Stability Analysis and Region in Control Parameter Space of Thermal System with Constant Delays

Sathish Babu P[*], Santhana Krishnan T[**], Senthil Kumar C[***], Sundarabalan C K[****], Venkatachalam V[*****]

[*]*Department of Electrical and Electronics Engineering, Anna University-University College of Engineering, Panruti, Tamil Nadu, India(email: psathishbabu@yahoo.co.in)*
[**]*Department of Electrical and Electronics Engineering, Rajalakshmi Engineering College, Chennai, Tamil Nadu, India (email:vishnu_santhan@yahoo.co.in)*
[***]*Department of Electrical and Electronics Engineering, AAA Engineering College, Sivakasi, Tamil Nadu, India (email: csenthilsaro@gmail.com)*
[****]*School of Electrical and Electronics Engineering, SASTRA Deemed University, Thanjavur, Tamil Nadu, India (email:ckseee@yahoo.com)*
[******]*Coimbatore Institute of Technology, Coimbatore-641014, Tamil Nadu, India (email:vikranth6488@gmail.com)*

Abstract: In this article, a comprehensive analysis of the Thermal Control System (TCS) with time-invariant communication delays is presented. The first method, a precise approach, taking an account of Gain and Phase margin (GM &PM) is introduced to determine the stable/delay margin based on TCS parameters. The technique implements a deletion process to change the Transcendental Characteristic Equation (TCE) into a novel polynomial root of the crossing frequency in a complex plane. The positive or real roots of the polynomial equation are rightly equivalent to completely complex roots of the original system characteristic equation. The second technique, a graphical approach, is executed to resolve the stability province of controlled parameters for a given delay. The technique is fundamentally an extracting stable province and Stability Frontier Line (SFL) in the control parameter space having user-evident gain & phase margin (GMPM). The extensive simulation analysis shows that the proposed techniques give an optimized performance of a thermal control system with time-invariant communication delays.

Keywords: Delay-dependent stability, thermal control system, time-invariant delay, gain margin and phase margin, stability region.

1. INTRODUCTION

The heat exchanger is a mechanical device which allows an exchange/transfer of the thermal energy from one medium (a liquid or gas) to another medium (liquid or gas) without coming into direct contact (Ramesh et al., 2003). However, heat exchangers are not only applicable to heating the medium, but are also used for cooling purposes (Ramesh et al., 2003; Donald, 2009). Heat-exchangers are widely used in various locations, generally working to heat or cool engines and machines to work with considerable efficiency. Different categories of heat exchangers can be classified based on their applications. They are (a) shell & tube heat-exchanger, (b) plate heat-exchanger, (c) plate& shell heat-exchanger, (d) Helical-coil heat-exchanger, (e) pillow plate heat exchanger, (f) Adiabatic wheel heat exchanger, (g) Plate fin heat-exchanger, (h) Spiral heat exchanger, (i) fluid heat exchanger, (j) waste heat recovery units. A shell & tube heat-exchanger is the most common type of unit the name of which is derived from construction (Sarabeevi and LailaBeebi, 2016). The heat exchanger device is very essential in industries such as (1) marine applications (Propulsion Plant (PP), Starting Air System (SAS), Fuel Injection System (FIS), Refrigeration System (RS), A.C system, fresh water system, and steam turbine unit), (2) Electrical applications (solar water heating, transformer oil cooler, generators and motors coolers, Air blast cooler for application HVDC, radiating cooler, nuclear power plant, thermal power plant, process air heat batteries), (3) bioprocess applications (heating or cooling of liquid foods, freezing & evaporation of fluids for the making of ice creams, sorbet and fruit based Juices) (Padmakshi et al., 2014), (4) medical applications (In this era, high-technology medical apparatus are required to be more effective for controlling the temperature. As an alternative to increasing the heat load, Original Equipment Manufacturers (OEMs) prefer liquid cooling to heat loads for medical lasers, imaging equipment. Recirculation chillers, cold plates, ambient cooling systems, liquid to liquid cooling systems are the cooling techniques used in medical systems, Cardioplegia apparatus), (5) Heat exchanger applications are found not only in the above said areas but also in pharmaceutical industry, alcohol production, metallurgical process, petroleum refinery process and biochemical processing. The mission of network based Thermal Control System (TCS) is to get the value of sensor measurement from the shell & tube heat exchanger and control signal/information from a controller, to terminate and to implement obligatory control measures for adjusting the outlet temperature (Venkatachalam et al., 2017). The TCS needs a dedicated network for communication to obtain measured data and to transmit signals from the outlet of the heat exchanger unit to the controller; as a consequence, delays get introduced in the feedback loop (Venkatachalam et al., 2017; Manikandan

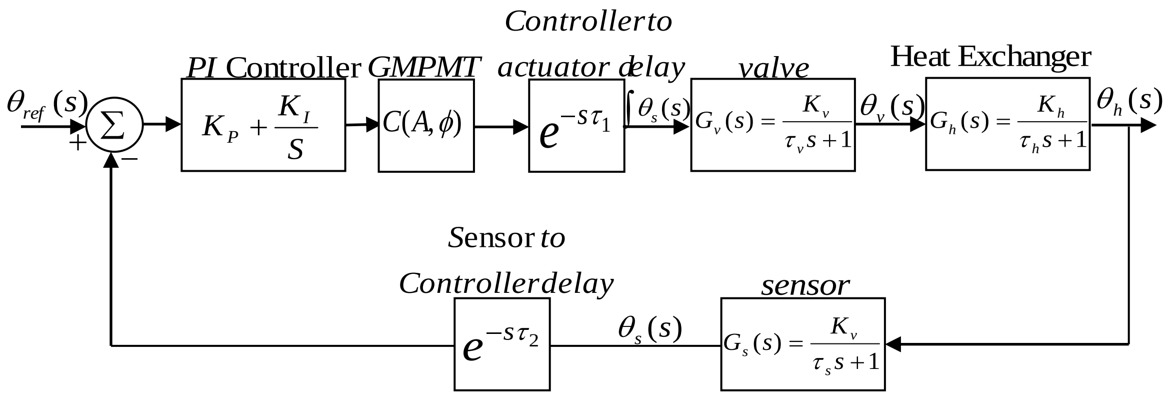

Fig. 1. Closed loop TCS with GMPMT.

and Kokil, 2017; Ana Paula Batista et al., 2014).

The interconnected closed-loop schematics of TCS contain a valve, controller, Heat Exchanger, and a sensor with delays τ_1 and τ_2 as shown in Fig.1. The τ_1 indicates the feed forward path time-delay, i.e. the delay between Gain margin/Phase Margin Tester (GMPMT) and valve. The τ_2 indicates the feedback path time-delay, i.e., the delay between the sensor and the Proportional-Integral (PI) controller. Thus, the time delays are normally not taken into account when designing the controller; however, these time delays occur in the closed-loop TCS when the PI-controller is integrated into the real-time system. Although these delays are not known for their characteristics, they are commonly approximated to be time-invariant; they can be merged into a single factor of delay $\tau = \tau_1 + \tau_2$. This paper works to quantify all stabilizing parameters of the PI controller which guarantee the desired dynamic TCS output with user-defined GMPMT. The solution is based on the locus of the stability limit: locus in the system theory means a set of points whose location is determined by specified conditions (e.g. root locus) which can be easily accomplished by equating the real and imaginary components of the characteristic equation with zero. For the synthesis of PI-controller and delayed TCS, the proposed approach has been effectively implemented. In inclusion to stability analysis, other relative stability specifications, performance such as GMPMT that gives assurance of desired performance is also considered in a delay margin computation process. The effect on the stability area of the user-defined GMPMT is evaluated. The stability zone has been shown to become smaller as the GMPMT increases.

2. TIME-DELAYED MODEL OF TCS

The linear state-space model of TCS is used to analyze the performance and to design a controller (Venkatachalam et al., 2017; Chhaya et al., 2011). Fig.1 represents the closed loop thermal control system with network-induced delay with GMPMT. The user described GMPMT as a ‘virtual compensator’ is incorporated to TCS in the feed forward path as depicted in Fig.1. The GMPMT is frequency independent as represented below (Sahin and Saffet, 2016a,b; Ramakrishnan and Swarnalakshmi, 2018; Sahin and Saffet, 2019; Sahin and Saffet, 2017; Hakan et al., 2017).

$$C(A,\emptyset) = Ae^{-j\emptyset} \tag{1}$$

where A and $\emptyset$ are marked as (GMPM), respectively. The original characteristic equation of the thermal control system without a GMPMT is first received (Sahin and Saffet, 2019; Chang and Han, 1990; Hamamci and Koksal, 2010; Olgac and Sipahi, 2002).

$$\Delta(s, \quad \tau) = P(s) + Q(s)e^{-s\tau} = 0 \tag{2}$$

where, the coefficients of $P(s)$ and $Q(s)$are:

$$\left.\begin{aligned} P(s) = p_3s^4 + p_2s^3 + p_1s^2 + p_0s^1 \\ Q(s) = q_1s + q_0 \end{aligned}\right\} \tag{3}$$

with

$p_3 = T_VT_HT_F; \quad p_2 = T_VT_F + T_VT_H + T_HT_F;$

$p_1 = T_V + T_F + T_H; \quad p_0 = 1;$

$q_1 = K_PK_VK_FK_H; \quad q_0 = K_IK_VK_FK_H;$

The new polynomial equation (1) of the modified TCS with GMPMT depicted in Fig. 1 is then obtained (V. Venkatachalam et al., 2019; V.Venkatachalam &D. Prabhakaran.,2018).

$$\Delta(s, \quad \tau') = P'(S) + Q'(S)e^{-s\tau'} = 0 \tag{4}$$

Where

$$\left.\begin{aligned} P(s) = P^{'}(s) = p_3s^4 + p_2s^3 + p_1s^2 + p_0s^1 \\ Q^{'}(s) = AQ(s) = Aq_1s + Aq_0 = q_1^{'}s + q_0^{'} \end{aligned}\right\} \tag{5}$$

An exponential term in equation (4) is $e^{-s\tau'}$instead of $e^{-s\tau}$ as in equation (4). It will be obtained by adding $e^{-s\tau}$& $e^{-j\emptyset}$ into a single form for $s = j\omega_c$ that is polynomial roots (4) on the s - plane. The correlation between τ' and τ is shown below

$$\tau' = \tau + \frac{\emptyset}{\omega_c} \tag{6}$$

In equation (6), τ' represents the stable margin of the modified TCS with GMPMT and τ represents the time-invariant delay for which the TCS without GMPMT (Sahin and Saffet, 2019; Chang and Han, 1990; Hamamci and Koksal, 2010; Olgac and Sipahi, 2002; Venkatachalam et al., 2019).

3. COMPUTATION OF DELAY MARGIN WITH GMPMT

It should be stressed that the maximum value of delay can be found using equation (4) of modified TCS roots on the $j\omega$ axis, (Chang and Han, 1990; Hamamci and Koksal, 2010). Therefore, the purely $j\omega$ roots of the modified TCS equation at $s = \pm j\omega_c$ and the corresponding value of maximum τ' have to be determined. Then applying these values, the delay margin for the original TCS gain (A) & phase $(\emptyset)$ could be simply obtained (Chang and Han,1990; Hamamci and Koksal, 2010).

The stability of GMPM based thermal control system is evaluated by the location of the modified system characteristic equation roots as given in (4). For a system to be asymptotically stable all roots must be in the left half of s-plane for a stated maximum time-invariant delay. Note that (4) is a Transcendental Characteristic Equation (TCE) due to term $e^{-j\omega_c\tau'}$. As a consequence of transcendental character, the modified equation (4) has infinitely much finite value of roots and to be quite difficult for the determination of the roots. Nevertheless, the main task is to obtain the maximum delay value for which the GMPM based Characteristic Equation (CE) roots on $j\omega$ axis. Let consider that $\Delta\ (j\omega_c\,,\tau') = 0$ has polynomial roots located on the line of the imaginary axis at $s = j\omega_c$, for some limited values of τ' and same for the Δ $(-j\omega_c, \tau') = 0$. As a consequence, the problem of determining the delay margin τ' such that both $\Delta\ (j\omega_c, \tau') = 0$ and Δ $(-j\omega_c, \tau') = 0$ have the same value of roots at $s = j\omega_c$ (Sahin and Saffet, 2017; Hakan et al., 2017; Chhaya et al., 2011; Sahin and Saffet, 2016). These results could be stated as

$$\left.\begin{aligned} P'(j\omega_c) + Q'(j\omega_c)e^{-j\omega_c\tau'} = 0 \\ P'(-j\omega_c) + Q'(-j\omega_c)e^{j\omega_c\tau'} = 0 \end{aligned}\right\} \tag{7}$$

In (7), the two equations are possibly one could remove effortlessly $e^{-j\omega_c\tau'}$ and $e^{j\omega_c\tau'}$ terms, to obtain a modified new polynomial equation without any transcendental terms as follows

$$W(\omega_c^2) = P(j\omega_c)P(-j\omega_c) - Q(j\omega_c)Q(-j\omega_c) = 0 \tag{8}$$

By substituting the $P'(s)$ and $Q'(s)$ polynomials given in (5) into (7), the new polynomial equation of the modified thermal control system with GMPMT is determined in terms of system parameters as given below

$$W(\omega_c^2) = t_4\omega_c^8 + t_3\omega_c^6 + t_2\omega_c^4 + t_1\omega_c^2 + t_0 = 0 \tag{9}$$

where

$$t_4 = p_3^2;\quad t_3 = p_2^2 - 2p_3p_1;\quad t_2 = p_1^2 - 2p_2p_0;\ t_1 = p_0^2 - {q_1'}^2;\ t_0 = -{q_0'}^2;$$

It clearly shows, that any approximation to the deletion process of the exponential terms is not be used. Therefore, real positive roots in (9) exactly coincide with the imaginary roots in (4). Based on the positive roots in (9), there are two possible categories of stability as described below:

1. Equation (9) does not have any real positive root of all finite value of delay $\tau' \geq 0$, so the GMPM based thermal control system is said to be delay-independent stable, indicating that all the roots of original characteristic equation (4) lie in the left of the s-plane (Sahin and Saffet, 2019).
2. If equation (9) has at least one real positive root, then the GMPM based thermal control system is said to be delay-dependent stable. In the presence, some roots in equation (4) transverse the $j\omega$ −axis at $s = j\omega_c$ for a finite value of communication delay τ', is called the stable delay margin (Sahin and Saffet, 2019).

If the real positive root in equation (9) is found, the corresponding delay margin of GMPM based thermal control system is obtained by using equation (4) as given below (Sahin and Saffet, 2019; Chang and Han, 1990, Hamamci and Koksal, 2010; Olgac and Sipahi, 2002; Venkatachalam et al., 2019).

$$\Delta(j\omega_c,\quad \tau') = P'(j\omega_c) + Q'(j\omega_c)e^{-j\omega_c\tau'} = 0$$

$$e^{-j\omega_c\tau'} = \cos(\omega_c\tau') - j\sin(\omega_c\tau') = -\frac{P'(j\omega_c)}{Q'(j\omega_c)} \tag{10}$$

$$\cos(\omega_c\tau') = Re\left\{-\frac{P'(j\omega_c)}{Q'(j\omega_c)}\right\};$$

$$\sin(\omega_c\tau') = Im\left\{\frac{P'(j\omega_c)}{Q'(j\omega_c)}\right\}$$

$$\tau' = \frac{1}{\omega_c}Tan^{-1}\left(\frac{Im\{P(j\omega_c)/Q(j\omega_c)\}}{Re\{-P(j\omega_c)/Q(j\omega_c)\}}\right) + \frac{2\pi r}{\omega_c}; \tag{11}$$

$$\tau' = \frac{1}{\omega_c}Tan^{-1}\left(\frac{a_5\omega_c^5 + a_3\omega_c^3 + a_1\omega_c^1}{a_4\omega_c^4 + a_2\omega_c^2 + a_0\omega_c^0}\right) + \frac{2\pi r}{\omega_c}; \tag{12}$$

$$r = 0,1,2,\dots,\infty$$

where the respective coefficients are given below:

$a_0 = 0$; $a_1 = -p_0q_0'$; $a_2 = p_0q_1' - p_1q_0'$; $a_3 = p_2q_0' - q_1'$; $a_4 = p_3q_0' - p_2q_1$; $a_5 = p_3$

It should be clearly expressed that the modified new polynomial equation given in (9) may have more than one real positive root whose set is given by

$$\{\omega_c\} = \{\omega_{c1}, \omega_{c2}, \dots, \omega_{cq}\} \tag{13}$$

To determine the corresponding value of delay by employing (12) for each positive root $\omega_{cm}, m = 1,2,\dots,q$. These values of maximum delay will be set of delay margins which are interval spaced and defined as shown in the repeated period.

$$\{\tau_m'\} = \{\tau_{m1}', \tau_{m2}', \dots, \tau_{m\infty}'\},\ m = 1,2,\dots,q \tag{14}$$

where

$$\tau_{m,r+1}' - \tau_{m,r}' = \frac{2\pi}{\omega_c} \tag{15}$$

Eventually, the minimum of $\tau_m', m = 1,2,\dots,q$ is stable delay of the GMPM based TCS.

$$\tau' = min(\tau_m') \tag{16}$$

3. The GMPM based TCS, it could easily obtain the time-delay of TCS without GMPM will have the desired gain margin and phase margin as follows (Sahin and Saffet, 2019).

$$\tau = \tau' - \frac{\emptyset}{\omega_c} \tag{17}$$

4. DETERMINATION OF STABILIZING REGION WITH GMPMT

To determine the stable frontier line for a given constant delay τ' and desired GMPMT, substitute $s = j\omega_c$ with $\omega_c > 0$ into the modified characteristic equation (4).

$$\Delta(j\omega_c, \quad \tau') = p_3(j\omega_c)^4 + p_2(j\omega_c)^3 + p_1(j\omega_c)^2 + p_0 j\omega_c + (q_1' j\omega_c + q_0')e^{-(j\omega_c)\tau'} = 0 \qquad (18)$$

Substituting $e^{-j\omega_c\tau'} = \cos(\omega_c\tau') - jsi\,n(\omega_c\tau')$into equation (18) and dividing by $(K_P,\ K_I)$controller gainsthe following equation is obtained (Olgac N & Sipahi R.,2002;V. Venkatachalam et al., 2019):

$$\begin{aligned}\Delta(j\omega_c, \quad \tau') = p_3\omega_c^4 - jp_2\omega_c^3 - p_1\omega_c^2 + p_0 j\omega_c \\ + jq_1'\omega_c\cos(\omega_c\tau') + q_1'\omega_c\sin(\omega_c\tau') \\ + q_0'\cos(\omega_c\tau') - jq_0'\sin(\omega_c\tau')\end{aligned}$$

$$\Delta(j\omega_c, \quad \tau') = [p_3\omega_c^4 - p_1\omega_c^2 + q_1'\omega_c\sin(\omega_c\tau') + q_0'\cos(\omega_c\tau')] + j[-p_2\omega_c^3 + p_0\omega_c + q_1'\omega_c\cos(\omega_c\tau') - q_0'\sin(\omega_c\tau')] \qquad (19)$$

Equating the real and the imaginary parts of $\Delta(-j\omega_c, \tau') = 0$ the following equation is obtained.

$$\left.\begin{aligned}K_P A_1(\omega_c) + K_I B_1(\omega_c) + C_1(\omega_c) = 0\\ K_P A_2(\omega_c) + K_I B_2(\omega_c) + C_2(\omega_c) = 0\end{aligned}\right\} \qquad (20)$$

where

$$A_1(\omega_c) = q_1'\omega_c\sin(\omega_c\tau');\ B_1(\omega_c) = q_0'\cos(\omega_c\tau');\ C_1(\omega_c) = p_3\omega_c^4 - p_1\omega_c^2$$

$$A_2(\omega_c) = q_1'\omega_c\cos(\omega_c\tau');\ B_2(\omega_c) = -q_0'\sin(\omega_c\tau');\ C_2(\omega_c) = -p_2\omega_c^3 + p_0\omega_c$$

To solve the equation (20) and to identify stable frontier locus (K_P, K_I, ω_c) on (K_P, K_I)plane.

$$\begin{aligned}K_P &= \frac{B_1(\omega_c)C_2(\omega_c) - B_2(\omega_c)C_1(\omega_c)}{A_1(\omega_c)B_2(\omega_c) - A_2(\omega_c)B_1(\omega_c)}\\ K_I &= \frac{A_2(\omega_c)C_1(\omega_c) - A_1(\omega_c)C_2(\omega_c)}{A_1(\omega_c)B_2(\omega_c) - A_2(\omega_c)B_1(\omega_c)}\end{aligned} \qquad (21)$$

It must be noted that the line $(K_I = 0)$ is also in a frontier curve because a positive root of $\Delta(j\omega_c, \quad \tau') = 0$ in (18) can cross the imaginary castle at $s = j\omega_c = 0$ for $K_I = 0$ (Venkatachalam and Prabhakaran, 2018; Sahaj and Yogesh, 2018; Saffet, 2008). As a result, the frontier locus(K_P, K_I, ω_c) and the line $K_I = 0$ split (K_I, K_P) plane into two territories which are unstable and stable. This area of the frontier locus is known as Real Root frontier (RRF) and also obtained in (21) and is described as Complex Root Frontier (CRF) of the stability regions (Sahin et al., 2016; Sahin et al., 2016; Olugbenga et al., 2017). That refers to the real root crossing over the imaginary axis at $s = \infty$ and might be noticed depending on the delayed system characteristics equation.

5. RESULTS AND DISCUSSIONS

5.1 GMPM Based Delay Margin Computation

In this division, the delay τ' for the stability of various sets of (K_I, K_P) and GMPMT is computed by using (12). Theoretical values of the delay are verified by Linear Matrix Inequalities (LMI) tool in MATLAB/Simulink (V. Venkatachalam et al., 2019). The parameters of TCS as given below (Venkatachalam et al., 2017; Venkatachalam and Prabhakaran, 2017; Venkatachalam and Prabhakaran, 2018).

Table 1. Parameters values of TCS.

System	Gain	Time-Constant
Heat Exchanger	$K_H = 34$	$T_H = 30$
Valve	$K_V = 1.25$	$T_V = 3$
Sensor	$K_F = 0.08$	$T_F = 2$

Table 2. Maximum value of delay τ' when A is varying

K_I	$A = 0.5;\ \emptyset = 0°$					
	$K_P = 0.5$	$K_P = 1.0$	$K_P = 1.5$	$K_P = 2.0$	$K_P = 2.5$	$K_P = 3.0$
0.02	42.3595	30.6206	18.8219	12.7610	9.3543	7.2192
0.04	20.2379	20.5495	15.7720	11.5785	8.7669	6.8760
0.06	12.3346	14.6560	13.0438	10.3769	8.1565	6.5200
0.08	8.2501	10.9378	10.7937	9.2267	7.5419	6.1567
0.10	5.7525	8.4044	8.9780	8.1664	6.9385	5.7917
K_I	$A = 1;\ \emptyset = 0°$					
0.02	20.5495	11.5785	6.8760	4.5766	3.2630	2.4252
0.04	10.9378	9.2267	6.1567	**4.2533**	3.0816	2.3092
0.06	6.5747	7.2104	5.4294	3.9208	2.8958	2.1909
0.08	4.1169	5.6017	4.7283	3.5858	2.7072	2.0710
0.10	5.2470	4.3355	4.0745	3.2540	2.5175	1.9500
K_I	$A = 2;\ \emptyset = 0°$					
0.02	9.2267	4.2533	2.3092	1.3704	0.8591	0.4806
0.04	5.6017	3.5858	2.0710	1.2490	0.7550	0.4304
0.06	3.3311	2.9304	1.8284	1.1257	0.6800	0.3796
0.08	1.8620	2.3202	1.5859	1.0014	0.6045	0.3286
0.10	0.8534	1.7709	1.3470	0.8768	0.5285	0.2773
K_I	$A = 3;\ \emptyset = 0°$					

0.02	6.9385	2.9891	1.5201	0.8106	0.4009	0.1371
0.04	4.3571	2.5175	1.3430	0.7176	0.3430	0.0973
0.06	2.5707	2.0483	1.1632	0.6234	0.2846	0.0572
0.08	1.3487	1.6006	0.9830	0.5285	0.2258	0.0170
0.10	0.4827	1.1858	0.8045	0.4335	0.1608	*

Table 3. Maximum value of delay τ' when A and $\emptyset$ is varying

K_I	$A = 0.5;\ \emptyset = 5°$					
	$K_P = 0.5$	$K_P = 1.0$	$K_P = 1.5$	$K_P = 2.0$	$K_P = 2.5$	$K_P = 3.0$
0.02	39.5693	28.8741	17.6766	11.9007	8.6541	6.6211
0.04	18.3234	19.0543	14.6734	10.7296	8.0705	6.2794
0.06	10.7836	13.3411	12.0015	9.5449	7.4661	5.9259
0.08	6.6099	9.7527	9.8066	8.4147	6.8592	5.5669
0.10	4.5539	7.3171	8.0414	7.3760	6.2649	5.2051
K_I	$A = 0.5;\ \emptyset = 10°$					
0.02	36.7790	27.1275	16.5312	11.0404	7.9540	6.0229
0.04	16.4089	17.5591	13.5747	9.8807	7.3742	5.6829
0.06	9.2326	12.0263	10.9591	8.7128	6.7757	5.3319
0.08	5.5697	8.5677	8.8195	7.6027	6.1765	4.9754
0.10	3.3553	6.2298	7.1048	6.5855	5.5913	4.6186
K_I	$A = 1;\ \emptyset = 5°$					
0.02	19.0543	10.7296	6.2794	4.1014	2.8588	2.0680
0.04	9.7527	8.4147	5.5661	3.7797	2.6780	1.9521
0.06	5.5640	6.4417	4.8477	3.4497	2.4931	1.8343
0.08	3.2195	4.8748	4.1576	3.1180	2.3058	1.7150
0.10	1.7305	3.6461	3.5159	2.7903	2.1178	1.5947
K_I	$A = 1;\ \emptyset = 10°$					
0.02	17.5591	9.8807	5.6829	3.6262	2.4547	1.7107
0.04	8.5677	7.6027	4.9754	3.3060	2.2744	1.5951
0.06	4.5533	5.6731	4.2660	2.9786	2.0905	1.4777
0.08	2.3222	4.1479	3.5869	2.6503	1.9045	1.3590
0.10	0.9140	2.9567	2.9573	2.3267	1.7187	1.2395
K_I	$A = 2;\ \emptyset = 5°$					
0.02	8.4147	3.7797	1.9521	1.0720	0.5666	0.2429
0.04	4.8748	3.1180	1.7150	0.9509	0.4926	0.1927
0.06	2.6747	2.4713	1.4741	0.8281	0.4179	0.1421
0.08	1.2599	1.8715	1.2338	0.7045	0.3426	0.0912
0.10	0.2936	1.3332	0.9976	0.5807	0.2671	0.0401
K_I	$A = 2;\ \emptyset = 10°$					
0.02	7.6027	3.3060	1.5651	0.7736	0.3041	0.0052
0.04	4.1479	2.6503	1.3590	0.6528	0.2303	*
0.06	2.0182	2.0122	1.1198	0.5305	0.1558	*
0.08	0.6577	1.4228	0.8816	0.4076	0.0808	*
0.10	*	0.8955	0.6481	0.2847	0.0056	*

Delay margin that assures the desired GMPM is computed by using (12) to (17) for a various sets of control parameter and for various GMPM. Table 2 provides a delay margin for $A = 1$ and $\emptyset = 0°$. Corresponds to this case, the conventional method of delay margin analysis, where the GM & PMs are not included. Therefore , $\tau' = \tau$ since $\emptyset = 0°$ as shown in (5). From the result at a fixed value of K_P , τ' decreases as K_I increases. As K_I increases, the system response would be less stable. Note that a fixed K_I and τ' decrease for every K_P. However, τ' is increases when K_P is (0-0.5) in range and τ' decreases when the K_P lies in the range of (0.5 - 8.50) and for a fixed value of $K_I = 0.02$. Next, the GMPM is selected as A=0.5; $\phi = 0°$; A=1; $\phi = 0°$; A=2; $\phi = 0°$; A=3; $\phi = 0°$ to analyze the effect of GM alone on the stable delay margin. The related results are depicted in Table 2. It is obvious from the Table 2 that the addition of GM notably delay margin has reduced for all values of control parameter as tabulated (Table 2). The reduction in delay results in significant increases of gain margin A. In graphical representation effects of GM with various values of PM are depicted in Fig. 2. From Fig. 2, it is very easy to infer the power of time-delay on the stability of TCS (Venkatachalam et al., 2017; Hernández-Pérez et al., 2018) and Fig. 3 vice versa.

The effect of PM ($\emptyset$) is also analyzed in Table 3 which gives the results of A=0.5; $\phi = 5°$; A=0.5; $\phi = 10°$; A=1;

ϕ = 5°; A=1; ϕ = 10°; A=2; ϕ = 5°; A=2; ϕ = 10°. Similar to the GM (A) case, the analytical results clearly bring out that the maximum delay margin reduces as for all $(K_P \text{ and } K_I)$ control parameters when the PM $(\emptyset)$ is considered. The reduction in delay is less than that of the GM case as tabulated (Table 3). Finally, both GMPMs are incorporated in the analytical computation of delay margin. The maximum delay margin for various values of GMPMs with $K_P = 0.5;\ K_I = 0.02$. It clearly indicates that the integrated effect of GMPM on maximum delay is more noticeable than their separate impacts as shown in Fig 4. Simulation results presented to demonstrate the causes of GMPM should be considered in the computations of delay margin. Fig. 5 shows the step response of thermal system for A=1; ϕ = 0°;A=1; ϕ = 10°;A=2; ϕ = 20°. From the Table 2, it can be observed that stable delay is found to be $\tau = 4.2533$, when the control parameters are $K_p = 2.0, K_I = 0.04$.

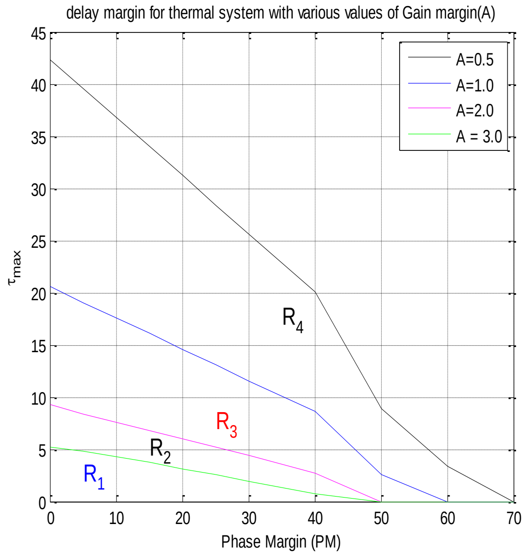

Fig. 2. Deviation of Network Delay versus Gain Margin.

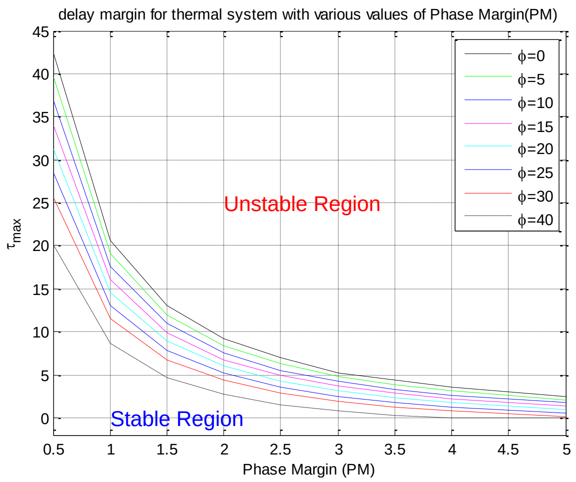

Fig. 3. Delay margin for various values of Phase Margin (PM).

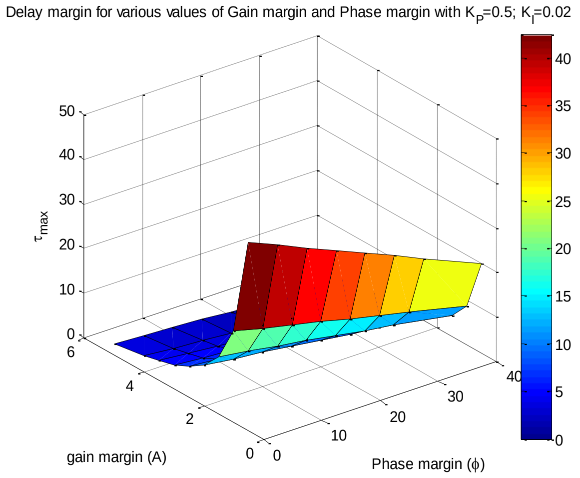

Fig. 4. Delay margin for various values of GMPMT with $K_p = 0.5;\ K_I = 0.02$.

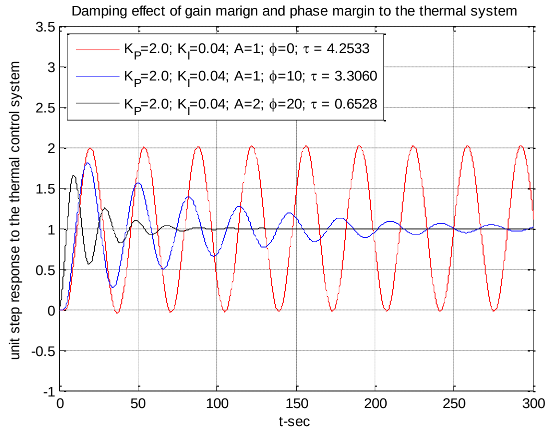

Fig. 5. Damping effect of GMPMT to the thermal control system.

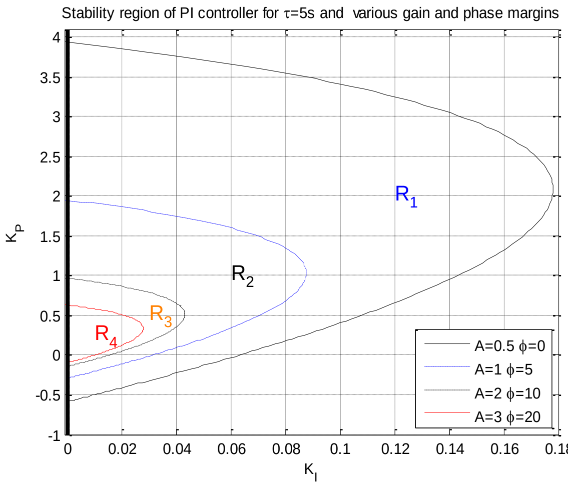

Fig. 6. Stability Region for various values of GMPMT.

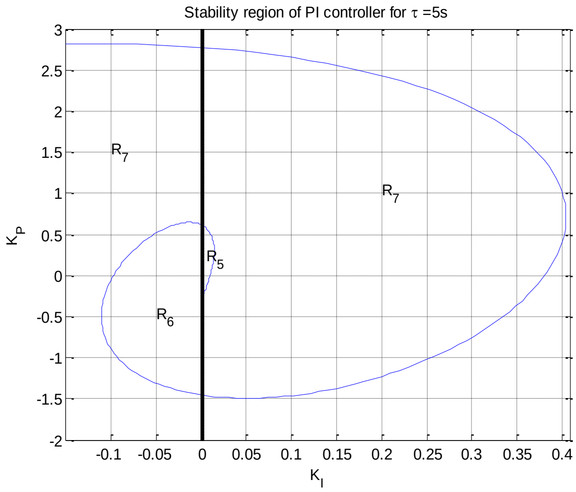

Fig. 7a. Stability region for without considering GMPMT.

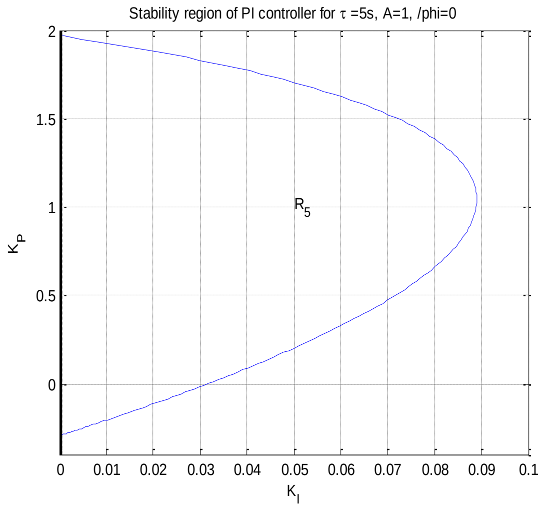

Fig. 7b. Stability region for without considering GMPMT.

As seen in Fig. 5, TCS shows the sustained oscillations for the maximum delay value. However, such sustained oscillations in the output response are not considerable for operational overview. To reduce such an unpleasant oscillation, it has to refer GMPM in computation process. As can be found in Table 3, delay bound based on GMPM is computed as $\tau = 3.3060$ for $A = 1$; $\emptyset = 10°$ and $\tau = 0.6528$ for $A = 2$; $\emptyset = 20°$. As compared with $A = 1$; $\emptyset = 0°$, it is seen clearly from Fig. 5 that the output response quickly shrinks for $\tau = 0.6528$ when $A = 2$; $\emptyset = 20°$ is examined. These results clearly indicate that GMPM must be incorporated into a computation process to need better dynamic response of TCS.

5.2 Stability Regions Based on GMPM

The networked-delay is elected as $\tau = 5\ secs$ and crossing frequency ω_c range is elected as $\omega \in [0,\ \ 5]$ for Stability Frontier Locus (SFL). The first task is to analyze the stability values of controller gains K_P and K_I such that modified characteristic equation (4) with (5) should be Hurwitz stable with GMPM. Suppose the desired GMPMs are $A = 0.5, 1, 2, 3$ and $\emptyset = 0°, 5°, 10°, 20°$ respectively, then put $A = 3$ and $\emptyset = 20°$ in equation (19) to (21) and could obtain the SFL as shown in Fig. 6. The corresponding region is marked on R_4.

Consequently, by putting various gain and phase margins it is indicated as R_1, R_2, R_3 respectively. Finally, the stable region of PI parameter without considering GMPM is obtained by putting $A = 1$ and $\emptyset = 0°$ in equations (19) to (21). The stability region is represented in Fig.7a and 7b. It can be noticed that the relative stability curve with the desired GMPM, R_2, R_3, R_4 is much smaller and R_1 is much greater. Next, three points are selected, ($K_P = 1$; $K_I = 0.085$) in R_5 (without considering GMPM case) ($K_P = 1$; $K_I = 0.07$) in R_2, ($K_P = 0.5$; $K_I = 0.04$) in R_3, ($K_P = 0.25$; $K_I = 0.02$) in R_4 regions. Fig. 8 depicts the performance of the TCS. It is clearly seen that the output responses are stable. Anyhow, the step functions for ($K_P = 1$; $K_I = 0.085$) carry undesirable performance compared with the other responses. From the controlling and operating point of view, such unpleasant deviations are not acceptable. It is obvious that the dynamic performance of TCS is rapid without oscillatory for control parameter (K_p, K_I) elected from the regions R_3, and R_4 with desired GMPM.

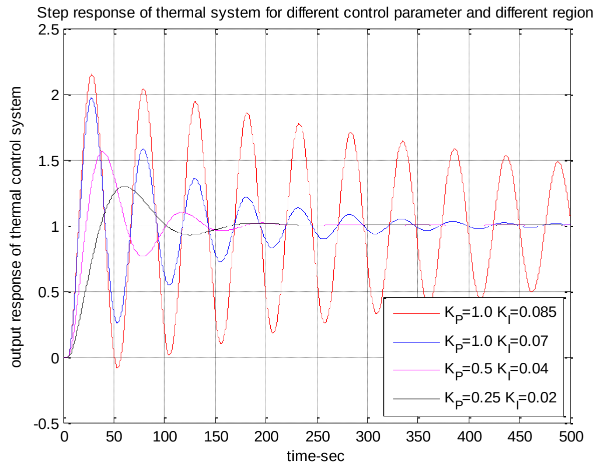

Fig. 8. Step response of TCS with GMPMT.

6. CONCLUSION

In this article, the networked-controlled TCS with delays has been investigated. The impact of GMPM on the stability margin of TCS is ascertained as an employed conventional TCE method. The delay almost takes places in the system feedback path owing to the uses of pneumatic valve, sensors, employment of data communication links for exchanges. The time-delay unchanged exerts an instability effect on the overall system performance. The implications of time delays on TCS stability can be obtained by employing the proposed criteria. The results drawn can be employed as guidelines for compilation control parameters for the networked TCS.

REFERENCES

Ana Paula Batista, Fabio Goncalves Jota (2014), “Effect of Time Delay Statistical Parameters on the Most Likely Regions of Stability in an NCS, *Control Engineering and Applied Informatics*, 16(1), 3-11.

Chang, C.H. and Han, K.W (1990). Gain margins and phase margins for control systems with adjustable parameters. *Journal of Guidance Control and Dynamics* 13(3), 404-408.

Chhaya, Sharma., Sanjeev, Gupta., Vipin, Kumar. (2011) Modeling and simulation of heat exchanger used in soda recovery, *proceeding of the World Congress on Engineering, London, UK, Vol II*, July 6-8.

Donald, R, Coughanowr, Steven, E, LeBlanc.(2009). *Process Systems Analysis and Control*, Third Edition, Published by *McGraw-Hill*, the McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY.

G, M, Sarabeevi., M, LailaBeebi.(2016). Temperature control of shell and tube heat exchanger system using internal model controller, *International conference on Next Generation Intelligent Systems*, 1-3.

Hakan, Gunduz., Sahin, Sonmez., Saffet, Ayasun.(2017), Comprehensive gain and phase margins based stability analysis of micro-grid frequency control system with constant communication time delay, *IET Generation, Transmission & Distribution*, 11(3), 719-729.

Hamamci SE and Koksal M.(2010) Calculation of all stabilizing fractional-order PD controllers for integrating time delays, *Computers and Mathematics with Applications*, 59(5),1621–1629.

K, Ramakrishnan. & G, Ray.(2015). Stability criteria for nonlinearly perturbed load frequency systems with time-delay, *IEEE Journal of Emerging and Selected Topics in Circuits and Systems*, 5(3), 383-392.

K. Ramakrishnan, and N. Swarnalakshmi. (2018). Impact of Gain and Phase Margins on Stability of Networked Micro-Grid Frequency Control System", *4th International Conference on Electrical Energy Systems (ICEES).*

M. A. Hernández-Pérez, B. del Muro-Cuéllar, M. Velasco-Villa, D. F. Novella-Rodríguez, R. A. Garrido-Moctezuma, P. J. García-Ramírez (2018), "An improvement on the PI controller for a class of high-order unstable delayed systems: Application to a thermal process, *Control Engineering and Applied Informatics*, 20(1), 25-35.

Olgac N and Sipahi R.(2002). An exact method for the stability analysis of time-delayed linear time-invariant (LTI) systems, *IEEE Transactions on Automatic Control* 47(5),793–797.

Olugbenga K. Ogidan, Carl Kriger, Raynitchka Tzoneva (2017), "Networked Control with Time Delay Compensation Scheme Based on a Smith Predictor for the Activated Sludge Process, *Control Engineering and Applied Informatics*, 19(3),79-87.

Padmakshi, Agarwal.,Adhirath, Sikand., Shanthi, V. (2014). Application of Heat Exchanger in Bioprocess Industry: A Review, *International Journal of Pharmacy and Pharmaceutical Sciences*, 6(1), 24-28.

Ramesh, K, Shan., Dusan, P, Sekulic. (2003).*Fundamentals of Heat Exchanger Design*, John Wiley & Sons, Inc., Hoboken, New Jersey.

SaffetAyasun. (2008). Computation of time delay margin for power system small signal stability, *European Transactions on Electrical Power*.19(7) ,949-968.

SahajSaxena, Yogesh V. Hote. (2018). PI Controller Based Load Frequency Control Approach for Single-Area Power System Having Communication Delay, *IFAC-Papers On Line*,51(4), 622-626.

Sahin, Sonmez., Saffet, Ayasun.(2016).Stability region in the parameter space of PI controller for a single-area load frequency control system with time-delay, *IEEE Transaction on Power Systems*, 31(1), 829-830.

Sahin, Sonmez., Saffet, Ayasun.(2016). Stability regions in controller parameter space of DC motor speed control system with communication delays, *International Journal of Scientific Engineering and Applied Science*,2(6), 86-93.

Sahin, Sonmez and Saffet, Ayasun.(2017). Gain and phase margins based delay-dependent stability analysis of single-area load frequency control system with constant communication time delay, *Transactions of the Institute of Measurement and Control,* 40(5),1701-1710.

SahinSonmez and SaffetAyasun. (2019). Gain and phase margins-based delay margin computation of load frequency control systems using Rekasius substitution, *Transactions of the Institute of Measurement and Control*, 41(12), 3385-3395.

S Manikandan and P Kokil (2017). A Discrete Delay N-decomposition approach for Delay-Dependent Stability of Generator Excitation Control system with Constant Communication Delays, *International Journal of Robotics and Automation,* 6(4),234-240.

V. Venkatachalam, D. Prabhakaran.(2018). Stability analysis of Network Controlled Temperature Control System with Additive Time-varying Delays, *Computer modelling in Engineering and Science*, Vol.114(3), 321-334.

V. Venkatachalam, D. Prabhakaran, K. Ramakrishnan (2019). Delay-dependent stability analysis for non-linearly perturbed and parametric uncertain heat-exchanger system with time varying-delay, *Journal of Applied research and technology*, Vol.17,.313-325.

V. Venkatachalam, D. Prabhakaran, M. Thirumarimurugan, K. Ramakrishnan. (2019). LMI based Stability analysis of Non-linearly perturbed DC Motor Speed Control System with Constant and Time-varying Delays" *International Journal of Power Electronics*, vol.½,1-17.

Venkatachalam, V., Prabhakaran, D., Thirumarimurugan, M., Ramakrishnan, K.(2017). Stability of networked based thermal control system with time-invariant delay, *International Research Journal of Pharmacy*, 8(10), 146-152.

Venkatachalam, Veeraragavan., Prabhakaran, Duraisamy., Thirumarimurugan, Marimuthu., Ramakrishnan, Krishnan. (2017).Stability analysis of uncertain temperature control system with two additive delays and nonlinear perturbation, *Journal of Advances in Chemistry*, 13(10), 5927- 5934.