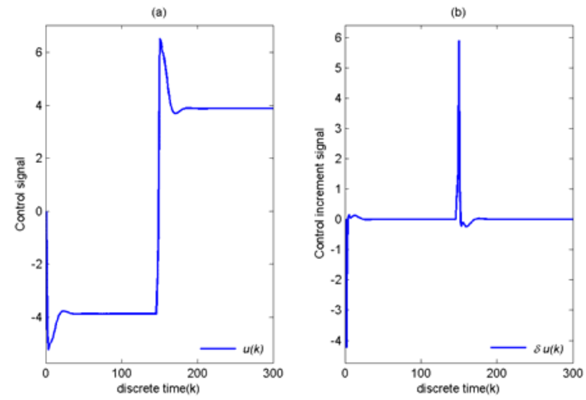

Fig. 6. Control and increment control signals.

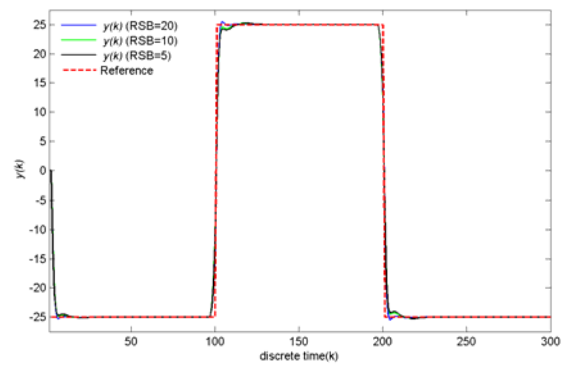

Fig. 7. Evolution of the predicted outputs for different SNR.

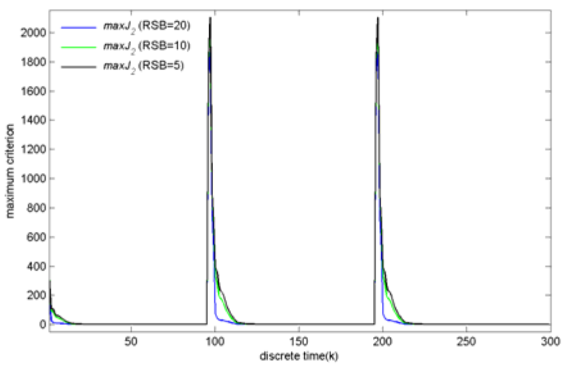

Fig. 8. Evolution of the maximum criteria for different SNR.

6. CONCLUSION

This paper has provided a robust predictive control for dynamical systems using a new Laguerre modelling. The optimal Laguerre pole is estimated using a new algorithm based on the Gradient method where new equations are derived to determine the filter sensitivities. The set of Fourier coefficients of the resulting Laguerre model is an ellipsoid updated using the UBBE approaches. A min-max optimization problem is formulated and solved under LMIs constraints resulting from the model uncertainty and the control signals. The future works will be focused on extending the result to the robust predictive controller of multivariable uncertain systems.

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