Fractional-Order Distributed Kalman Filter in Virtualized Sensor Networks by Diffusion Strategies

Mahmoud Ghanbari Firouzabadi[1], Mohammad Ali Nekoui[1], Ehsan Mohammadzadeh[2], Amir Hooshang Mazinan[1]

[1] Department of Control Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran;
(Tel: +9821-33722831; e-mail: Mahmoudghanbari.fa@ gmail.com; manekoui@eetd.kntu.ac.ir; mazinan@azad.ac.ir)
[2]Department of Electrical Engineering, Khorasan Institute of Higher Education, Mashhad, Iran (e-mail: emzadeh@gmail.com)

Abstract: Wireless sensor networks are applied in a broad range of applications such as medical, industrial and military fields.Therefore, a precise Kalman filter mechanism is needed for fixed configuration of topologies in sensor networks in order to ensure accuracy and precision of sensor measurements.On the other hand, fractional calculus as a generalization of integer order operators enables modeling of physical systems with high accuracy.Hence, a new fractional-order distributed Kalman filter algorithm is presented in this study to estimate the states in sensor networks.Therefore, as a generalization of filtering algorithm a fractional order Kalman filter algorithm is proposed.For this purpose, fractional-order distributed Kalman filter algorithms and fractional diffusion Kalman filters are formulated and their performance is evaluated based on mean squares for algorithm feasibility analysis. Simulations show that performance of the proposed algorithm is improved in terms of accuracy and efficiency compared to previous methods such as conventional fractional Kalman filter.

Keywords: Fractional-Order Calculations, Sensor Networks, Distributed Kalman Filter, Diffusion Kalman Filter.

1. INTRODUCTION

Recently, our lives have been transformed via increased data analysis with applications such as smart city requiring multi-path data transfer (Xu et al., 2015b) target tracking (Bhuiyan et al., 2015; Kuo et al., 2018) environmental monitoring (Kar et al., 2018) video-on-demand services (Xu et al., 2015a) and distributed data storage. In this regard, Wireless Sensor Networks (WSNs) have attracted the attention of academic and industrial researchers as a practical and key technology (Boselin Prabhu et al., 2016; Biswas et al., 2018; Jagadesh et al., 2018). Given the different applications of wireless sensor networks, each node can consist of different components based on its expected task.

However, usually each node consists of a number of core components, including central processing unit, radio transmitter-receiver, power supply and one or more sensors that collect data from the environment. Wireless sensor networks may be divided into homogeneous and heterogeneous groups. In the homogeneous mode, all nodes have similar properties in terms of size, power, spectrum, connectivity, etc and the heterogeneous mode includes different types of sensor nodes. Traditional WSNs are unable to share infrastructures and reach a general agreement on large-scale systems due to the natural characteristics of heterogeneous wireless networks. Fortunately, the rapid development of virtual sensor networks (VSNs) has provided a new opportunity to solve the problems of traditional wireless sensor networks (Liang and Yu, 2015; Khan et al., 2016; Nkomo et al., 2018; Luo et al., 2018). A prominent feature of VSNs is their ability to provide a suitable framework for coordinating heterogeneous WSNs in a shared physical structure. This feature makes VSNs scalable and flexible. Thus, these networks offer a promising solution for monitoring large-scale systems. Despite the unique opportunities offered by VSNs, it is still vital to ensure accurate and precise sampling since mobile sensors in VSNs are highly dynamic, fully distributed and have no central controllers. Therefore, fractional-order distributed Kalman filter algorithms which consist of a combination of distributed algorithms and Kalman algorithms based on fractional-order computation are proposed in this research study to solve this problem.

In distributed algorithms, a set of nodes can accurately estimate target state through a collaboration method. These nodes can be personal computers, laptops, cell phones, sensors, or triggers (Abadi and Shafiee, 2018).

Distributed estimation algorithms with desirable features such as robustness, ease of development and low consumption are applied in various fields (Azpicueta-Ruiz et al., 2017; Al-Sayed et al., 2018).

Kalman filter algorithms are among the most popular methods for estimating dynamic system states through measurement. Kalman filter must be implemented with little computation and memory space by recursive algorithms for use in real-time systems. This type of filter was first introduced in the 1960s and it has since been widely used in many fields such as navigation, signal processing, control systems and information integration (Hong et al., 2018).

References (Li et al., 2015; Wu et al., 2018) suggest distributed Kalman filters where agents are related with each other using the probable protocol (Wu et al., 2015). A communication cycle is established between a pair of connected sensors at each measurement instance in a probabilistic Kalman filter and they change their current state estimates. This change in mode estimation results in presentation of the covariance matrix of current error by the Riccati equation of the local Kalman filter. Although probabilistic filters require very little bandwidth, their mean square deviations (MSDs) are higher and their convergence rates are lower. This is the reason they are less popular and have a relatively low efficiency.

As a strategy of distributed Kalman filter algorithm, the distributed Kalman filter algorithm provides better performance in state estimation using data diffusion by creating a sequence of Kalman repetitions and a data set. Diffusion Kalman filter is introduced which includes two incremental and intermediate update phases introduced in (Cattivelli et al., 2008; Cattivelli et al., 2010). At the incremental update stage, each node receives observations from its neighboring nodes and combines these observations to update the existing estimate and obtain an intermediate value. At the diffusion update stage, each node aggregates the average estimates of its neighbors generated by the last step in order to update its estimate. It should be noted that all nodes perform these two steps simultaneously. Therefore, this algorithm has excellent performance in tracking a mobile target and good convergence performance (Yang et al., 2016).

Fractional computing has been the focus of interest for many researchers as an extended model of derivatives and integer differentials for use in practical applications in the last decade (Sierociuk and Dzieliński, 2006; Stanisławski et al., 2015). It has been shown over the years that the use of this emerging tool in description and modeling of most real systems increases accuracy of modeling and reduces model order (Kailath et al., 2000; Sayed, 2003). In addition, some systems, such as the lithium-ion battery model or the Bertrand super capacitor, cannot be modeled with proper-order derivatives. However, they can only be modeled by using fractional derivatives. Therefore, fractional order Kalman filter algorithm was proposed for mode estimation in linear fractional systems due to the weak performance of Kalman filter algorithms in estimating the state of such systems (Stanisławski et al., 2015; Xue et al., 2018).

Thereby, two fractional-order distributed Kalman filter algorithms and fractional-order diffusion Kalman filter algorithms with fixed topologies are proposed in this study to solve them. However, the mean performance of the algorithm must be analyzed under reasonable assumptions in order to reasonably analyze the algorithm. Besides proposing a new algorithm, performance of diffusion strategies in different scenarios is investigated in this study.

Therefore, performance of steady-state mean and mean squares of the order Fractional Kalman filter algorithm is proposed in this paper.

In order to compare performance of the proposed algorithm with existing algorithms, position tracking model of a projectile is presented. Simulation results show significant improvement in the mean squared error of the proposed fractional order Kalman filter algorithm compared to the common order fractional Kalman filter algorithm without cooperation.

The paper is organized as follows. The linear fractional-order Kalman filter algorithm is presented in Section 2. The fractional-order distributed Kalman filter algorithm is reviewed in Section 3. In Section 4, the fractional-order diffusion Kalman filter algorithm is presented. Analysis of fractional-order distributed Kalman filter algorithm based on mean and performance of square mean of estimation errors is presented in Section 5. Section 6 presents the simulation results. Conclusions and suggestions for future studies are described in Section 7.

2. LINEAR FRACTIONAL-ORDER OF KALMAN FILTER ALGORITHM

Fractional dynamics system, local observations, and modeling assumptions are described in this section. Then, the linear fractional order Kalman filter algorithm (Sierociuk and Dzieliński, 2006) is presented.

Consider tracking a moving target in WSNs. We use x_i^b to define the state of this object with the characteristic b at time i, b∈M, M is one of the properties (such as position coordinates, speed, or direction). Then, a discrete control process of the system will be introduced in order to describe this. The system can be derived from a linear stochastic difference equation with M-stack variables in a state-position vector $x_k = [x_k^1, \dots, x_k^M]^T$ based on the Grunwald–Letnikov fractional derivative (Stanisławski et al., 2015; Xue et al., 2018) as follows:

$$\begin{cases} \Delta^{\Upsilon} \mathrm{x}_{k+1} = \mathrm{F}_k \mathrm{x}_k + \mathrm{G}_k \mathrm{u}_k + \mathrm{w}_k \\ \mathrm{x}_{k+1} = \Delta^{\Upsilon} \mathrm{x}_{k+1} - \sum_{j=1}^{k+1} (-1)^j \Upsilon_j \mathrm{x}_{k+1-j} \end{cases} \tag{1}$$

$$\mathrm{y}_k = \mathrm{H}_k \mathrm{x}_k + \mathrm{v}_k \tag{2}$$

Where

$$\begin{cases} \Upsilon_k = \mathrm{diag}\left[\binom{n_1}{k} \dots \binom{n_M}{k}\right] \\ \Delta^{\Upsilon} \mathrm{x}_{k+1} = \begin{bmatrix} \Delta^{n_1} \mathrm{x}_{1,k+1} \\ \vdots \\ \Delta^{n_M} \mathrm{x}_{M,k+1} \end{bmatrix} \end{cases} \tag{3}$$

Moreover, $\Delta^{\Upsilon} x_k$is the fractional order difference of γ for the state vector of $x_k \in \mathbb{C}^M$.In addition, k is the sampling time, $x_k \in \mathbb{C}^M$is the system vector, $y_k \in \mathbb{C}^q$is a measurement vector, $F_k \in \mathbb{C}^{M \times M}$ is a sparse localized model matrix, $u_k = [u_k^1, \dots, u_k^M] \in \mathbb{C}^M$ is the state noise vector and $G_k \in$

$\mathbb{C}^{M\times M} is$ is the noise matrix of the state, $n_1, \dots, n_M$ is the order of system equations, M is the number of system equations and q is the number of system outputs. The input signal w_k is usually equal to zero in system 1.

Assuming that noise signals u_k and v_k are white and independent, the corresponding covariance matrices are as follows (Cattivelli et al., 2010):

$$E\begin{bmatrix}u_k\\v_k\end{bmatrix}\begin{bmatrix}u_k\\v_k\end{bmatrix}^* = \begin{bmatrix}Q_k & 0\\0 & R_k\end{bmatrix}\delta_{kl} \tag{4}$$

where * indicates mixed conjugate transposition and δ_{kl} is the Kronecker delta function. The initial mode vector x_0 is measured noise and uncorrelated state with an average of zero and covariance matrix $\Pi_0 > 0$. Q_k and R_k are also diametric matrices with size M and q .

Linear fractional order Kalman filter algorithm: Based on the above analysis, $\hat{x}_{k|l}$ estimates the minimum error of linear mean square x_k with respect to the given observations before time l and $P_{k|l}$. The covariance matrix of the estimated error is $\tilde{x}_{k|l} \triangleq x_k - \hat{x}_{k|l}$. Therefore, the fractional order Kalman filter algorithm, considering the initial values of the estimate of state $\hat{x}_{0|-1} = 0$, and the covariance of the initial state estimate error $P_{0|-1} = \Pi_0$ will be as the following equations (5) and (6) (Sierociuk and Dzieliński, 2006):

1. Measure update

$$\begin{cases} K_k = P_{k|k-1}H_k^*\left(R_k + H_kP_{k|k-1}H_k^*\right)^{-1} \\ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(y_k - H_k\hat{x}_{k|k-1}\right) \\ P_{k|k} = P_{k|k-1} - K_kH_kP_{k|k-1} \end{cases} \tag{5}$$

2. Time update

$$\begin{cases} \Delta^{\Upsilon}\hat{x}_{k+1|k} = F_k\hat{x}_{k|k} \\ \hat{x}_{k+1|k} = \Delta^{\Upsilon}\hat{x}_{k+1|k} - \sum_{j=1}^{k+1}(-1)^j\Upsilon_j\,\hat{x}_{k+1-j} \\ P_{k+1|k} = (F_k + \Upsilon_1)P_{k|k}(F_k + \Upsilon_1)^* \\ +G_kQ_kG_k^* + \sum_{j=2}^{k}\Upsilon_jP_{k-j}\Upsilon_j^T \end{cases} \tag{6}$$

where K_k is a fraction of the Kalman filter.

3. FRACTIONAL-ORDER DISTRIBUTED OF KALMAN FILTER ALGORITHM

Consider a set of N nodes (or sensors) distributed in a region. If two nodes can communicate directly, they nodes are connected. Thus, each node is always connected to itself. Set of nodes connected to node i are called the i-th neighbors and it is represented by $\mathcal{N}_i$ $(i \in \mathcal{N}_i)$. Therefore, the adjacency matrix Ω with elements $\Omega_{i,l}$ is defined as follows:

$$\Omega = \{\Omega_{i,l}\} = \begin{cases}1, & l \in \mathcal{N}_i\\ 0, & otherwise\end{cases} \tag{7}$$

Therefore, the elements on the main diameter Ω indicate that the sensor i is in constant contact with itself.

Now, assume that the output of system 1 is observed by N sensors, so that each sensor only observes a limited number of properties. This process is schematically illustrated in Fig. 1. If B_i represents the number of properties observed by the i[th]-sensor and M is the number of system equations, then the observations made by the i[th]-sensor at moment k can be represented by the linear model (8):

$$y_{i,k} = H_{i,k}x_k + v_{i,k}\,, B_i << M, i = 1,..,k \tag{8}$$

where $y_{i,k} \in \mathbb{C}^q$ represents measurements made by the sensor i at moment k, $H_{i,k} \in \mathbb{C}^{B_i\times M}$ is the local observation matrix and $v_{i,k} \in \mathbb{C}^{B_i}$, is the noise of local observations to reflect measurement inaccuracies with respect to sensor accuracy and other inevitable limitations.

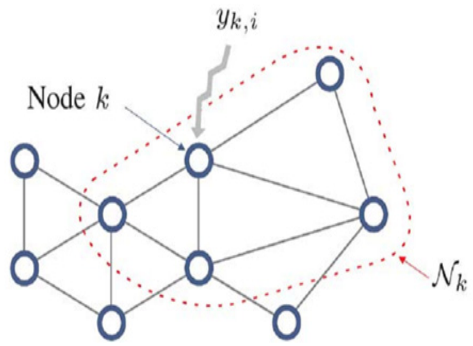

Fig. 1. System output measurement $y_{i,k}$ by node i at moment k.

The model of global observation is obtained by collecting observations as follows:

$$y_k = \begin{bmatrix}y_{1,k}\\ \vdots \\ y_{N,k}\end{bmatrix},\ H_k = \begin{bmatrix}H_{1,k}\\ \vdots \\ H_{N,k}\end{bmatrix}, v_k = \begin{bmatrix}v_{1,k}\\ \vdots \\ v_{N,k}\end{bmatrix} \tag{9}$$

Therefore, the general observation of matrix $y_k \in \mathbb{C}^{\sum_{i=1}^{N} B_i}$ is given as follows:

$$y_k = H_kx_k + v_k \tag{10}$$

Assume that measurement noise $v_{i,k}$ is unconnected, i.e.:

$$E\left[v_{i,k}v_{j,l}^*\right] = R_{i,k}\delta_{j,i}\delta_{l,k} \tag{11}$$

Where $R_i, k > 0$ for all k, i.

The goal of implementing a fractional-order distributed Kalman filter is to estimate the uncertain x_k state for each node i of the network. It should be noted that in this network, nodes can only share their data with their neighbors $\{l \in \mathcal{N}_i\}$.

The main challenge is to ensure that an accurate estimate of the system state is obtained such that if each node has access to all measurements across the network nodes, accuracy of the overall state estimate does not increase. In order to address this issue, the Kalman filter algorithm and fractional-order distributed Kalman filter algorithms were used and presented in Table 1.

In the above algorithm, the term "incremental updating" is used instead of the update term. This is due to the fact that at this stage, the optimal local estimate at node i is produced iteratively by incremental measurements at its neighboring nodes $\{y_{l,k}, l \in \mathcal{N}_i\}$.

Table 1. Fractional-order distributed Kalman filter algorithm.

Consider the space-state model of the order-fraction (1):
For each node i we have: $\hat{x}_{i,0|-1} = E(x_0), P_{i,0|-1} = \Pi_0$

In each sampling period k, repeat the following two steps:

Step 1: Incremental update

$$P_{i,k|k}^{-1} = P_{i,k|k-1}^{-1} + \sum_{l\in\mathcal{N}_i} H_{l,k}^{*} R_{l,k}^{-1} H_{l,k} \tag{12}$$

$$\phi_{i,k|k} = \hat{x}_{i,k|k-1} + P_{i,k|k} \sum_{l\in\mathcal{N}_i} H_{l,k}^{*} R_{l,k}^{-1} \left(y_{l,k} - H_{l,k}\hat{x}_{i,k|k-1}\right) \tag{13}$$

Step 2: Update time

$$\hat{x}_{i,k|k} = \phi_{i,k|k}$$
$$\Delta^{\Upsilon}\hat{x}_{i,k+1|k} = F_k \hat{x}_{i,k|k}$$
$$\hat{x}_{i,k+1|k} = \Delta^{\Upsilon}\hat{x}_{i,k+1|k} - \sum_{j=1}^{k+1}(-1)^j \Upsilon_j\, \hat{x}_{i,k+1-j}$$
$$P_{i,k+1|k} = (F_k + \Upsilon_1)P_{i,k|k}(F_k + \Upsilon_1)^* + G_k Q_k G_k^* + + \sum_{j=2}^{k} \Upsilon_j P_{i,k-j} \Upsilon_j^T \tag{14}$$

Assume that a node i has access to measurements of its neighbors $\mathcal{N}_i$. The local estimate for node i can be calculated from Eq. (5) by performing multiple measurement updates. Each update is calculated for each neighbor i. These repetitions are shown in Eq. (15):

- In the i^{th} node at time k:

$$\phi_{i,k} \leftarrow \hat{x}_{i,k|k-1}$$
$$P_{i,k} \leftarrow P_{i,k|k-1}$$

Where $\phi_{i,k}$ is the local estimate.

- Then, it is repeated for each $l \in \mathcal{N}_i$.

$$\begin{cases} K_k \leftarrow P_{i,k}H_{l,k}^{*}\left(R_{l,k} + H_{l,k}P_{i,k}H_{l,k}^{*}\right)^{-1} \\ \phi_{i,k} \leftarrow \phi_{i,k} + K_k\left(y_{l,k} - H_{l,k}\phi_{i,k}\right) \\ P_{i,k} \leftarrow P_{i,k} - K_k H_{l,k} P_{i,k} \end{cases} \tag{15}$$

- Running time update

$$\begin{cases} \hat{x}_{i,k|k} \leftarrow \phi_{i,k|k} \\ P_{i,k|k} \leftarrow P_{i,k} \\ \Delta^{\Upsilon}\hat{x}_{i,k+1|k} = F_k\hat{x}_{i,k|k} \\ \hat{x}_{i,k+1|k} = \Delta^{\Upsilon}\hat{x}_{i,k+1|k} - \sum_{j=1}^{k+1}(-1)^j \Upsilon_j\, \hat{x}_{i,k+1-j} \\ P_{i,k+1|k} = (F_k + \Upsilon_1)P_{i,k|k}(F_k + \Upsilon_1)^* \\ \quad +G_k Q_k G_k^* + \sum_{j=2}^{k} \Upsilon_j P_{i,k-j} \Upsilon_j^T \end{cases} \tag{16}$$

Now we show that the incremental update of the fractional-order distributed Kalman filter algorithm can be calculated using equations (12) and (13).

Assume $\{i_1, \dots, i_{n_i}\}$ shows the set of neighbors of node i and $\phi_{i,k}^{(m)}$ and $P_{i,k}^{(m)}$ represent the values of $\phi_{i,k}$ and $P_{i,k}$ after the m[th] repetition of the following equations:

$$\begin{aligned} &\phi_{i,k}^{(0)} = \hat{x}_{i,k|k-1} \\ &P_{i,k}^{(0)} = P_{i,k|k-1} \\ &for\ m = 1\ to\ n_i,\ repeat: \\ &K_k^{(m)} = P_{i,k}^{(m-1)}H_{i_m,k}^{*}\left(R_{i_m,k} + H_{i_m,k}P_{i,k}^{(m-1)}H_{i_m,k}^{*}\right)^{-1} \\ &\phi_{i,k}^{(m)} = \phi_{i,k}^{(m-1)} + K_k^{(m)}\left(y_{i_m,k} - H_{i_m,k}\phi_{i,k}^{(m-1)}\right) \\ &P_{i,k}^{(m)} = P_{i,k}^{(m-1)} - K_k^{(m)}H_{i_m,k}P_{i,k}^{(m-1)} \\ &end \\ &\hat{x}_{i,k|k} = \phi_{i,k|k}^{(n_k)} \\ &P_{i,k|k} = P_{i,k}^{n_{(k)}} \end{aligned} \tag{17}$$

Now, by applying the inverse matrix lemma (Tylavsky and Sohie, 1986) to Eq. (12), we have the covariance matrix of the posterior estimation error after m repetitions:

$$\left(P_{i,k}^{(m)}\right)^{-1} = \left(P_{i,k}^{(m-1)}\right)^{-1} + H_{i_m,k}^{*}R_{i_m,k}^{-1}H_{i_m,k} \tag{18}$$

This may be defined for all m since $P_{i,k}^{(0)}$ is reversible. By repeating (18) an update for $P_{i,k|k}$, is obtained as follows:

$$P_{i,k|k}^{-1} = P_{i,k|k-1}^{-1} + \sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}H_{l,k} \tag{19}$$

Moreover, if $P_{i,k}^{(m-1)}$ is invariant, then we have:

$$\begin{aligned} &\phi_{i,k}^{(m)} = \\ &[P_{i,k}^{(m-1)} - P_{i,k}^{(m-1)}H_{i_m,k}^{*}\left(R_{i_m,k} + H_{i_m,k}P_{i,k}^{(m-1)}H_{i_m,k}^{*}\right)^{-1}] \times \\ &\left(P_{i,k}^{(m-1)}\right)^{-1}\phi_{i,k}^{(m-1)} + \\ &P_{i,k|k-1}^{(m-1)}H_{i_m,k}^{*}\left(R_{i_m,k} + H_{i_m,k}P_{i,k}^{(m-1)}H_{i_m,k}^{*}\right)^{-1}y_{i_m,k} = \\ &P_{i,k}^{(m)}\left(P_{i,k}^{(m-1)}\right)^{-1}\phi_{i,k}^{(m-1)} + P_{i,k|k-1}^{(m-1)}H_{i_m,k}^{*}\left(R_{i_m,k} + \right. \\ &\left. H_{i_m,k}P_{i,k}^{(m-1)}H_{i_m,k}^{*}\right)^{-1}y_{i_m,k} \end{aligned} \tag{20}$$

Eq. (20) can be rewritten by using matrix inversion lemma as follows:

$$\begin{aligned} &P_{i,k}^{(m-1)}H_{i_m,k}^{*}\left(R_{i_m,k} + H_{i_m,k}P_{i,k}^{(m-1)}H_{i_m,k}^{*}\right)^{-1} = \\ &P_{i,k}^{(m)}H_{i_m,k}^{*}R_{i_m,k}^{-1} \end{aligned} \tag{21}$$

$$\left(P_{i,k}^{(m)}\right)^{-1}\phi_{i,k}^{(m)} = \left(P_{i,k}^{(m-1)}\right)^{-1}\phi_{i,k}^{(m-1)} + H_{i_m,k}^{*}R_{i_m,k}^{-1}y_{i_m,k} \tag{22}$$

By iterating Eq. (22), we obtain (23):

$$\left(P_{i,k|k}\right)^{-1}\hat{x}_{i,k|k} = \left(P_{i,k|k-1}\right)^{-1}\hat{x}_{i,k|k-1} + \sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}y_{l,k} \tag{23}$$

Using Eq. (19) and multiplying both sides of Eq. (23) by $P_{i,k|k}$, the measurement update for $\hat{x}_{i,k|k}$ is obtained as follows:

$$\hat{x}_{i,k|k} = \hat{x}_{i,k|k-1} + P_{i,k|k}\sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}\left(y_{l,k} - H_{l,k}\hat{x}_{i,k|k-1}\right) \tag{24}$$

The local estimate is computed in the incremental updating stage of fractional order distributed Kalman filter algorithm according to Eq. (24). Moreover, the relationships between update time of this algorithm are the same as those mentioned in the regular update of the ordinary Kalman filter algorithm

(Sierociuk and Dzieliński, 2006). Therefore, the Kalman filter fractional-order algorithm is formulated and proved.

4. THE FRACTIONAL-ORDER DIFFUSION KALMAN FILTER ALGORITHM

It can be seen in the fractional-order distributed Kalman filter algorithm that the incremental update step in each node i ends with an optimal local estimate of $\phi_{i,k|k}$ after combining all measurements of its neighbors $\mathcal{N}_i$ are completed. Therefore, this estimate is converted to an estimate of the new updated state $\hat{x}_{i,k|k}$ which is the variance matrix of error $P_{i,k|k}$. Now the problem of estimating $\hat{x}_{i,k|k}$ is defined such that in addition to terms $\phi_{i,k|k}$, local estimation terms $\{\phi_{l,k|k}\}$ in the neighborhood Ni also affect its calculation. This is meant to approximate the results of the local estimate $\hat{x}_{i,k|k}$ and global estimate $\hat{x}_{k|k}$ (estimate based on access to all data across the network). The relation between $\{\phi_{l,k|k}\}$ in the neighborhood of $\mathcal{N}_i$ and the general estimate $\hat{x}_{k|k}$ must be investigated first in order to achieve this objective.

Therefore, assume that the adjacency matrix Ω meet the requirements of the following Equation:

$$\sum_{i=1}^{N}[\Omega]_{l,i}\,\gamma_i = 1 \quad l = 1,\ldots,N \tag{25}$$

where γ_i are weights. For example, if Ω is reversible, then the vector of weights $\gamma = col\{\gamma_k\}$ is obtained as $\gamma = \Omega^{-1}1$ and condition (12) is always met. In general, any given matrix Ω may not satisfy condition (12). Fortunately, there is a solution to this problem without changing network topology. Each node is connected to itself according to the definition of the network. In this method, if Ω is not reversible, it could be reversed by appropriately changing the values of some elements of the original diameter Ω to zero (Cattivelli et al., 2010). Therefore, condition (12) would be met without affecting network topology.

However, assumption (12) is only used to motivate diffusion updates as mentioned before while it is not necessary for diffusion algorithms. Three different optimal solution methods for the Kalman filter may be considered depending on the data access for each node.

The individual estimation of mode x_k in node i, represented by $\hat{x}_{i,k|k}^{ind}$, depends only on the observations $y_{i,j}$ in node i for j = 0, ..., k. The local estimation of the state of x_k in node i, represented by $\hat{x}_{i,k|k}^{loc}$, depends on observations $y_{l,j}$ for j = 0, ..., k on the whole neighborhood i, $where\ l \in \mathcal{N}_i$ is dependent. Finally, the global estimate of mode x_k represented by $\hat{x}_{k|k}$ depends on observations $y_{i,j}$ for j = 0, ..., k among all nodes i = 1, ..., N. Now, the relationship between local estimates of $\hat{x}_{i,k|k}^{loc}$ and their inclusive estimates is examined.

The covariance matrices for individual, local and learning estimation errors are represented by $P_{i,k|k}^{ind}$ $P_{i,k|k}^{loc}$ and $P_{k|k}$ respectively. Since measurement noise $v_{i,k}$ is assumed to be with zero mean in various unconnected nodes, it can be shown that individual and inclusive estimates are related as follows (Sayed, 2003):

$$\begin{cases} P_{k|k}^{-1}\hat{x}_{k|k} = \sum_{i=1}^{N}\left(P_{i,k|k}^{ind}\right)^{-1}\hat{x}_{i,k|k}^{ind} \\ P_{k|k}^{-1} = \sum_{i=1}^{N}\left(P_{i,k|k}^{ind}\right)^{-1} - (N-1)\Pi_k^{-1} \end{cases} \tag{26}$$

where Π_k is the covariance matrix x_k. According to the above, local estimates will be related to individual estimates:

$$\begin{cases} \left(P_{i,k|k}^{loc}\right)^{-1}\hat{x}_{i,k|k}^{loc} = \sum_{l=1}^{N}[A]_{l,i}\left(P_{l,k|k}^{ind}\right)^{-1}\hat{x}_{l,k|k}^{ind} \\ \left(P_{i,k|k}^{loc}\right)^{-1} = \sum_{l=1}^{N}[A]_{l,i}\left(P_{l,k|k}^{ind}\right)^{-1} - \left(\sum_{l=1}^{N}[A]_{l,i} - 1\right)\Pi_k^{-1} \end{cases} \tag{27}$$

Now consider a set of real weights $\gamma_i, i = 1, \ldots, N$ with the following combinations:

$$\sum_{i=1}^{N}\gamma_i\left(P_{i,k|k}^{loc}\right)^{-1}\hat{x}_{i,k|k}^{loc} = \sum_{l=1}^{N}\sum_{i=1}^{N}[A]_{l,i}\left(P_{l,k|k}^{ind}\right)^{-1}\hat{x}_{l,k|k}^{ind} \tag{28}$$

$$\sum_{i=1}^{N}\gamma_i\left(P_{i,k|k}^{loc}\right)^{-1} = \sum_{l=1}^{N}\sum_{i=1}^{N}\gamma_i[A]_{l,i}\left(P_{l,k|k}^{ind}\right)^{-1} - \left(\sum_{l=1}^{N}\sum_{i=1}^{N}\gamma_i[A]_{l,i} - \sum_{i=1}^{N}\gamma_i\right)\Pi_k^{-1} \tag{29}$$

If a set of weights is found such that the condition $\sum_{i=1}^{N}\gamma_i[\Omega]_{l,k} = 1$ holds true for all l s, then equations (28) and (29) are reduced to equations (30) and (31) after changing the symbol $\hat{x}_{i,k|k}^{loc} = \phi_{i,k}$,

$$P_{k|k}^{-1}\hat{x}_{k|k} = \sum_{i=1}^{N}\gamma_i P_{i,k|k}^{-1}\phi_{i,k} \tag{30}$$

$$P_{k|k}^{-1} = \sum_{i=1}^{N}\gamma_i P_{i,k|k}^{-1} - \left(\sum_{i=1}^{N}\gamma_i - 1\right)\Pi_k^{-1} \tag{31}$$

Therefore, local and inclusive estimates are related by the above relationship. Now, with a distributed Kalman filter algorithm in a distributed fraction, it is expected that the covariance matrix of individual errors $P_{i,k|k}$ will decrease over time. Therefore, the first term of the right-hand side of (31) dominates. Hence, the relation (32) can be approximated as follows:

$$P_{k|k}^{-1} \approx \sum_{i=1}^{N}\gamma_i P_{i,k|k}^{-1} \tag{32}$$

Thus:

$$\hat{x}_{k|k} \approx \left[\sum_{i=1}^{N}\gamma_i P_{i,k|k}^{-1}\right]^{-1}\sum_{i=1}^{N}\gamma_i P_{i,k|k}^{-1}\phi_{i,k} \tag{33}$$

Eq. (33) can be used to rewrite the average weighted value as follows:

$$\hat{x}_{k|k} \approx \sum_{i=1}^{N}\Gamma_i\phi_{i,k} \tag{34}$$

where the matrices meet the average of Eq. (35):

$$\sum_{i=1}^{N}\Gamma_i = I \tag{35}$$

The result of Eq. (34) provides an approximation method such that estimates $\{\phi_{l,k}\}$ in all nodes in the neighborhood k can be localized. For example, for two connected nodes l and

i, we can assign non-negative weight $c_{l,i}$ and if two nodes l and i are not connected, $c_{l,i} = 0$. Weights are chosen to satisfy Eq. (36):

$$\sum_{l\in\mathcal{N}_i} c_{l,i} = 1 \tag{36}$$

Therefore, according to Eq. (34), the relation $\hat{x}_{i,k|k} = \phi_{i,k|k}$ in the time update section in the Kalman filter of the distributed fraction is replaced by Eq. (37):

$$\hat{x}_{i,k|k} = \sum_{l\in\mathcal{N}_i} c_{l,i}\phi_{l,k} \tag{37}$$

Eq. (37) is called diffusion update.

At any particular moment, each sensor collects information viewed by its neighbors in order to make accurate estimates of target tracking. Nevertheless, considering the effect of observation noise, $v_{k,i}$, we consider a conversion factor for each neighbor in order to adjust the effect of noise. The matrix of the coefficient $C = [c_{l,k}]$ is defined by the following property:

$$\begin{gathered} c_{l,i} \geq 0,\ \ \sum_{l\in\mathcal{N}_i} c_{l,i} = 1 \\ c_{l,i} = 0,\ if\ l \notin \mathcal{N}_i\ \ for\ \ k = 1,\ldots,N \end{gathered} \tag{38}$$

Matrix C is called diffusion matrix because it manages the diffusion process and plays an important role in the steady state performance of the network. The elements in C represent the weights used by the diffusion algorithm to combine estimates from neighboring sensors and there are different rules for combining these values. In this paper, the metropolis rule for the expression of a set of non-negative coefficients $c_{l,i}$, is obtained as follows (Sayed, 2003):

$$\begin{gathered} c_{l,i} = \frac{1}{\max(n_l,n_i)},\ if\ l \neq k\ are\ linked \\ c_{l,i} = 0,\ if\ l,k\ not\ linked \\ c_{i,i} = 1 - \sum_{l\in\mathcal{N}_i/i} c_{l,i},\ if\ l = k \end{gathered} \tag{39}$$

where n_i represents the degree of node i.

In implementing diffusion, the nodes only cooperate with their immediate neighbors and propagate the information across the network through a sequence of Kalman repetitions and data aggregation. According to the above explanation, the fractional order Kalman filtering algorithm which consists of two stages of information exchange (local information exchange and global information exchange) is summarized in Table 2.

Table 2: Fractional-order diffusion Kalman filter algorithm

Consider the space-state model of the order-fraction (1):
For each node i we have: $\hat{x}_{i,0|-1} = E(x_0), P_{i,0|-1} = \Pi_0$
In each sampling period k, repeat the following two steps:
Step 1: Incremental update

$$\begin{cases} P_{i,k|k}^{-1} = P_{i,k|k-1}^{-1} + \sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}H_{l,k} \\ \phi_{i,k|k} = \hat{x}_{i,k|k-1} + P_{i,k|k}\sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}\left(y_{l,k} - H_{l,k}\hat{x}_{i,k|k-1}\right) \end{cases}$$

Step 2: Diffusion update

$$\hat{x}_{i,k|k} = \sum_{l\in\mathcal{N}_i} c_{l,i}\phi_{l,k}$$

$$\Delta^{\Upsilon}\hat{x}_{i,k+1|k} = F_k\hat{x}_{i,k|k}$$

$$\hat{x}_{i,k+1|k} = \Delta^{\Upsilon}\hat{x}_{i,k+1|k} - \sum_{j=1}^{k+1}(-1)^j\Upsilon_j\,\hat{x}_{i,k+1-j}$$

$$P_{i,k+1|k} = (F_k + \Upsilon_1)P_{i,k|k}(F_k + \Upsilon_1)^* + G_kQ_kG_k^* + \sum_{j=2}^{k}\Upsilon_jP_{i,k-j}\Upsilon_j^T$$

5. AVERAGE PERFORMANCE ANALYSIS

In this section, convergence of diffusion Kalman filter algorithms are analyzed. The main results of this section are as follows. The estimation error at each node and at any moment is expressed through a proper expression (46). Then, using this result it is shown that the Kalman filter distribution estimates for all moments of $k \geq 0$are without bias.

Hypothesis 1: Consider the discrete-time linear fractional order system given by mode Eq. (1), output observation Eq. (8) and initial estimation conditions $\hat{x}_{i,0|-1} = E(x_0), P_{i,0|-1} = \Pi_0$ for each node i. Global state estimates obtained by the fractional-order diffusion Kalman filter for the system at all $k \geq 0$ moments are without bias.

Proof: To prove hypothesis 1, the estimation error at the end of the incremental update is calculated by $\tilde{\phi}_{i,k} = x_k - \phi_{i,k}$. Also, n_i is the degree of node i and the set $\{i_j\}, j = 1,..,n_i$ represents neighbors indices of node i.

Consider the incremental step in the algorithm mentioned in Table 2 and $P_{i,k}^{(j)}$, $P_{i,k}$ values in the j^th iteration where $j = 1,..,n_i$, $P_{i,k}^{(0)} = P_{i,k|k-1}$ and $P_{i,k}^{(n_i)} = P_{i,k|k}$. By repeating the incremental step on the neighbors of node i we can write:

$$\begin{aligned} \tilde{\phi}_{i,k} = &\left[I - P_{i,k}^{(n_i-1)}H_{i_{n_i},k}^{*}\left(R_{i_{n_i},k} + H_{i_{n_i},k}P_{i,k}^{(n_i-1)}H_{i_{n_i},k}^{*}\right)^{-1}H_{i_{n_i},k}\right] \\ &\times\left[I - P_{i,k}^{(0)}H_{i_1,k}^{*}\left(R_{i_1,k} + H_{i_1,k}P_{i,k}^{(0)}H_{i_1,k}^{*}\right)^{-1}H_{i_1,k}\right]\tilde{x}_{i,k|k-1} \\ &+\left[I - P_{i,k}^{(n_i-1)}H_{i_{n_i},k}^{*}\left(R_{i_{n_i},k} + H_{i_{n_i},k}P_{i,k}^{(n_i-1)}H_{i_{n_i},k}^{*}\right)^{-1}H_{i_{n_i},k}\right] \\ &\times\left[I - P_{i,k}^{(1)}H_{i_2,k}^{*}\left(R_{i_2,k} + H_{i_2,k}P_{i,k}^{(1)}H_{i_2,k}^{*}\right)^{-1}H_{i_2,k}\right] \\ &\times P_{i,k}^{(0)}H_{i_1,k}^{*}\left(R_{i_1,k} + H_{i_1,k}P_{i,k}^{(0)}H_{i_1,k}^{*}\right)^{-1}v_{i_1,k} \\ &\vdots \\ &+P_{i,k}^{(n_i-1)}H_{i_{n_i},k}^{*}\left(R_{i_{n_i},k} + H_{i_{n_i},k}P_{i,k}^{(n_i-1)}H_{i_{n_i},k}^{*}\right)^{-1}v_{i_{n_i},k} \end{aligned} \tag{40}$$

Notice that $P_{i,k}^{(j)}H_{i_j,k}^{*}R_{i_j,k}^{-1} = P_{i,k}^{(j-1)}H_{i_j,k}^{*}\left(R_{i_j,k} + H_{i_j,k}P_{i,k}^{(j-1)}H_{i_j,k}^{*}\right)^{-1}$, therefore the above equation can be rewritten as:

$$\begin{aligned} \tilde{\phi}_{i,k} &= \tilde{x}_{i,k|k-1} - P_{i,k|k}\sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}\left(H_{l,i}\tilde{x}_{i,k|k-1} + v_{l,k}\right) \\ &= \left(I - P_{i,k|k}S_{i,k}\right)\tilde{x}_{i,k|k-1} - P_{i,k|k}\sum_{l\in\mathcal{N}_i} H_{l,k}^{*}R_{l,k}^{-1}v_{l,k} \end{aligned} \tag{41}$$

where $\tilde{x}_{i,k|k-1} = x_k - \hat{x}_{i,k|k-1}$ indicates estimation error in node i at the end of the diffusion update and $S_{i,k} = \sum_{l\in\mathcal{N}_i} H^*_{l,k} R^{-1}_{l,k} H_{l,k}$, so it is:

$$\tilde{x}_{i,k|k-1} = F_{k-1}\tilde{x}_{i,k-1|k-1} + G_{k-1}u_{k-1} - \sum_{j=1}^{k}(-1)^j \Upsilon_j \tilde{x}_{i,k-j} \tag{42}$$

Combining Eq. (43) with the diffusion update step of the mentioned algorithm in Table 2 results in Eq. (43):

$$\tilde{x}_{i,k|k-1} = \sum_{l\in\mathcal{N}_i} c_{l,i}\tilde{\Phi}_{l,k}$$
$$= \sum_{l\in\mathcal{N}_i} c_{l,i}\left[(I - P_{l,k|k}S_{l,k})\tilde{x}_{l,k|k-1} - P_{l,k|k}\sum_{m\in\mathcal{N}_i} H^*_{m,k}R^{-1}_{m,k}v_{m,k}\right] \tag{43}$$

By obtaining the expected value from both sides of equations (44) and (45) the recursive formula for the expected value of the estimates is obtained by the fractional-order Kalman filter algorithm as follows:

$$E[\tilde{x}_{i,k|k-1}] = F_{k-1}E[\tilde{x}_{i,k-1|k-1}] - \sum_{j=1}^{k}(-1)^j \Upsilon_j E[\tilde{x}_{i,k-j}] \tag{44}$$

$$E(\tilde{x}_{i,k|k-1}) = \sum_{l\in\mathcal{N}_i} c_{l,i}(I - P_{l,k|k}S_{l,k})E(\tilde{x}_{l,k|k-1}) \tag{45}$$

Since the $\hat{x}_{i,0|-1} = 0$ is $(x_0) = 0$. For all i, $(\tilde{x}_{i,0|-1}) = 0$. Thus,

$$E(\tilde{x}_{i,0|0}) = \sum_{l\in\mathcal{N}_i} c_{l,i}(I - P_{l,0|0}S_{l,0})E(\tilde{x}_{l,0|-1}) = 0 \tag{46}$$

Therefore, we can conclude from repeating equations (44) and (45) that the fractional-order diffusion Kalman filter estimation for all k≥0 is without bias.

6. SIMULATION

In this section, a projectile path tracking scenario is implemented in a wireless sensor network in order to numerically evaluate performance of the fractional-order distributed Kalman filter algorithms and the fractional-order diffusion Kalman filter algorithm. Then, by estimation of the state of a projectile performance of the proposed fractional-order diffusion Kalman filter algorithm, the fractional-order distributed Kalman filter algorithm and conventional fractional Kalman filter algorithm are compared. The results obtained from simulations, as confirmed in the previous sections, prove the validity of the proposed algorithms in terms of convergence error of estimation error due to the proposed fractional order Kalman filter algorithm. The accuracy of the estimation obtained from the proposed fractional-order Kalman filter algorithms also shows a significant improvement over the conventional fractional-order Kalman filter algorithm.

Consider a set of sensors in a wireless sensor network that try to predict and track the trajectory of a projectile. Assume that the projectile is adjacent to an adaptive network in which the sensors observe the position of the projectile affected by noise. This network consists of 20 sensors or agents with topology shown in Fig. 2, where the connecting lines between the nodes represent information communication between the agents. At the same time, each sensor node can independently detect the position of the projectile and communicate with its neighbors.

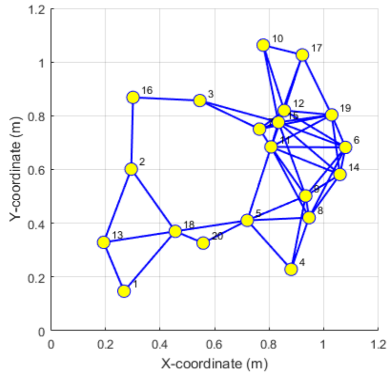

Fig. 2. Network topology with N = 20 nodes.

Now acceleration a, velocity v, and projectile position p are as follows:

$$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \tag{47}$$

For the projectile move (Ebaid, 2011):

$$\begin{aligned} &D^{n_1}v(t) = a(t) \\ &D^{n_2}p(t) = v(t) \\ &a_1 = a_2 = 0,\ a_3 = -g \end{aligned} \tag{48}$$

where g is the acceleration of gravity on earth. x mode is a 6-dimensional vector system consisting of projectile's velocity and position collection as follows. Therefore, the dynamics of the process with respect to Eq. (49) are considered as follows:

$$\begin{aligned} &f_k(x_k, u_k) = [D^n v \quad D^n p]^T \times h^n \\ &\Delta^{\Upsilon} x_{k+1} = f_k(x_k, u_k) + w_k,\ k \geq 0 \quad w_k = (0, Q_k) \\ &x_{k+1} = \Delta^{\Upsilon} x_{k+1} - \sum_{j=1}^{k+1}(-1)^j \Upsilon_j x_{k+1-j},\ k \geq 0 \end{aligned} \tag{49}$$

where $n = 0.99$ is the fractional-order system, $x_k = [v_k, \mathrm{p_k}]^T$ system modes with default values. Moreover, $x_0 = [0.7,0.1,0.2,0.8,0.2]^T$ and $u_k = [0]$ are input system. Assume each node measures an unspecified target position in one of the following two situations: $H_{i,k} = [0, diag([1\ 1\ 0])]$, for the case where only horizontal dimensions are seen and $H_{i,k} = [0\ \ , diag([1\ 0\ 1])]$, for the case where both a horizontal and a vertical dimension are seen. Therefore, nodes can't directly measure projectile position in three dimensions. The assignment of the observable pair is performed at random by each node.

The values of the parameters are: $h = 0.1$, $G_k = I_6$, $Q_k = 0.001I_6$, $S_i = 0$ and $R_{i,k} = \sqrt{i}PR_0P^T$ with $R_0 = 0.5 \times diag([1\ 4\ 7])$ and P is a permutation matrix that is randomly selected for each node. The coefficient $\sqrt{k}$ makes variation of noise conditions for each node possible. The initial conditions are $x_0 = (10,2,8,0.1,0.1,0.1)^T$ and $P_0 = I_6$.

Fig. 3 shows actual vertical path (continuous curve) and noise measurements of vertical position at node 5 (dashed curve). Fig. 4 also shows the mode estimation function when estimating vertical position for different algorithms across the entire network. The three remaining curves are related to conventional Kalman filter, fractional-order distributed and fractional-order diffusion Kalman filter. It is observed that the estimates obtained by fractional-order diffusion Kalman filter are closer to the actual path than that of the other two algorithms.

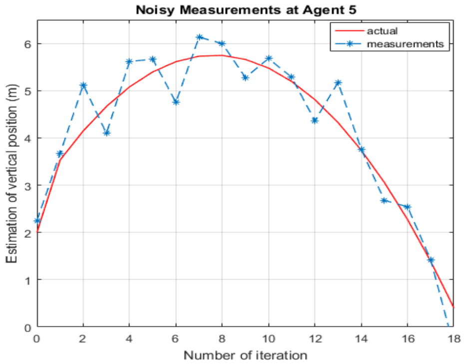

Fig. 3. True vertical path and noise measurement of vertical position in node 5

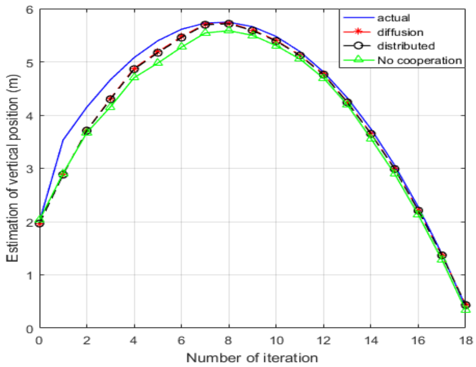

Fig. 4. Average estimates of vertical position of all nodes by different algorithms.

The mean square deviation (MSD) criterion for performance of the fractional-order Kalman filter is evaluated. It should be noted that the mean MSD for all nodes is defined as follows:

$$MSD_{i,k} = E\left\|x_i - \hat{x}_{i,k|k}\right\|^2$$
$$MSD_k^{avg} = \frac{1}{N}\sum_{i=1}^{N} MSD_{i,k} \tag{50}$$

where the time index k and node i are considered for MSD, because different nodes generally produce different estimates for diffusion algorithms.

The MSD Kalman filter function is evaluated numerically and is compared with conventional and distributed Kalman filters. In Fig. 5 and Table 3, a comprehensive MSD evaluation for the three algorithms is presented.

In this Figure, the x-axis represents the number of repetitions and the y-axis shows MSD.

As you can see, the error in the conventional fractional Kalman filter is high, since the nodes do not have access to three-dimensional measurements of projectile movement, and the pair $\{F, H_i^{loc}\}$ is indistinguishable. It can be concluded from this Figure that the two distributed and diffusion fractional order Kalman filter algorithms converge to constant values of MSD with different convergence speeds over time. By comparing and analyzing simulations of the three different algorithms, it can be concluded that the estimated order fractional diffusion Kalman filter shows significant improvement over the distributed state. These results confirm the benefits of using diffusion strategies for adaptive networks.

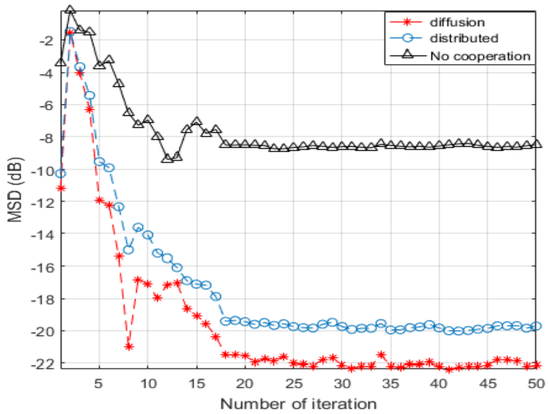

Fig. 5. Inclusive MSD function for Fractional Order diffusion Kalman Filter algorithms, Fractional-Order distributed Kalman Filter Algorithm and Ordinary Kalman Filter Algorithm.

Table 3. Global MSD performance and computing time for diffusion fractional-order, distributed fractional-order, and conventional Kalman filter.

Algorithm	MSD performance	Computing time (s)
diffusion fractional-order Kalman filter	0.63	5.25
fractional-order distributed Kalman filter	0.66	5.22
conventional Kalman filter	0.87	5.109

The comparison of the methods was investigated in terms of computing time and was run by hardware the following specifications " (processor: Intel, core i7(TM), cpu 2 GHz, RAM: 8 GB)", was included in table 3 and the descriptions of the table were added to line 3 of the last paragraph of simulation section. Table 3 shows that, in the implemented method, as the estimation accuracy increases, computations take longer and MSD decreases, and this indicates that even though the computing time has increased, the error have decreased and a greater accuracy is yielded. All items mentioned in the last paragraph of the article simulation section were added. Table 3 shows that, in the implemented method, as the estimation accuracy increases, computations take longer and MSD decreases, and this indicates that even though the computing time has increased, the error have decreased and a greater accuracy is yielded.

7. CONCLUSIONS

In this paper, strategies of fractional-order diffusion Kalman filter for estimating distributed mode in linear systems are presented. In this algorithm, each node only needs to communicate with its neighbors in order to share data and estimates. Also, the proposed diffusion method guarantees diffusion of information across the entire network. Then steady-mode mean analysis was presented and it was proved that the computational estimation calculated by Kalman filter is a non-bias emission fraction. Finally, the performance of the proposed diffusion and distributed fractional-order Kalman filter algorithms were compared with conventional Kalman algorithm to estimate the state of a projectile. Simulation results show that the generalized estimation error of the fractional-order diffusion Kalman filter algorithm can converge to a desirable single value. Also, the accuracy of the estimation obtained by the fractional-order diffusion Kalman filter differs significantly from the other two algorithms.

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