A Robust Fuzzy Observer-Based Control with Reference State Model and Unmeasurable Premise Variables: Application to a Biological Process

Abdelmounaim Khallouq, Asma Karama, Mohamed Abyad

LAEPT-URAC 28, Faculty of Sciences Semlalia, P.B. 2390, University Cadi Ayyad, Marrakech, Morocco
abdelmounaim.khallouq@gmail.com, karama@uca.ac.ma, abymedlmp@gmail.com

Abstract: This paper proposes a new procedure for the design of a robust fuzzy observer-based tracking controller for nonlinear systems using Takagi-Sugeno (TS) formalism. A reference state model is considered and the premise variables are considered inaccessible to measurement. In order to ensure the global stability and to minimize efficiently the effect of the disturbance affecting the tracking performances of the closed loop system and the observer, in addition to the Lyapunov approach, the H _∞ norm is used. The controller and the observer design are developed in a single step and new sufficient conditions are obtained and given in terms of linear matrix inequalities (LMIs). The application on a biological process in simulation studies is provided to explain the tracking control design procedure and to prove the efficiency of the proposed approach.

Keywords: Bioprocesses, Multiple equilibrium, Stabilization, Fuzzy observer, PDC control, Takagi-Sugeno models, Unmeasurable premise variables.

1. INTRODUCTION

The study of model following and tracking problems keeps considerable attentions due to demands from practical dynamical processes in mechanics, economics and biology (Benattia et al., 2016; Haidegger et al., 2011; Haidegger et al., 2011). A variety of approaches have been proposed. The output feedback linearization technique and adaptive scheme are incorporated to achieve the tracking purpose (Benattia et al., 2020; Owczarkowski et al., 2019). The robust control is also a frequently used method in case of existence of parameter variation and disturbances (Miranda-Colorado and Aguilar, 2020; Owczarkowski et al., 2019; Szelitzky et al., 2011). To attenuate the effect of parameter variation and external disturbance, the H1 tracking methods also studied in the literature.

Furthermore, actual systems are generally nonlinear which makes observers and controllers design more difficult. Using the fuzzy Takagi-Sugeno (TS) formalism makes the task easier to handle. Fuzzy technique is recognized as a powerful tool in approximating complex nonlinear system. Various techniques have been developed and successfully used in nonlinear modeling and control (Benattia et al., 2016; Benzaouia and El Hajjaji, 2014; Haidegger et al., 2011; Haidegger et al., 2011; Tanaka and Wang, 2001; Abyad et al., 2020; Benzaouia and El Hajjaji, 2018; W.-J. Chang et al., 2019; Saifia et al., 2020; Tong and Li, 2002; J.-W. Wang et al., 2013).

We are interested by the design of a fuzzy observer based tracking controller. However, on the one hand, the problem of the state fuzzy tracking control didn't get enough importance. Few studies have addressed the tracking problem of all the output/state variables, for example (Abyad et al., 2018a; Chung-Chun Kung and Hai-Huang Li, 2002; Chung-Shi Tseng et al., 2001). On another side, fuzzy observer-based control problem is in general difficult to deal with. Most often a two-step procedure for the observer and the controller design is used. As examples one can cite: an adaptive fuzzy tracking control problem for uncertain systems despite the existence of the external disturbances, the controller design is based on the state estimation was presented in (Liu et al., 2011). A robust fuzzy observer based H_∞ control problem is addressed in (Lo and Lin, 2004). A two step procedure is also given in (Chung-Shi Tseng et al., 2001) to achieve the tracking purpose and state estimation. In these cited works, the stability conditions are expressed by bilinear matrix inequalities (BMI) difficult to solve. The solution needs to recast the BMI into two LMIs where the obtained solution of the first LMI is substituted as a known variable when resolving the second LMI. In (Y. Wang et al., 2018) an H1 fuzzy control was designed for a class of nonlinear disturbed systems with uncertain parameters. In (G.-H. Chang and Wu, 2012), a fuzzy controller based on the state estimation is synthesized to reduce the tracking error. Authors in (Tong and Li, 2002) present a sufficient conditions in terms of LMIs for robust output tracking controllers based on the state estimation. New conditions are given in (Xie et al., 2019) for the stabilization of TS fuzzy systems based on the observer with unknown premise variables.

On the other hand, control and monitoring of biological processes is an active area of research. Due to the presence of living microorganisms, these processes can exhibit a wide range of dynamic behaviors and are typically described by complex nonlinear systems with time varying characteristics. Moreover, due to the lack of direct online sensors, these processes are also known by the difficulty to measure chemical and biological variables. Automatic control of biological processes has been extensively the subject of lot of studies, one can cite especially, for the nonlinear modeling (Bastin and Dochain, 1990; Bernard and Quelinnec, 2008; Denis Dochain and Vanrolleghem, 2015; Karama et al., 2010) for the control (Alcaraz-González et al., 2012; Bastin and Dochain, 1990; Dahhou et al., 1992; Evans et al., 2003; Ioan and Mihai, 2001; Teanu and Petre, 2004) and the estimation (Bastin and Dochain, 1990; Ben Youssef et al., 1996; Bernard and Gouzé, n.d.; Montiel-Escobar et al., 2012; Zhang et al., 2017). Based on the TS formulation and the Parallel Distributed Compensation (PDC), in two of our previous works we were interested by the stabilisation and the fault tolerant control issues. In (Khallouq and Karama, 2017), a fuzzy statefeedback controller has been developed for the stabilization of the state variables of a bacterial growth process around a specific equilibrium point. In (Abyad et al., 2018b) a Fault Tolerant Control (FTC) has been investigated using a fuzzy linear quadratic integral controller to compute the nominal controller of the free fault system, and a proportional-integral observer to estimate faults.

In (Ghorbel et al., 2014) a robust fuzzy observer-based tracking controller is developed to reduce both the tracking error and the estimation error and gives in one step procedure the controller and observer gains. It is certainly more advantageous when the controller and the observer are designed in one step than in the two step approach where they are synthesized independently. Nevertheless, requiring the controller and observer stability conditions to be resolved simultaneously can be more restrictive and therefore can lead to constraining LMIs that are more difficult to resolve. It is in this context that the contribution of this article can be stated. Our objective is to propose an approach aimed at considerably simplifying the conditions for stability.

This paper addresses a similar problem studied in (Ghorbel et al., 2014). Based on an algebraic transformation, we propose a different procedure to design a fuzzy robust observer based tracking controller using a reference model and with non measurable premise variables. At first, the nonlinear system is tranformed into a TS fuzzy model and a TS fuzzy observer is built. Then a PDC control law depending on the reference states and the observer states is considered. To study the stability conditions, an augmented system using the estimation error and the tracking error is constructed. Using the H_∞ norm, the controller and the observer synthesis is achieved in a single step. The novelty of this paper is by introducing a variable change modifying the considered Lyapunov function, more relaxed sufficient conditions are obtained compared to those in (Ghorbel et al., 2014). These new conditions are given in terms of linear matrix inequalities (LMIs). To show the new design features of the current work, an application on a biological process in simulation studies is provided. To check the efficiency of our method, a comparison with the work in (Ghorbel et al., 2014) is realized. First, the simulations of the two methods are compared . Later on, by mean of a feasibility test, the comparison of the obtained conditions and those in (Ghorbel et al., 2014) is performed.

The paper is organized as follows. In section 2, the problem formulation is presented. A detailed description of the proposed procedure to design the robust observer based fuzzy tracking controller is given in section 3. In section 4, the bioprocess model is described. Its transformation to a TS model is illustrated in Section 5. Section 6 presents the simulation results to illustrate ended by a conclusion

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following dynamic model:

$$\begin{aligned} \dot{x}(t) &= f(x(t))x(t) + g(x(t))u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^q$ is the output vector, $C \in \mathbb{R}^{q \times n}$ and $f(\cdot)$ and $g(\cdot)$ are two nonlinear functions.

2.1. The TS fuzzy model

The fuzzy dynamic model had been proposed by Takagi-Sugeno to represent the nonlinear systems (Takagi and Sugeno, 1985). The TS fuzzy model is defined by the IF-THEN fuzzy rules describing the local input-output relations of a nonlinear system. The fundamental idea of the TS fuzzy model is to represent the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy system model is generated through fuzzy "blending" of the linear system models. It is proved that Takagi-Sugeno fuzzy models are universal approximators of any smooth nonlinear system (Buckley, 1992; Fantuzzi and Rovatti, 1996).There are three methods to construct T-S fuzzy models (Tanaka and Wang, 2001): black box identification, linearization method, and sector nonlinearity methods. We are interested in the third one, this technique gives an exact T-S representation of nonlinear system without information loss. the model (1) is transformed to a T-S fuzzy model following the rules below:

Rule i :

if $\xi_1(t)$ is M_i^1 and . . . and $\xi_p(t)$ is M_i^p **THEN**

$$\dot{x}(t) = A_i x(t) + B_i u(t) \ldots \{i = 1, 2, \ldots, n_r\} \tag{2}$$

Where $\xi_1(t), \ldots, \xi_p(t)$ are the premise variables that may be functions of state variables, external disturbances and/or time. M_i^j are the fuzzy sets ($j \in \{1, 2, ..., p\}$), $n_r = 2^p$ denotes the number of if-then rules $A_i \in^{n \times n}$, $B_i \in^{n \times m}$ are constant matrices of sub-models.

The final variables of the fuzzy systems are inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{n_r} h_i(\xi)(A_i x(t) + B_i u(t)) \tag{3}$$

Where:

$$h_i(\xi) = \frac{\prod_{j=1}^{p} M_i^j(\xi_j(t))}{\sum_{i=1}^{nr} \prod_{j=1}^{p} M_i^j(\xi_j(t))} \tag{4}$$

with $\xi(t)=[\xi_1(t)\ldots\xi_p(t)]M_i^j(\xi_j(t))$. The term $M_i^j(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_i^j is the $h_i(\xi(t))$ normalized membership function which denotes the weight of the associated sub-model calculated from membership functions in the premise parts. By definition we have:

$$\begin{cases} 0 \le h_i(\xi(t)) \le 1, \quad \forall i \in \{1,2,\ldots,n_r\} \\ \sum_{i=1}^{n_r} h_i(\xi(t)) = 1 \end{cases}$$

2.2. The TS fuzzy observer

We consider the case of an observer with unknown premise variable. The TS fuzzy observer rules are as below:

Rule i :

if $\hat{\xi}_1(t)$ is M_{i1} and . . . and $\hat{\xi}_p(t)$ is M_{ip} **THEN**

$$\begin{aligned} &\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i(y(t)-\hat{y}(t)) \\ &i = 1,2,\ldots,n_r \end{aligned} \tag{5}$$

where $\hat{\xi}$, $\hat{x}$ and $\hat{y}$ are the estimated respectively of the premise variable, the state vector and the output vector. L_i is the unknown observer gain of the i^{th} observer rule that should be determined.

The global TS fuzzy observer is so given by:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{n_r} h_i(\hat{\xi}(t))(A_i\hat{x}(t) + B_iu(t) + L_i(y(t)-\hat{y}(t))) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{6}$$

2.3. Problem statement of a TS observer-based control with reference state model

We consider a linear reference model given by the following equation:

$$\dot{x}_r = A_{ref}x_r + r(t) \tag{7}$$

where x_r is the reference state which should be tracked by the system, A_{ref} is a stable matrix and $r(t)$ is a bounded input reference.

Our goal is to synthesize a control law based on the state estimation capable to reduce the error between the reference trajectory $x_r(t)$ and the state $x(t)$ for all $t \ge 0$.

The controller can then be expressed by an observer-based PDC control with reference model as follows:

$$u(t) = \sum_{i=1}^{n_r} h_i(\hat{\xi}(t))K_i(\hat{x}(t) - x_r(t)) \tag{8}$$

where $K_i \in \mathbb{R}^{m\times p}$ is the controller gain that should be determined.

The scheme of the simulation model is given in the Figure 1.

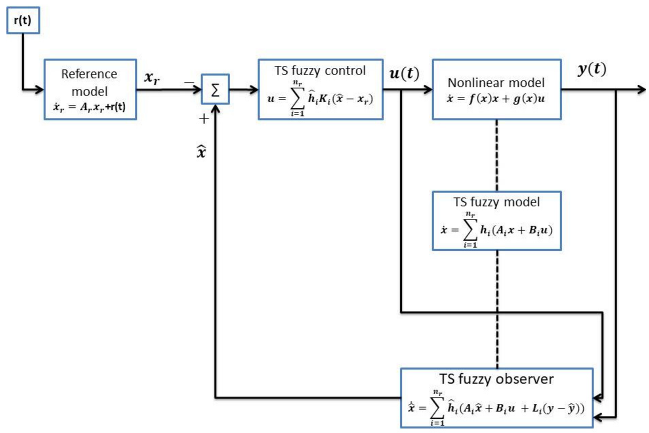

Fig.1. The scheme of the simulation model.

The following notation will be adopted

$$\sum_{i=1}^{n_r} h_i(\xi(t)) \sum_{j=1}^{n_r} h_j(\hat{\xi}(t)) = \sum_{i,j=1}^{n_r} h_i\hat{h}_j$$

Let us define by $e_r(t) = x(t) - x_r(t)$ the tracking error and $e_o(t) = x(t) - \hat{x}(t)$ the state estimation error. Their dynamics are defined by:

$$\begin{aligned} \dot{e}_r(t) = \sum_{i,j=1}^{n_r} h_i\hat{h}_j((A_i + B_iK_j)e_r(t) + (A_i - A_{ref})x_r(t) \\ -B_iK_je_o(t) - r(t)) \end{aligned} \tag{9}$$

$$\dot{e}_0(t) = \sum_{i=1}^{n_r}(\hat{h}_i)((A_i - L_iC)e_o(t) + \omega(t)) \tag{10}$$

where

$$\omega(t) = \sum_{i=1}^{n_r}(h_i - \hat{h}_i)(A_ix(t) + B_iu(t)) \tag{11}$$

The state and the control input are supposed to be bounded, the membership functions are Lipschitz hence the terms $\omega(t)$ is bounded and acting like a bounded perturbation.

3. THE MAIN CONTRIBUTION

Our aim is to determine in a single step unlike the works presented in(Tong and Li, 2002), (G.-H. Chang and Wu, 2012), (Xie et al., 2019) the controller gains K_i and the observer gains L_i ensuring the convergence of both the tracking error $e_r(t)$ and the observer error $e_o(t)$ to zero.

Let consider the variable change $z_r = P_re_r$ which will help us to simplify the theoretical calculus and avoid several matrix nonlinearities:

$$\dot{z}_r = \sum_{i,j=1}^{n_r} h_i\hat{h}_j(P_r(A_i + B_iK_j)P_r^{-1}z_r(t) + E_{i,j}\Omega(t)) \tag{12}$$

where

$$\begin{aligned} E_{i,j} &= \begin{bmatrix} P_r(A_i - A_{ref}) & -P_rB_iK_j & -P_r \end{bmatrix} \\ \Omega(t) &= \begin{bmatrix} x_r(t)^T & e_o(t)^T & r(t)^T \end{bmatrix}^T \end{aligned} \tag{13}$$

The controller performance (12) is affected by the presence of the disturbance vector $\Omega(t)$. In order to minimize efficiently its effect on $z_r(t)$, we suggest the use of the H_∞ performance criteria given by:

$$\int_0^{t_f} z_r^T(t)Qz_r(t)dt \le \gamma^2 \int_0^{t_f} \Omega^T(t)\Omega(t)dt \tag{14}$$

The same reasoning is done for the observer performance (10), which can be improved by the introduction of the H_∞ performance criteria as follows:

$$\int_0^{t_f} e_o^T(t)Re_o(t)dt \le \nu^2 \int_0^{t_f} \omega(t)^T\omega(t)dt \tag{15}$$

t_f defines the terminal control time, Q and R are two definite, positive matrices. γ and ν are two prescribed positive scalars that define the attenuation level of the disturbances $\Omega(t)$ and $\omega(t)$.

Remark 1 *The stability of (12) implies the stability of (9).*

In fact, consider the following candidate Lyapunov functions for the systems *(12)* and *(9)*:

$$\begin{cases} V_{z_r} = z_r^T(t)P_r^{-1}z_r(t) \\ V_{e_r} = e_r^T(t)P_re_r(t) \end{cases} \tag{16}$$

The stability of *(12)* is guaranteed if the derivative of V_{z_r} is definite negative. We can easily show that $V_{z_r} = V_{e_r}$ and $\dot{V}_{z_r} = \dot{V}_{e_r}$. So if $\dot{V}_{z_r}$ is negative-definite $\dot{V}_{e_r}$ is also.

Remark 2 *The attenuation level γ of the disturbances of $z_r(t)$ and $e_r(t)$ is identical.*

In fact we have:

$$\begin{aligned} &\int_0^{t_f} z_r^T(t)Qz_r(t)dt = \int_0^{t_f} e_r^T(t)\bar{Q}e_r(t)dt \\ &\le \gamma^2 \int_0^{t_f} \Omega^T(t)\Omega(t)dt \end{aligned} \tag{17}$$

where $\bar{Q} = P_rQP_r$. So for $z_r(t)$ and $e_r(t)$, we have the same attenuation level γ.

Theorem 1 *There exists a TS observer-based controller with reference model of type (12) guaranteeing H_∞ norms less than ν and γ respectively for the state observer error and the tracking reference error models (10) and (12), if there exist matrices $P_o = P_o^T > 0, X = X^T > 0, Y_j, J_i, i, j = 1,\cdots,n_r$ and matrices $Q > 0, R > 0$ and a prescribed positive scalars $\eta > 0$, $\zeta > 0$ and λ such that the following conditions hold:*

$$\begin{bmatrix} P_oA_i - J_iC + (P_oA_i - J_iC)^T + R & P_o \\ P_o & -\eta I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} M_{i,j} & \Delta A_{i,ref} & -B_iY_j & I & 0 \\ \Delta A_{i,ref}^T & -\zeta I & 0 & 0 & 0 \\ -(B_iY_j)^T & 0 & -2\lambda X & 0 & \lambda I \\ I & 0 & 0 & -\zeta I & 0 \\ 0 & 0 & \lambda I & 0 & -\zeta I \end{bmatrix} < 0 \tag{19}$$

with

$$M_{i,j} = A_iX + B_iY_j + (A_iX + B_iY_j)^T + Q$$

and

$$\Delta A_{i,ref} = A_i - A_{ref}$$

By solving LMIs *(18)* and *(19)*, the observer and controller gains L_i, K_i are given by:

$$L_i = P_o^{-1}J_i, K_i = Y_iP_r$$
$$\textit{where } P_r = X^{-1}, \textit{ for } i = 1,\cdots,n_r$$

The scalars verifying the H_∞ norms for the observer error and the tracking reference error are given respectively by: $\nu = \sqrt{\eta}$ and $\gamma = \sqrt{\zeta}$.

Before starting the proof of the theorem, some useful lemmas (Guerra et al., 2006) are recalled.

Lemma 1 For any matrices X, Y of appropriate dimensions and for any positive scalar η the following inequality hold:

$$X^TY + Y^TX \le \eta X^TX + \eta^{-1}Y^TY \tag{20}$$

Lemma 2 Consider matrices $\Pi = \Pi^T < 0$ and X and a scalar λ, the following holds:

$$\begin{gathered} (X + \lambda\Pi^{-1})^T\Pi(X + \lambda\Pi^{-1}) \le 0 \\ \Leftrightarrow \\ X^T\Pi X \le -\lambda(X^T + X) - \lambda^2\Pi^{-1} \end{gathered} \tag{21}$$

Proof:

Let consider the following candidate Lyapunov function:

$$V(e_o(t), z_r(t)) = \begin{bmatrix} e_o(t) \\ z_r(t) \end{bmatrix}^T \begin{bmatrix} P_o & 0 \\ 0 & P_r^{-1} \end{bmatrix} \begin{bmatrix} e_o(t) \\ z_r(t) \end{bmatrix} \tag{22}$$

By defining the following Lyapunov function $V_{e_o} = e_o^T(t)P_oe_o(t)$, $V(e_o(t), z_r(t))$ can be rewritten as a sum of V_{e_o} and V_{z_r}

Our goal is to find out sufficient conditions for which

$\dot{V}(e_o(t), z_r(t))$ is negative definite. This can be reached if $\dot{V}_{e_o} < 0$ and $\dot{V}_{z_r} < 0$

Let's start by checking for the sufficient conditions to have $\dot{V}_{e_o} < 0$:

$$\begin{aligned}\dot{V}_{e_o} &= \sum_{i=1}^{n_r} \hat{h}_i [((A_i - L_i C) e_o(t) + \omega(t))^T P_o e_o(t) \\ &\quad + e_o^T(t) P_o ((A_i - L_i C) e_o(t) + \omega(t))] \\ &\le \\ &e_o^T(t)((A_i - L_i C)^T P_o + P_o (A_i - L_i C)) e_o(t) \\ &\quad + \omega(t)^T P_o e_o(t) + e_o^T(t) P_o \omega(t) < 0\end{aligned} \tag{23}$$

Applying Lemma 1 to the crossed terms in *(23)* we get:

$$\begin{aligned}&\omega(t)^T P_o e_o(t) + e_o^T(t) P_o \omega(t) \le \\ &\eta \omega(t)^T \omega(t) + \eta^{-1} e_o^T(t) P_o P_o e_o(t)\end{aligned} \tag{24}$$

The observer dynamical error *(10)* is stable and the H_∞ norm *(15)*is bounded by ν if and only if:

$$\dot{V}_{e_o} + e_o^T(t) R e_o(t) - \nu^2 \omega(t)^T \omega(t) < 0 \tag{25}$$

By choosing $\eta = \nu^2$, this condition can becomes:

$$\begin{aligned}&e_o^T(t)((A_i - L_i C)^T P_o + P_o (A_i - L_i C)) e_o(t) \\ &+ \eta^{-1} e_o^T(t) P_o P_o e_o(t) + e_o^T(t) R e_o(t) < 0\end{aligned} \tag{26}$$

and which holds if:

$$(A_i - L_i C)^T P_o + P_o (A_i - L_i C) + R + \eta^{-1} P_o P_o < 0 \tag{27}$$

Applying Schur complement to *(27)* we get:

$$\begin{bmatrix} P_o (A_i - L_i C) + (A_i - L_i C)^T P_o + R & P_o \\ P_o & -\eta I \end{bmatrix} < 0 \tag{28}$$

Now let's look for the second sufficient condition to have $\dot{V}_{z_r} < 0$:

$$\begin{aligned}\dot{V}_{z_r} &= \sum_{i,j=1}^{n_r} h_i \hat{h}_j ([P_r (A_i + B_i K_j) P_r^{-1} z_r(t) + E_{i,j} \Omega(t)]^T P_r^{-1} z_r(t) \\ &\quad + z_r^T(t) P_r^{-1} [P_r (A_i + B_i K_j) P_r^{-1} z_r(t) + E_{i,j} \Omega(t)]) \\ &\le \\ &z_r^T(t)(P_r^{-1}(A_i + B_i K_j)^T + (A_i + B_i K_j) P_r^{-1}) z_r(t) \\ &\quad + \Omega^T(t) E_{i,j}^T P_r^{-1} z_r(t) + z_r^T(t) P_r^{-1} E_{i,j} \Omega(t)\end{aligned} \tag{29}$$

Using the Lemma 1 for the crossed terms leads to:

$$\begin{aligned}\dot{V}_{z_r} &\le z_r^T(t)(P_r^{-1}(A_i + B_i K_j)^T + (A_i + B_i K_j) P_r^{-1}) z_r(t) \\ &\quad + \Omega^T(t) \zeta \Omega(t) + z_r^T P_r^{-1} E_{i,j} \zeta^{-1} E_{i,j}^T P_r^{-1} z_r\end{aligned} \tag{30}$$

The controlled model *(12)* is stable and the H_∞ norm *(14)* is bounded by γ if and only if:

$$\dot{V}_{z_r} + z_r^T(t) Q z_r(t) - \gamma^2 \Omega^T(t) \Omega(t) < 0 \tag{31}$$

It follows that:

$$\begin{aligned}&z_r^T(t)(P_r^{-1}(A_i + B_i K_j)^T + (A_i + B_i K_j) P_r^{-1}) z_r(t) + \Omega^T(t) \zeta \Omega(t) \\ &+ z_r^T P_r^{-1} E_{i,j} \zeta^{-1} E_{i,j}^T P_r^{-1} z_r + z_r^T(t) Q z_r(t) - \gamma^2 \Omega^T(t) \Omega(t) < 0\end{aligned} \tag{32}$$

which can be simplified by choosing $\zeta = \gamma^2$ and yields:

$$\begin{aligned}\dot{V}_{z_r} &\le z_r^T(t)(P_r^{-1}(A_i + B_i K_j)^T + (A_i + B_i K_j) P_r^{-1}) z_r(t) \\ &\quad + z_r^T P_r^{-1} E_{i,j} \zeta^{-1} E_{i,j}^T P_r^{-1} z_r + z_r^T(t) Q z_r(t) < 0\end{aligned} \tag{33}$$

This holds if:

$$\begin{aligned}&P_r^{-1}(A_i + B_i K_j)^T + (A_i + B_i K_j) P_r^{-1} \\ &\quad + P_r^{-1} E_{i,j} \zeta^{-1} E_{i,j}^T P_r^{-1} + Q < 0\end{aligned} \tag{34}$$

Using the Schur lemma on *(34)*, we get:

$$\begin{bmatrix} \Theta_{i,j} & P_r^{-1} E_{i,j} \\ E_{i,j}^T P_r^{-1} & -\zeta I \end{bmatrix} < 0 \tag{35}$$

with

$$\Theta_{i,j} = P_r^{-1}(A_i + B_i K_j)^T + (A_i + B_i K_j) P_r^{-1} + Q$$

Replacing $E_{i,j}$ by its expression, *(35)* becomes:

$$\begin{bmatrix} \Theta_{i,j} & \Delta A_{i,ref} & -B_i K_j & I \\ \Delta A_{i,ref}^T & -\zeta I & 0 & 0 \\ -(B_i K_j)^T & 0 & -\zeta I & 0 \\ I & 0 & 0 & -\zeta I \end{bmatrix} < 0 \tag{36}$$

with

$$\Delta A_{i,ref} = A_i - A_{ref}$$

Multiplying *(36)* left and right by $diag(\begin{bmatrix} I & I & P_r^{-1} & I \end{bmatrix})$ leads to:

$$\begin{bmatrix} \Theta_{i,j} & \Delta A_{i,ref} & -B_i K_j P_r^{-1} & I \\ \Delta A_{i,ref}^T & -\zeta I & 0 & 0 \\ -P_r^{-1}(B_i K_j)^T & 0 & P_r^{-1}(-\zeta) P_r^{-1} & 0 \\ I & 0 & 0 & -\zeta I \end{bmatrix} < 0 \tag{37}$$

By using the Lemma 2 with $X = P_r^{-1} = P_r^{-T}$

$$\Pi = -\zeta I = -\zeta^T I < 0$$

we have:

$$(P_r^{-1}+\lambda(-\zeta)^{-1}I)^T(-\zeta)(P_r^{-1}+\lambda(-\zeta)^{-1}I)\le 0$$
$$\Leftrightarrow$$
$$P_r^{-1T}(-\zeta)P_r^{-1}\le -\lambda(P_r^{-1T}+P_r^{-1})-\lambda^2(-\zeta)^{-1}I \qquad (38)$$
$$P_r^{-1}(-\zeta)P_r^{-1}\le -2\lambda P_r^{-1}+\lambda^2(\zeta)^{-1}I$$

and with the Schur complement and the following variable change: $Y_j = K_j P_r^{-1}$ *(41)* becomes:

$$\begin{bmatrix} \Theta_{i,j} & \Delta A_{i,ref} & -B_iY_j & I & 0 \\ \Delta A_{i,ref}^T & -\zeta I & 0 & 0 & 0 \\ -(B_iY_j)^T & 0 & -2\lambda P_r^{-1} & 0 & \lambda I \\ I & 0 & 0 & -\zeta I & 0 \\ 0 & 0 & \lambda I & 0 & -\zeta I \end{bmatrix} < 0 \qquad (39)$$

The bilinear matrix inequalities (BMIs) *(28)* and *(37)* can be transformed into linear matrix inequalities (LMIs) respectively *(18)* and *(19)* by using the following variable change $J_i = P_iL_i, X = P_r^{-1}, Y_i = K_iX$. This ends the proof of the theorem1.

Remark 3 *Note that Theorem 1 allows us to obtain both controller gains K_i and observer gains L_i using one step procedure by resolving the LMI constraints given in theorem1 unlike the two-step procedure in*(Zaidi et al., 2013).

Algorithm 1 *The following algorithm gives the necessary steps to use the result of Theorem1:*

- Construct the TS fuzzy model *(3)*.
- Define the reference model *(7)*.
- Solve LMIs *(18)* and *(19)* of the theorem1 by finding out the minimal values of η and ζ .
- Get the matrices $P_o, P_r = X^{-1}, R, Q$ then compute the gains $K_i = Y_iP_r$ and $L_i = P_o^{-1}J_i$
- Compute the fuzzy observer given by *(6)* and the fuzzy controller given by *(8)*.

4. BIOLOGICAL PROCESS DESCRIPTION

We consider a generic model of biomass growth. It is a process of microorganisms cultures (yeasts, bacterias, mushrooms,...) fed with carbonated substrate that take place in a bioreactor. The evolution of the biomass and the substrate concentrations respectively $X(t)$ and $S(t)$ in a Continuous stirred-tank reactor(CSTR) is described by the following mass balance model(Bastin and Dochain, 1990), (Denis Dochain & Vanrolleghem, 2015)

$$\begin{aligned} \dot{X}(t) &= \mu(S)X(t) - D(t)X(t) \\ \dot{S}(t) &= -k_1\mu(S)X(t) + D(t)(S_{in} - S(t)) \end{aligned} \qquad (40)$$

where : $D(t)$ the dilution rate, S_{in} is the substrate influent concentration, $\frac{1}{k_1}$ is the biomass/substrate yield and $\mu(S)$ is the specific growth rate.

In the following, the Haldane model will be considered to express the relationship $\mu(S)$. It describes the inhibition phenomenon occurring at high substrate concentration.

$$\mu(S) = \mu^* \frac{S}{K_s + S + S^2 / K_I} \qquad (41)$$

where K_s is Monod's constant, K_I is the inhibition and μ^* is related to the maximum of the specific growth rate μ_{max} such that

$$\mu^* = \mu_{max}(1 + 2\sqrt{\frac{K_s}{K_I}}) \qquad (42)$$

5. TRANSFORMATION OF THE MASS BALANCE MODEL INTO A T-S MODEL

To rewrite the biomass growth model *(40)* in the affine form *(1)*, we consider that:

$$\begin{aligned} D &= D_1 + D_2 \\ S_{in} &= \frac{D_2}{D_1 + D_2} S_{in}^{max} \end{aligned} \qquad (43)$$

where $D_1(t)$ and $D_2(t)$ are respectively the water and the substrate dilution rates. Replacing $D(t)$ and $S_{in}(t)$ by their expressions *(43)* in *(40)*, leads to the affine form as follows:

$$\begin{bmatrix} \dot{X}(t) \\ \dot{S}(t) \end{bmatrix} = \begin{bmatrix} \mu(S) & 0 \\ -k_1\mu(S) & 0 \end{bmatrix} \begin{bmatrix} X \\ S \end{bmatrix} + \begin{bmatrix} -X & -X \\ -S & S_{in}^{max} - S \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \qquad (44)$$

Where

$$x = \begin{bmatrix} X \\ S \end{bmatrix},\ u = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},\ f(x(t)) = \begin{bmatrix} \mu(S) & 0 \\ -k_1\mu(S) & 0 \end{bmatrix}$$

and $g(x(t)) = \begin{bmatrix} -X & -X \\ -S & S_{in}^{max} - S \end{bmatrix}$ The following non linearities are considered and construct the vector ξ :

$$\begin{cases} \xi_1(x(t)) = \mu(S) \\ \xi_2(x(t)) = X \\ \xi_3(x(t)) = S \end{cases} \qquad (45)$$

The nonlinear model (44) can be written in the form:

$$\begin{cases} \dot{x}(t) = A(\xi_1)x(t) + B(\xi_2, \xi_3)u(t) \\ y(t) = Cx(t) \end{cases} \qquad (46)$$

where the obtained matrices have the general form:

$$A(\xi_1) = \begin{bmatrix} \xi_1 & 0 \\ -k_1\xi_1 & 0 \end{bmatrix};\quad B(\xi_2, \xi_3) = \begin{bmatrix} -\xi_2 & -\xi_2 \\ -\xi_3 & S_{in}^{max} - \xi_3 \end{bmatrix}$$

Each time varying parameter ξ_j, $j=1...3$ can be expressed with two models as follows:

$$\xi_j = M_1^j(\xi_j)\xi_j^{min} + M_2^j(\xi_j)\xi_j^{max}$$

$$M_1^j(\xi_j) = \frac{\xi_j - \xi_j^{min}}{\xi_j^{max} - \xi_j^{min}}, \quad M_2^j(\xi_j) = \frac{\xi_j^{max} - \xi_j}{\xi_j^{max} - \xi_j^{min}}$$

where $\xi_j^{min} = \min\{\xi_j\}$ and $\xi_j^{max} = \max\{\xi_j\}$.

3 premise variables are considered. Therefore the model (46) can be represented by $n_r = 2^3 = 8$ sub-models of the form (2) having respectively as a membership function $h_i(\xi)$, $i \in \{1,2...,8\}$ defined by:

$$h_i(\xi) = M_{k_1}^1(\xi_1)M_{k_2}^2(\xi_2)M_{k_3}^3(\xi_3)$$

with $k_1 \in \{1,2\}$, $k_2 \in \{1,2\}$ and $k_3 \in \{1,2\}$. For more details, the reader can find the systematic multimodeling methodology in (Nagy et al., 2010).

6. SIMULATION STUDIES

6.1. Problem statement of a TS observer-based control with reference state model

Using the simulation parameters summarized in Table 1, the matrices (A_i, B_i) of the multimodel are obtained:

Table 1. Simulation parameters.

Parameter	μ^*	K_s	K_I	S_{in}^{max}
Value	6.3	10	0.1	12
Unit	h^{-1}	g/l		g/l

$$A_1 = A_2 = A_3 = A_4 = \begin{bmatrix} 0.3 & 0 \\ -0.6 & 0 \end{bmatrix}$$

$$A_5 = A_6 = A_7 = A_8 = \begin{bmatrix} 0.01 & 0 \\ -0.02 & 0 \end{bmatrix}$$

$$B_1 = B_5 = \begin{bmatrix} -12 & -12 \\ -4 & 8 \end{bmatrix}, B_2 = B_6 = \begin{bmatrix} -12 & -12 \\ -0.2 & 11.8 \end{bmatrix} \tag{47}$$

$$B_3 = B_7 = \begin{bmatrix} -0.2 & -0.2 \\ -4 & 8 \end{bmatrix}, B_4 = B_8 = \begin{bmatrix} -0.2 & -0.2 \\ -0.2 & 11.8 \end{bmatrix}$$

For the simulation, it is considered that the substrate is the output of the system and the only variable accessible to the measurement ($C = \begin{bmatrix} 0 & 1 \end{bmatrix}$) and the matrix $A_{ref} = diag\begin{pmatrix} -0.13 & -0.33 \end{pmatrix}$ in the reference model to generate the reference trajectories.

By using the Algorithm 1, the resolution of the LMIs given in Theorem 1 leads to the following matrices with $\lambda = 23, \eta = 0.6,$ and $\zeta = 0.6$

$$P_o = \begin{bmatrix} 200.6387 & 4.0355 \\ 4.0355 & 204.2312 \end{bmatrix}, \quad P_r = \begin{bmatrix} 0.0839 & 0.0001 \\ 0.0001 & 0.0320 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.72 & 0 \\ 0 & 0.72 \end{bmatrix}, \quad Q = \begin{bmatrix} 1.9083 & -0.3956 \\ -0.3956 & 67.7284 \end{bmatrix}$$

$$\nu = \sqrt{\eta} = 0.7746, \text{ and } \gamma = \sqrt{\zeta} = 0.7746$$

The controller gains:

$$K_1 = \begin{bmatrix} 2.7176 & 1.6494 \\ 0.6280 & -1.6638 \end{bmatrix}, K_2 = \begin{bmatrix} 2.7173 & 1.6493 \\ 0.6279 & -1.6640 \end{bmatrix}$$

$$K_3 = K_4 = K_5 = \begin{bmatrix} 2.7173 & 1.6501 \\ 0.6279 & -1.6640 \end{bmatrix}$$

$$K_6 = \begin{bmatrix} 2.7174 & 1.6496 \\ 0.6280 & -1.6640 \end{bmatrix}, K_7 = K_8 = \begin{bmatrix} 2.7175 & 1.6496 \\ 0.6280 & -1.6640 \end{bmatrix}$$

The observer gains:

$$L_1 = L_2 = L_3 = L_4 = \begin{bmatrix} -0.6425 \\ 2.3155 \end{bmatrix}$$

$$L_5 = L_6 = L_7 = L_8 = \begin{bmatrix} -0.0599 \\ 2.3032 \end{bmatrix}$$

6.2. The simulation results

All the simulations are applied on the nonlinear mass balance model (40) used as a simulator for the real process. The evolution of the state variables, respectively the biomass $X(t)$ and the substrate $S(t)$, their estimated values $\hat{X}(t)$ and $\hat{S}(t)$ and the reference model state $X_r(t)$ and $S_r(t)$ are compared and shown in the figure 2. We can easily notice that the observer produces a good estimation and the proposed controller allows to obtain a satisfying trajectory tracking.

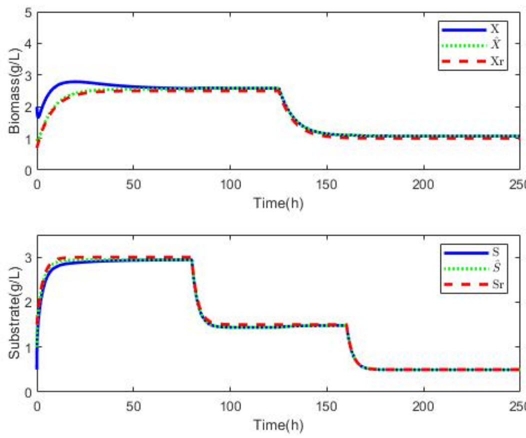

Fig. 2. Comparison between the state variables, their estimates and the reference model.

The manipulated variables, the dilution rate $D(t)$ and the influent substrate $S_{in}(t)$ are presented in figure 3. It is clearly shown that the controller outputs change smoothly whenever the setpoints change to keep the state variables close as possible to the reference trajectories.

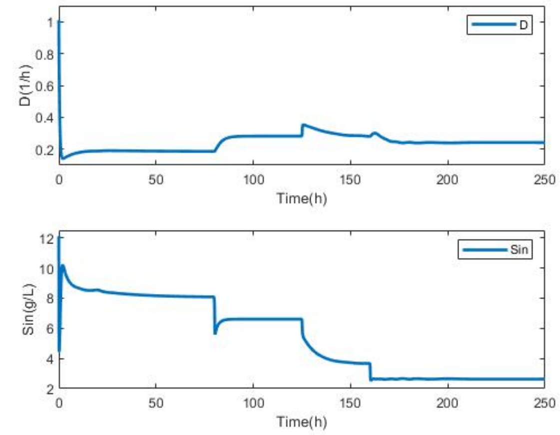

Fig. 3. The evolution of the control variables.

The figure 4 shows the evolution of the ratio of the energy of $e_r^T(t)\bar{Q}e_r(t)$ to the energy of $\Omega^T(t)\Omega(t)$ corresponding to controller and the ratio of the energy of $e_o^T(t)Re_o(t)$ to the energy of $\omega^T(t)\omega(t)$ corresponding to the observer.

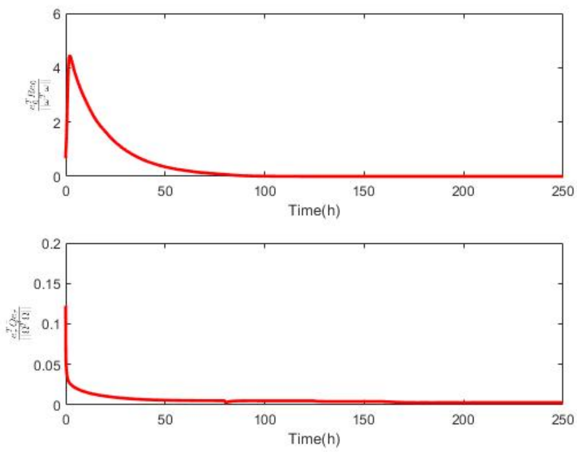

Fig. 4. The ratio of the energy of $e_r^T(t)\bar{Q}e_r(t)$ to the energy of $\Omega^T(t)\Omega(t)$ corresponding to controller and the ratio of the energy of $e_o^T(t)Re_o(t)$ to the energy of $\omega^T(t)\omega(t)$ corresponding to the observer.

Clearly both of the energy ratio of the controller and the observer converge toward zero which means that the obtained observer and controller ensure a good H_∞ performance. For comparison purposes, the same simulations are carried by applying the method proposed in (Ghorbel et al., 2014). The obtained results are shown in figures 5 and 6 respectively for the control and state variables.

It is clear that even if the controller and the observer converge and the tracking objective is achieved (figure 6), the cost in term of control is too high (max $(\boldsymbol{D(t)}) = \boldsymbol{20h^{-1}}$) see the transient regime in figures 5). The values of the dilution rate $\boldsymbol{D(t)}$ and the influent substrate $\boldsymbol{S_{in}(t)}$ concentration are too high exceeding the physical constraints and become aberrant ($\boldsymbol{min}(\boldsymbol{S_{in}(t)}) = \boldsymbol{-14g/l}$ negative). There is a lost of the physical meaning of the manipulated variables, the positivity constraint are no longer respected.

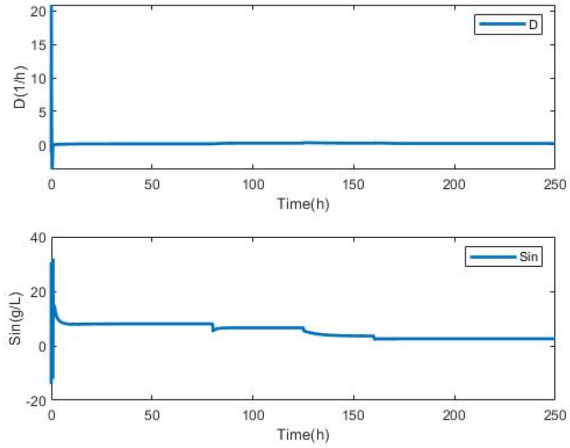

Fig. 5. The evolution of the control variables obtained by application of theorem in (Ghorbel et al., 2014).

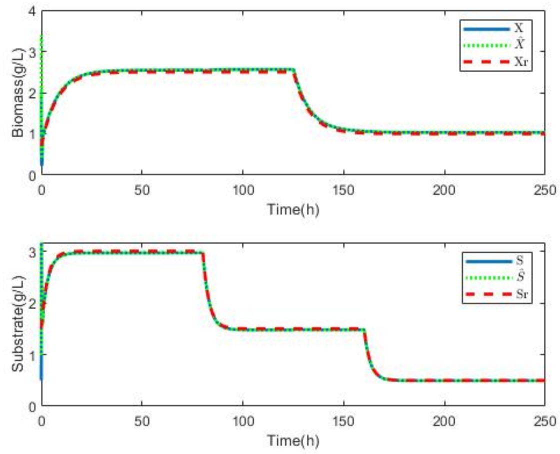

Fig. 6. Comparison between the state variables, their estimates and the reference model obtained by application of the theorem in (Ghorbel et al., 2014).

The attenuation level of the disturbance obtained by application of the method in (Ghorbel et al., 2014) $\gamma = 4.0845$ is too high compared to the obtained value with our proposed method $\gamma = 0.7746$. We can conclude that the stability conditions developed in the proposed approach are less difficult to satisfy, leading to more realistic results and also better in terms of disturbance rejection.

6.3. Feasibility test

An exhaustive numerical comparison through a feasibility test among the condition proposed in (Ghorbel et al., 2014) and the condition stated in Theorem 1 has been performed for the obtained TS model. The matrices A_i and B_i given in (47) are considered with two free parameters. The entries A_{i21} and B_{i22} are respectively multiplied by two real numbers a and b uniformly distributed in the interval $[-3 \quad 3]$. The resolution of the LMIs is done using the YALMIP tool.

Figure 7 shows the obtained results. It is clearly seen that the feasibility of LMIs given in theorem1 is more expanded than the ones in (Ghorbel et al., 2014). This demonstrates that the conditions given in theorem1 are less conservative comparing to those given (Ghorbel et al., 2014) and of the effectiveness of the presented method.

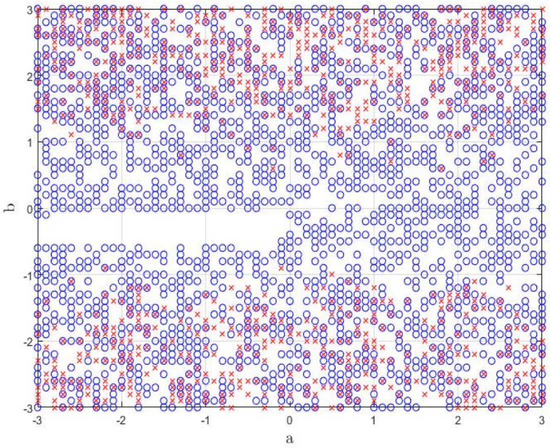

Fig. 7. Test of feasibility: blue circles for Theorem 1 and red stars for Theorem of (Ghorbel et al., 2014).

7. CONCLUSION

In this work, we've proposed a reference model fuzzy tracking controller based on a state observer using the TS formulation with unmeasurable premise variables. Using Lyapunov theory and the H_∞ performance criteria for the stability analysis, we've elaborated less restrictive sufficient conditions. The proposed method allows calculating in a one-step procedure the observer and the controller gains by solving a set of new relaxed LMIs. The efficiency of the suggested method is shown via simulation subject to a biological process where the goal is achieved regarding the tracking control and the state estimation.

REFERENCES

Abyad, M., Karama, A., and Khallouq, A. (2018a). A comparative analysis of two controllers for trajectory tracking control: Application to a biological process. *Advances in Science, Technology and Engineering Systems*, *3*(4), 318–326.

Abyad, M., Karama, A., and Khallouq, A. (2020). Takagi-Sugeno tracking control of a fermentation process with respect to asymmetric constraints. *International Journal of Adaptive Control and Signal Processing*, *34*(2), 266–282.

Abyad, M., Karama, A., and Khallouq, A. (2018b). Fault-Tolerant Tracking Control for a Biological Process: Multiple Lyapunov Functions Approach. *2018 7th International Conference on Systems and Control, ICSC 2018*, 391–396.

Alcaraz-González, V., López-Bañuelos, R. H., Steyer, J.-P., Méndez-Acosta, H. O., González-Álvarez, V., and Pelayo-Ortiz, C. (2012). Interval-Based Diagnosis of Biological Systems - a Powerful Tool for Highly Uncertain Anaerobic Digestion Processes. *CLEAN - Soil, Air, Water*, *40*(9), 941–949.

Bastin, G., and Dochain, D. (1990). *On-line Estimation and Adaptive Control of Bioreactors*. Elsevier.

Ben Youssef, C., Dahhou, B., Zeng, F. Y., and Rols, J. L. (1996). Estimation and filtering of nonlinear systems: application to a wastewater treatment process. *International Journal of Systems Science*, *27*(5), 497–505.

Benattia, S. E., Tebbani, S., and Dumur, D. (2020). Linearized min-max robust model predictive control: Application to the control of a bioprocess. *International Journal of Robust and Nonlinear Control*, *30*(1), 100–120.

Benattia, S. E., Tebbani, S., and Dumur, D. (2016). A linearized robust model predictive control applied to bioprocess. *2016 IEEE 55th Conference on Decision and Control (CDC)*, 4046–4052.

Benzaouia, A., and El Hajjaji, A. (2014). Advanced Takagi–Sugeno Fuzzy Systems. In *Studies in Systems, Decision and Control* (Vol. 8). Springer International Publishing.

Benzaouia, A., and El Hajjaji, A. (2018). Conditions of Stabilization of Positive Continuous Takagi–Sugeno Fuzzy Systems with Delay. *International Journal of Fuzzy Systems*, *20*(3), 750–758.

Bernard, O., and Gouzé, J.-L. (n.d.). State Estimation for Bioprocesses. *In Dochain, D., Editor, Automatic Control of Bioprocesses, 2008, Wiley-ISTE.*

Bernard, O., and Quelinnec, I. (2008). Dynamic Models of Biochemical Processes: Properties of Models. In D. Dochain (Ed.), *Bioprocess Control* (pp. 17–45). ISTE.

Buckley, J. J. (1992). Universal fuzzy controllers. *Automatica*, *28*(6), 1245–1248.

Chang, G.-H., and Wu, J.-C. (2012). Robust Tracking Control Design for Nonlinear Systems via Fuzzy Observer. *2012 Fifth International Symposium on Computational Intelligence and Design*, 366–369.

Chang, W.-J., Lin, Y.-H., Du, J., and Chang, C.-M. (2019). Fuzzy Control with Pole Assignment and Variance Constraints for Continuous-time Perturbed Takagi-Sugeno Fuzzy Models: Application to Ship Steering Systems. *International Journal of Control, Automation and Systems*, *17*(10), 2677–2692.

Chung-Chun Kung, and Hai-Huang Li. (2002). Tracking control of nonlinear systems by fuzzy model-based controller. *Proceedings of 6th International Fuzzy Systems Conference*, *2*, 623–628.

Chung-Shi Tseng, Bor-Sen Chen, and Huey-Jian Uang. (2001). Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model. *IEEE Transactions on Fuzzy Systems*, *9*(3), 381–392.

Dahhou, B., Roux, G., and Chamilothoris, G. (1992). Modelling and adaptive predictive control of a continuous fermentation process. *Applied Mathematical Modelling*, *16*(10), 545–552.

Dochain, Denis, and Vanrolleghem, P. (2015). Dynamical Modelling & Estimation in Wastewater Treatment Processes. In *Water Intelligence Online* (Vol. 4). IWA publishing.

Evans, J. L., Goldfine, I. D., Maddux, B. A., and Grodsky, G. M. (2003). Are Oxidative Stress-Activated Signaling Pathways Mediators of Insulin Resistance and -Cell Dysfunction? *Diabetes*, *52*(1), 1–8.

Fantuzzi, C., and Rovatti, R. (1996). On the approximation capabilities of the homogeneous Takagi-Sugeno model. *IEEE International Conference on Fuzzy Systems*, *2*, 1067–1072.

Ghorbel, H., El Hajjaji, A., Souissi, M., and Chaabane, M. (2014). Robust Tracking Control for Takagi–Sugeno Fuzzy Systems With Unmeasurable Premise Variables: Application to Tank System. *Journal of Dynamic Systems, Measurement, and Control*, *136*(4).

Guerra, T. M., Kruszewski, A., Vermeiren, L., and Tirmant, H. (2006). Conditions of output stabilization for nonlinear models in the Takagi–Sugeno's form. *Fuzzy Sets and Systems*, *157*(9), 1248–1259.

Haidegger, T., Kovács, L., Precup, R. E., Preitl, S., Benyó, B., and Benyó, Z. (2011). Cascade control for telerobotic systems serving space medicine. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, *44*(1 PART 1), 3759–3764.

Haidegger, T., Kovács, L., Preitl, S., Precup, R. E., Benyó, B., and Benyó, Z. (2011). Controller design solutions for long distance telesurgical applications. *International Journal of Artificial Intelligence*, *6*(11 S), 48–71.

Ioan, D., and Mihai, C. (2001). Optimal De-Batch Bioprocess Control Via Intelligent System Based on Hybrid Techniques. *Journal of Control Engineering and Applied Informatics*, *3*(1), 5–14.

Karama, A., Bernard, O., and Gouzé, J.-L. (2010). Constrained Hybrid Neural Modelling of Biotechnological Processes. *International Journal of Chemical Reactor Engineering*, *8*(1).

Khallouq, A., and Karama, A. (2017). Multimodel modelization and stabilization of a bioreactor. *2017 International Conference on Electrical and Information Technologies (ICEIT)*, 1–5.

Liu, Y.-J., Tong, S.-C., and Li, T.-S. (2011). Observer-based adaptive fuzzy tracking control for a class of uncertain nonlinear MIMO systems. *Fuzzy Sets and Systems*, *164*(1), 25–44.

Lo, J. C., and Lin, M. L. (2004). Observer-based robust H∞ control for fuzzy systems using two-step procedure. *IEEE Transactions on Fuzzy Systems*, *12*(3), 350–359.

Miranda-Colorado, R., and Aguilar, L. T. (2020). Robust PID control of quadrotors with power reduction analysis. *ISA Transactions*, *98*, 47–62.

Montiel-Escobar, J. L., Alcaraz-González, V., Méndez-Acosta, H. O., and González-Álvarez, V. (2012). ADM1-Based Robust Interval Observer for Anaerobic Digestion Processes. *CLEAN - Soil, Air, Water*, *40*(9), 933–940.

Nagy, A. M., Mourot, G., Marx, B., Ragot, J., and Schutz, G. (2010). Systematic Multimodeling Methodology Applied to an Activated Sludge Reactor Model. *Industrial & Engineering Chemistry Research*, *49*(6), 2790–2799.

Owczarkowski, A., Horla, D., and Zietkiewicz, J. (2019). Introduction of Feedback Linearization to Robust LQR and LQI Control – Analysis of Results from an Unmanned Bicycle Robot with Reaction Wheel. *Asian Journal of Control*, *21*(2), 1028–1040.

Saifia, D., Chadli, M., Labiod, S., and Guerra, T. M. (2020). Robust H ∞ static output-feedback control for discrete-time fuzzy systems with actuator saturation via fuzzy Lyapunov functions. *Asian Journal of Control*, *22*(2), 611–623.

Szelitzky, T., Inoan, I., and Dumitrache, D. C. (2011). Advantages of robust control for series load frequency controlled induction heating inverters. *Control Engineering and Applied Informatics*, *13*(1), 62–68.

Takagi, T., and Sugeno, M. (1985). Fuzzy Identification of Systems and Its Applications to Modeling and Control. *IEEE Transactions on Systems, Man and Cybernetics*, *SMC-15*(1), 116–132.

Tanaka, K., and Wang, H. O. (2001). Fuzzy Control Systems Design and Analysis. In *Fuzzy Control Systems Design and Analysis*. John Wiley & Sons, Inc.

Teanu, D. S. Ş., and Petre, E. (2004). Some on adaptive control of a wastewater biodegradation process. *Journal of Control Engineering and Applied Informatics*, *6*(3), 48–56.

Tong, S., and Li, H.-H. (2002). Observer-based robust fuzzy control of nonlinear systems with parametric uncertainties. *Fuzzy Sets and Systems*, *131*(2), 165–184.

Wang, J.-W., Wu, H.-N., Guo, L., and Luo, Y.-S. (2013). Robust fuzzy control for uncertain nonlinear Markovian jump systems with time-varying delay. *Fuzzy Sets and Systems*, *212*, 41–61.

Wang, Y., Wang, R., Xie, X., and Zhang, H. (2018). Observer-based H ∞ fuzzy control for modified repetitive control systems. *Neurocomputing*, *286*, 141–149.

Xie, W.-B., Li, H., Wang, Z.-H., and Zhang, J. (2019). Observer-based Controller Design for A T-S Fuzzy System with Unknown Premise Variables. *International Journal of Control, Automation and Systems*, *17*(4), 907–915.

Zaidi, I., Tadeo, F., and Chaabane, M. (2013). Positive observer based tracking controller design for Takagi-Sugeno systems. *10th International Multi-Conferences on Systems, Signals & Devices 2013 (SSD13)*, 1–8.

Zhang, H., Yan, H., Yang, J., Hu, Z., and Wang, X. (2017). The Properties of MRFs Based on Carbonyl Iron Particles Modified by Nano-Sized Silica and Phosphate Coating Layer. *Arabian Journal for Science and Engineering*, *42*(11), 4713–4723.