Active Control Strategy of High-Speed Elevator Horizontal Vibration Based on LMI Optimization

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Abstract: In order to solve the problem of horizontal vibration of high-speed elevator caused by rail roughness, in this paper, the active control strategy of horizontal vibration suppression of high-speed elevator based on linear matrix inequality (LMI) is studied by considering the constraints of stroke and power of the actuator of active guide shoe. Firstly, considering the constraints of the stroke and power, a 6-DOFs horizontal vibration model of the high-speed elevator car system is established, then its dynamic equation, state space equation, and the calculation formula of the working stroke and the force of the actuator are derived. Secondly, the generalized H2 norm is used to describe the above constraints, and the H2 norm is selected to minimize the vibration acceleration output of the car system. Based on LMI optimization technology, a H2/generalized H2 hybrid control strategy is proposed, which attributes the control law of car system to solving the semi-definite programming problem with LMI constraints. Finally, by MATLAB, the vibration acceleration of elevator is simulated and analyzed under three conditions: no-load, medium-load and full-load. The results show that, after using the H2/generalized H2 control method, the mean, maximum and root mean square values of the horizontal vibration acceleration of the car system have a decrease of more than 40%, and the constraints of stroke and power are satisfied. Therefore, the H2/generalized H2 hybrid control strategy proposed in this study can effectively suppress the horizontal vibration of high-speed elevators.

Keywords: High-speed elevator, Horizontal vibration, Linear matrix inequality, H2/generalized H2 hybrid control, Actuator

1. INTRODUCTION

As one of the important components of elevator lifting system, the guide shoe plays an important role in reducing the vibration caused by roughness of the excitation guide rail and improving the ride comfort of the elevator (Tao et al., 2017). The guide shoe is mainly divided into two types: passive guide shoe and active guide shoe. The passive guide shoe suppresses vibration by installing a suitable damping spring between the palanquin frame and the guide shoe. Active guide shoe is one of the most advanced technology products in modern active vibration control technology. It can be weaken the vibration with a force which is generated by an actuator, mounted on the passive guide shoe, and opposite to the direction of vibration (Feng et al., 2007a; Bao et al., 2017). With the elevator speed increasing, the horizontal vibration of the elevator caused by guide rail roughness intensifies (Noguchi et al., 2011). The passive guide shoe, as a traditional method of vibration reduction, cannot meet the vibration reduction requirements of the high-speed elevator because it only adapts to small external disturbance input and cannot provide large damping force. Active guide shoe, as one of the representatives of active control mode, has become a new way to solve elevator vibration because of its good effect and strong adaptability (Ai et al., 2007). Many scholars have done a lot of research on this. For the horizontal vibration problem of high-speed elevator, (Feng et al., 2007b) designed an elevator hydraulic active guide shoe, established the mathematical model of elevator car with hydraulic guide shoe, and designed a fuzzy controller to suppress the horizontal vibration of elevator. (Wang, 2011) established the dynamic model of active control of horizontal vibration of elevator car with 6-DOFs. Meanwhile, a kind of structures of hydraulic active guide shoe is put forward, and the controller is designed with adaptive fuzzy control algorithm, so as to achieve the purpose of vibration reduction. (Xue et al., 2012) took a guide shoe of elevator as the research object, established the horizontal vibration model of 1/4 elevator car, given the differential equation and state space equation in X direction and Y direction of the system. Based on this, a generalized predictive PID controller was designed to suppress horizontal vibration by combining generalized predictive control with PID strategy. (Song et al., 2014) used voice coil motor as actuator, established the mathematical model of horizontal vibration of voice coil motor and elevator, and combined the proportional feedback of displacement, velocity, and acceleration into PID controller by arranging the position of sensor and actuator, so as to restrain the horizontal vibration of elevator. (Santo et al., 2018) established a 3-DOF elevator car model and derived the dynamic equation of the influence of guide rail deformation on it. Meanwhile, a control strategy based on the state-dependent Riccati equation (SDRE) was proposed to reduce the vibration of the elevator. The scholars mentioned above have effectively reduced the horizontal vibration acceleration of high-speed elevator through the study of the elevator vibration model and the active control method, but they all neglected the hard constraint problem of the mechanical part of the active guide shoe, that is, the mechanical stroke constraint perpendicular to the direction of the guide rail and the power constraint of the actuator on the active guide shoe under the working state of the active guide shoe. During the operation of the elevator, excessive roughness excitation of the guide rail leads to the problem that the roller impact the guide shoe, which will aggravate the vibration of the elevator. Meanwhile, the actuator can only provide limited force, and the excessive vibration will lead to the damage of the actuator.

Thus, in view of the shortcomings that the existing active vibration reduction schemes do not consider the stroke constraint of the active guide and the force constraint of the actuator, firstly, a 6-DOF horizontal vibration model of the high-speed elevator car system with active guide shoes is established in this paper, and its dynamic equation and state equation are given. Secondly, considering that there is no deduction of the calculating formula for the stroke and maximum force of the actuator of active guide shoes in the present research, based on the design method of the active suspension of the automobile, the calculating formula for the stroke and maximum force of the actuator of active guide shoes are derived, and use the calculation results in the design of the active vibration reduction controller considering the active guide shoe time-domain hard constraint. Moreover, based on the validity of the H2 norm verified in the previous study (Cao et al., 2019) for optimizing the horizontal vibration acceleration of the elevator car, the H2 norm is chosen to minimize the vibration acceleration of the elevator car system, and the generalized H2 is applied to describe the time-domain hard constraint of the active guide shoes. Based on the LMI optimization technique, a H2/generalized H2 hybrid control strategy is proposed. Finally, the simulation analysis is carried out by MATLB.

2. HORIZONTAL VIBRATION DYNAMICS MODEL AND ITS CONTROL PROBLEM

2.1 Horizontal vibration dynamic model of car system with active control

In this paper, only the horizontal vibration between the elevator car and the guide rail, that is, the vibration parallel to the elevator car door, is considered. To simplify the model, the following reasonable assumptions are made according to the elevator structure and its movement law：

1) Because of the higher strength and stiffness of the elevator car and its frame, and the smaller deformation of the elevator car and its frame in the process of vibration compared with its structural size, the elevator car and its frame can be regarded as a rigid connector in the process of modeling and called the elevator car system.

2) Due to the guide wheel clinging to the guideway, the relative displacement between them is small, so the guide-shoe can be simplified as mass-spring damping system.

3) The relative displacement between the elevator car system and the guide wheel is small, so the elevator car system is simplified as a mass spring damping system.

4) The structure and parameters of each guide wheel are identical.

5) Only considering the roughness excitation of guide rail.

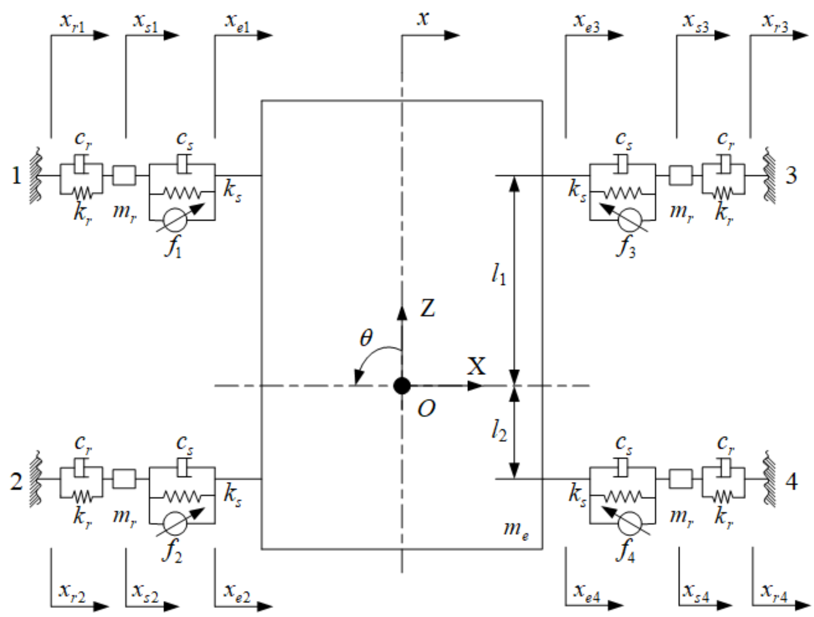

Fig. 1. Horizontal vibration dynamic model of elevator car with active control.

Based on the horizontal vibration model of high-speed elevator established in the references (Zhang et al., 2018; Zhang et al., 2018), considering the above assumptions and the time-domain hard constraints of active guide shoes, a horizontal vibration model of high-speed elevator car system with active control is established in this paper. As shown in Fig. 1, the centroid O of the car system is the original point, the horizontal direction is the X-axis, the rotation angle of the car system around the centroid O is θ. The elevator model established in this paper has 6-DOFs, which are the degree of freedom of movement of the car in the X-axis direction, the degree of freedom of rotation of the car around the centroid, and the freedom of movement of each shoe in the X-axis direction. The mass of the elevator car is m_e. The moment of inertia is J. The horizontal displacement of the centroid of the elevator car is x. The horizontal displacements of the guide shoe are x_{ei} (i = 1, 2, 3, 4). The horizontal displacement of the roller is x_{si} (i = 1, 2, 3, 4). The stiffness coefficient and damping coefficient between the roller and the guide rail are k_r, c_r. The stiffness coefficient and damping coefficient between the roller and the elevator car are k_s, c_s. The vertical distances from active guide shoes 1, 3 to the centroid of the car system is l_1. The vertical distances from active guide shoes 2, 4 to the centroid of the car system is l_2.The roughness excitation of guide rail are x_{ri} (i = 1, 2, 3, 4). The active control force is f_i (i = 1, 2, 3, 4). The positive direction of all displacements and forces in this model is along the X-axis direction.

The dynamic equations of the model are as follows:

$$\begin{cases} M_e\ddot{z}+LC_s(\dot{x}_e-\dot{x}_s)+LK_s(x_e-x_s)=L_1F \\ M_r\ddot{x}_s+L_2C_s(\dot{x}_e-\dot{x}_s)+C_r(\dot{x}_s-\dot{x}_r) \\ +L_2K_s(x_e-x_s)+K_r(x_s-x_r)=L_2F \end{cases} \tag{1}$$

where,

$$x_e=L^{\mathrm{T}}z\,,\ z=\begin{bmatrix}x & \theta\end{bmatrix}^{\mathrm{T}}$$

$$,\ x_s=\begin{bmatrix}x_{s1} & x_{s2} & x_{s3} & x_{s4}\end{bmatrix}^{\mathrm{T}},\ x_e=\begin{bmatrix}x_{e1} & x_{e2} & x_{e3} & x_{e4}\end{bmatrix}^{\mathrm{T}},$$

$$F=\begin{bmatrix}f_1 & f_2 & f_3 & f_4\end{bmatrix}^{\mathrm{T}},\ x_r=\begin{bmatrix}x_{r1} & x_{r2} & x_{r3} & x_{r4}\end{bmatrix}^{\mathrm{T}},$$

$$L=\begin{bmatrix}1 & 1 & 1 & 1\\ -l_1 & l_2 & -l_1 & l_2\end{bmatrix},\ L_1=\begin{bmatrix}1 & 1 & 1 & 1\\ -l_1 & l_2 & l_1 & -l_2\end{bmatrix}$$

$$M_r=diag(m_r,m_r,m_r,m_r)\,,\ K_s=diag(k_s,k_s,k_s,k_s)\,,$$

$$K_r=diag(k_r,k_r,k_r,k_r)\,,\ C_s=diag(c_s,c_s,c_s,c_s)\,,$$

$$C_r=diag(c_r,c_r,c_r,c_r)\,,\ L_2=diag(-1,-1,-1,-1)$$

$$M_e=diag(m_e,J)\,.$$

2.2 Control problem description

The state vector is selected as

$$X=\begin{bmatrix}(x_e-x_s)^{\mathrm{T}} & \dot{x}_e^{\mathrm{T}} & (x_s-x_r)^{\mathrm{T}} & \dot{x}_s^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\in R^{16\times 1}\,.$$

The control vector is $U=\begin{bmatrix}f_1 & f_2 & f_3 & f_4\end{bmatrix}^{\mathrm{T}}$.

The disturbance input vector is $W=\begin{bmatrix}\dot{x}_{r1} & \dot{x}_{r2} & \dot{x}_{r3} & \dot{x}_{r4}\end{bmatrix}^{\mathrm{T}}$.

Then, the state space equation of the system can be expressed as:

$$\dot{X}=AX+BU+RW \tag{2}$$

where,

$$A=\begin{bmatrix}0_4 & I_4 & 0_4 & -I_4\\ -L^TM_e^{-1}LK_s & -L^TM_e^{-1}LC_s & 0_4 & L^TM_e^{-1}LC_s\\ 0_4 & 0_4 & 0_4 & I_4\\ M_r^{-1}K_s & M_r^{-1}C_s & -M_r^{-1}K_r & -M_r^{-1}C_s-M_r^{-1}C_r\end{bmatrix}$$

$$B=\begin{bmatrix}0_4\\ L^TM_e^{-1}L_1\\ 0_4\\ -M_r^{-1}\end{bmatrix},\ R=\begin{bmatrix}0_4\\ 0_4\\ -I_4\\ M_r^{-1}C_r\end{bmatrix}.$$

The research of active guide shoe is mainly used to solve the problem of horizontal vibration of high-speed elevator in operation. The main evaluation index of horizontal vibration of elevator is the horizontal vibration acceleration of the car system. In this paper, the main research is the vibration in the direction which parallel to the car door (X direction), so the vibration acceleration $\ddot{x}$ along the X direction of the car and the rotation acceleration $\ddot{\theta}$ around the centroid O of the car system are the main indicators to evaluate the elevator horizontal vibration. The equation is:

$$\ddot{z}=\begin{bmatrix}-M_e^{-1}LK_s & -M_e^{-1}LC_s & 0_{2\times 4} & M_e^{-1}LC_s\end{bmatrix}X \\ +M_e^{-1}L_1U \tag{3}$$

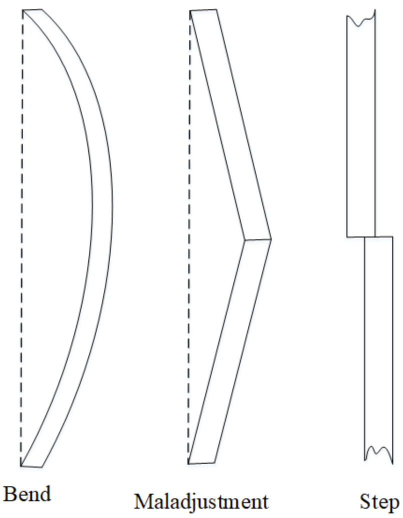

Fig. 2. Guide rail roughness.

Due to the manufacturing and installation errors of the elevator guide rail, and the loosening of the guide rail bracket caused by the vibration during the operation of the elevator, the guide rail roughness as shown in Fig. 2 will be formed (Zhang et al., 2018; Zhang et al., 2018). When the elevator runs on the guide rail, the dynamic deflection of the active guide shoe can be calculated as follows:

$$x_d=x_e-x_s \tag{4}$$

where,

x_d - The dynamic deflection of the active guild shoe, (m).

As a result of the guide rail roughness excitation mentioned above (especially, the step guide rail roughness excitation), the vibration acceleration of the elevator car system is too large. This will increase the roller displacement x_e and the guide shoe displacement x_s, results in an increase in the dynamic deflection x_d, even exceeds the dynamic deflection limit $[x_d]$, that is, the roller impact the guide shoe base, damage the active guide shoe. It results in the deformation of the elevator car, the failure of safety devices and so on. Moreover, the actuator of the active guide shoe also has a power limitation problem and can only provide limited force. Therefore, for suppressing the horizontal vibration of the high-speed elevator car system, the power limitation of the main actuator should be taken into consideration. Therefore, in order to improve the ride comfort and safety of the high-speed elevator, the above constraint problem must be considered.

So, the constraints problem of active guide shoe can be summed up as follows:

1) Since the mechanical structure of the active guide shoe has a stroke limitation, the movement stroke of the guide shoe needs to be limited to a certain range, so as to prevent the guide wheel from striking the guide shoe base, increase the horizontal vibration of the car system and causing danger, that is,

$$\left|x_{ei}-x_{si}\right| \le S_{\max}, i=1,2,3,4 \tag{5}$$

where,

$S_{\max}$ - The maximum stroke of the active guide shoe, (m).

2) Considering the power limitation of the actuator, it can only provide limited force, that is,

$$\left|f_a\right| \le F_{\max}, a=1,2,3,4 \tag{6}$$

where,

$F_{\max}$ - The maximum force of the active guide shoe, (N).

In view of the fact that there is no mention of the mechanical stroke of the active guide shoe and the dynamic range of the active force generator in the existing data, in this paper, considering the similarity between the elevator car system running on the guide rail and the vehicle running on the road (the former is mainly caused by the guide rail roughness, the latter is mainly caused by the road roughness), according to the parameter design principle of active suspension, the mechanical stroke and the maximum force of the active guide shoe are deduced (Wang et al., 2004; Zhang et al., 2017).

(1) Mechanical stroke of active guide shoes

Distribution of the supporting quality of the guide shoe 1, 3 and guide shoe 2, 4

Refer to vehicle suspension mass distribution coefficient:

$$\varepsilon=\rho^2/ab=0.8\sim1.2 \tag{7}$$

where,

ε - Suspension mass distribution coefficient.

ρ - The rotation radius of the body around the transverse axis

a - The distance from front suspension system to body centroid.

b - The distance from rear suspension system to body centroid.

Considering the independence and same parameters of the four guide shoes, the support mass distribution coefficient of the guide shoes is approximately 1. The horizontal vibration of the guide shoes 1, 3 and 2, 4 is independent from each other. The frequencies n_1 and n_2 are used to denote the horizontal free vibration frequencies of the two parts respectively. Since the human body is sensitive to horizontal vibration with a frequency band of 1-2 Hz, it is necessary to avoid this frequency band when choosing the free vibration frequency of the guide shoe, and take $n_1 = n_2$.

The solution of the static deflection x_{rr} of the guide shoe

When the mass distribution coefficient of the guide shoe is 1, the relationship between the horizontal free vibration frequencies n_1 and n_2 of the upper and lower parts of the car (the centroid O is the separation point of the upper and lower parts), and the corresponding stiffness k_s of the guide shoe and the support mass m is as follows:

$$\begin{cases} n_1=n_2=\dfrac{1}{2\pi}\sqrt{\dfrac{k_s}{m}} \\ m=\dfrac{F_N}{g} \end{cases} \tag{8}$$

where,

k_s - The stiffness coefficient between the guide shoe and the elevator car, ($N\cdot m^{-1}$).

m - Guide shoe support mass, (kg).

F_N - Pre-tightening force of guide shoes, (N).

g - Acceleration of gravity, $g=9.8m/s^2$.

From the formula of static deflection, the static deflection of the guide shoe can be obtained as follows,

$$x_{rr}=\frac{mg}{k_s} \tag{9}$$

where,

x_{rr} - The static deflection of the guide shoe, (m)

m - The support mass of guide shoe, i.e. the equivalent mass of pre-tightening force F_N. (kg)

The direction of F_N is the horizontal direction of car system.

Therefore, equation (8) can be expressed as:

$$n_1=n_2=\frac{1}{2\pi}\sqrt{\frac{g}{x_{rr}}} \tag{10}$$

From this, the static deflection x_{rr} can be obtained as follows:

$$x_{rr}=\frac{g}{4\pi^2 n_1^2} \tag{11}$$

The solution of the dynamic deflection $[x_d]$ of the guide shoe

Taking into account the comfort requirements of the car and the similarity with the design of the suspension stroke, it is:

$$[x_d] = (0.5 \sim 0.7)x_{rr}, \text{ take } [x_d]{=}0.5x_{rr} \tag{12}$$

Therefore, the working stroke of the active guide shoe is the sum of the static deflection and the dynamic deflection, that is:

$$S_{\max} = x_{rr} + [x_d] \tag{13}$$

Maximum operating force of actuator

In the existing literature, the actuators used in the study of active guided shoes are mostly hydraulic actuators. In this paper, the maximum actuation of a hydraulic actuator is derived.

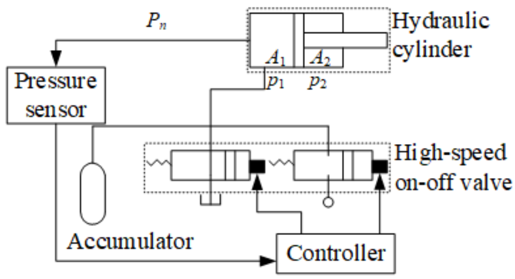

Fig. 3. Schematic diagram of hydraulic actuator.

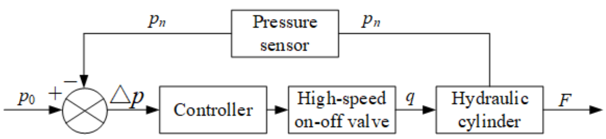

Fig. 4. Control system block diagram of hydraulic actuator.

The working principle of the hydraulic actuator is shown in Fig. 3, and its control system block diagram is shown in the Fig. 4. The pressure sensor detects the pressure p_n of the hydraulic actuator and feeds the p_n back to the controller. The controller compares the detected pressure p_n with the preset pressure p_0, the pressure difference $\triangle p$ is obtained, which is used to control the action of the high-speed on-off valve. Then, hydraulic oil flows into the hydraulic cylinder for pressure control, so that the pressure of the guide shoe system is stable. Therefore, the maximum flow rate q_1 of the high-speed on-off valve and the size of the hydraulic cylinder determine the maximum operating force of the hydraulic actuator. And the determination of the size of the hydraulic cylinder should consider the size of the guide shoe and the degree of difficulty in installation.

Suppose that the maximum flow rate of the high-speed on-off valve is q_1, the pressure of the rodless cavity and the rod cavity of the hydraulic cylinder are p_1 and p_2, differently, and the area of the rodless cavity and the rod cavity of the hydraulic cylinder are A_1 and A_2, respectively, the maximum force $F_{\max}$ of the system is:

$$F_{\max} = p_1 A_1 - p_2 A_2 \tag{14}$$

In summary, the performance output and the normalized constraint output of the control system of the active guide shoe can be expressed as follows:

$$y_1 = \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix}, \quad y_2 = \begin{bmatrix} \dfrac{x_e - x_s}{S_{\max}} \\ \dfrac{f_a}{F_{\max}} \end{bmatrix} \tag{15}$$

Therefore, the control problem of the active guide shoe can be summed up as follows:

Finding a controller to obtain the minimum horizontal vibration acceleration of the car system (minimize the value of each element in performance output y_1) while satisfying the working stroke constraint and the power constraint of the active guide shoe (the absolute value of each element in the normalized constraint output y_2 is less than or equal to 1).

3. H2 / GENERALIZED H2 HYBRID CONTROL

3.1 H2 performance and generalized H2 performance

The H2 norm from disturbance input W to performance output y_1 is defined as (Baghbani et al., 2016; Kim et al., 2016; Lin et al., 2018):

$$\|y_1\|_2 \overset{def}{=} \left(\int_{-\infty}^{\infty} y_1(t)^2 dt \right)^{1/2} \tag{16}$$

It has two physical meanings:

1) Representing the square root of the total energy carried by the system signal y_1;

2) The H2 norm is the root mean square (RMS) value of the system output when the system input signal is independent of each other.

In this paper, random excitation is used to describe the guide rail roughness, and the input excitation signals are independent of each other, and the H2 norm is the RMS of its output. Meanwhile, vibration acceleration is often used to describe the vibration of elevator car in engineering, so H2 norm is selected as a performance index to describe the horizontal vibration acceleration of elevator.

The generalized H2 norm (Kim et al., 2017) describes the maximum output in time domain when the input signal is a unit energy signal. For the active guide shoe in this paper, there is a danger of violating the time-domain constraint such that the roller of the car system impacts the base of the guide shoe. Therefore, the generalized H2 norm is introduced to describe the time-domain hard constraint of the elevator car system.

3.2 H2 / generalized H2 control method

In view of the horizontal vibration problem of elevator car system, considering the time-domain hard constraints, according to the dynamic model of elevator car system established above, the controlled object (elevator car system) can be described by the following linear time-invariant system state equation:

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t) + RW(t) \\ y_1(t) = C_1X(t) + D_1U(t) \\ y_2(t) = C_2X(t) + D_2U(t) \end{cases} \tag{17}$$

where,

X - State vector, $X \in R^{16\times 1}$

W - Disturbance input, i.e. the random excitation of the guide rails, $W \in R^{4\times 1}$

y_1 - System performance output, $y_1 \in R^{2\times 1}$

y_2 - Constrained output of the system, $y_2 \in R^{8\times 1}$

The control block diagram of this system is shown in Figure 5.

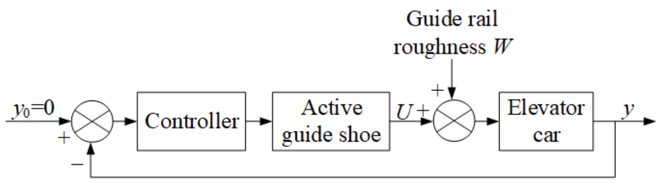

Fig. 5. Block diagram of the elevator car system vibration reduction control system.

Therefore, the active control problem of horizontal vibration of high speed elevator studied in this paper is expressed as follows:

Finding a controller to make the elevator car system satisfies the generalized H2 norm from the random excitation signal W to the system constraint output y_2 less than or equal to a given positive number ρ (i.e. satisfying the time-domain hard constraints of the elevator car system), and make the H2 norm from the random signal of the rails W to the system performance output y_1 is minimized. That means the RMS of horizontal vibration acceleration of elevator car system is the smallest. Let G_1 denote the transfer function from the interference input signal $W(t)$ to the performance output $y_1(t)$, and G_2 denote the transfer function from the interference input signal $W(t)$ to the constraint output $y_2(t)$, then the above proposition can be described for:

$$\min \|G_1\|_2 \text{, satisfy } \|G_2\|_g \le \rho, \rho > 0 \tag{18}$$

where,

$\|G_1\|_2$ - The H2 norm of transfer function G_1

$\|G_2\|_g$ - The generalized H2 norm of transfer function G_2

min (•) - The minimum value of (•)

satisfy (•) - Satisfy the condition of (•)

Suppose that the state feedback gain of the system is K, then U=KX, and the corresponding closed loop system is:

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t) + RW(t) \\ \qquad = (A + BK)X(t) + RW(t) \\ y_1(t) = C_1X(t) + D_1U(t) \\ \qquad = (C_1 + D_1K)X(t) \\ y(t) = C_2X(t) + D_2U(t) \\ \qquad = (C_2 + D_2K)X(t) \end{cases} \tag{19}$$

That is,

$$\begin{pmatrix} A_{cl} & B_{cl} \\ C_{cl1} & D_{cl1} \\ C_{cl2} & D_{cl2} \end{pmatrix} = \begin{pmatrix} A + BK & R \\ C_1 + D_1K & 0 \\ C_2 + D_2K & 0 \end{pmatrix} \tag{20}$$

In the case of multi-objective control, the linear time invariant system has a conclusion (Chen et al., 2007; Kim et al., 2016): If the system state matrix A_{cl} is stable and $\|G_1\|_2^2 < v$, $\|G_2\|_g^2 < \rho$, when and only if there is $P=P^{\mathrm{T}}>0$, $S>0$, so that,

$$\begin{pmatrix} A_{cl}^{\mathrm{T}}P + PA_{cl} & PB_{cl} \\ B_{cl}^{\mathrm{T}}P & -I_{4\times 4} \end{pmatrix} < 0, \begin{pmatrix} P & C_{cl1}^{\mathrm{T}} \\ C_{cl1} & S \end{pmatrix} > 0, \begin{pmatrix} P & C_{cl2}^{\mathrm{T}} \\ C_{cl2} & \rho I_{8\times 8} \end{pmatrix} > 0,$$

$$Trace(S) < v, D_{cl1} = D_{cl2} = 0.$$

Where $I_{n\times n}$ is the unit matrix, the subscript is its dimension. Let $Q=P^{-1}$ and $Y=KQ$, using S-Procedure and Schur complement formula to get LMI of v, Q, Y, S:

$$\begin{pmatrix} QA^{\mathrm{T}} + AQ + Y^{\mathrm{T}}B^{\mathrm{T}} + BY & R \\ R^{\mathrm{T}} & -I_{8\times 8} \end{pmatrix} < 0, Trace(S) < v,$$

$$\begin{pmatrix} Q & QC_1^{\mathrm{T}} + Y^{\mathrm{T}}D_1^{\mathrm{T}} \\ C_1Q + D_1Y & S \end{pmatrix} > 0$$

$$, R_1 = 0, \begin{pmatrix} Q & QC_2^{\mathrm{T}} + Y^{\mathrm{T}}D_2^{\mathrm{T}} \\ C_2Q + D_2Y & \rho I_{8\times 8} \end{pmatrix} > 0.$$

At this point, the above proposition can be reduced to solving a set of LMI:

$$\min_{v,Q,Y,S} v \text{, satisfy the above LMI.} \tag{21}$$

If the above semidefinite programming problem has an optimal solution such that:

1) The closed-loop stability of elevator car system (i.e. the stability of state matrix A_{cl});

2) Under the action of the random excitation signal of the guide rail roughness, the elevator car satisfies the working stroke constraint condition of the active guide shoe and the power constraint condition of the actuator.

3) Under the action of random excitation signal of guide rail roughness, the RMS value of horizontal vibration acceleration of elevator car system is not greater than $\sqrt{v^*}$ ($\sqrt{v^*}$ is the RMS of performance output under the action of

unit white noise excitation input).

4. SOLUTION AND SIMULATION OF H2/ GENERALIZED H2 ACTIVE GUIDE SHOE CONTROLLER

4.1 Controller solution

Some parameters of the selected 4m/s high-speed elevator car system are shown in Table 1.

According to the parameters in Table 1, the LMI toolbox in MATLAB is used to solve the above LMI, and the optimal gain matrix K of the system state feedback is obtained,

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \end{bmatrix}$$

where,

$$K_{11} = \begin{bmatrix} \text{-8.8848e3} & \text{4.4401e2} & \text{-8.9242e3} & \text{1.6356e2} \\ \text{3.9366e2} & \text{-7.6764e3} & \text{9.0835e1} & \text{-7.9206e3} \\ \text{1.5475e3} & \text{-8.9551e3} & \text{1.5368e3} & \text{-8.6405e3} \\ \text{-7.0266e3} & \text{-1.7186e3} & \text{-6.7344e3} & \text{-1.4672e3} \end{bmatrix}$$

$$K_{12} = \begin{bmatrix} \text{2.4627e2} & \text{3.1619e2} & \text{-6.9701e2} & \text{-3.1332e2} \\ \text{3.3960e2} & \text{4.2908e2} & \text{-3.4848e2} & \text{-9.6833e2} \\ \text{-3.3856e2} & \text{-7.0574e2} & \text{4.3845e2} & \text{3.7133e1} \\ \text{-5.3431e3} & \text{-7.4071e2} & \text{1.4999e2} & \text{6.5928e2} \end{bmatrix}$$

$$K_{13} = \begin{bmatrix} \text{-1.3753e4} & \text{-1.5454e2} & \text{-1.3548e4} & \text{1.2084e3} \\ \text{4.2063e1} & \text{-1.2051e4} & \text{1.1454e3} & \text{-1.1052e4} \\ \text{2.2552e3} & \text{-1.2009e4} & \text{2.1915e3} & \text{-1.3159e4} \\ \text{-9.6788e3} & \text{-2.0064e3} & \text{-1.0780e4} & \text{-3.0067e3} \end{bmatrix}$$

$$K_{14} = \begin{bmatrix} \text{-5.8017} & \text{4.5830e-2} & \text{-6.8065} & \text{3.3074e-1} \\ \text{1.0886e-2} & \text{-4.7715} & \text{2.5089e-1} & \text{-5.5221} \\ \text{1.0808} & \text{-6.1428} & \text{2.0741} & \text{-6.4198} \\ \text{-5.0819} & \text{-1.1390} & \text{-5.4039} & \text{-3.3251e-1} \end{bmatrix}$$

Table 1. Simulation parameters of elevator car system.

Parameter	Unit	Numerical value
The mass of elevator car m_e	kg	2.0e3
Rated load $m_{\max}$	kg	2.0e3
The mass of guide wheel m_r	kg	10
Stiffness coefficient k_r	N/m	6.71e5
Damping coefficient c_r	N·s/m	134
Stiffness coefficient k_s	N/m	4.27e4
Damping coefficient c_s	N·s/m	920
Moment of inertia J	kg·m^2	6.69e3
Distance l_1	m	2.2
Distance l_2	m	1.5
Derivation of maximum operating Force $F_{\max}$	N	10
Maximum actuating distance $S_{\max}$	m	0.008
Running speed V	m/s	4.0

4.2 Performance output simulation experiment

According to the elevator car system parameters in Table 1 and the optimal system state feedback gain obtained, the simulation analysis is carried out in MATLAB. Taking the disturbance input of car system as random excitation of guide rail roughness (essentially a stationary random process with zero expectation and obeying normal distribution (Wang, 2016)).

In order to verify the correctness and validity of the control strategy proposed in this paper, a comparative analysis of the horizontal vibration acceleration response of the car system under three control modes (no control, traditional PID control and H2/Generalized H2 control) is carried out under three kinds of operating conditions of elevator (no load, medium load and full load).

Since the evaluation index of the ride comfort of the car is the vibration acceleration at the centre of the car floor, the vibration acceleration y at the centre of the car floor is used as the output during the simulation. Therefore, the system state equation can be described as:

$$\begin{cases} \dot{X} = AX + BU + RW \\ y = C_N X \end{cases} \tag{22}$$

where,

the y is the horizontal vibration acceleration at the centre of the car floor, $U=KX$, $C_N = L_3 (C_1+D_1) K$, $L_3=[1 \; l_2]$.

Under the conditions of no load, medium load, and full load, the H2 / generalized H2 control algorithm, traditional PID Control and No-Control are used to control the horizontal vibration acceleration of the car system under the same random excitation respectively. The horizontal vibration acceleration images under the three conditions are shown in Figs. 6-8.

As can be seen from Figs. 6-8, under these three operating conditions of high-speed elevator, the traditional PID control and H2/generalized H2 control can effectively reduce the horizontal vibration of the car system. However, in the process of reducing vibration, the vibration acceleration curve under the action of traditional PID control is characterized by jagged irregular curve, the oscillation phenomenon is obvious. This is mainly because the traditional PID control needs to use error feedback to reduce the vibration of the car system, and this "error feedback to eliminate the error" way will often cause the initial control force is too large and the system behavior overshoot, therefore, under the traditional PID control of the car system horizontal vibration acceleration oscillation phenomenon is more obvious. The H2/ generalized H2 control is based on the feedback control of the current state amount of the system, and the optimal state feedback gain has been determined, so the suppression effect is better in the vibration process of the car system, and the acceleration curve is smoother.

Since the main evaluation index of the high-speed elevator vibration reduction effects are the mean, maximum, and RMS values of the vibration acceleration, these values under the three operation conditions are listed in Table 2. Under the no load condition, compared with the no control mode, the mean, maximum, and RMS values of the horizontal vibration acceleration of the car system are reduced by 41.2%, 32.0%, and 41.8%, respectively with the traditional PID control. Meanwhile, under the action of the H2/ generalized H2 control mode, these values are reduced by 51.0%, 64.9%, and 52.2%, respectively. Under the medium load condition, compared with the no control mode, the mean, maximum, and RMS values of the horizontal vibration acceleration of the car system are reduced by 49.0%, 32.2%, and 49.2%, respectively with the traditional PID control. Meanwhile, under the action of the H2/ generalized H2 control mode, these values are reduced by 61.2%, 44.8%, and 59.0%, respectively. Under the full load condition, compared with the no control mode, the mean, maximum, and RMS values of the horizontal vibration acceleration of the car system are reduced by 42.1%, 36.5%, and 40.4%, respectively with the traditional PID control. Meanwhile, under the action of the H2/ generalized H2 control mode, these values are reduced by50.0%, 46.1%, and 51.1%, respectively. From comparing the acceleration evaluation index under the three load conditions, we see that the H2/generalized H2 control mode can effectively suppress the horizontal vibration of the car system, and the suppression effect is better than the traditional PID control.

Table 2. Mean, Maximum and RMS value of horizontal vibration acceleration of the car system.

	Control Mode	Mean	Maximum	RMS
No Load	H_2/ generalized H_2 Control	0.0025	0.0078	0.0032
	Traditional PID Control	0.0030	0. 0151	0.0039
	NO Control	0.0051	0.0222	0.0067
Medium Load	H_2/ generalized H_2 Control	0.0019	0.0079	0.0025
	Traditional PID Control	0.0025	0.0097	0.0031
	NO Control	0.0049	0.0143	0.0061
Full Load	H_2/ generalized H_2 Control	0.0019	0.0062	0.0023
	Traditional PID Control	0.0022	0.0073	0.0028
	NO Control	0.0038	0. 0115	0.0047

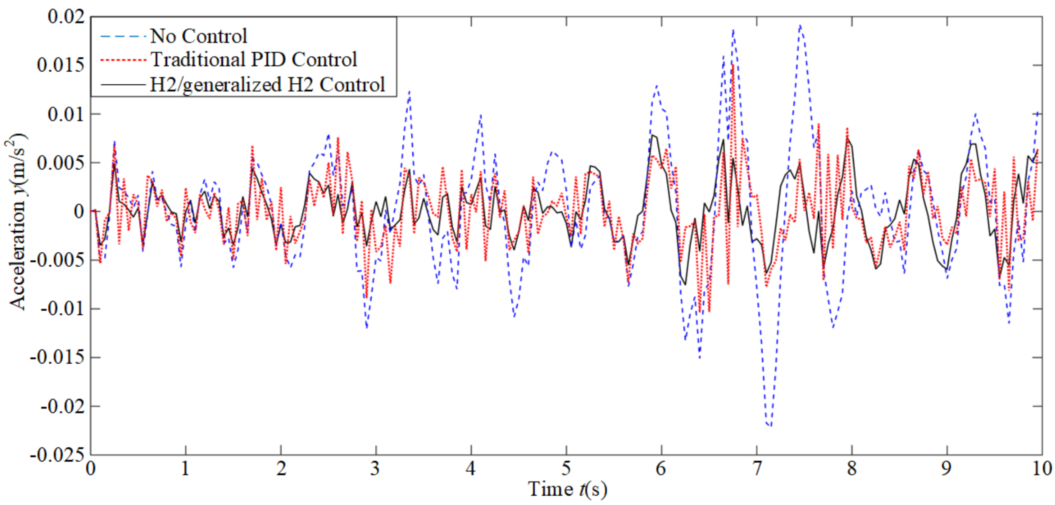

Fig. 6. Horizontal vibration acceleration response of elevator car system under no load condition.

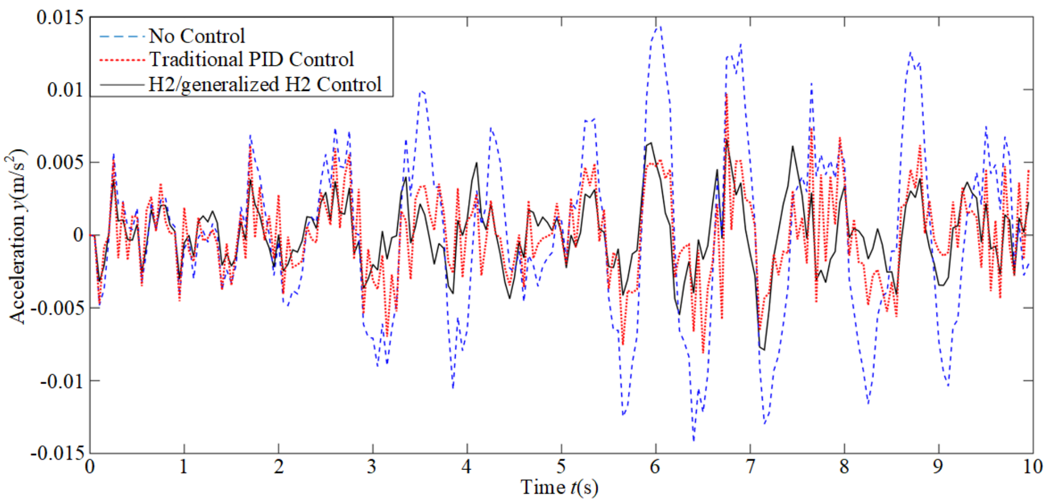

Fig. 7. Horizontal vibration acceleration response of elevator car system under medium load condition.

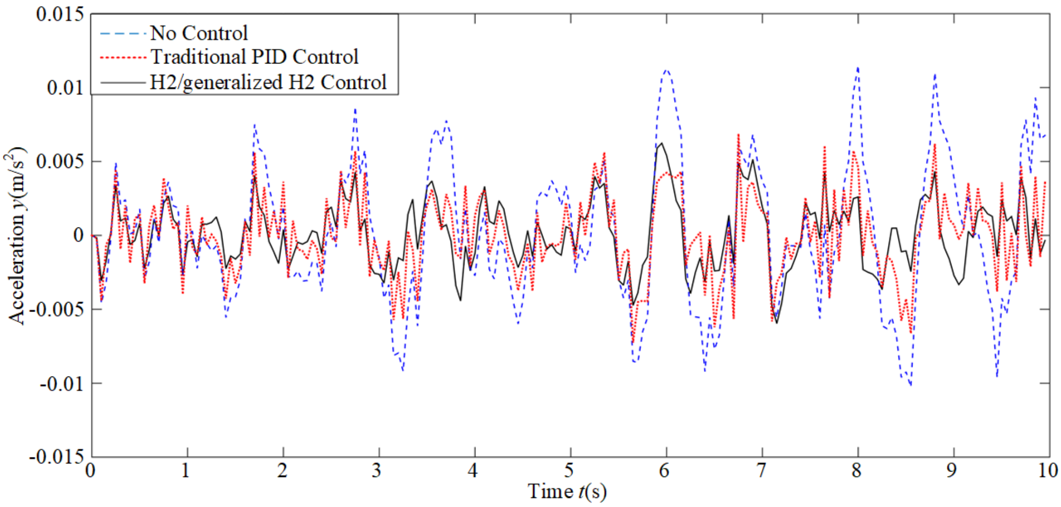

Fig. 8. Horizontal vibration acceleration response of elevator car system under full load condition.

4.3 Constraint output Simulation experiment

Due to the influence of manufacturing, installation and other factors, there will be step type and other pulse excitation at the rail joint (Fu et al., 2003). When the high speed elevator passes through the steps of the rail joint, the impact vibration is generated, which causes the constraint problem described in Section 1.2. Therefore, this section uses the pulse signal to simulate the guide rail impact excitation as the input of the car system, analyzes the suppression effect of the H2/generalized H2 control method and the traditional PID control method on the horizontal vibration acceleration response of the car system, and verifies the effectiveness of the control strategy proposed further in this paper under the action of the constraint conditions. Figure 9 is the rail pulse excitation, figure 10 is the acceleration response of three control modes under the action of pulse excitation, figures 11-14 are the control force output of actuator 1-4, respectively, and figures 15-18 are the stroke output of actuator 1-4, respectively.

In the analysis of the horizontal vibration acceleration response curve of elevator under pulse excitation, it can be seen from Fig. 10 that the traditional PID control method and H2 generalized H2 control method can better suppress the vibration of the car than the uncontrolled control method. Comparing the change trend of acceleration curve in the Fig. 10, it can be seen that when the car system encounters pulse excitation, the car acceleration response curve under the controlling of the traditional PID control method presents irregular jagged shape, which means that under the controlling of the traditional PID control method, the vibration suppression of the car system will aggravate the oscillation of the car system. Under the action of H2 generalized H2 control method, the horizontal vibration acceleration curve of the car system is shown as a smooth curve, and the oscillation is small. In the aspect of control force constraint and stroke constraint, compared with figs. 11-18, it can be seen that under the controlling of the traditional PID control method, the actuator 2 has an output saturation problem, and the optimal control cannot be achieved. Moreover, in the process of reducing the vibration of the car system in the traditional PID control method, when the pulse excitation is input, there is an abrupt increase of vibration acceleration of car system, as shown in the curve peak and trough position in Fig.10, which reduces the ride comfort of the elevator. Compared with the traditional PID control, under the action of the H2/generalized H2 control method, the four actuators satisfy the control force constraint and the stroke displacement constraint, and the curve peak and trough position in Fig. 8 has better vibration reduction effect on the car. In terms of response speed, as you know from Figure 10, in the moment of pulse excitation input, the traditional PID control method has hysteresis for the suppression of horizontal vibration of the car system, while the H2/generalized H2 control method responds instantaneously, effectively reduces the horizontal vibration of the car system and improves the ride comfort.

In summary, under the action of pulse impact excitation, H2/generalized H2 control method can effectively suppress the horizontal vibration of the car system under the condition of satisfying the constraints problem of the actuator, and the inhibition effect is better than the traditional PID control method.

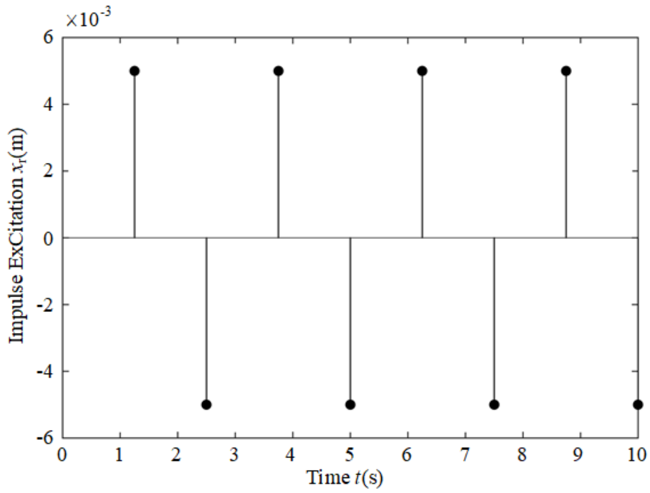

Fig. 9. Pulse excitation at guide rail joint.

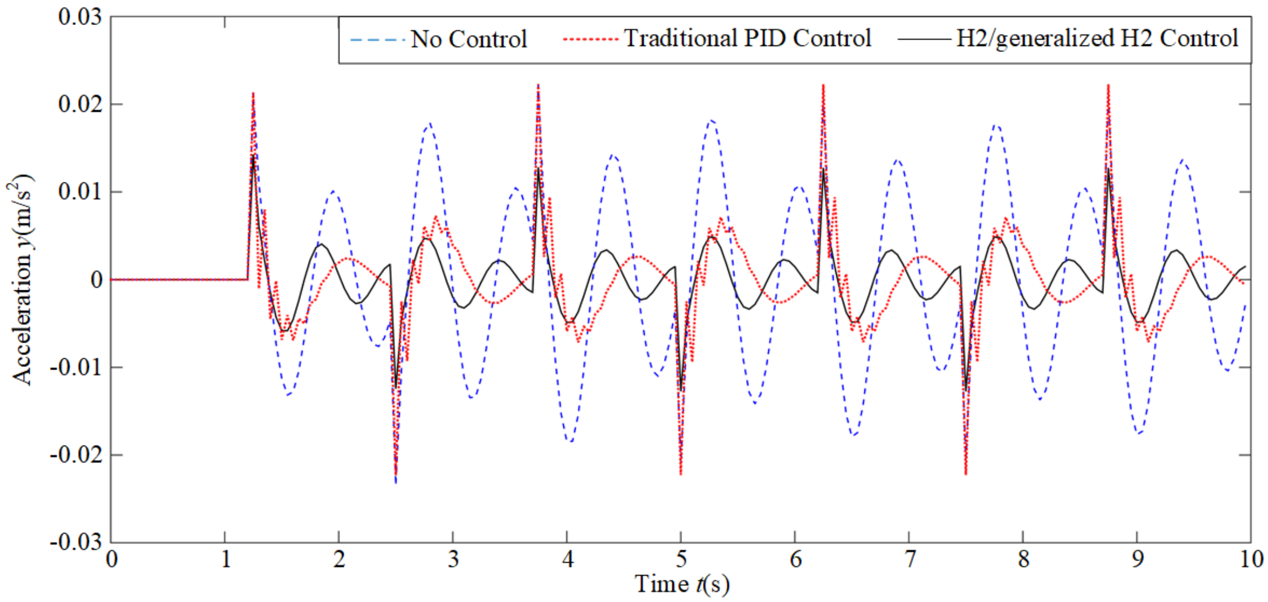

Fig. 10. Horizontal vibration acceleration response of elevator car system under pulse excitation.

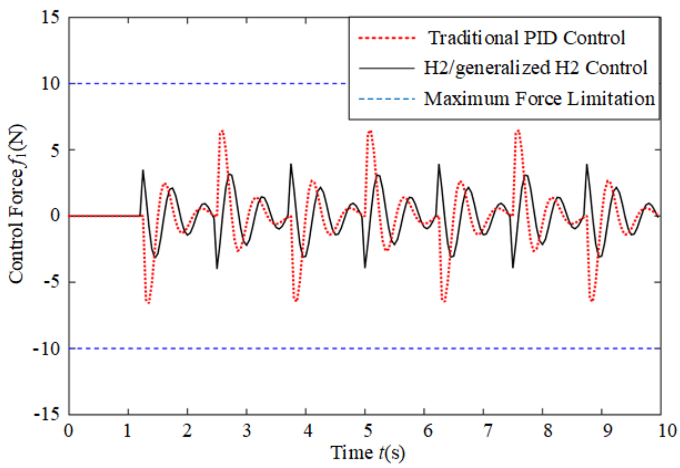

Fig. 11. The control force of actuator 1.

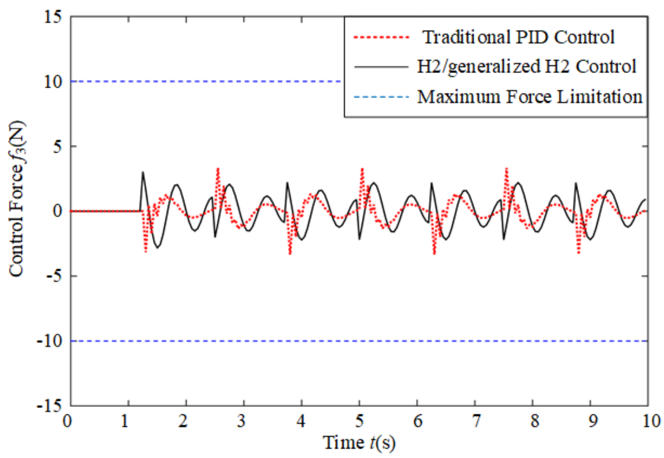

Fig. 13. The control force of actuator 3.

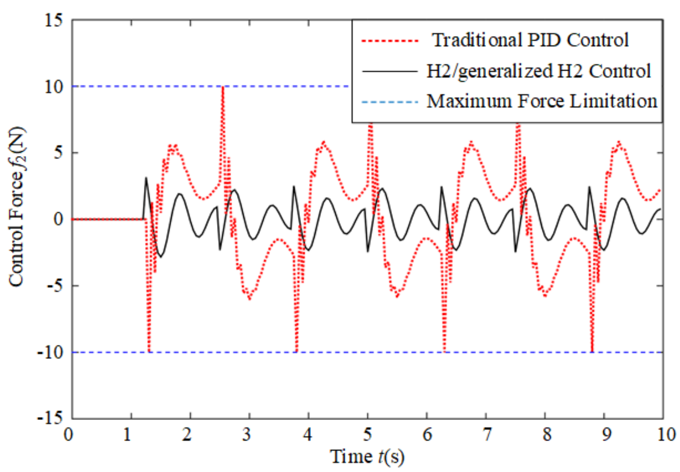

Fig. 12. The control force of actuator 2.

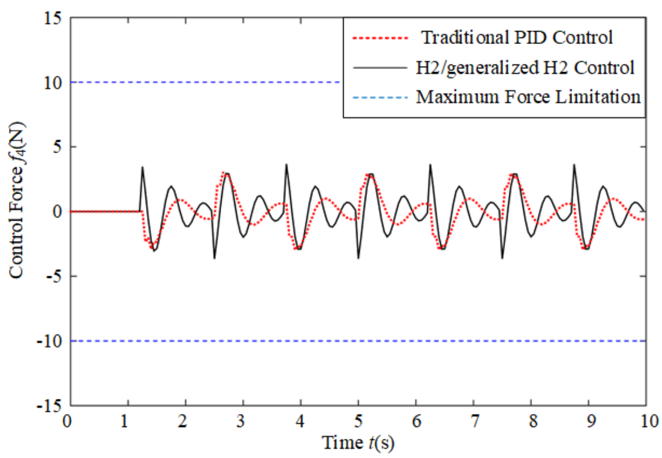

Fig. 14. The control force of actuator 4.

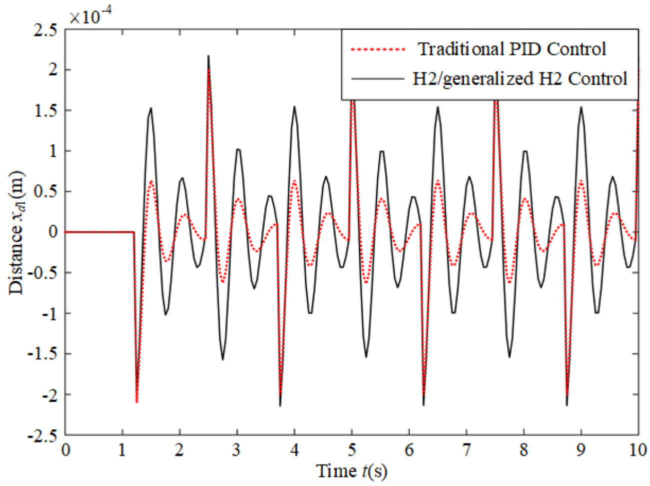

Fig. 15. The stroke of actuator 1.

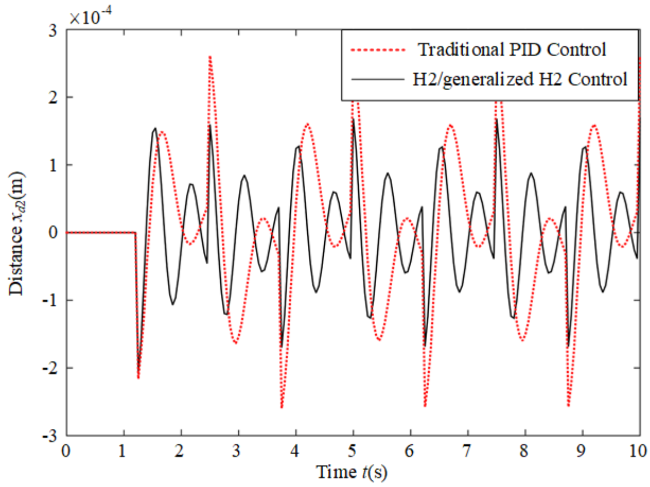

Fig. 16. The stroke of actuator 2.

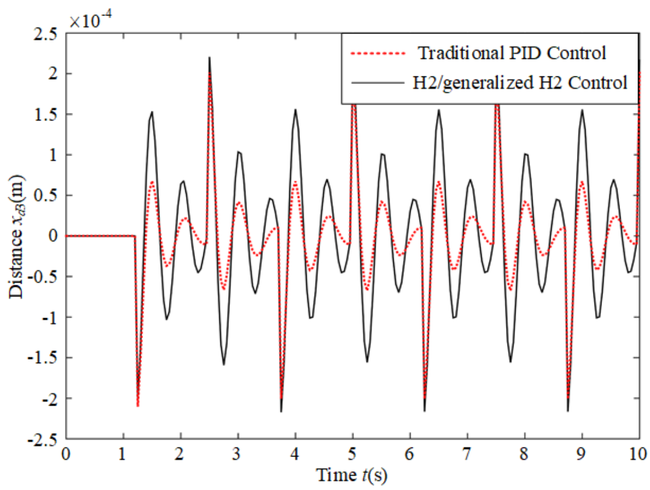

Fig. 17. The stroke of actuator 3.

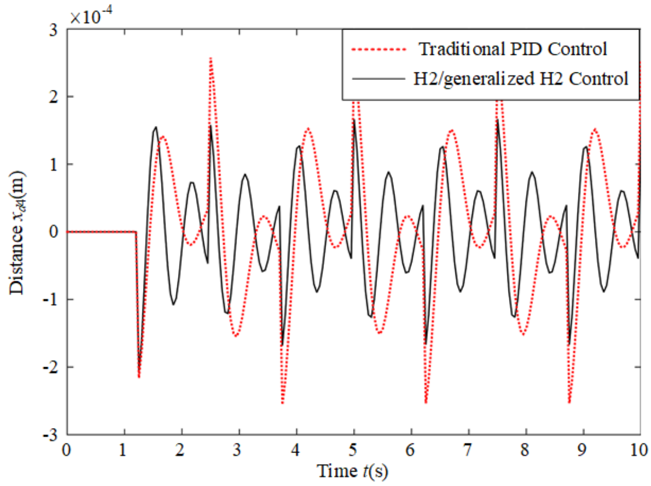

Fig. 18. The stroke of actuator 4.

5. CONCLUSIONS

(1) Aiming at the horizontal vibration model of high speed elevator car system with active control, on the basis of considering the actuator stroke and power constraint problem of active guide shoes, the calculation formula of actuator stroke and force of active guide shoes is derived, which provides the data basis for the controller design considering constraints.

(2) The control problem of horizontal vibration of car system is reduced to two parts, performance constraint and output constraint, taking H2 norm as performance index and generalized H2 norm as constraint index, a hybrid control algorithm for interference suppression of constrained system is proposed. Under the multi-objective LMI constraint, the complex design problem of the system controller is simplified to solve the optimization problem of a set of LMI as constraints, and the design of the active guide shoe controller is completed.

(3) By considering the output constraint, the car system from three kinds of operating conditions of elevator, two kinds of guide rail excitation and three control modes are simulated and analysed in this paper, and the results show that the H2/generalized H2 hybrid control strategy proposed in this paper can effectively suppress the horizontal vibration of high-speed elevators under the condition of meeting the hard constraints of active guide shoes, and the suppression effect is better than traditional PID control.

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