Stabilisation of Dynamic Positioning Ships Based on Sampled-data Control

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Abstract: This study investigates the sampled-data stabilization problem for a dynamic positioning ship (DPS). Firstly, the DPS is converted to a system with time delay, and the upper and lower bounds of the delay are considered. By introducing a new Lyapunov-Krasovskii functional (LKF), some delay dependent stability criteria are obtained by using linear matrix inequalities (LMIs). When estimating the derivative of LKF, a relaxation variable is introduced by the method of reciprocally convex inequality. Thus, it is proved that the conservatism reduction is obvious. Finally, a specific example is given to prove that the designed sampled-data controller can ensure the states of the DPS under the influence of external interference.

Keywords: dynamic positioning ship; sampled-data control; input delay approach; reciprocally convex inequality

1. INTRODUCTION

Dynamic positioning ship (DPS) is a vessel which uses propulsion device to resist the influence of wind, wave, current without the aid of mooring system; maintain a certain target position on the sea surface with a certain attitude and complete various operation functions. It has stronger mobility and higher positioning accuracy than the vessel with anchor moored. It has widely used in offshore oil drilling platforms, salvage ships, engineering ship and so on (see T. I. Fossen, 2002). At present, the issue about PID control, H_∞ control, linear quadratic optimal control, backstep integral control, fuzzy control, hybrid switching control and so on have been reported for DPS (see Sorensen, 2011; Ngongi et al., 2015; Du et al., 2016; Bidikli et al., 2017; Fu et al., 2018; Zhang et al., 2018; Peng et al., 2019).

In the past few years, sampled-data systems have become a hot topic (see Delchev et al., 2014; Abedi, 2015; Wang et al., 2018). The outstanding feature of the systems is the coexistence of continuous signal and discrete signal, which is difficult to be analyzed and designed. Recently, the control analysis of sampled-data systems have attracted much attention, and various viewpoints have been given to solve the control problems (see Fridman, 2006; Rubagotti et al., 2011; Lee, 2012; Wu, 2013; Chen et al., 2014; Abedi, 2015; Chen, 2015; Moarref, 2016; Liaquat, 2016; Wang et al., 2019).

Recently, the main design methods for analysing the sampled-data systems can be divided into three kinds. The first one is lifting technique (see Ding et al., 2005), which converted a continuous system to a discrete system of equal finite dimension. The second one is jump system methods (see Hu et al., 2006), where the performance analysis of sampling system is transformed into the solution of two Riccati equations with mutual jump. However, the two methods mention above can’t solve the uncertainty problems caused by the sampling time or system matrix. The third one is input delay method (see Liu et al., 2012), which is expressed as a continuous time control model with time-varying delay, and then the stability analysis can be accomplished using the method of delay system. Besides, the input delay approach can deal with the uncertainty of system parameters, and has been widely adopted in analysing the sampled-data system (see Shen et al., 2012; Wu et al., 2013; Yucel et al., 2017) and practical engineering field such as autonomous airship, high speed train, etc (see Li et al., 2013; Wang et al., 2014).

The DPS is also a sampled-data system (see Zheng et al., 2017), first of all, it uses various digital sensors, such as DGPS to obtain more accurate and reliable positioning data; the electric compass to determine the ship's heading; the wind speed sensor to predict the change of wind speed, etc., then the signals produced by the sensors have been sampled by the digital computer to determine the motion of the ship. In recent year, sampled-data control theory has been adopted for solving the DP problem. In (Katayama, 2010), by using integrator backstepping technique, the nonlinear sampled-data control problem for DPS is discussed, and the output and state feedback controllers are designed. In (Katayama et al., 2014), the nonlinear sampled-data control theory is used for DPS to deal with the problem about straight-line trajectory tracking. In (Zheng et al., 2017), the sampled-data controllers for DPS are designed to make the system exponentially stable and achieve good H_∞ performance. In (Zheng et al., 2018), the fuzzy control issues for the nonlinear sampled-data DPS are discussed, and the free-weighting matrices are used for guaranteeing the system’s stability. (Yang et al., 2018) discusses the robust fault-tolerant control issue for sampled-data DPS with actuator-failure. And a controller’s error integral is designed in the fault-tolerant model.

However, there still exist enough research room for sampled-data DPS. Firstly, in these papers, the lower bounds of the delay have strict limitations which will lead to considerable conservatism, because the value of the delay may change in a range. Therefore, the delay bound of the lower and upper should be both considered. On the other hand, the LKF for sampled-data DPS have enough room for improving to get less conservative result, such as adding novel item to LKF or adopting new approaches to estimate the derivative of the LKF.

In this paper, the stabilization of sampled-data DPS is studied. Firstly, the motion model of DPS is established, and then it is transformed into a time-delay system by using input delay method. Then the upper and lower bounds of delay are considered. LMI and Lyapunov theorem are introduced for stability analysis. By introducing the convex reciprocal inequality, the less conservative results can be obtained. Then the design method of sampled-data controller is introduced, which makes the system achieve good performance under external interference. Finally, a specific example is given to prove the effectiveness of this method.

2. PROBLEM FORMULATION

2.1 System modelling

To describe the horizontal plane motion of DPS, the coordinates are established (see Fig.1). X_oOY_o is the earth-fixed inertial frame. The OX_O axis points to the north and the OY_O axis points to the East. XaY is body-fixed frame, and aX axis points to bow and aY axis points to starboard. The XY plane coincides with the static water surface.

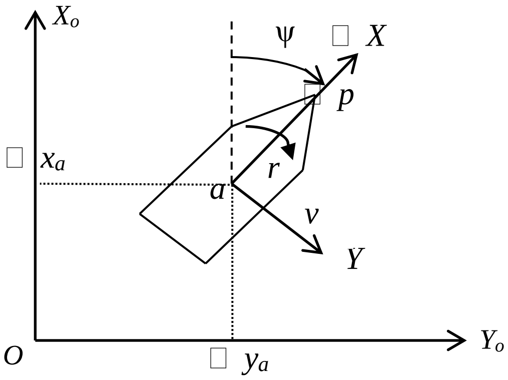


Fig. 1. The earth-fixed and body-fixed frame.

(x_a, y_a) represents the position vector and ψ is yaw angle in earth-fixed frame. p, v, r denote the velocity of surge, sway and yaw velocity respectively in body-fixed frame.

In this paper, only the DPS motion in the horizontal plane is considered, which means that only three degrees of freedom motion need to be considered: surge, sway and yaw. The effects of heave, roll and pitch motions are ignored, because they have little effect on the motion of the horizontal plane.

Define the input variable $\eta = [x_a, y_a, \psi]^T$ and $\upsilon = [p, v, r]^T$. Then the relationship can be obtained as follow

$$\dot{\eta} = J(\psi)\upsilon \tag{1}$$

where $J(\psi)$ denotes the transformation matrix,

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

According to (Sorensen et al., 2011), the motion equations of the DPS are considered as follow.

$$M\dot{\upsilon} + D\upsilon = \tau + w \tag{3}$$

where τ denotes the control input; w denotes the extern disturbance; M represent the inertia matrix, D represent damping matrix, which are given that

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix}$$

$$D = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix}$$

where m represents the mass of the ship; I_z is the moment of inertia; x_G is the distance from the origin of the ship's coordinate system to the center of gravity; $X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}}$ are the added mass of hydrodynamic forces in the three directions of surge, sway and yaw; $Y_{\dot{r}}$ is the added mass due to the coupling of sway and yaw;

Assumed that yaw angle ψ is small enough, which means that

$\cos(\psi) \approx 1, \sin(\psi) \approx 0$, then

$$J(\psi) \approx I. \tag{4}$$

Under the assumption (4), and define

$$x(t) = [\eta, \upsilon]^T = [x_a, y_a, \psi, p, v, r]^T,$$

then the equations (1) and (3) are converted to

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{5}$$

where

$$A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times3} \\ M^{-1} \end{bmatrix}, \quad B_w = \begin{bmatrix} 0_{3\times3} \\ M^{-1} \end{bmatrix},$$

Choosed the controlled output $y(t)$ as

$$y(t) = \eta, \tag{6}$$

From (5) and (6), one has

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_w w(t) \\ y(t) &= Cx(t) \end{aligned} \tag{7}$$

where $C = [I_{3\times3} \quad 0_{3\times3}]$.

2.2 Sampled-data process

Assumed that the signal of DPS can be obtained at sampling instant t_k with $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$ .The proposed sampled-data scheme is depicted in Fig. 2.

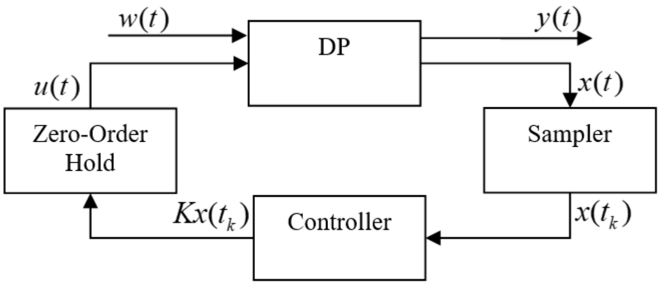


Fig. 2. The schematic of sampled-data scheme for DP.

The sampling period is assumed that

$$h_1 \le t_{k+1} - t_k \le h_2, \quad \forall k \ge 0, h_2 > h_1 \ge 0, \tag{8}$$

and the reliable controller is considered as follow

$$u(t) = Kx(t_k), \quad t_k \le t < t_{k+1}, \tag{9}$$

where K is a compensator matrix. Combining (9) and (1), then

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t_k) + B_w w(t), \quad t > 0 \\ y(t) = Cx(t) \end{cases} \tag{10}$$

The paper's purpose is designing a sampled-data controller to satisfy the following additions:

1) The system (10) with $w(t) = 0$ is asymptotically stable.

2) Despite the extern disturbances, the output signal $y(t)$ satisfies that $\| y(t) \|_2 \le \gamma \| w(t) \|_2$ for all nonzero $w(t) \in L_2[0, \infty)$ under zero condition, where $\gamma > 0$.

2.3 System conversion based on the input delay approach

To solve the sampled-data stabilization problem of system (10), the input delay approach has been used in this section. Firstly, the controller (9) can be converted

$$u(t) = Kx(t - (t - t_k)), \tag{11}$$

Let

$$d(t) = t - t_k,$$

then

$$u(t) = u(t - d(t)), \tag{12}$$

where $d(t)$ is piecewise-linear and satisfies that

$$\begin{aligned} &h_1 \le d(t) \le h_2, \quad h_1 \ge 0, h_2 \ge h_1, \\ &\dot{d}(t) = 1 \quad ,t \ne t_k \end{aligned} \tag{13}$$

Then the sampled-data system (10) is transformed into the following time-varying delay system:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - d(t)) + B_w w(t), \quad t > 0 \\ y(t) = Cx(t) \end{cases} \tag{14}$$

Remark 1: Input delay approach is an effective way to solve the sampled-data stabilization problem for DPS (see Fridman, 2010), the advantage is that the sampling distances could be non-constant, and it can solve the stabilization problem with uncertainty of system parameters easily.

Remark 2: The upper and lower bounds of the time delay are considered, and it is more significant than (Zheng et al., 2017; Fridman, 2010), which have strict limitations on the lower bound of the delay. Let $h_1 = h_2 = h,$, Theorem 1 can deal with the sampled-data stabilization problem for the references. Thus, the proposed method has wider application scopes than the references.

3. STABILITY ANALYSIS AND CONTROLLER DESIGN FOR SAMPLED-DATA DPS

In this section, the sufficient stability criteria for sampled-data DPS is exhibited by establishing LKF and formulating in terms of LMI. Then the sampled-data controller is designed.

3.1 Linear matrix inequality

Most of the control problems are solved by Riccati equation, but there are a lot of parameters need to be adjusted in advance, and LMI can make up for the above short comings of Riccati equation. When solving LMI, it is not necessary to adjust any parameters in advance.

The general form of LMIs is that:

$$F(x) = F_0 + x_1 F_1 + \cdots + x_n F_n < 0 \tag{15}$$

where $x_1, \cdots, x_m$ are decision variables , column vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$ is the decision vector , $F_i = F_i^T \in \mathbb{R}^{n \times n}, i = 0, 1, \cdots, n$ are given real symmetric matrix, the sign '<' indicates that the function is negative definite, and all the *x* which satisfies LMI (15) constitute a convex set.

3.2 Stability analysis

To analyse the stability for the sampled-data DPS, the LKF is constructed as

$$V(t) = \sum_{i=1}^{3} V_i(t), \quad t \in [t_k, t_{k+1}) \tag{16}$$

where

$$
\begin{aligned}
&V_1(t) = x(t)^T Px(t) \\
&V_2(t) = \int_{t-h_1}^{t} x(s)^T Q_1 x(s)ds + \int_{t-d(t)}^{t} x(s)^T Q_2 x(s)ds \\
&\quad + \int_{t-h_2}^{t} x(s)^T Q_3 x(s)ds \\
&V_3(t) = h_1 \int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s)dsd\theta \\
&\quad + h_2 \int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s)dsd\theta \\
&\quad + \delta \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s) R_3 \dot{x}(s)dsd\theta
\end{aligned} \tag{17}
$$

Where $P, Q_1, Q_2, Q_3, R_1, R_2, R_3$ are symmetric matrices with appropriate dimensions. Then the following stability criterion for system (14) is established based on the theorem 1.

Theorem 1: For constant delay $h_2 > h_1 \geq 0$, the system (14) is asymptotically stable when $w(t) = 0$, if there exist matrices $Z, P > 0, Q_i > 0, R_i > 0, i = 1,2,3$, such that the following LMIs satisfy

$$
\begin{bmatrix} R_3 & Z \\ * & R_3 \end{bmatrix} > 0, \tag{18}
$$

$$
\begin{bmatrix}
\Xi_{11} & P^T BK & R_1 & R_2 & A^T \\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & K^T B^T \\
* & * & \Xi_{33} & R & 0 \\
* & * & * & \Xi_{44} & 0 \\
* & * & * & * & -R^{-1}
\end{bmatrix} < 0 \tag{19}
$$

where

$$
\begin{aligned}
&\Xi_{11} = P^T A + A^T P + Q_1 + Q_2 + Q_3 - R_1 - R_2 \\
&\Xi_{22} = -Q_2 - 2R_3 + Z + Z^T \\
&\Xi_{23} = R_3 - Z \\
&\Xi_{24} = R_3 - Z \\
&\Xi_{33} = -Q_1 - R_1 - R_3 \\
&\Xi_{44} = -Q_3 - R_2 - R_3 \\
&R = h_1^2 R_1 + h_2^2 R_2 + \delta^2 R_3 \\
&\delta = h_2 - h_1
\end{aligned}
$$

Proof: See the Appendix.

Remark 3 Compared with (Zheng et al., 2017), if $h_1 = 0, Q_1 = Q_2 = 0, R_2 = R_3 = 0$, then LKF is similar with the one in (Zheng et al., 2017). therefore, the LKF in (Zheng et al., 2017) is a special case of (9).

Remark 4 Compared with (Yoneyama, 2012) and (Zhang et al., 2007), which use more relaxed defining techniques to estimate some integral terms, such as $-\delta\int_{t-h_2}^{t-h_1} \dot{x}^T(s) R_3 \dot{x}(s)ds$, the paper directly uses tighter convex combination inequality to define them. Therefore, less conservative result can be obtained.

3.3 H_∞ performance analysis

Firstly, for all nonzero $w(t) \in L_2[0,\infty)$, the H_∞ performance index function is considered:

$$
J_{yw} = \int_0^t \left[y^T(s)y(s) - \gamma^2 w^T(s)w(s) \right] ds, \quad \gamma > 0 \tag{20}
$$

Then, based on the theorem 2, it will be proved that the system (14) satisfies the H_∞ performance index γ under external disturbances.

Theorem 2: For constant delay $h_2 > h_1 \geq 0$, the system (14) is asymptotically stable and satisfies the H_∞ performance index γ, if there exist matrices $P > 0, Q_i > 0, R_i > 0, i = 1,2,3$ such that the following LMI hold

$$
\begin{bmatrix} R_3 & Z \\ * & R_3 \end{bmatrix} > 0, \tag{21}
$$

$$
\begin{bmatrix}
\Xi_{11} & P^T BK & R_1 & R_2 & P^T B_w & C^T & A^T \\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & K^T B^T \\
* & * & \Xi_{33} & R & 0 & 0 & 0 \\
* & * & * & \Xi_{44} & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & 0 & B_w^T \\
* & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & -R^{-1}
\end{bmatrix} < 0 \tag{22}
$$

where

$$
\begin{aligned}
&\Xi_{11} = P^T A + A^T P + Q_1 + Q_2 + Q_3 - R_1 - R_2 \\
&\Xi_{22} = -Q_2 - 2R_3 + Z + Z^T \\
&\Xi_{23} = R_3 - Z \\
&\Xi_{24} = R_3 - Z \\
&\Xi_{33} = -Q_1 - R_1 - R_3 \\
&\Xi_{44} = -Q_3 - R_2 - R_3 \\
&R = h_1^2 R_1 + h_2^2 R_2 + \delta^2 R_3 \\
&\delta = h_2 - h_1
\end{aligned}
$$

Proof: See the Appendix.

Remark 5: Theorem 1 and 2 provide the stability conditions for the sample-data DPS. These conditions are expressed by LMI, and they are easily to be calculated by MATLAB toolbox. Besides, the method proposed in this paper is also suitable for the other ships.

3.4 Sampled-data controller design for DPS

The sampled-data controller (9) is designed for stabilizing the system (14) according to the following theorem.

Theorem 3: For scales $h_2 > h_1 \geq 0, \gamma > 0$, the system (14) is asymptotically stable with H_∞ performance index γ, if there exist matrices $P > 0, Q_i > 0, R_i > 0, i = 1,2,3$, such that the following LMI are satisfied:

$$\begin{bmatrix} \bar{R}_3 & \bar{Z} \\ * & \bar{R}_3 \end{bmatrix} > 0, \tag{23}$$

$$\begin{bmatrix} \bar{\Xi}_{11} & B\bar{K} & \bar{R}_1 & \bar{R}_2 & B_w & \bar{P}C^T & \bar{P}A^T \\ * & \bar{\Xi}_{22} & \bar{\Xi}_{23} & \bar{\Xi}_{24} & 0 & 0 & \bar{K}^T\bar{B}^T \\ * & * & \bar{\Xi}_{33} & \bar{R} & 0 & 0 & 0 \\ * & * & * & \bar{\Xi}_{44} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & \bar{P}B_w^T \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\bar{P}R^{-1}\bar{P} \end{bmatrix} < 0 \tag{24}$$

where

$$\bar{\Xi}_{11} = A\bar{P} + \bar{P}A^T + \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 - \bar{R}_1 - \bar{R}_2$$
$$\bar{\Xi}_{22} = -\bar{Q}_2 - 2\bar{R}_3 + \bar{Z} + \bar{Z}^T$$
$$\bar{\Xi}_{23} = \bar{R}_3 - \bar{Z}$$
$$\bar{\Xi}_{24} = \bar{R}_3 - \bar{Z}$$
$$\bar{\Xi}_{33} = -\bar{Q}_1 - \bar{R}_1 - \bar{R}_3$$
$$\bar{\Xi}_{44} = -\bar{Q}_3 - \bar{R}_2 - \bar{R}_3$$
$$\bar{R} = h_1^2\bar{R}_1 + h_2^2\bar{R}_2 + \delta^2\bar{R}_3$$
$$\delta = h_2 - h_1$$

Then, the controller gain matrix K can be obtained

$$K = \bar{K}\bar{P}^{-1} \tag{25}$$

Proof: By noticing that $-\bar{P}\bar{R}^{-1}\bar{P} \le \bar{R} - 2\bar{P}$, let $\eta = diag\left\{P^{-T}, P^{-T}, P^{-T}, P^{-T}, I, I, I\right\}$. Defining

$$\bar{P} = P^{-1}, \bar{K} = KP^{-1}, \bar{R} = P^{-T}RP^{-1}, \bar{Z} = P^{-T}ZP^{-1},$$
$$\bar{R}_i = P^{-T}R_iP^{-1}, \bar{Q}_i = P^{-T}Q_iP^{-1}, i = 1,2,3$$

Pre- and post-multiplying (22) by η and η^T respectively, then according to Schur complement, (24) can be obtained. This completed the proof.

Remark 6: Theorem 3 applies the designed sampled-data control laws to the DPS system (14). According to Theorem 3, when the sampled-data controller matrix K and the design parameter matrices $P, Z, Q_1, Q_2, Q_3, R_1, R_2, R_3$ are selected appropriately, the sampled-data DPS system (14) is globally asymptotically stable. Besides, the designed sampled-data controller will make the position, yaw angle and velocity of the DPS globally asymptotically stable, which accomplishes the sampled-data control target of the DPS.

4. NUMERICAL EXAMPLES

To verify the effectiveness of the developed strategy, a DPS model is taken as the simulation object. Considered the ship model parameters in Table1 (see Sorensen, 2011), which can be shown in Fig.3.

Table 1. Ship model parameters.

Parameter	Unit	Value
Length overall (L_{OA})	m	3.65
Length waterline (L_{WA})	m	3.645
Length perpendicular (L_{PP})	m	3.40
Breadth moulded (B)	m	0.86
Depth moulded (D)	m	0.40
Draught (T)	m	0.33
Displacement	kg	710
Center of gravity position	m	0.20

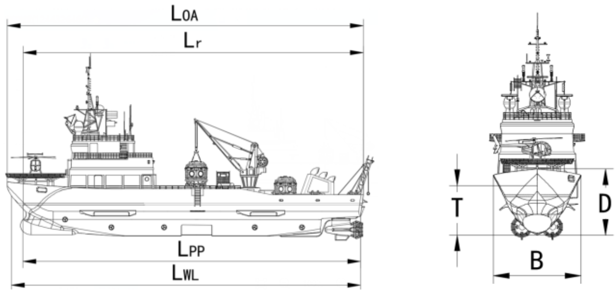


Fig. 3. Main dimensions of ship model.

The inertia matrix M and damping coefficient matrix D are

$$M = \begin{bmatrix} 0.754 & 0 & 0 \\ 0 & 1.199 & 0.211 \\ 0 & 0.029 & 0.524 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.014 & 0 & 0 \\ 0 & 0.102 & -0.024 \\ 0 & 0.192 & 0.095 \end{bmatrix}.$$

Noted that

$$A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$

Remark 7: In actual DPS system, the control signal vector $\boldsymbol{\tau} = \left[\tau_1, \tau_2, \tau_3\right]^T$, which is provided by the DP controller, is composed of surge force τ_1 (unit: N), sway force τ_2 (unit: N), and yaw moment τ_3 (unit: Nm). Fig.4 shows the relation of thrust control. From the fig.4, the scenario can be shown that:

Firstly, the DP controller gives the thrust command vectors of pitch, sway and yaw to thrust distribution; Secondly, the thrust distribution assigns these command vectors to each propeller and obtains the thrust command T_d (unit: N) of each propeller; Finally, according to the thrust characteristics of the thruster, the speed (pitch) n_d (unit: RPS (revolutions per second)) corresponding to the thrust command is obtained, and then the thruster is controlled and the desired position and velocity are produced.

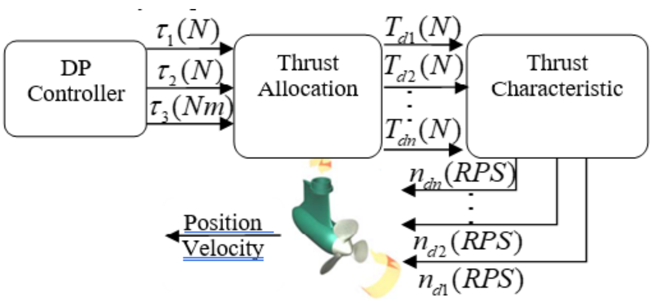


Fig. 4. the relation of thrust control.

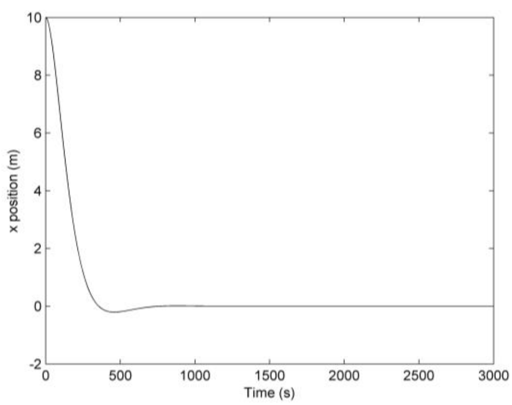
$$A=\begin{bmatrix} 0 & 0 & 0 & 1 & -0.0349 & 0 \\ 0 & 0 & 0 & 0.0349 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0186 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0208 & 0.0342 \\ 0 & 0 & 0 & 0 & -0.3653 & -0.1832 \end{bmatrix},$$

$$B=B_w=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1.3263 & 0 & 0 \\ 0 & 0.8422 & -0.3391 \\ 0 & -0.0466 & 1.9272 \end{bmatrix}.$$

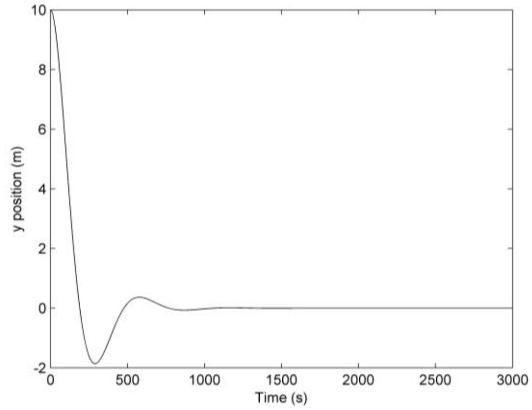
The ship's initial state is $x_s(t)=\begin{bmatrix}1 & 1 & 0.1 & 0 & 0 & 0\end{bmatrix}^T$. The expected target of the ship is $x_d(t)=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$. Assuming the sampling interval of computer is $d_1=0.8s, d_2=1.6s,$ then the H_∞ performance index is obtained that $\gamma_{\min}=2.945$ .Then, the controller gain can be computed that

$$K=\begin{bmatrix} -0.2438 & -0.0109 & -0.0008 & -0.5459 & -0.0105 & 0.0002 \\ 0.0072 & -0.4429 & -0.1018 & 0.0186 & -0.5794 & -0.1722 \\ -0.0011 & 0.0088 & -0.2082 & -0.0022 & 0.2673 & -0.5638 \end{bmatrix}$$

The simulation results are shown in Fig. 5-6., Fig.5 is the responses curve of the position (x, y) and yaw angle, Fig.6 is the responses curve of the ship's velocity, which show that the proposed control strategy can make the DPS stabile and reach the desired target without extern disturbance.



(a)



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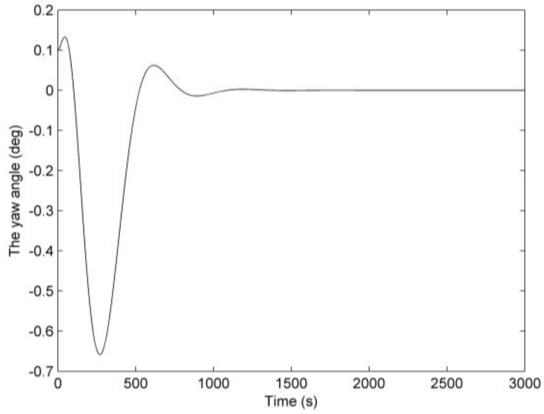
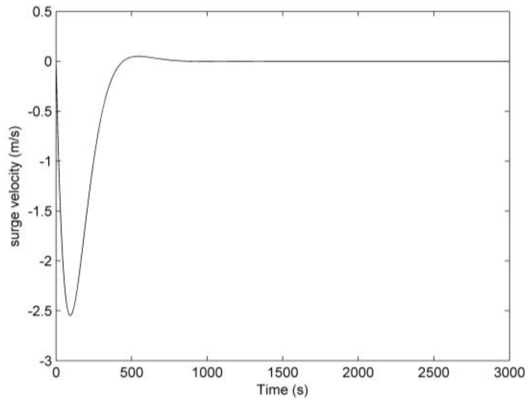
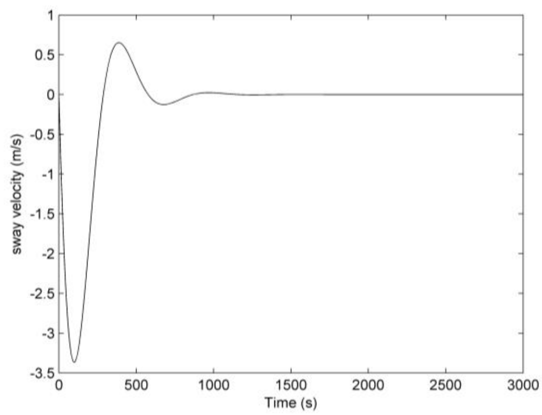


(c)

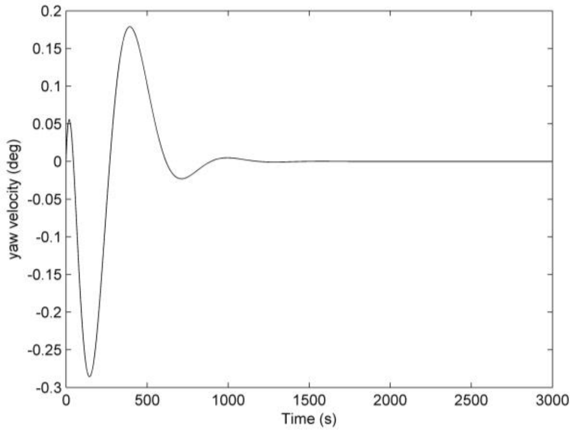
Fig. 5. Responses of the DPS positions and yaw angle.



(a)



(b)



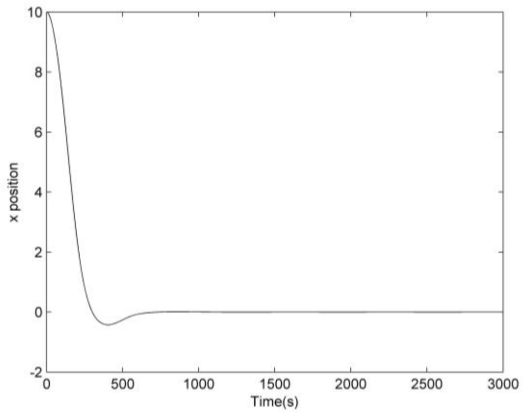
(c)

Fig. 6. Responses of the DPS velocities.

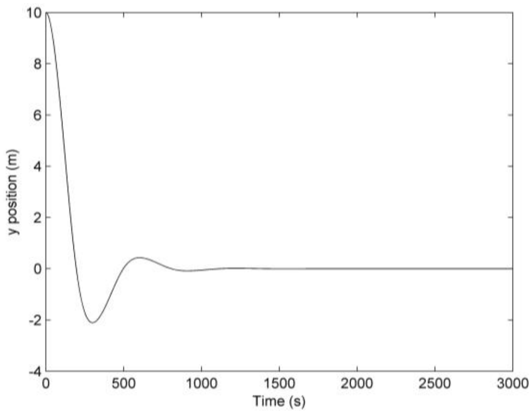
The value of extern disturbance is considered that

$$w(t) = 0.1\sin(t)\begin{bmatrix}1 & 1 & 1 & 0 & 0 & 0\end{bmatrix}^T \qquad (26)$$

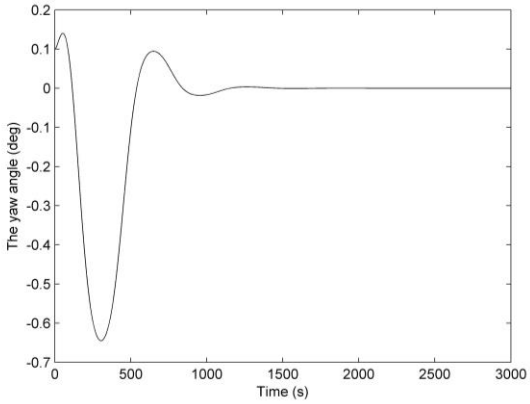
and the responses curve of the $\eta(t)$ and velocities $\upsilon(t)$ are shown in Fig.7-8. It can be seen that the $\eta(t)$ and $\upsilon(t)$ is stable and reach the desired target with any accuracy, which further illustrated that the designed controller can make the DPS overcome the environmental disturbance and keep the DPS stabile.



(a)

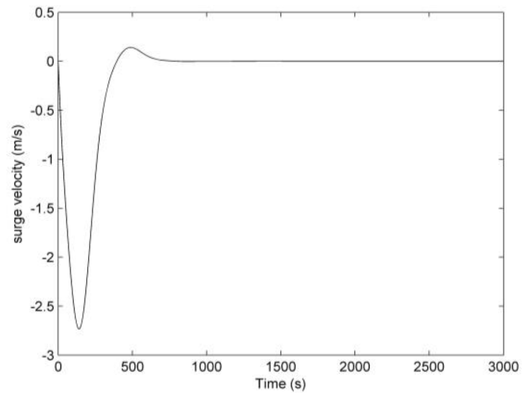


(b)

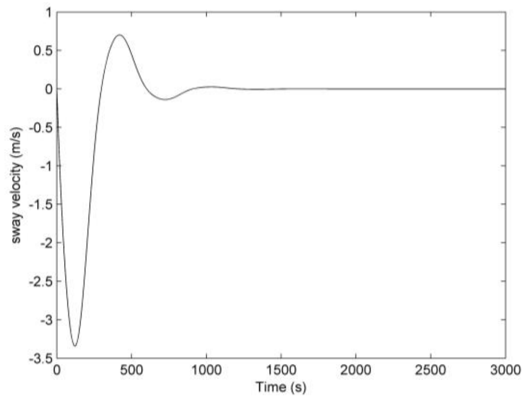


(c)

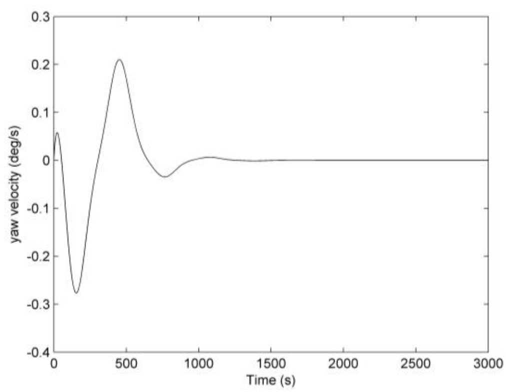
Fig. 7. Responses of the DPS positions and yaw angle.



(a)



(b)



(c)

Fig. 8. Responses of the DPS velocities.

5. CONCLUSIONS

The main contributions of article are summarized as.

1) The sampled-data stabilization problem for DPS is discussed based on LMIs.

2) The upper and lower bounds of the time delay are considered, and it has wider application scopes than the references which have strict limitations on the lower bound of the delay.

3) The tighter reciprocally convex inequalities are used to estimate some integral term for LKF, and less conservative result can be obtained.

The next step will continue to improve the control algorithm to further improve the stabilization and robustness of the DPS.

Appendix A. PROOF OF THEOREM 1

Calculating the derivative of $V(t)$, it can be obtained that:

$$\begin{aligned}
\dot{V}_1(t) &= 2x(t)^T P^T \dot{x}(t) \\
\dot{V}_2(t) &= x(t)^T Q_1 x(t) - x(t-h_1)^T Q_1 x(t-h_1) + \\
&\quad x(t)^T Q_2 x(t) - x(t-d(t))^T Q_2 x(t-d(t)) \\
&\quad + x(t)^T Q_3 x(t) - x(t-h_2)^T Q_3 x(t-h_2)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(t) &= h_1^2 \dot{x}(t)^T R_1 x(t) + h_2^2 \dot{x}(t)^T R_2 \dot{x}(t) + \delta^2 \dot{x}(t)^T R_3 \dot{x}(t) \\
&\quad - h_1 \int_{t-h_1}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds - h_2 \int_{t-h_2}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds \\
&\quad - \delta \int_{t-h_2}^{t-h_1} \dot{x}^T(s) R_3 \dot{x}(s) ds
\end{aligned} \tag{27}$$

According to Jensen inequation (see Gu et al., 2003), it can be obtained that

$$\begin{aligned}
&-h_1 \int_{t-h_1}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds \le \\
&-\left[x(t) - x(t-h_1)\right]^T R_1 \left[x(t) - x(t-h_1)\right],
\end{aligned} \tag{28}$$

$$\begin{aligned}
&-h_2 \int_{t-h_2}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds \le \\
&-\left[x(t) - x(t-h_2)\right]^T R_2 \left[x(t) - x(t-h_2)\right].
\end{aligned} \tag{29}$$

Thus, according to the reciprocally convex approach, it can be obtained that

$$\begin{aligned}
&-\delta \int_{t-h_2}^{t-h_1} \dot{x}^T(s) R_3 \dot{x}(s) ds = \\
&-\delta \int_{t-h_2}^{t-d(t)} \dot{x}^T(s) R_3 \dot{x}(s) ds - \delta \int_{t-d(t)}^{t-h_1} \dot{x}^T(s) R_3 \dot{x}(s) ds \\
&\le -\frac{\delta}{d(t) - h_1} \left[x(t-h_1) - x(t-d(t))\right]^T \\
&R_3 \left[x(t-h_1) - x(t-d(t))\right] \\
&-\frac{\delta}{h_2 - d(t)} \left[x(t-d(t)) - x(t-h_2)\right]^T \\
&R_3 \left[x(t-d(t)) - x(t-h_2)\right] \\
&\le -\begin{bmatrix} x(t-h_1) - x(t-d(t)) \\ x(t-d(t)) - x(t-h_2) \end{bmatrix}^T \begin{bmatrix} R_3 & Z \\ * & R_3 \end{bmatrix} \\
&\quad -\begin{bmatrix} x(t-h_1) - x(t-d(t)) \\ x(t-d(t)) - x(t-h_2) \end{bmatrix}
\end{aligned} \tag{30}$$

Substituting (25) and (26) into (24) that

$$\begin{aligned}
&\dot{V}(t) \le \varsigma^T(t)(\Phi + \begin{bmatrix} A & BK & 0 & 0 \end{bmatrix}^T \\
&\left(h_1^2 R_1 + h_2^2 R_2 + \delta^2 R_3\right) \begin{bmatrix} A & BK & 0 & 0 \end{bmatrix}) \varsigma(t)
\end{aligned} \tag{31}$$

where

$$\Phi = \begin{bmatrix} \Xi_{11} & P^T BK & R_1 & R_2 \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & Z \\ * & * & * & \Xi_{44} \end{bmatrix}$$

$$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-h_1) & x^T(t-h_2) \end{bmatrix}^T$$

By Schur complement, (16) guarantee that

$$\begin{aligned}
&\Phi + \begin{bmatrix} A & BK & 0 & 0 \end{bmatrix}^T \\
&\left(h_1^2 R_1 + h_2^2 R_2 + \delta^2 R_3\right) \begin{bmatrix} A & BK & 0 & 0 \end{bmatrix} < 0
\end{aligned} \tag{32}$$

Therefore, it can be obtained that $\dot{V}(t) < -\sigma \left\| x(t) \right\|^2$ when $x(t) \ne 0, \sigma > 0$. Then, the system (14) is asymptotically stable. The proof is completed.

Appendix B. PROOF OF THEOREM 2

Under zero initial conditions, it can be obtained that

$$\begin{aligned}
J_{yw} &= \int_0^t \left[y^T(s) y(s) - \gamma^2 w^T(s) w(s) \right] ds, \\
&= \int_0^t \left[y^T(s) y(s) - \gamma^2 w^T(s) w(s) + \dot{V}(s) \right] ds - V(t) \\
&\le \int_0^t \left[y^T(s) y(s) - \gamma^2 w^T(s) w(s) + \dot{V}(s) \right] ds \\
&= \int_0^t \zeta^T(t) \Big(\Theta + \begin{bmatrix} C & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
&\quad \begin{bmatrix} C & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} A & BK & 0 & 0 & B_w \end{bmatrix}^T \\
&\quad \left(h_1^2 R_1 + h_2^2 R_2 + \delta^2 R_3\right) \begin{bmatrix} A & BK & 0 & 0 & B_w \end{bmatrix} \Big) \zeta(t)
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\zeta(t) = \big[& x^T(t) \quad x^T(t-d(t)) \quad x^T(t-h_1) \\
& x^T(t-h_2) \quad w^T(t) \big]^T
\end{aligned}$$

$$\Theta = \begin{bmatrix} \Xi_{11} & P^T BK & R_1 & R_2 & PB_w \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 \\ * & * & \Xi_{33} & Z & 0 \\ * & * & * & \Xi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

By Schur complement, (19) guarantees

$$\begin{aligned} &\Theta + \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix} + \\ &\begin{bmatrix} A & BK & 0 & B_w \end{bmatrix}^T \left(h_1^2 R_1 + h_2^2 R_2 + \delta^2 R_3 \right) \\ &\begin{bmatrix} A & BK & 0 & B_w \end{bmatrix} < 0 \end{aligned} \tag{34}$$

From (34), it can be obtained that:

$$\int_0^t \left[y^T(s) y(s) - \gamma^2 w^T(s) w(s) + \dot{V}(t) \right] ds < 0 \tag{35}$$

Under zero initial conditions, we have $V(0) = 0$, $V(\infty) \geq 0$, and

$$V(t) + \int_0^t \left[y^T(s) y(s) \right] ds < \int_0^t \left[\gamma^2 w^T(s) w(s) \right] ds \tag{36}$$

Let $t \to \infty$ for both sides of (36), it can be got that

$\| y(t) \|_2 \leq \gamma \| w(t) \|_2$ for all nonzero $w(t) \in L_2 [0, \infty)$.

Then according to the condition, the system (14) is not only asymptotically stable, but also satisfies the H_∞ performance index γ.This completed the proof.

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