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Tracking Control for Unmanned Aerial Vehicles with Time-Delays Based on Event-Triggered Mechanism

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Abstract: A position tracking control approach is presented for quad-rotor unmanned aerial vehicles (UAVs) with multiple state time-delays and external disturbances based on an event-triggered mechanism. This approach achieves the desired performance by using less control execution, which results in a reduction in the usage rate of the onboard embedded microprocessor. First, the basic control structure is formed by a weighted multiple-model method. Next, the event-triggered mechanism is designed in such a way to achieve an aperiodic control approach. This is done by embedding the mechanism into the existing control structure. Additionally, the negative influence of time-delays and external disturbances are considered using a linear matrix inequality (LMI) technique based on a Lyapunov function. This allows the coefficients of the controller and event-triggered mechanism to be obtained. Consequently, the system can be stabilized and performs robustly. The position tracking control approach is applied to a quad-rotor UAV and the simulation results confirm its effectiveness.

Keywords: event-triggered control; tracking control; UAV; time-delay; robust control

1. INTRODUCTION

Quad-rotor unmanned aerial vehicles (UAVs) are used extensively in aerial photography, exploratory operations, and disaster relief, since they have the advantages of these contraptions: high manoeuvrability, capability for controlled vertical ascent and descent, and long hover times. On the one hand, UAV systems generally display nonlinearity, uncertainties and coupling. On the other hand, the control performance is often influenced by external disturbances, signal transmission time-delays, and actuation failures during flight (Amin et al., 2016; Amoozgar et al., 2013; Bateman et al., 2011). So numerous control methods have been presented in the academic literatures to deal with the problem mentioned of UAVs. In addition, another challenge is to improve the usage rate of computation and communication resources to satisfy the operational requirements with limited power. This problem is generally solved by using a high-performance onboard microprocessor and a higher power system (Yang and Liu, 2017; Idres et al., 2015). Alternatively, an event-triggered mechanism can be used, which is able to reduce usage rate of computation and communication resources with less control execution.

Control of UAVs is achieved with an onboard embedded microprocessor unit. When higher control performance is required, more complex control algorithms must be implemented. This results in the onboard microprocessor having to deal with larger volumes of data and being limited to do more missions. Consequently, a method to improve control performance with low computation resource consumption in the embedded or networked system must be realized. For example, event-triggered control (ETC) for networked and embedded control systems is widely recognized for its aperiodic control mechanism, which optimizes the usage of communication and computation resources in the system (Mazo and Tabuada, 2008; Heemels et al., 2012).

ETC differs from traditional control because its control execution depends on the occurrence of an event, which is triggered under certain conditions. The number of control execution can be reduced along with satisfactory closed loop response (Dhar et al., 2018). ETC problems have been widely researched. The ETC model-based approach was proposed by (Heemels and Donkers, 2013) for linear system's stability. (Tabuada, 2007) proposed an event-triggered algorithm for nonlinear systems to ensure global asymptotic stability. ETC was applied in decentralized systems by (Mazo and Tabuada, 2011). The application of ETC in hybrid systems was presented by (Postoyan et al., 2011). ETC is very popular for networked control systems (NCSs). For example, (Wang and Lemmon, 2011) use ETC for NCSs to guarantee the asymptotic stability of the entire system. Application of ETC is investigated for systems with time-delays and uncertainties, as presented by (Wu et al., 2015; Sahoo et al., 2016). In this paper, ETC is used to guarantee satisfactory control performance of a quad-rotor UAV and reduce computation demands on the onboard embedded microcomputer.

The dynamic characteristics of UAVs result in nonlinearity and uncertainties in the control system (Besnard et al., 2012). Generally, UAVs are controlled based on linear models of their dynamics, as obtained at the equilibrium points (within allowable approximation error). The linear model is sufficient for control of the UAV when cruising or hovering. However, the linear model would not ensure the desired control performance when the UAV is accelerating from certain equilibrium (because of increased nonlinearity). When the model nonlinearity cannot be ignored (in certain cases), position tracking control for the UAV becomes difficult. We adopt a weighted multiple-model control method to address the model nonlinearity. The control algorithm, designed based on a weighted multiple-model structure, is formed by combining several linear models at different equilibrium points with the corresponding weight functions (Zhang, 2013; Xiao et al., 2010). The control system can be stabilized by adjusting the control parameters, with the flight state altering to match the current dynamic model.

In fact, the whole UAV control system consists of one or more UAV bodies, a base station computer and a set of space position sensors. The base station computer monitors the flight status and sets the flight path. The position sensors measure and transmit the position signals via a wireless network. The flight states are directly controlled by the onboard embedded microcomputer, which receives and processes the signals from the base station computer and the position sensors to determine the necessary control command. This is computed using the control algorithm and then sent to the actuator (Yang et al., 2017). The UAV control system is different from a traditional flight control system. The former must use a wireless network to transmit signals during flight, while the latter does not. Wireless communication causes time-delays in the control system, which can influence the control performance and even cause the system instability. It would be difficult to achieve the satisfactory control performance for an actual UAV without accounting for the time-delays in the control system design. For a practical control approach, it is necessary to design the control law to reduce the negative influence of time-delays. Additionally, the UAV control must overcome external disturbances, which occur for any flight vehicle (Bouadi et al., 2015; Xiong and Zheng, 2014). In this paper, a robust control method based on LMI technology is used to overcome both external disturbances and time-delays (Mobayen, 2015; Zhang et al., 2015).

Specifically, a position tracking control approach for quad-rotor UAVs based on an event-triggered mechanism is proposed in this paper. A weighted multiple-model control structure is designed for nonlinear tracking control systems. The LMI technology is used to obtain the coefficients of the controller and the event-triggered mechanism (using Lyapunov functions) to stabilize the tracking system within the given bounds for the external disturbances and state time-delays. Lower computation resources with less control output updating than that of traditional periodic control is attained. Consequently, the usage rate of onboard microprocessors is improved, resulting in more powerful data-handling capacity.

This paper is divided into four parts and is organized as follows. Section 2 presents the problem formulation. Section 3 describes the design of tracking controller based on the event-triggered mechanism. Finally the simulation results are presented followed by the conclusions in Section 4.

2. PROBLEM FORMULATION

2.1 System modelling

This paper focuses on the position tracking control of quad-rotor UAVs, and their mathematics model is discussed first of all. Without generality, some hypotheses are listed as below.

(1) The UAV has a symmetric homogeneous rigid body, and keeps mass constant during its flight;
(2) The air resistance and gyroscopic effect can be ignored;
(3) The distance between the center of aircrew and that of body mass can be ignored;
(4) The aircrew is light enough and its rotary inertia can be ignored;
(5) Earth-fixed frame is the inertia frame, and the effect of earth curvature and gravitational acceleration alteration with altitude can be ignored;
(6) The centre-mass is the origin of body-fixed frame.

According to the above hypotheses of flight dynamics, the dynamic position system of UAVs can be obtained as follows,

$$\begin{cases} m\ddot{X} = (\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)U \\ m\ddot{Y} = (\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi)U \\ m\ddot{Z} = mg - (\cos\varphi\cos\theta)U \end{cases}, \tag{1}$$

where m = body mass, g =gravitational constant, ψ = yaw angle, θ = pitch angle, and φ = roll angle. (X,Y,Z) = body location in the earth-fixed frame; U = total thrust force generated by four rotors and each of thrust force F_e can be defined as follows,

$$F_e = L\frac{\omega}{s+\omega}u\ , \tag{2}$$

where s = Laplace transform variable, u = input of each actuator, ω = actuator bandwidth, and L = positive gain. v = state variable represented the actuator dynamics of each rotor, which can be described as follows,

$$v = \frac{\omega}{s+\omega}u\ . \tag{3}$$

The actuators are four brushless DC motor whose speed is controlled by the duty cycle of PWM, and so the inputs of these motors are the inputs of the UAV control system. Here, this paper takes X-axis position as the controlled variable, designing the specific control algorithm. According to the dynamic analysis, the body movement on X-axis is mainly influenced by the total thrust and the pitch angle θ. Hence, it assumes that the yaw angle and roll angle are both zero, i.e. $\psi = \varphi = 0$, only considering the influence of the pitch angle

θ to obtain the following dynamic description of X-axis position from equations (1)-(3),

$$\begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{4L}{m}\sin\theta \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} u \,. \tag{4}$$

Choose the position and velocity in X-axis, and the actuator dynamics to form the state vector, i.e. $\boldsymbol{x} = [X, \dot{X}, v]^{\mathrm{T}}$; Letting u = control input and y = output, the position control system(4) can be rewritten in standard form,

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}, \tag{5}$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{4L}{m}\sin\theta \\ 0 & 0 & -\omega \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^{\mathrm{T}}$.

2.2 Time-delays in the control system

The control system of quad-rotor UAVs contains: one or more UAV bodies, the space position sensors, and a control based station. When the UAV flights, it has to receive the signals of space position, flight commands and control parameter settings via a wireless network, so that to generate the control input by the onboard embedded microprocessor computing designed control algorithm. At the same time, it sends all kinds of the flight state signals to the ground control base station via the wireless network. Obviously, the wireless network is necessary, and the time-delays caused by the network transmission cannot be ignored. In most cases, the delay signals can be described as certain state time-delay variables in the system mathematic description.

Define the output error as $e(t) = y(t) - y_d(t) = Cx(t) - y_d(t)$, where $y_d(t)$ = given desired output. Taking the external disturbances $d(t)$ and multiple state time-delays τ_k into consideration, the following augmented system as the tracking control system can be obtained from (5),

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \sum_{k=1}^{h} \bar{A}_k \bar{x}(t-\tau_k) + \bar{B}u(t) + \phi(t)\,, \tag{6}$$

where $\bar{A} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}$, $\bar{A}_k = \begin{bmatrix} 0 & 0 \\ 0 & A_k \end{bmatrix}$, $\phi(t) = \begin{bmatrix} -y_d \\ d(t) \end{bmatrix}$, $\bar{x} = \left[(\int_0^t e(s)ds)^{\mathrm{T}} \quad x(t)^{\mathrm{T}} \right]^{\mathrm{T}}$, and $d(t)$ is the bounded external disturbance.

3. DESIGN of TRACKING CONTROLLER BASED ON EVENT-TRIGGERED MECHANISM

This section presents the position tracking control design base on the method that combines the weighted multiple-model control and the event-triggered mechanism. The block diagram of the overall control system is shown in Fig. 1.

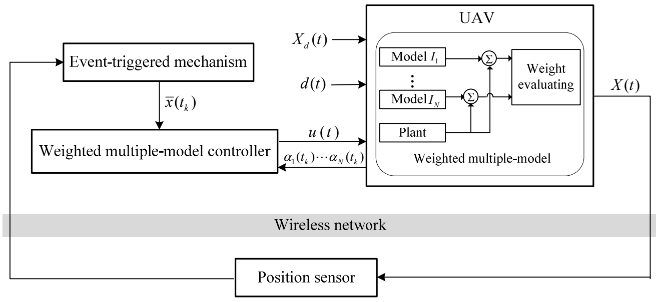

Fig. 1. Block diagram of the control system.

3.1 Weighted multiple-model control structure

In many literatures about UAV control, it is used to adopt the approximation $\sin\theta \approx \theta$ to make the system linearized. This paper focuses on the position tracking control in X-axis, and in this case, the pitch angle θ has the possibility to alter within a wide range. Consequently, the model nonlinearity becomes more powerful, which cannot be ignored in the design of the tracking control law. In order to address this problem, a weighted multiple-model approach is utilized as the basic control structure for the tracking system. First of all, several system models at different pitch angles are established, and among them, the i-th model at pitch angle θ_i can be described as follows.

$$I_i: \quad \dot{\bar{x}}(t) = \bar{A}_i\bar{x}(t) + \sum_{k=1}^{h} \bar{A}_{ik}\bar{x}(t-\tau_k) + \bar{B}u(t) + \phi(t)\,, \tag{7}$$

where $\bar{A}_i = \begin{bmatrix} 0 & C \\ 0 & A_i \end{bmatrix} \in \mathbb{R}^{4\times 4}$, $\boldsymbol{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{4L}{m}\sin\theta_i \\ 0 & 0 & -\omega \end{bmatrix}$, $\bar{A}_{ik} = \begin{bmatrix} 0 & 0 \\ 0 & A_{ik} \end{bmatrix} \in \mathbb{R}^{4\times 4}$, and $\bar{B} \in \mathbb{R}^4$.

The weighted multiple-model structure adopted in the paper includes N subsystems. Each subsystem is denoted as $\{I_i\}_{i=1}^{N}$, and all of them are operating simultaneously. At every instant, the error $\tilde{e}_i = y - y_c$ caused by comparing the output y of the model I_i with that y_c of the current controlled system is evaluated to generate i-th quadric performance index J_i that denotes the match level between the model I_i and the current system.

Obviously, the weighted multiple-model approach can update the weight coefficient of each model adaptively via weight evaluating. Different from the normal multiple-model structure which has several controllers switched, the weighted multiple-model approach has only one controller to make the system controlled continuously, avoiding the performance deterioration due to high frequent switching.

Define the following quadratic index to evaluate the matching degree between the model I_i and the current system,

$$J_i(t) = a\left\|\tilde{e}_i(t)\right\|^2 + b\int_0^t \exp(-\eta(t-s))\left\|\tilde{e}_i(s)\right\|^2 ds\,, \tag{8}$$

where ($a \geq 0$, $b > 0$) = given constants, and ($\eta > 0$) = memory factor. Then the weight coefficient of the model I_i is designed as,

$$\alpha_i(t) = \frac{\exp(-\left|J_i(t)\right|^2 / \delta)}{\sum_{i=1}^{N}\exp(-\left|J_i(t)\right|^2 / \delta)}\,, \tag{9}$$

where δ = given positive scalar. Obviously, $0 < \alpha_i(t) < 1$, and $\sum_{i=1}^{N}\alpha_i(t) = 1$. When $\alpha_i(t)$ achieves the maximal value, the model I_i matches the current actual system most perfectly. According to (7) and (9), the whole controlled system can be expressed as follows,

$$\dot{x}(t) = \sum_{i=1}^{N}\alpha_i(t)\left[\bar{A}_i\bar{x}(t) + \sum_{k=1}^{h}\bar{A}_{ik}\bar{x}(t-\tau_k)\right] + \bar{B}u(t) + \phi(t)\,. \tag{10}$$

For (10), the weighted multiple-model controller is presented,

$$u_f(t) = \sum_{i=1}^{N}\alpha_i(t)K_i\bar{x}(t)\,, \tag{11}$$

where K_i = control gain matrix.

3.2 Tracking control based on the event-triggered mechanism

In this section, an event-triggered mechanism is presented to improve the weighted multiple-model control structure, forming the aperiodic tracking control law. The event-triggered mechanism can judge whether the triggering condition is met or not through using the received state information. If the condition was met, the event-triggered mechanism would release the signal to the controller, triggering the control execution, otherwise, it would not release the triggering signal and the control output would remain unchanged with the zero-order holder. Assuming that Δ = sampling period and $\{t_0, t_1, t_2, \cdots\}$ = triggering instant set , t_k = current triggering instant at which the control output updates, therefore, the next triggering instant t_{k+1} can be designed as follows,

$$t_{k+1} = \inf\{t > t_k \mid \; \bar{e}^{\mathrm{T}}(t_k)\Phi\bar{e}(t_k) > \kappa\bar{x}^{\mathrm{T}}(t)\Phi\bar{x}(t)\}\,, \tag{12}$$

where $t = t_k + n\Delta$, $n = 0,1,2,3\cdots$, κ = given bounded positive real number, and Φ = symmetric positive matrix. Comparing the state vector at the last triggering instant with that at the current instant, the error is defined as $\bar{e}(t_k) = \bar{x}(t_k) - \bar{x}(t)$. The event-triggered mechanism proposed as (12), makes the outputs of the controller being updated at the time t_k that the error $\bar{e}(t_k)$ becomes "too large" beyond the state-dependent range for the first time. Otherwise, it remains the outputs unchanged.

It is clear that the control execution time set, i.e. the event-triggering instant set $\{t_0, t_1, t_2, \cdots\}$, is the subset of the sampling time set $\{0, \Delta, 2\Delta, \cdots\}$. Since the control executes according to the event rather than the sampling period, the control computation can be reduced under the presented event-triggered mechanism. Especially, when $\kappa = 0$, the event-triggered mechanism becomes the traditional periodic control mechanism, i.e. $\{t_0, t_1, t_2, \cdots\}$ = $\{0, \Delta, 2\Delta, \cdots\}$. According to (11), the controller based on the event-triggered mechanism (12) can be expressed as,

$$u(t) = \sum_{i=1}^{N}\alpha_i(t_k)K_i\bar{x}(t_k)\,. \tag{13}$$

If $t \in [t_k, t_{k+1})$, the closed loop expression of the system can be described as follows from (10) and (13),

$$\dot{\bar{x}}(t) = \sum_{i=1}^{N}\alpha_i(t)\left[\bar{A}_i\bar{x}(t) + \sum_{k=1}^{h}\bar{A}_{ik}\bar{x}(t-\tau_k)\right] + \sum_{i=1}^{N}\alpha_i(t_k)\bar{B}K_i\bar{x}(t_k) + \phi(t)\,. \tag{14}$$

Based on the approach proposed by Peng et al. (2014), the following equation (15) holds, assuming that there are real numbers ρ_j and Δ_j ,

$$\alpha_j(t_k) = \rho_j\alpha_j(t)\,, \left|\alpha_j(t_k) - \alpha_j(t)\right| \leq \Delta_j\,. \tag{15}$$

So that, if $t \in [t_k, t_{k+1})$, the controller (13) can be further converted to,

$$u(t) = \sum_{i=1}^{N}\rho_i\alpha_i(t)K_i\bar{x}(t_k)\,. \tag{16}$$

And the system (14) can be rewritten as

$$\dot{\bar{x}}(t) = \sum_{i=1}^{N}\alpha_i(t)\left[\bar{A}_i\bar{x}(t) + \bar{B}\rho_iK_i\bar{x}(t_k) + \sum_{k=1}^{h}\bar{A}_{ik}\bar{x}(t-\tau_k)\right] + \phi(t)\,. \tag{17}$$

Letting $\bar{K}_i = \rho_iK_i$, (17) becomes

$$\dot{\bar{x}}(t) = \sum_{i=1}^{N}\alpha_i(t)\left[\bar{A}_i\bar{x}(t) + \bar{B}\bar{K}_i\bar{x}(t) + \sum_{k=1}^{h}\bar{A}_{ik}\bar{x}(t-\tau_k) + \bar{B}\bar{K}_i\bar{e}(t_k)\right] + \phi(t)\,. \tag{18}$$

Lemma 1 (Gu et al., 2001): For arbitrary $x_1 \in \mathbb{R}^m$, $x_2 \in \mathbb{R}^n$, $M \in \mathbb{R}^{m\times n}$, and arbitrary positive definite matrix $N \in \mathbb{R}^{n\times n}$, there is

$$x_1^{\mathrm{T}}Mx_2 + x_2^{\mathrm{T}}M^{\mathrm{T}}x_1 \leq x_1^{\mathrm{T}}MN^{-1}M^{\mathrm{T}}x_1 + x_2^{\mathrm{T}}Nx_2\,. \tag{19}$$

Next, the influence of the external disturbances and state time-delays will be considered in the following content. Here choose the following H_∞ performance index,

$$J = \frac{1}{\gamma}\int_0^{t_f} \bar{x}(t)^{\mathrm{T}} R\bar{x}(t)dt - \gamma\int_0^{t_f} \phi(t)^{\mathrm{T}}\phi(t)dt < 0\,, \tag{20}$$

where t_f = finished time of control, γ = given constant which denotes the effect of $\phi(t)$ on $x(t)$, and R = given positive definite weighted matrix.

Theorem 1: If there are a symmetric positive definite matrix $P \in \mathbb{R}^{4\times 4}$, a set of matrix $\bar{K}_i^{\mathrm{T}} \in \mathbb{R}^4$ ($i = 1,\ldots,q$), and a set of scalars $\sigma_k > 0$ ($k = 1,\ldots,h$), so that the following LMI (21) holding, the controller (13) can make the system (10) stable within the given H_∞ bound γ.

$$\begin{bmatrix} \Gamma_i & * & * & * \\ A_{di}^T & -\sigma & * & * \\ Q & 0 & -\Theta^{-1} & * \\ E_i^T & 0 & 0 & -\Phi \end{bmatrix} < 0\,, \tag{21}$$

where $\Gamma_i = (\bar{A}_i + \bar{B}\bar{K}_i)Q + Q(\bar{A}_i + \bar{B}\bar{K}_i)^{\mathrm{T}} + (1/\gamma)I_{4\times 4}$, $Q = P^{-1}$, $\Theta = (1/\gamma)R + I\sigma I^{\mathrm{T}} + \kappa\Phi$, $\sigma = \mathrm{diag}(\sigma_1 I_{4\times 4},\ldots,\sigma_h I_{4\times 4})_{4h\times 4h}$, $A_{di} = \left[\bar{A}_{i1},\ldots,\bar{A}_{ih}\right]_{4\times 4h}$, $I = \left[I_{4\times 4},\ldots,I_{4\times 4}\right]_{4\times 4h}$, $E_i = \bar{B}\bar{K}_i$. $I_{n\times n}$ = n -dimensional unit matrix, and $*$ = transposed elements corresponding to the symmetric position in the matrix.

Proof: Choose the Lyapunov function as

$$V(\bar{x}) = \bar{x}(t)^{\mathrm{T}} P\bar{x}(t) + \sum_{k=1}^{h}\int_{t-\tau_k}^{t} \sigma_k \bar{x}(s)^{\mathrm{T}}\bar{x}(s)ds\,. \tag{22}$$

Obviously, $V(\bar{x}) > 0$, $\forall \bar{x} \neq 0$. The derivative of the Lyapunov function (22) is

$$\begin{aligned}
\dot{V}(\bar{x}) &= \dot{\bar{x}}(t)^{\mathrm{T}} P\bar{x}(t) + \bar{x}(t)^{\mathrm{T}} P\dot{\bar{x}}(t) + \sum_{k=1}^{h}\sigma_k\bar{x}(t)^{\mathrm{T}}\bar{x}(t) - \sum_{k=1}^{h}\sigma_k\bar{x}(t-\tau_k)^{\mathrm{T}}\bar{x}(t-\tau_k) \\
&= \sum_{i=1}^{N}\alpha_i(t)\left\{\begin{aligned}&\bar{x}(t)^{\mathrm{T}}(H_i^{\mathrm{T}}P + PH_i)\bar{x}(t) + \bar{e}^{\mathrm{T}}(t_k)E_i^{\mathrm{T}}P\bar{x}(t) + \bar{x}^{\mathrm{T}}P\bar{E}_i\bar{e}(t_k) \\ &+ \sum_{k=1}^{h}\left[\bar{x}(t-\tau_k)^{\mathrm{T}}\bar{A}_{ik}^{\mathrm{T}}P\bar{x}(t) + \bar{x}(t)^{\mathrm{T}}P\bar{A}_{ik}\bar{x}(t-\tau_k)\right]\end{aligned}\right\} \\
&\quad + \bar{x}(t)^{\mathrm{T}}P\phi(t) + \phi(t)^{\mathrm{T}}P\bar{x}(t) + \sum_{k=1}^{h}\sigma_k\bar{x}(t)^{\mathrm{T}}\bar{x}(t) \\
&\quad - \sum_{k=1}^{h}\sigma_k\bar{x}(t-\tau_k)^{\mathrm{T}}\bar{x}(t-\tau_k)\,,
\end{aligned} \tag{23}$$

where $H_i = \bar{A}_i + \bar{B}\bar{K}_i$. According to Lemma 1 and the event-triggered condition (12), the following inequality can be obtained from (23),

$$\begin{aligned}
\dot{V}(x) \le \sum_{i=1}^{N}\alpha_i(t)\{&\bar{x}(t)^{\mathrm{T}}(H_i^{\mathrm{T}}P + PH_i + PA_{di}\sigma^{-1}A_{di}^{\mathrm{T}}P + \kappa\Phi \\
&+ PB_i\Phi^{-1}E_i^{\mathrm{T}}P + I\sigma I^{\mathrm{T}} + \frac{1}{\gamma}PP)\bar{x}(t) + \gamma\phi(t)^{\mathrm{T}}\phi(t)\}\,.
\end{aligned} \tag{24}$$

Then consider the performance index (20),

$$\begin{aligned}
J &= \int_0^{t_f}(\frac{1}{\gamma}\bar{x}(t)^{\mathrm{T}}R\bar{x}(t) - \gamma\phi(t)^{\mathrm{T}}\phi(t))dt \\
&= \int_0^{t_f}(\frac{1}{\gamma}\bar{x}(t)^{\mathrm{T}}R\bar{x}(t) - \gamma\phi(t)^{\mathrm{T}}\phi(t) + \dot{V})dt - V(t_f) + V(0)\,.
\end{aligned} \tag{25}$$

It is obvious that $V(0) = 0$, $V(t_f) > 0$. Substituting (24) into (25), it yields

$$\begin{aligned}
J &< \int_0^{t_f}(\frac{1}{\gamma}\bar{x}(t)^{\mathrm{T}}R\bar{x}(t) - \gamma\phi(t)^{\mathrm{T}}\phi(t) + \dot{V})dt \\
&= \sum_{i=1}^{N}\alpha_i(t)\int_0^{t_f}\bar{x}(t)^{\mathrm{T}}(S + PE_i\Phi^{-1}E_i^{\mathrm{T}}P + \frac{1}{\gamma}PP + \frac{1}{\gamma}R)\bar{x}(t)dt\,,
\end{aligned}$$

where $S = H_i^{\mathrm{T}}P + PH_i + PA_{di}\sigma^{-1}A_{di}^{\mathrm{T}}P + I\sigma I^{\mathrm{T}} + \kappa\Phi$. If

$$S + PE_i\Phi^{-1}E_i^{\mathrm{T}}P + \frac{1}{\gamma}PP + \frac{1}{\gamma}R < 0\,, \tag{26}$$

the inequality $J < 0$ holds.

Consequently, multiplying (26) by Q both in left and right, it yields

$$\begin{aligned}
&QSQ + E_i\Phi^{-1}E_i + \frac{1}{\gamma}I_{n\times n} + \frac{1}{\gamma}QRQ \\
&= QH_i^{\mathrm{T}} + H_iQ + A_{di}\sigma^{-1}A_{di}^{\mathrm{T}} + QI\sigma I^{\mathrm{T}}Q - \kappa Q\Phi Q - E_i\Phi^{-1}E_i^{\mathrm{T}} \\
&+ \frac{1}{\gamma}I_{n\times n} + \frac{1}{\gamma}QRQ \\
&= (\bar{A}_i + \bar{B}\bar{K}_i)Q + Q(\bar{A}_i + \bar{B}\bar{K}_i)^{\mathrm{T}} + \frac{1}{\gamma}I_{n\times n} + \frac{1}{\gamma}QRQ \\
&+ A_{di}\sigma^{-1}A_{di}^{\mathrm{T}} + QI\sigma I^{\mathrm{T}}Q + \kappa Q\Phi Q + E_i\Phi^{-1}E_i^{\mathrm{T}} \\
&= (\bar{A}_i + \bar{B}\bar{K}_i)Q + Q(\bar{A}_i + \bar{B}\bar{K}_i)^{\mathrm{T}} + \frac{1}{\gamma}I_{n\times n} + A_{di}\sigma^{-1}A_{di}^{\mathrm{T}} \\
&+ Q(\frac{1}{\gamma}R + I\sigma I^{\mathrm{T}} + \kappa\Phi)Q + E_i\Phi^{-1}E_i^{\mathrm{T}} < 0\,.
\end{aligned} \tag{27}$$

Let $\Gamma_i = (\bar{A}_i + \bar{B}\bar{K}_i)Q + Q(\bar{A}_i + \bar{B}\bar{K}_i)^{\mathrm{T}} + (1/\gamma)I_{n\times n}$, $\Theta = (1/\gamma)R + I\sigma I^{\mathrm{T}} + \kappa\Phi$. Based on Schur complement theorem, (27) can be converted to $\begin{bmatrix} \Gamma_i & * & * & * \\ A_{di}^{\mathrm{T}} & -\sigma & * & * \\ Q & 0 & -\Theta^{-1} & * \\ E_i^{\mathrm{T}} & 0 & 0 & -\Phi \end{bmatrix} < 0$, which is the LMI in Theorem 1. The proof is completed.

4. SIMULATION RESULTS

This paper applies the proposed tracking control to the quad-rotor “Qball-X4” developed by Quanser Inc, and carries out the simulation of the position tracking along X axis, in order to illustrate the effectiveness of the control approach.

Set the original states are: $X = 0$, $\dot{X} = 0$, $v = 1$, and the external disturbance $d(t)$ is the random noise with the mean value is 0 and the variance is 0.1. The desired position in X-axis is given as $X_d = 5$, i.e. $y_d = \begin{bmatrix} 5 & 0 & 1 \end{bmatrix}$. The other parameter values are given as follows:

$\tau_1 = 0.8$, $\tau_2 = 1$, $\gamma = 1$, $\kappa = 0.1$, $a = b = 1$, $\eta = 6$, $\delta = 0.01$,

$$A_{11} = A_{12} = A_{21} = A_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 15 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^{T},$$

$$R = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}.$$ According to the parameters of Qball-X4, the following matrix can be obtained,

When $\theta = 5^\circ$, $A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 43 \\ 0 & 0 & -15 \end{bmatrix}$; When $\theta = 15^\circ$, $A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 128.99 \\ 0 & 0 & -15 \end{bmatrix}$. Therefore, the weighted multiple-model structure can be formed by using the model in the case of $\theta = 5^\circ$ and that in the case of $\theta = 15^\circ$.

Through solving the LMI in Theorem 1, the values of the control feedback matrixes and the symmetric positive matrix in (12) can be derived as follows,

$$K_1 = \begin{bmatrix} -3.78 & -6.89 & -5.5 & 0.73 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -11.33 & -20.68 & -16.51 & 0.38 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 1.92 & 0 & 0 & 0 \\ 0 & 1.92 & 0 & 0 \\ 0 & 0 & 1.92 & 0 \\ 0 & 0 & 0 & 1.92 \end{bmatrix}.$$

Consequently, the control algorithm (16) and the event-triggered mechanism (12) applied to the control system of Qball-X4 can be determined with the above coefficients. The simulation results are shown as follows.

From Fig. 2, it is shown that the X-axis position can achieve tracking the control command within 5 seconds and keep the position with the satisfied control performance. Fig. 3 shows the response of the actuator dynamic and Fig. 4 shows the response of the control output during the tracking period. There are a little vibration in these two figures, which may be caused by the external disturbances and the long interval between neighbouring triggering instants.

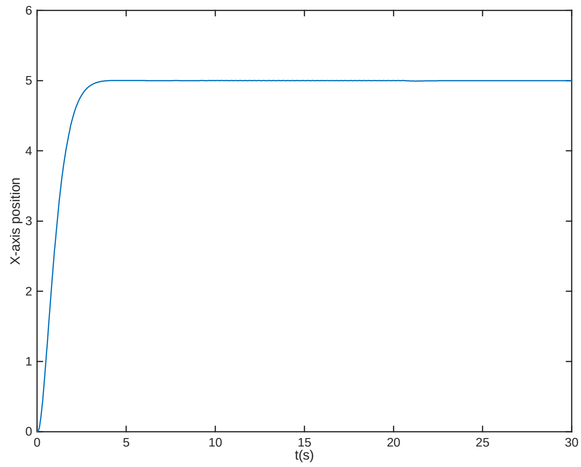

Fig. 2. X-axis position response curve.

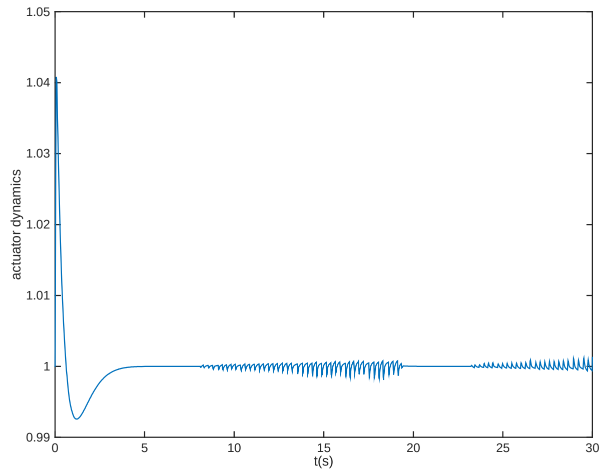

Fig. 3. Actuator dynamic response curve.

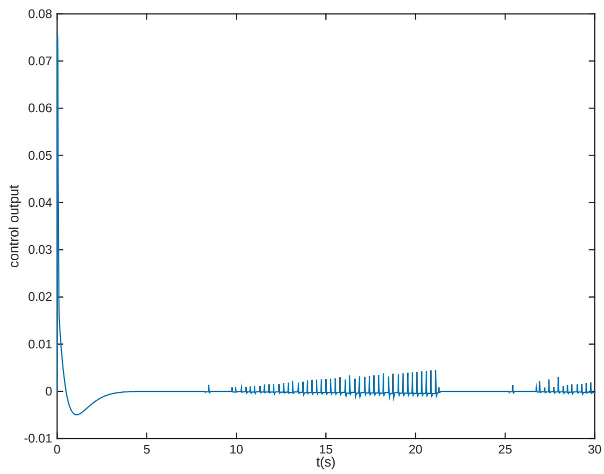

Fig. 4. Control output.

Fig. 5 shows the event-triggering instants by X-axis and the interval of neighbouring triggering instants by Y-axis. Furthermore, Fig. 6 and Fig. 7 give the enlarged details for the two parts in Fig. 5. They illustrate clearly that the event-triggering instants do not appear periodically, i.e., the control output does not update periodically. And sometimes the control algorithm executes even after a long interval from the last execution.

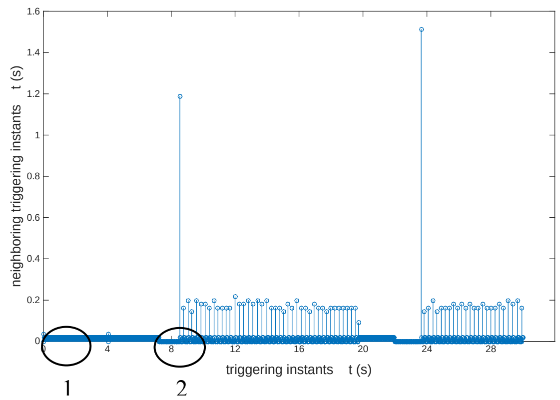

Fig. 5. Event-triggering instants.

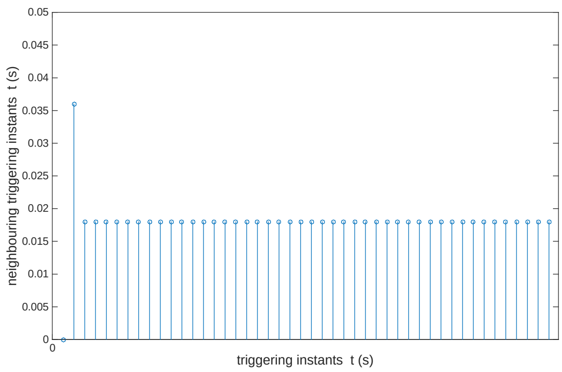

Fig. 6. Enlarged detail for the first circle part in Fig.4.

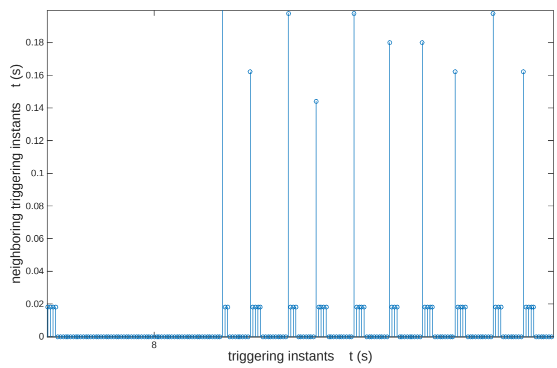

Fig. 7. Enlarged detail for the second circle part in Fig.5.

Though the system is controlled aperiodically, the desired robust control performance can be still achieved with less control executions, which is shown in the simulation result figures. That means the onboard embedded microprocessor could do less computation for the satisfactory control performance, if the proposed control method has been used in actual.

5. CONCLUSIONS

This paper presents a tracking control approach for the quad-rotor UAV system with the time-delays based on the even-triggered mechanism. For the nonlinear dynamic feature, the weighted multiple-model control method is used as the basic system structure. And then the event-triggered mechanism is augmented to the structure, forming the aperiodic tracking control system. The robust stability is analyzed for the external disturbances and state time-delays with the LMI technology based on a Lyapunov function, making the system meet the desired robust performance index. Finally, the simulation is taken by applying the proposed control approach to Qball-X4. From the simulation results, it is shown that the UAV can track the given position signal and achieve the satisfactory performance with less control executions. That is to say, this control law based on the event-triggered mechanism is feasible and effective, which can not only meet the satisfactory control performance but also save the computation resources consumption.

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