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SOLUTIONS FOR MODELING AND CONTROL IN MOBILE ROBOTICS

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Abstract: *This work presents some considerations regarding mathematical models and control solutions for a class of mobile robots namely two-wheel differential drive mobile robots, one of the most utilized mechanical structures now in mobile robotics practice. The closed loop control diagrams for position control and respectively for direction control in tracking along imposed trajectories are developed, analyzed and included in this paper. For these control solutions, the paper presents therefore some analyses regarding the stability in different circumstances. In addition, direct and inverse cinematic models for this class of mobile robots are included.*

Keywords: *mobile robots, modeling, control, stability.*

1. INTRODUCTION

Usually, the mechanical mobile robot solution namely "two-wheel differential drive mobile robot" has minimum three wheels. The two "drive wheels" have a common horizontal axis, fixed on the robot body. One or more free wheels (or "castor" wheels) assure the robot equilibrium [12]. Therefore, while three wheels introduce isostatic equilibrium for robot body, more that three wheels introduce hyperstatic equilibrium, which ensure a better stability on complex trajectories including curve segments [2]. Each castor wheel is independently mounted on a vertical not drive axis of the body and it is automatically and free aligned on the route as result of the forces developed by the two "drive wheels" [3].

The entire control of the mobile robot on the trajectories is assured controlling the angular velocities of the two drive wheels [8]. There are three fundamental cases:

- If the angular velocities are identical as values and relative senses, the robot makes a “spin” motion. The spin motion produces a rotation of the robot body around a vertical axis passing through the geometrical symmetry point (or centre of gravity). There is a particularity of this mechanical configuration because only the two-wheel differential drive mobile robot can do this type of motion.

- If the angular velocities are identical as values but opposite as senses, the robot makes a linear motion; the direction on the linear motion, forward or backwards, depends of the opposite group of sense of the driven wheels angular velocities.
- If the angular velocities are different as value, the robot makes a curve motion. Of course, the characteristics of the curve motion, i.e. the curvature coefficient k of the curve-segment trajectory depend of the differences between values and senses of the two drive wheels.

This mechanical mobile robot solution namely "two-wheel differential drive mobile robot" is extensively used now in practice. The explanation is that it assures a good balance between large capabilities in locomotion (or tracking possibilities) and mechanical complexity (or construction associated costs) [4].

2. MODELS FOR THE TWO-WHEEL DIFFERENTIAL DRIVE MOBILE ROBOT

To characterize the current localization of the mobile robot in its operational space of evolution, we must define first its position and its orientation.

The position of the mobile robot on a plane surface is given by the vector (x, y), which contains the Cartesian coordinates of its characteristic point P (see Figure 1). Usually, this characteristic point P is placed in the middle

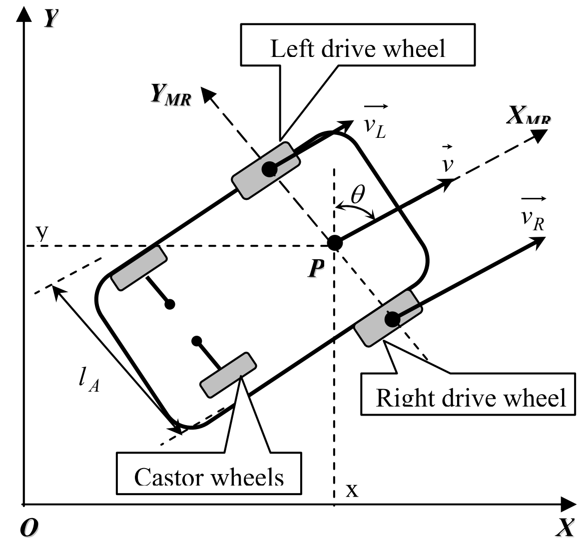

Fig. 1. The co-ordinate systems and notations for the two-wheel differential drive mobile robot.

of the common axis of the driven wheels.

As we can see in Figure 1, the orientation (or direction) of the mobile robot is given by the angle θ between the instant linear velocity of the mobile robot $\vec{v}$ (or the X_{RM} axis) and the local vertical axis.

The instant linear velocity of the mobile robot $\vec{v}$ is attached and defined relative to the characteristic point P. As equation (1) denotes, this velocity is a result of the linear velocities of the left driven wheel $\vec{v}_L$ and respectively the right driven wheel $\vec{v}_R$. These two drive velocities $\vec{v}_L$ and $\vec{v}_R$ are permanently two parallel vectors and, in the same time, they are permanently perpendicular on the common mechanical axis of these two driven wheels.

$$v = \frac{v_L + v_R}{2} \tag{1}$$

Equations (2) and (3) give the two Cartesian components of linear velocity:

$$v_x = \dot{x} = v \cdot \sin\theta \tag{2}$$

$$v_y = \dot{y} = v \cdot \cos\theta \tag{3}$$

The position, the orientation and the linear velocities of the two driven wheels define the robot state as a five elements vector:

$$(x, y, \theta, v_L, v_R)^T \tag{4}$$

The input vector contains the two accelerations of the left $\vec{a}_L$ and respectively the right $\vec{a}_R$ driven wheels.

Using equation (1) into equations (2) and (3), the next (5) and (6) equations are immediately. They give finally the first two state equations (for the linear velocity components of the mobile robot):

$$\dot{x} = \frac{v_L + v_R}{2} \cdot \sin\theta \tag{5}$$

$$\dot{y} = \frac{v_L + v_R}{2} \cdot \cos\theta \tag{6}$$

If we note by x_L, y_L, x_R, y_R the Cartesian positions of the driven wheels in the global references attached to the operational space, we can write the next two equations:

$$x_L - x_R = -l_A \cdot \cos\theta \tag{7}$$

$$y_L - y_R = l_A \cdot \sin\theta \tag{8}$$

and respectively the associate equations:

$$\dot{x}_L - \dot{x}_R = l_A \cdot \dot{\theta} \cdot \sin\theta \tag{9}$$

$$\dot{y}_L - \dot{y}_R = l_A \cdot \dot{\theta} \cdot \cos\theta \tag{10}$$

Because the vectors for linear speed of wheels $\vec{v_L}$ and $\vec{v_R}$ are orthogonal on the common axis of the driven wheels (see Figure 1), we can write the third state equation (11), representing the angular velocity of the robot:

$$\dot{\theta} = \frac{v_L - v_R}{l_A} \tag{11}$$

The last two state equations denoting the linear accelerations of the two drive wheels are evident:

$$\dot{v}_L = a_L \tag{12}$$

$$\dot{v}_R = a_R \tag{13}$$

The curvature coefficient (k) associated on a specific trajectory-segment is defined as the inverse ratio of the radius of that trajectory–segment. The equation for the curvature can be obtained because the radius of the trajectory–segment can be writing as a ratio between the linear velocity and the angular velocity of the robot body. Therefore, dividing equation (11) by equation (1) we obtain finally the equation for the curvature coefficient (k) of a segment-trajectory as:

$$k = \frac{1}{\rho} = \frac{\dot{\theta}}{v} = \frac{v_L - v_R}{v_L + v_R} \cdot \frac{2}{l_A} \tag{14}$$

Because the equations (5) and (6) are nonlinear, we must introduce some assumptions to obtain a linear model for the mobile robot. There are some different solutions. A possible method is to introduce the hypothesis that the two instant drive wheel accelerations, a_L and respectively a_R, are equals in module. If the sign of them is the same, the mobile robot executes a linear motion and if the sign is opposite, the mobile robot executes a special curve namely "clotoide" [5].

The most common actuator used to energize the locomotion system of the mobile robots is the DC motor. An associated encoder, as common speed a position sensor, is currently attached. In same normal hypothesis (electrical constants are smaller those mechanical constants), the DC servomotor is a first order system with a transfer function:

$$H_S(s) = \frac{\omega(s)}{U(s)} = \frac{K}{1 + T \cdot s} \tag{15}$$

were ω represents the angular speed of the DC servomotor and U is the applied voltage.

So, considering two DC servomotors, as right (R) and left (L) actuators for the two driven wheels of the mobile robot, and the associated simplest transfer function:

$$H_L(s) = \frac{v_L}{v_{Lc}} = \frac{K_L}{1 + T_L \cdot s} \tag{16}$$

$$H_R(s) = \frac{v_R}{v_{Rc}} = \frac{K_R}{1 + T_R \cdot s} \tag{17}$$

we can obtain, finally, a first cinematic model for the two-wheel differential drive mobile robot, which is depicted in Figure 2.

If our target is to simplify the mathematical model, we can introduce the evident assumption that the two DC servomotors are practically identical in their behavior.

So, in addition, some equalities betweens the parameters of their transfer functions (16) and (17) can be write as:

$$K_L = K_R = K_a \tag{18}$$

$$T_L = T_R = T \tag{19}$$

Using the equation for the linear velocity (1) and the equation for the angular velocity (11) of this type of mobile robot, we can obtain (after same bloc-diagram reductions and associate transformations) a new bloc-diagram [9]. The Figure 3 depicts this new bloc-diagram.

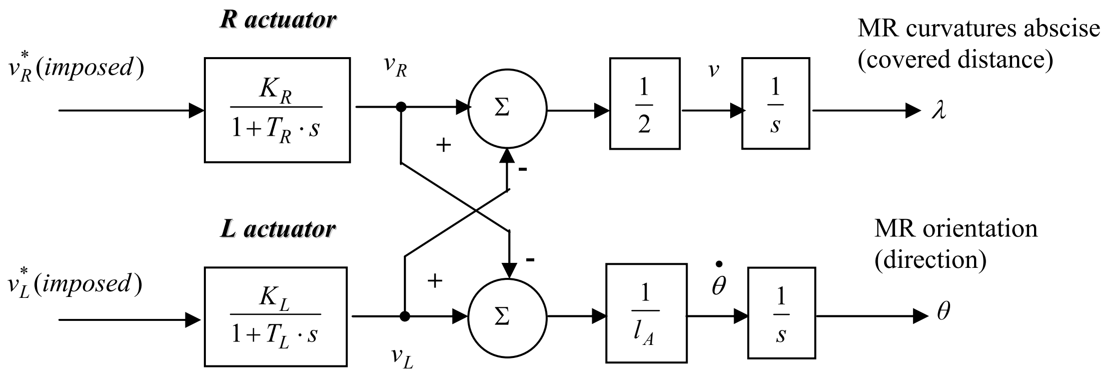

Fig. 2. A primary model for the two-wheel differential drive mobile robot, considering two DC servo-motors as actuators in the locomotion system.

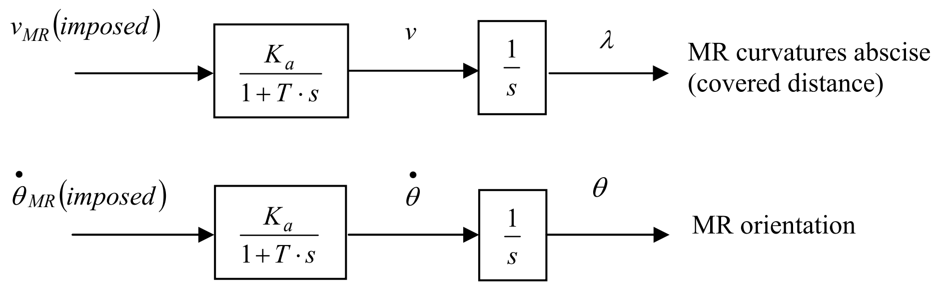

Fig. 3. The simplified model for the two-wheel differential drive mobile robot, considering the same behavior for the two actuators of the locomotion system.

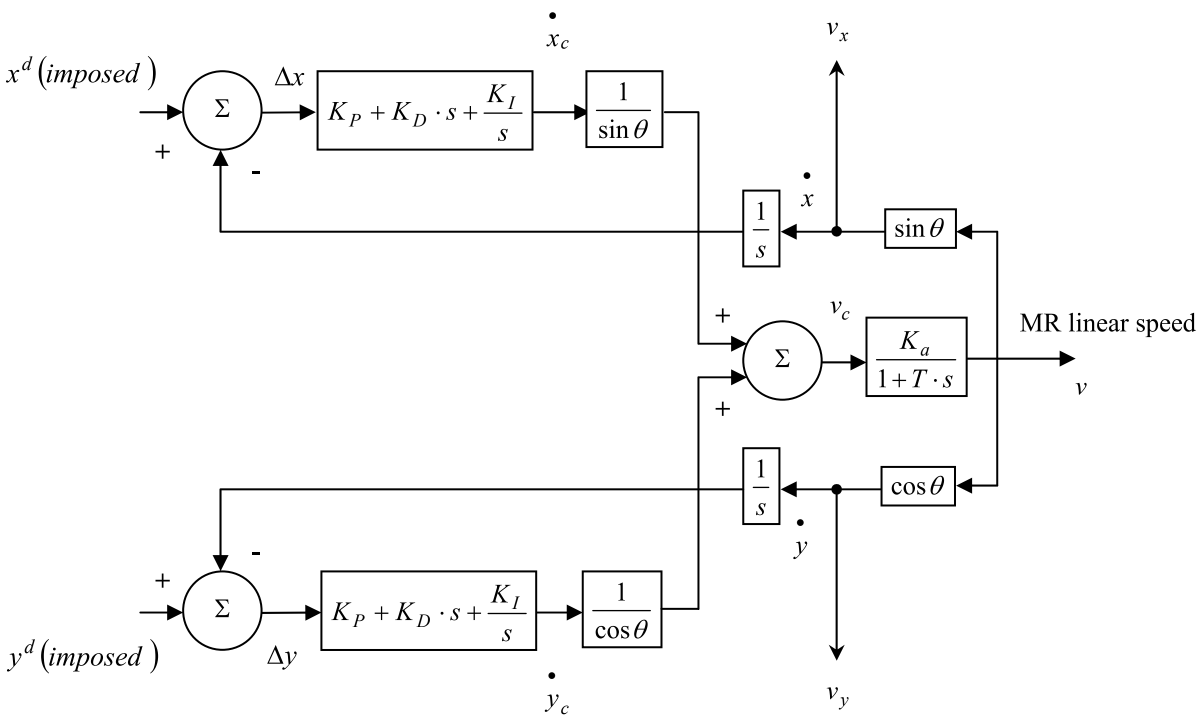

Fig. 4. The position control for the two-wheel differential drive mobile robot.

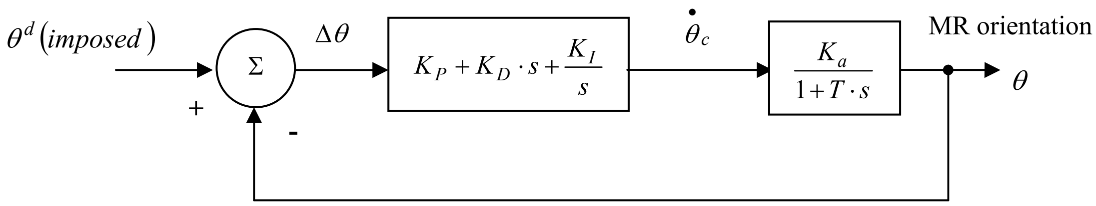

Fig. 5. The orientation or direction control for the two-wheel differential drive mobile robot.

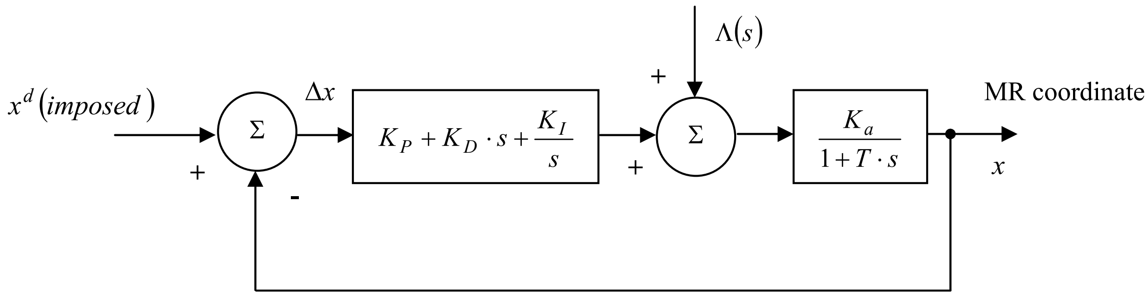

Fig. 6. The simplified model of the two-wheel differential drive mobile robot for stability analyzes.

This new control diagram is still not satisfactory. The explanation is that substantial tracking errors can occur between an imposed (or desired) trajectory for the mobile robot and the real trajectory developed by them. If these errors exceed acceptable and predefined limits, different impacts with obstacles prezented in the mobile robot scene can occur and the entire functionality is compromise.

This is a serious reason to introduce two closed loops control for our mobile robot. The first one is for the curvatures abscise λ (or covered distance by the robot) and the second closed loops control is for the robot orientation. Each of them uses a PID controller, depicted by the equation (20) an respectively (21):

$$\dot{\theta}_c = K_{P\theta}\cdot\Delta\theta + K_{D\theta}\cdot\Delta\dot{\theta} + K_{I\theta}\cdot\int\Delta\theta\cdot dt \qquad (20)$$

$$\dot{\bar{x}}_c = K_P\cdot\Delta\bar{x} + K_D\cdot\Delta\dot{\bar{x}} + K_I\cdot\int\Delta\bar{x}\cdot dt \qquad (21)$$

where $\dot{\theta}_c$ represents the imposed angular velocity, $\Delta\theta$ represents the orientation (or direction) error of the mobile robot, $\dot{\bar{x}}_c$ represents the imposed linear velocity and $\Delta\dot{\bar{x}}$ represents the position error of the mobile robot.

3. CONTROL SOLUTION FOR THE MOBILE ROBOT

Figure 4 and Figure 5 present the final solutions proposed to control the two-wheel differential drive mobile robot.

Figure 4 presents the closed loop control for the position of the two-wheel differential drive mobile robot.

Figure 5 includes the closed loop control proposed for the position control of this type of mobile robot.

4. CONTROL STABILITY

To evaluate the stability of the control for the mobile robot we consider a single channel $(x^d \to x)$ while the influence of the second channel $(y^d \to y)$ is integrated in the perturbation $\Lambda(s)$ as in Figure 6.

The open lop transfer function is:

$$H(j\omega) = \frac{K_a\left[K_I + \omega^2(TK_P - K_D)\right]}{\omega^2\left(1+T^2\omega^2\right)} - j\frac{K_a\left[(K_P - TK_I) + K_D T\omega^2\right]}{\omega\left(1+T^2\omega^2\right)} \qquad (22)$$

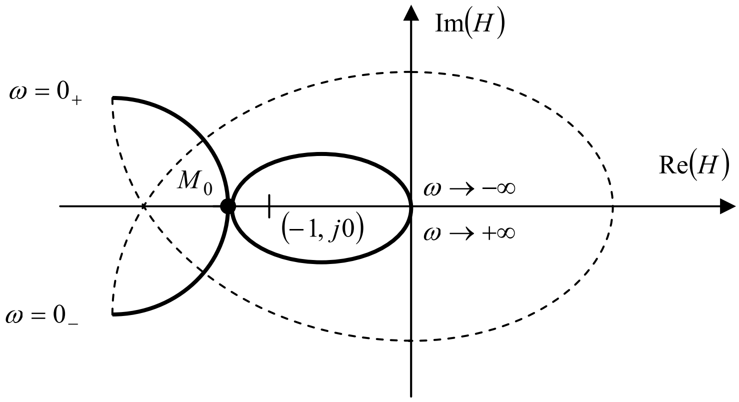

Fig. 7. The root locus method diagram for stability analyzes of the two-wheel differential drive mobile robot.

A first case is: $K_P - T \cdot K_I \geq 0$. Using Nyquist criteria, the conclusion is that the stability is assured if $K_D > 0$. If $K_P = T \cdot K_I$ and $K_D = 0$, some oscillations with constant amplitude are produced.

A second case is $K_P - T \cdot K_I < 0$. In this situation, the system is stable if the point M_0 is placed in the left of the point $(-1, j0)$ in the root locus method diagram depicted in Figure 7. If the system is stable, the residual error is zero for an input step of position or velocity and constant for an input step of acceleration.

Concerning the perturbation, the residual error is zero for an input step of position and constant for an input step of velocity.

5. DIRECT AND INVERSE CINEMATIC EQUATIONS

Using the notations introduced by Figure 8, the arcs lengths decrypted by the two driven wheels are:

$$\Delta L_L = \left(\rho - \frac{l_A}{2}\right) \cdot \Delta\theta$$
$$\Delta L_R = \left(\rho + \frac{l_A}{2}\right) \cdot \Delta\theta \qquad (23)$$

The length of the arc evolution describe by the characteristic point *P* is:

$$\Delta L = \frac{\Delta L_L + \Delta L_R}{2} \qquad (24)$$

and the center angle of the trajectory $\Delta\theta$ is:

$$\Delta\theta = \frac{\Delta L_L - \Delta L_R}{l_A} \qquad (25)$$

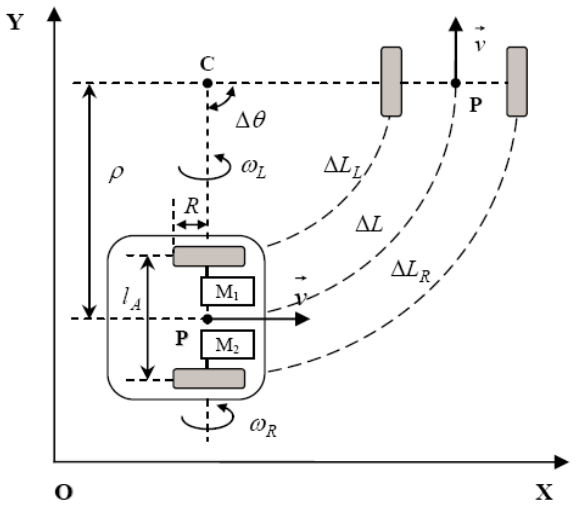

Fig. 8. Notations for the two-wheel differential drive mobile robot equations.

The linear velocities for the two driven wheels can be calculated in function of angular velocities of the driven wheels:

$$v_L = R \cdot \omega_L$$
$$v_R = R \cdot \omega_R \qquad (26)$$

Finally, the direct cinematic model for the two-wheel differential drive mobile robot is represented by the next two equations, where the first one is for the linear velocity and the second is for the angular velocity of the mobile robot. They give this information in function of two variables: angular speed of the left and right driven wheels ω_L, ω_R and others two constant parameters, radius of the driven wheels R and the distance between them l_A:

$$v = \frac{v_L + v_R}{2} = \frac{(\omega_L + \omega_R) \cdot R}{2} \quad (27)$$

$$\dot{\theta} = \frac{v_L - v_R}{l_A} = \frac{(\omega_L - \omega_R) \cdot R}{l_A} \quad (28)$$

From other point of view, the angular velocity can be calculated as:

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{d\theta}{dL} \cdot \frac{dL}{dt} = \frac{v}{\rho} \quad (29)$$

The equality between the equations (28) and (29) gives the values for the angular velocities of the driven wheels:

$$\omega_L = \left(1 - \frac{l_A}{2 \cdot \rho}\right) \cdot \frac{v}{R} \quad (30)$$

$$\omega_R = \left(1 + \frac{l_A}{2 \cdot \rho}\right) \cdot \frac{v}{R} \quad (31)$$

The last two equations were obtained for the counter-clockwise turn of the mobile robot. For the opposite clockwise turn of the mobile robot, these equations can be easily translated into the next two:

$$\omega'_L = \left(1 + \frac{l_A}{2 \cdot \rho}\right) \cdot \frac{v}{R} \quad (32)$$

$$\omega'_R = \left(1 - \frac{l_A}{2 \cdot \rho}\right) \cdot \frac{v}{R} \quad (33)$$

Combining equations (30) - (33), we can obtain finally the inverse cinematic model of the two-wheel differential drive mobile robot. The model is depicted by the next two equations of angular velocities for the driven left and right wheels:

$$\omega_L = \left(1 - sign(\rho) \frac{l_A}{2 \cdot \rho}\right) \cdot \frac{v}{R} \quad (34)$$

$$\omega_R = \left(1 + sign(\rho) \frac{l_A}{2 \cdot \rho}\right) \cdot \frac{v}{R} \quad (35)$$

6. EXPERIMENTS

A program was developed to compare two control solutions for the "two-wheel differential drive mobile robot". The robot is characterized in both by its mechanical parameters $R = 5cm$, $l_A = 0.4m$. The desired performances in linear velocity and acceleration are $v_{\max} = 0.2m/s$ and $a_{\max} = 0.2m/s^2$, while in angular velocity and acceleration are $\omega_{\max} = \pi/4 rad/s$ and respectively $\alpha_{\max} = \pi/4 rad/s^2$.

The experiments were performed for different complex global trajectories as shown Figs. 9-11. In "Method 1" we use the control for position and orientation included in Figs. 4-5, while in "Method 2" we use a classical control solution based on the primary model presented in Fig. 2.

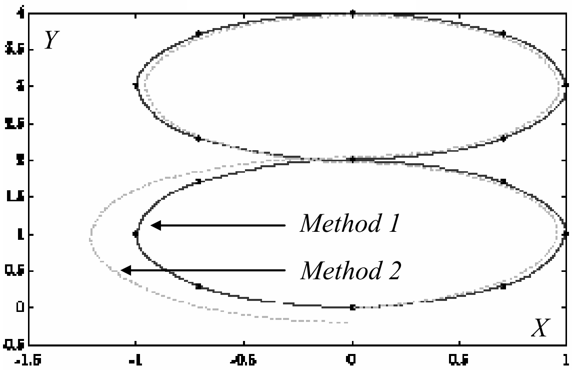

Fig. 9. Complex trajectory based on circular arcs.

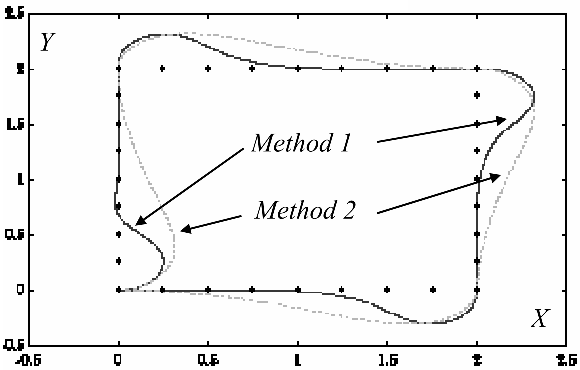

Fig. 10. Complex trajectory based on straight lines.

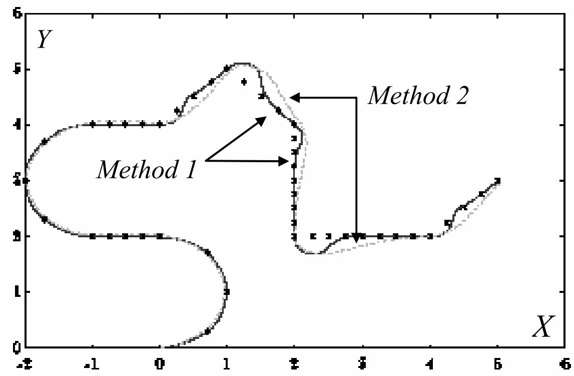

Fig. 11. Complex trajectory with all cases.

Figure 9 resumes the results in the case of the plane trajectory planned by circular arcs like an eight number.

Figure 10 resumes the results in the case of the plane trajectory planned by a combination of four straight lines and 90^0 quick turning motions, like a rectangle.

Figure 11 resumes the results in the case of the real complex plane trajectory containing straight lines, circular arcs and quick turning motions.

7. CONCLUSION

This paper presents some results regarding mathematical models for one kind of mobile robots, namely two-wheel differential drive mobile robot. The closed loop control diagrams for position control and respectively for direction control in tracking along imposed trajectories are also analyzed. Afterwards, for these control solutions, the paper presents therefore some analyses regarding the stability for different type of inputs and alternativs experiments to evaluate performances on tipical test trajectories. Direct and inverse cinematic models for this class of mobile robots are also included. Finally, to evaluate the control performances, some experimental and comparative results on complex trajectories are so indicated.

REFERENCES

[1]Ailon A., Berman N., Arogeti S., Robot controller design for achieving global asymptotic stability and local prescribed performance, IEEE Transactions on Robotics and Automation, Vol. 20, No. 5, p. 790, 2004.

[2] Andrea, B., Bastin G., Modeling and control of non-holonomic wheeled mobile robots, Proceedings of IEEE International Conference on Robotics and Automation, Sacramento, SUA, p.1130, 1991.

[3]Bicchi, A., Casalino C., Santilli C., Planning shortest bounded curvature paths for a class of nonholonomic vehicles among obstacles, Proceedings of IEEE International Conference on Robotics and Automation, Nagoya, Japan, p.1349, 1995.

[4]Campion G., Bastin G., Dandrea B., Structural properties and classification of kinematic and dynamic models of wheeled mobile robots, IEEE Transactions on Robotics and Automation, Vol. 12, No. 1, p.47, 1996.

[5]De Santis R., Modelling and path tracking control of a mobile wheeled robot with a differrential drive, Robotica, 13, Part 4, p. 401, 1995.

[6]Dong W., Huo W., Tso S.K., Xu W.L., Tracking control of uncertain dynamic nonholonomic system and its application to wheeled mobile robots, IEEE Transactions on Robotics and Automation, Vol. 16, No. 6, p. 870, 2000.

[7]Koh K., Cho. H., A path tracking control system for autonomous mobile robots, Mechatronics, 4, p.799, 1994.

[8]Kriegman D., Triendl E., A mobile robot: sensing, planning and locomotion, IEEE Transaction on Robotics and Automation, p. 402, 1987.

[9]Krogh B., Thorpe C., Integrated path planning and dynamic steering control for autonomous vehicles, Proceedings of IEEE International Conference on Robotics and Automation, San Francisco, USA, p.1664, 1986.

[10]Latombe J. C., Robot motion planning, Ed. Kluwer Academic Publishers, USA, p.42, 1993.

[11]Nishizawa T., Ohia A., An implementation of on board position estimation for a mobile robot, Proceedings of IEEE International Conference on Robotics and Automation, Nagoya, Japan, p.395, 1995.

[12]Nițulescu M., Robotic systems with navigation capabilities (in Romanian), Ed. Universitaria, Craiova, Romania, ISBN: 973 -8043-143-3, p.151, 2002.

[13]Tsai P., Wang L., Chang F., Modeling and hierarchical tracking control of tri-wheeled mobile robots, IEEE Transactions on Robotics, Vol. 22, No. 5, p.1055, 2006.

[14]Wenjie D., Kuhnert K., Robust adaptive control of nonholonomic mobile robot with parameter and nonparameter uncertainties, IEEE Transactions on Robotics and Automation, Vol. 21, No. 2, p.261, 2005.