Improved Fractional-Order Distributed Kalman Filter For Use In Time-Delay Sensor Networks

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Abstract: Presently, distributed network systems are extensively used in a wide range of applications such as war field supervision, target tracing and positioning, error recognition, etc. However, a mechanism such as Kalman is needed to resolve issues such as configuration of topologies at the physical layer of sensor networks and delay in measurement time and data transmission in order to guarantee correctness and accuracy of parameter measured by the sensors. On the other hand, fractional calculus which is a generalization of integer order operators allows for highly precise modelling of physical systems. Thus, a new fractional-order distributed Kalman filter algorithm is presented for state estimation in measurement time-delay sensor networks in this paper. Therefore, at first fractional distributed Kalman filter algorithms and then their performance metrics such as means squared deviation and average will be evaluated to investigate feasibility of the algorithm. Finally, simulations show that performance of the proposed algorithm in terms of accuracy and efficiency has considerably improved as compared with previously proposed approaches such as conventional fractional-order Kalman filter.

Keywords: Fractional-order calculus, sensor networks, distributed Kalman filter, time delay data measurement

1. INTRODUCTION

During the past decade, wireless sensor networks have appeared as a powerful low cost platform for connecting large sensor networks. These networks which serve as a new technology are built of a large number of small sensors which are spread in the physical environment. Each sensor is able to perform a limited amount of calculations, establish radio communications and perform measurements.

Wireless sensor networks have been applied to a wide range of applications in recent years including environment supervision (Mainwaring et al., 2002), hygiene (Noel et al., 2017), military (Azzabi et al., 2017), war field supervision, target tracing and positioning, error recognition, etc. (Li et al., 2015; Song et al., 2014; Jiang et al., 2016; Liu et al., 2017). Considering the variety of different functions of sensor networks, each node can be made up of different components based on its defined duties. However, in general each node is built of a series of main components which are as follows: central processor unit, radio sender-receiver, power source and one or a few sensors which gather needed data from the environment. In distributed algorithms, a set of nodes can exactly estimate target state by cooperation. These nodes can be computers, cell phones or sensors (Abadi and Shafiee, 2018).

Distributed estimate algorithms are used in different fields such as sensor and wireless networks which are simple to expand, robust and low in energy consumption as some of their desirable characteristics (Al-Sayed et al., 2018; Fernandez-Bes et al., 2017).

Different approaches have been proposed for state estimation including Bayesian estimation approaches (Lainiotis, 1971; Särkkä, 2010) and Kalman filters (Kalman, 1960). Kalman filter algorithms are among the most favourite approaches for state estimation of dynamic systems by measurement. A small amount of memory and calculations is needed to implement a Kalman filter as a recursive algorithm. This provides for using this algorithm in real time systems.

Fractional calculus has attracted attention of many researchers as an expanded model of integer order differentials and derivatives due to their practical applications (Petras, 2011; Podlubny, 1998). Moreover, some systems such as Lithium-Ion batteries (Nasser-Eddine, 2018) cannot be modeled with integer order derivatives. Rather, they can only be modelled by means of fractional derivatives.

Thus, fractional-order Kalman filter algorithm was proposed for state estimation of fractional-order linear systems due to the importance of fractional-order systems and the severe weak performance of integer order Kalman filters in state estimation of such systems (Sierociuk and Dzieliński, 2006).

In recent years, more attention has been paid to fractional-order Kalman filters and the research done on this issue has been mainly focused on systems without time-delay (Sierociuk et al., 2011; Sadeghian and Salarieh, 2011; Sun and Yan, 2011).

Time-delays are usually encountered in industrial applications such as heat exchange, mining processes, steel production and so on. Time-delay processes exist in biological systems and mechanical systems including economic or electric fields. However, they are not limited to industrial applications. Physical phenomena which need information transmission, energy or different masses produce time delays.

For example, when sensors measurement and receive signals or when microcontrollers (or other machines) generate control signals to actuate upon processes and become active in the process a time delay is created (Birs et al., 2019). Yet, time delays cannot be ignored for a large class of practical applications. For example, sensor networks are built of a group of intelligible sensors which communicate with each other. Each intelligent sensor communicates with its neighboring intelligent sensor by means of time-delayed wireless networks (Yang H. et al., 2019). If time delays are not considered in sensor networks, there will be serious degradation of state estimation (Yang H. et al., 2020).

Many research studies on delay in fractional-order systems have been reported in recent years given the above mentioned importance of fractional-order systems and solving state estimation problems in time-delayed systems (Azami et al., 2017; Torabi et al., 2016; Yan and Kou, 2012; Ding and Ye, 2009).

Time-delay fractional-order systems may also be found in practice. For example, motion control systems with actuator limitations may be modeled with time-delay fractional systems (Tang et al., 2017; Marzban and Razzaghi, 2005). Thus, development of state estimation approaches for time-delay fractional-order systems is important. In this paper an improved fractional-order distributed Kalman filter is presented for use in time-delay sensor networks. Here the fractional-order model used for tracing position of a projectile is presented in order to evaluate performance of the proposed algorithm.

Simulation results verify considerable improvement in performance of mean-squared deviation of the proposed improved time-delayed fractional-order distributed Kalman filter algorithm in comparison with unimproved fractional-order distributed Kalman filters in sensor networks.

The paper is organized as follows. The problem study is presented in section 2. The fractional-order distributed Kalman filter algorithm is discussed in section 3. The fractional-order distributed Kalman filter algorithm for estimating sensor networks with time delay is proposed in section 4. In section 5, numerical simulation for analyzing the algorithm is presented and in section 6, the conclusion is discussed.

2. PROBLEM FORMULATION

At first fractional-order random linear discrete time state space equations are defined in this section. Then, the concept of time delay is defined and dynamic equations of random discrete time linear fractional-order random system plus measurement equations for delayed sensors in state space are discussed. For this purpose, lets first consider a linear discrete time fractional-order system with M groups of variables from a position state vector based on the fractional-order Litinikuf-Granold $x_k = [x_k^1, \dots, x_k^M]^T$ (Stanisławski et al., 2015).

Definition 1: State space equations for discrete time fractional-order random linear systems are generally defined by equations (1) and (2) (Cattivelli et al., 2010; Sierociuk and Dzieliński, 2006):

$$\begin{cases} \Delta^{\psi} x_{k+1} = F_k x_k + G_k w_k + w_k \quad u_k \sim (0, Q_k) \\ x_{k+1} = \Delta^{\psi} x_{k+1} - \sum_{j=1}^{k+1} (-1)^j \psi_j x_{k+1-j} \end{cases} \tag{1}$$

$$y_k = H_k x_k + v_k \qquad v_k \sim (0, R_k) \tag{2}$$

where $x_k \in R^M$ is a state vector, $u_k \in R^d$ is a system input, $y_k \in R^q$ is a system output, $F_k \in R^{M\times M}$, $G_k \in R^{M\times d}$and $H_k \in R^{q\times M}$ are the state system input and system output matrices, respectively, and w_k, v_k denote the state space noise with zero mean and covariance Q_k and system measurement noise with zero mean and covariance R_k, respectively. $\Delta^{\psi} x_k$ Of fractional-order differential with respect to ψ is for the system state vector of x_k and k is sampling time.

with

$$\begin{cases} \psi_k = \operatorname{diag}\left[\binom{n_1}{k} \dots \binom{n_M}{k}\right] \\ \Delta^{\psi} x_{k+1} = \begin{bmatrix} \Delta^{n_1} x_{1,k+1} \\ \vdots \\ \Delta^{n_M} x_{M,k+1} \end{bmatrix} \end{cases} \tag{3}$$

Where n1, n2, . . . , nM are the system equations orders and M denotes the number of these equations.

Assuming that noise signals u_k and v_k are white and independent, their covariance matrices are shown in Eq. (4), (Cattivelli et al., 2010):

$$E \begin{bmatrix} u_k \\ v_k \end{bmatrix} \begin{bmatrix} u_l \\ v_l \end{bmatrix}^* = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix} \delta_{kl} \tag{4}$$

where * shows transposed complex conjugate and δ_{kl} is the Kronecher delta function. Initial state vector x_0 is measurement noise and uncorrelated state with zero mean and covariance matrix $\Pi_0 > 0$. Moreover, Q_k *and* R_k are diagonal matrices with dimensions M and q.

Assume a set of N nodes (or sensors) distributed over an area. If two nodes directly communicate with each other, then they are two connected nodes. Thus, each node is always connected to itself. A collection of nodes connected to node i is called neighbor of the *ith* node and is shown by $\mathcal{N}_i$ $(i \in \mathcal{N}_i)$. Thus, adjacency matrix A is defined with elements $A_{i,l}$ as Eq. (5):

$$A = \{A_{i,l}\} = \begin{cases} 1, & l \in \mathcal{N}_i \\ 0, & otherwise \end{cases} \tag{5}$$

Assume that system output 1 (Eq. 2) is seen by N sensors such that each sensor just observes a limited number of considered characteristics as shown in Fig. 1.

If B_i shows the number of characteristics observed by the ith sensor and M is the number of system equations, then we can express observations made by sensor i at time k by the linear model in Eq. (6) (Ghanbari Firouzabadi et al., 2020):

$$y_{i,k} = H_{i,k}x_k + v_{i,k}, \quad B_i << M, \ i = 1, \dots, k \tag{6}$$

Where $y_{i,k} \in R^q$ shows measurements by the ith sensor at time k, $H_{i,k} \in R^{B_i \times M}$ is local observations matrix and $v_{i,k} \in R^{B_i}$ is local observations noise for reflecting measurement inaccuracy considering sensor accuracy and other unavoidable limitations.

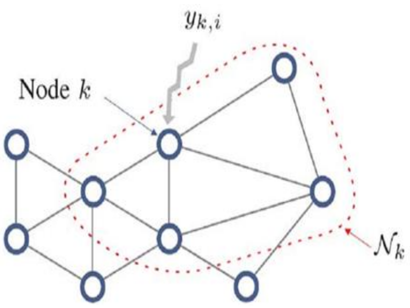

Fig. 1. Showing the measurement of system output $y_{i,k}$ by node i at moment k.

Widespread observations model is obtained by gathering the observations as Eq. (7):

$$y_k = \begin{bmatrix} y_{1,k} \\ \vdots \\ y_{N,k} \end{bmatrix}, \ H_k = \begin{bmatrix} H_{1,k} \\ \vdots \\ H_{N,k} \end{bmatrix}, v_k = \begin{bmatrix} v_{1,k} \\ \vdots \\ v_{N,k} \end{bmatrix} \tag{7}$$

The global observations matrix $y_k \in R^{\sum_{i=1}^{N} B_i}$ is assumed to be as shown in Eq. (8):

$$\mathrm{y_k = H_k x_k + v_k} \tag{8}$$

Assume that measurement noise $v_{i,k}$ is unconnected, Then we can write:

$$E\left[v_{i,k}v_{j,l}^*\right] = R_{i,k}\delta_{j,i}\delta_{l,k} \tag{9}$$

Where $R_{i,k} > 0$ for all i, k.

Fig. 2 shows the effect of time-delay sensor network in time sequence.

N different buffers exist for storing related local estimation signals and they store the latest data in time (Liu et al., 2017).

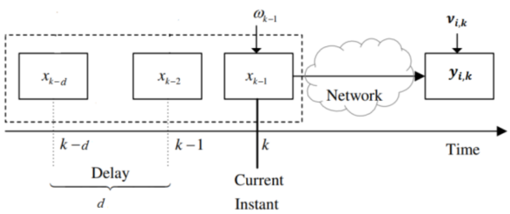

Fig. 2. Structure of a distributed system.

Definition 2: Discrete time linear fractional-order random system dynamic equation and the measurement equations of delayed sensors in state space based upon Granold-Litinikuf fractional derivative are presented as the collection of Eq. (10):

$$\begin{aligned} &\Delta^{\psi}x_{k+1} = F_k x_k + G_k w_k + w_k, \qquad u_k \sim (0, Q_k) \\ &x_{k+1} = \Delta^{\psi}x_{k+1} - \textstyle\sum_{j=1}^{k+1}(-1)^j \psi_j x_{k+1-j} \\ &y_{i,k} = H_{i,k}x_{i,k-d_i} + v_{i,k}, \qquad v_{i,k} \sim \left(0, R_{i,k}\right) \end{aligned} \tag{10}$$

3. DISTRIBUTED FRACTIONAL-ORDER KALMAN FILTER ALGORITHM

State estimation is very important in such systems due to the fact that describing systems with fractional-order models is closer to real systems. Authors of (Sierociuk and Dzieliński, 2006) in 2006 proposed a generalization of Kalman filters for discrete linear fractional order systems which makes it possible to use fractional order Kalman filter algorithm for parameter estimation and fractional order systems.

The aim of implementing distributed fractional-order Kalman filter, x_k is uncertain state estimation in system 1 (Eq. 2) along with measurement Eq. (6) for each node i of the network. It should be mentioned that in the network shown in Fig. 1 the nodes are only able to share their data with their neighboring nodes $\{l \in \mathcal{N}_i\}$.

The main challenge is to assure an exact estimation of system's state such that even if each node has access to all measurements in the whole network, accuracy of state estimation does not increase. The distributed fractional-order Kalman filter used to overcome this challenge is shown in Table 1, (Ghanbari Firouzabadi et al., 2020).

Table 1. Fractional-order distributed Kalman filter algorithm.

Consider the fractional-order state space model (1):
For each node i we have: $\hat{x}_{i,0|-1} = E(x_0), P_{i,0|-1} = \Pi_0$
In each sampling period k, repeat the following two phases:
phases 1: Incremental update

$$P_{i,k|k}^{-1} = P_{i,k|k-1}^{-1} + \sum\nolimits_{l \in \mathcal{N}_i} H_{l,k}^* R_{l,k}^{-1} H_{l,k} \tag{12}$$

$$\phi_{i,k|k} = \hat{x}_{i,k|k-1} + P_{i,k|k} \sum_{l \in \mathcal{N}_i} H_{l,k}^* R_{l,k}^{-1}\left(y_{l,k} - H_{l,k}\hat{x}_{i,k|k-1}\right) \tag{13}$$

phases 2: Update time

$$\begin{cases} \hat{x}_{i,k|k} = \phi_{i,k|k} \\ \Delta^{\Upsilon}\hat{x}_{i,k+1|k} = F_k \hat{x}_{i,k|k} \\ \hat{x}_{i,k+1|k} = \Delta^{\Upsilon}\hat{x}_{i,k+1|k} - \sum_{j=1}^{k+1}(-1)^j \Upsilon_j \, \hat{x}_{i,k+1-j} \\ P_{i,k+1|k} = (F_k + \Upsilon_1)P_{i,k|k}(F_k + \Upsilon_1)^* + G_k Q_k G_k^* + \sum_{j=2}^{k} \Upsilon_j P_{i,k-j} \Upsilon_j^T \end{cases} \tag{14}$$

Instead of updating measurements sequential updating is used in the above algorithm since in this phase local optimized estimation at node i is done by gradual sequential addition of measurements at neighboring nodes $\{y_{l,k}, l \in \mathcal{N}_i\}$.

4. IMPROVED DISTRIBUTED FRACTIONAL-ORDER KALMAN FILTER ALGORITHM FOR ESTIMATION IN TIME-DELAY SENSOR NETWORKS

Each subsystem shares its local information with that of its neighboring sensors to obtain additional information about system dynamics with introduction of delayed distributed complex structure in Eq. (10). Now assume that there is some time delay at the time of sending data from each sensor to the processor.

Thus, each data processor can coordinate its behavior by receiving data from other sensors in a special area. Each sensor communicates with its neighbors to exchange data at the estimation coordination center. Thus, distribution coordination strategy has flexibility as advantages.

Theorem 1. If we have discrete time fractional-order random systems with equations as stated in definition 1 which have time delay in sensor networks, then the simplified Kalman filter (which is named improved fractional-order distributed Kalman filter) is obtained in two phases as stated below.

A: Sequential updating

$$\begin{aligned}&\hat{x}_{i,k|k-1}\\&=(\Pi_{l=1}^{d_i}F_{k-l})\hat{x}_{i,k-d_i|k-d_i}\\&-\sum_{m=1}^{d_i-1}\left((\Pi_{l=1}^{m}F_{k-l})\sum_{j=1}^{k-m}(-1)^j\psi_j\hat{x}_{i,k-m-j|k-m-j}\right)\\&-\sum_{j=1}^{k}(-1)^j\psi_j\hat{x}_{i,k-j|k-j}\end{aligned} \tag{11}$$

$$P_{i,k|k}^{-1}=P_{i,k|k-1}^{-1}+\sum_{l\in\mathcal{N}_i}H_{l,k}^*R_{l,k}^{-1}H_{l,k}$$

$$\phi_{i,k|k}=\hat{x}_{i,k|k-1}+P_{i,k|k}\sum_{l\in\mathcal{N}_i}H_{l,k}^*R_{l,k}^{-1}(y_{l,k}-H_{l,k}\hat{x}_{i,k|k-1})$$

B: Time updating

$$\hat{x}_{i,k|k}=\phi_{i,k|k} \tag{12}$$

$$\Delta^{\psi}\hat{x}_{i,k+1|k}=F_k\hat{x}_{i,k|k}$$

$$\hat{x}_{i,k+1|k}=\Delta^{\psi}\hat{x}_{i,k+1|k}-\sum_{j=1}^{k+1}(-1)^j\psi_j\,\hat{x}_{i,k+1-j}$$

$$\begin{aligned}P_{i,k+1|k}&=(\Pi_{l=1}^{d_i}F_{k-l})\,P_{k|k}(\Pi_{l=1}^{d_i}F_{k-l})^T+Q_{k-d_i}\\&+\sum_{m=1}^{d_i-1}\left((\Pi_{l=1}^{m}F_{k-l})\sum_{j=1}^{k-m}\psi_jP_{i,k-m-j|k-m-j}\,\psi_j{}^T(\Pi_{l=1}^{m}F_{k-l})^T\right)\\&+\sum_{j=1}^{k}\psi_j\,P_{k-j|k-j}\psi_j{}^T\end{aligned}$$

With initial conditions $\hat{x}_{i,0|-1}=E(x_0),P_{i,0|-1}=\Pi_0$

End of Theorem 1. □

Proof. Measurement output $y_{i,k}$ in Eq. (10) is expressed by the ith sensor by means of state $x_{i,k-d_i}$ at time k and the local state estimation of the fractional order Kalman filter is shown as $\hat{x}_{i,k}$.

In equations (13) and (15), local state estimations of fractional-order Kalman filter at times k and k-1, respectively are shown:

$$\hat{x}_{i,k}=F_{k-1}\hat{x}_{i,k-1}-\sum_{j=1}^{k}(-1)^j\psi_j\hat{x}_{i,k+1-j} \tag{13}$$

Where Eq. (13) can be written as follows:

$$\begin{aligned}\hat{x}_{i,k}=F_{k-2}(F_{k-1}\hat{x}_{i,k-1}&\\-\sum_{j=1}^{k-2}(-1)^j\psi_j\hat{x}_{i,k-2-j})&\\-\sum_{j=1}^{k-1}(-1)^j\psi_j\hat{x}_{i,k-1-j}&\end{aligned} \tag{14}$$

$$\hat{x}_{i,k-1}=F_{k-2}\hat{x}_{i,k-2}-\sum_{j=1}^{k-1}(-1)^j\psi_j\hat{x}_{i,k-j} \tag{15}$$

Using reorganizing approach changes the systems to measurement without delay equivalent systems by measuring the delay time.

Remark 1. Especially when time delays are long, using the above mentioned approach for solving different kinds of Kalman filters of the same dimensions is proposed for the main system for calculations load.

In Eq. (10), time-delay measurement is investigated from repeated measurement output $\{y_{i,k},y_{i,k-1},\dots,y_{i,k-d_i}\}$ and measurement noise $\{v_{i,k},v_{i,k-1},\dots,v_{i,k-d_i}\}$.

Note that measurement noise sequence is also white noise with zero mean and covariance $R_{i,k}$.

Estimating the minimum mean squared error,$\hat{x}_{i,k}$ is defined by Eq. (16):

$$\begin{aligned}\hat{x}_{i,k}&=Proj\{x_k|y_{i,k},\dots,y_{i,k-d},\dots,y_{i,0}\}\\&=(\Pi_{l=1}^{d_i}F_{k-l})\hat{x}_{i,k-d_i}\\&-\sum_{m=1}^{d_i-1}\left((\Pi_{l=1}^{m}F_{k-l})\sum_{j=1}^{k-m}(-1)^j\psi_j\hat{x}_{i,k-m-j}\right)\\&-\sum_{j=1}^{k}(-1)^j\psi_j\hat{x}_{i,k-j}\end{aligned} \tag{16}$$

That is equal to compensating the filtered amount $\hat{x}_{i,k-d_i|k-d_i}$ (for example $(\Pi_{l=1}^{d_i}F_{k-l})\hat{x}_{i,k-d_i|k-d_i}$).

Now by placing in Eq. (17), (estimating the previous state) we have:

$$\begin{aligned}&\hat{x}_{i,k|k-1}=Proj\{x_k|y_{i,k},\dots,y_{i,k-d},\dots,y_{i,0}\}\\&=(\Pi_{l=1}^{d_i}F_{k-l})\hat{x}_{i,k-d_i|k-d_i}\\&-\sum_{m=1}^{d_i-1}\left((\Pi_{l=1}^{m}F_{k-l})\sum_{j=1}^{k-m}(-1)^j\psi_j\hat{x}_{i,k-m-j|k-m-j}\right)\\&-\sum_{j=1}^{k}(-1)^j\psi_j\hat{x}_{i,k-j|k-j}\end{aligned} \tag{17}$$

Since the measurement sequence includes d-step time delays for estimating local state $\hat{x}_{i,k}$, the repeated measurement order is proposed for designing a predictor $\hat{x}_{i,k-d_i}$.

Remark 2. Predictor error and estimation error have been shown in equations (18) and (19):

$$\tilde{x}_{i,k|k-d_i}=x_k-\hat{x}_{i,k|k-d_i} \tag{18}$$

$$\tilde{x}_{i,k}=x_k-\hat{x}_{i,k} \tag{19}$$

Estimation error covariance matrix is obtained from Eq. (20):

$$P_{i,k+1|k}=E\left[(x_{i,k+1-d_i|k-d_i}-\hat{x}_{i,k-d_i|k-d_i})(x_{i,k+1-d_i|k-d_i}-\hat{x}_{i,k-d_i|k-d_i})^T\right] \tag{20}$$

In Eq. (20), the term $x_{i,k+1-d_i|k-d_i}-\hat{x}_{i,k-d_i|k-d_i}$ is obtained from Eq. (21):

$$\begin{aligned}&\tilde{x}_{i,k-d_i}=x_{i,k+1-d_i|k-d_i}-\hat{x}_{i,k-d_i|k-d_i}\\&=(\Pi_{l=1}^{d_i}F_{k-l})(x_{i,k-d_i|k-d_i}-\hat{x}_{i,k-d_i|k-d_i})\\&-\sum_{m=1}^{d_i-1}\left((\Pi_{l=1}^{m}F_{k-l})\sum_{j=1}^{k-m}(-1)^j\psi_j(x_{i,k-m-j|k-m-j}-\hat{x}_{i,k-m-j|k-m-j})\right)\\&-\sum_{j=1}^{k}(-1)^j\psi_j(x_{i,k-m-j|k-m-j}-\hat{x}_{i,k-m-j|k-m-j})+\mathrm{u}_{\mathrm{k-d_i}}\end{aligned} \tag{21}$$

By placing relation Eq. (21), in Eq. (20), we have:

$$
\begin{aligned}
P_{i,k+1|k} = {} & \\
& \left(\Pi_{l=1}^{d_i} F_{k-l}\right) E\Big[\big(x_{i,k-d_i|k-d_i} - \hat{x}_{i,k-d_i|k-d_i}\big)\big(x_{i,k-d_i|k-d_i} - \hat{x}_{i,k-d_i|k-d_i}\big)^T\Big] \left(\Pi_{l=1}^{d_i} F_{k-l}\right)^T \\
& + E\left[u_{k-d_i} {u_{k-d_i}}^T\right] + \sum_{m=1}^{d_i-1} \Bigg(\left(\Pi_{l=1}^{m} F_{k-l}\right) \sum_{j=1}^{k-m} (-1)^j \psi_j E\Big[\big(x_{i,k-m-j|k-m-j} - \hat{x}_{i,k-m-j|k-m-j}\big)\big(x_{i,k-m-j|k-m-j} - \hat{x}_{i,k-m-j|k-m-j}\big)^T {\psi_j}^T \left(\Pi_{l=1}^{m} F_{k-l}\right)^T \Bigg) \\
& + \sum_{j=1}^{k} \psi_j E\Big[\big(x_{i,k-j|k-j} - \hat{x}_{i,k-j|k-j}\big)\big(x_{i,k-j|k-j} - \hat{x}_{i,k-j|k-j}\big)^T\Big] {\psi_j}^T
\end{aligned}
\tag{22}
$$

Eq. (22) can be written briefly as:

$$
\begin{aligned}
P_{i,k+1|k} = {} & \left(\Pi_{l=1}^{d_i} F_{k-l}\right) P_{k|k} \left(\Pi_{l=1}^{d_i} F_{k-l}\right)^T + Q_{k-d_i} \\
& + \sum_{m=1}^{d_i-1} \left(\left(\Pi_{l=1}^{m} F_{k-l}\right) \sum_{j=1}^{k-m} \psi_j P_{i,k-m-j|k-m-j} {\psi_j}^T \left(\Pi_{l=1}^{m} F_{k-l}\right)^T \right) \\
& + \sum_{j=1}^{k} \psi_j P_{k-j|k-j} {\psi_j}^T
\end{aligned}
\tag{23}
$$

Assume that node i has access to its neighbors' measurements $\mathcal{N}_i$. Local estimation at node i can be calculated by performing some measurement updating (each updating is done for every neighbor i). We have the following using matrix inversion lemma (Tylavsky and Sohie, 1986):

$$
P_{i,k|k}^{-1} = P_{i,k|k-1}^{-1} + \sum_{l \in N_i} H_{l,k}^{*} R_{l,k}^{-1} H_{l,k}^{T} \tag{24}
$$

Thus, measurement updating for $\hat{x}_{i,k|k}$ is obtained by Eq. (25):

$$
\hat{x}_{i,k|k} = \hat{x}_{i,k|k-1} + P_{i,k|k} \sum_{l \in N_i} H_{l,k}^{*} R_{l,k}^{-1} \left(y_{l,k} - H_{l,k} \hat{x}_{i,k|k-1}\right) \tag{25}
$$

Local estimations in the sequential updating phase of distributed fractional-order Kalman filter is done by considering Eq. (25).

That proves Theorem. □

In Table 2, the improved distributed fractional-order Kalman filter algorithm for estimation in time delay sensor networks is briefly shown in Table 2.

Table 2. Improved distributed fractional-order Kalman filter algorithm for estimation in time-delay sensor networks.

Consider fractional order state space model (10) and (11):
For each node i we have: $\hat{x}_{i,0|-1} = E(x_0), P_{i,0|-1} = \Pi_0$
In each sampling period k, repeat the following two phases:
phases 1: Sequential updating

$$
\begin{aligned}
& \hat{x}_{i,k|k-1} = \left(\Pi_{l=1}^{d_i} F_{k-l}\right) \hat{x}_{i,k-d_i|k-d_i} \\
& - \sum_{m=1}^{d_i-1} \left(\left(\Pi_{l=1}^{m} F_{k-l}\right) \sum_{j=1}^{k-m} (-1)^j \psi_j \hat{x}_{i,k-m-j|k-m-j} \right) \\
& - \sum_{j=1}^{k} (-1)^j \psi_j \hat{x}_{i,k-j|k-j} \\
& P_{i,k|k}^{-1} = P_{i,k|k-1}^{-1} + \sum_{l \in N_i} H_{l,k}^{*} R_{l,k}^{-1} H_{l,k} \\
& \phi_{i,k|k} = \hat{x}_{i,k|k-1} + P_{i,k|k} \sum_{l \in N_i} H_{l,k}^{*} R_{l,k}^{-1} \left(y_{l,k} - H_{l,k} \hat{x}_{i,k|k-1}\right)
\end{aligned}
$$

phases 2: Update time

$$
\begin{gathered}
\hat{x}_{i,k|k} = \phi_{i,k|k} \\
\Delta^{\psi} \hat{x}_{i,k+1|k} = F_k \hat{x}_{i,k|k} \\
\hat{x}_{i,k+1|k} = \Delta^{\psi} \hat{x}_{i,k+1|k} - \sum_{j=1}^{k+1} (-1)^j \psi_j \hat{x}_{i,k+1-j} \\
P_{i,k+1|k} = \left(\Pi_{l=1}^{d_i} F_{k-l}\right) P_{k|k} \left(\Pi_{l=1}^{d_i} F_{k-l}\right)^T + Q_{k-d_i} \\
+ \sum_{m=1}^{d_i-1} \left(\left(\Pi_{l=1}^{m} F_{k-l}\right) \sum_{j=1}^{k-m} \psi_j P_{i,k-m-j|k-m-j} {\psi_j}^T \left(\Pi_{l=1}^{m} F_{k-l}\right)^T \right) + \sum_{j=1}^{k} \psi_j P_{k-j|k-j} {\psi_j}^T
\end{gathered}
$$

6. SIMULATION

In this section, a projectile path tracing measurement scenario in a wireless time-delay sensor network is implemented in order to numerically evaluate operation of the improved fractional-order distributed Kalman filter algorithm for estimation in time-delay sensor networks. Then, performance of the proposed improved fractional-order distributed Kalman filter algorithm is compared with that of conventional fractional-order distributed Kalman filter algorithm for projectile state estimation. The results obtained from simulations verify proper performance and estimation error convergence of the proposed fractional-order Kalman filter algorithm.

Moreover, estimation precision of the proposed fractional-order distributed Kalman filter algorithm shows considerable improvement compared with that of conventional fractional-order distributed Kalman filters. Consider a set of sensors in a time-delay wireless sensor network which tries to estimate and trace path of a projectile. Assume that the projectile is near an adaptive network where the sensors observe projectile position subject to noise. This network includes 20 agents or sensors with topology as shown in Figure 3 where the branches show the communication lines between the agents. At the same time, each sensor node can independently obtain projectile's position and communicate with its neighbors.

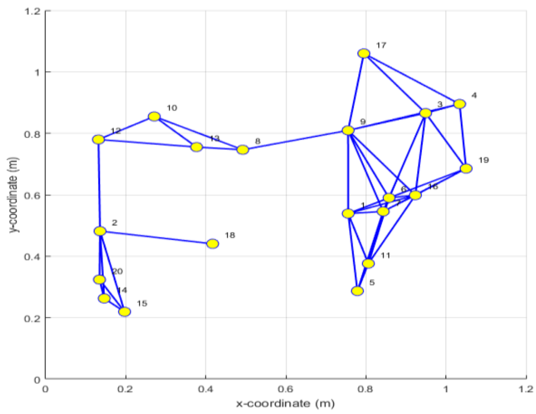

Fig. 3. Network topology with N = 20 nodes.

Acceleration a, speed v, and projectile's position p may be written as follows:

$$
\mathrm{a} = \begin{bmatrix} \mathrm{a_x} \\ \mathrm{a_y} \\ \mathrm{a_z} \end{bmatrix}, \mathrm{v} = \begin{bmatrix} \mathrm{v_x} \\ \mathrm{v_y} \\ \mathrm{v_z} \end{bmatrix}, \mathrm{p} = \begin{bmatrix} \mathrm{p_x} \\ \mathrm{p_y} \\ \mathrm{p_z} \end{bmatrix} \tag{26}
$$

We can write Eq. (27), for projectile's motion (Ebaid, 2011):

$$
\begin{aligned}
& \mathrm{D^{n_1} v(t) = a(t)} \\
& \mathrm{D^{n_2} p(t) = v(t)} \\
& \mathrm{a_x = a_y = 0,\ a_z = -g}
\end{aligned}
\tag{27}
$$

Where g is the acceleration of gravity on earth.

The system's state vector is a six-dimensional matrix which consists of speed and position of the projectile as follows:

Thus, process dynamic is as follows by considering Eq. (28):

$$\begin{aligned} & f_k(x_k, u_k) = [D^n v \quad D^n p]^T \times h^n \\ & \Delta^{\Psi} x_{k+1} = f_k(x_k, w_k) + w_k,\ k \geq 0 \quad u_k = (0, Q_k) \\ & x_{k+1} = \Delta^{\Psi} x_{k+1} - \sum_{j=1}^{k+1} (-1)^j \psi_j x_{k+1-j},\ k \geq 0 \end{aligned} \tag{28}$$

Where $n = 0.99$ is the fractional-order system, $x_k = [v_k, p_k]^T$ system modes with default values. Moreover, $x_0 = [0.7, 0.1, 0.2, 0.8, 0.2]^T$ and $w_k = [0]$ are input system. Assume that each node measures the position of an uncertain target in one of the following two states:

$H_{i,k} = [0, \mathrm{diag}([1\ 1\ 0])]$, is for the case in which just the horizontal dimensions are seen and $H_{i,k} = [0\ \ , \mathrm{diag}([1\ 0\ 1])]$, for the case in which only a horizontal dimension and one vertical dimension are seen.

Thus, the nodes do not have the capability of direct measurement of projectile position in three dimensions. Also, time delay at each node is a randomly chosen number between zeros to three sampling periods. Creating an observable pair is randomly done by each node.

The parameters are $h = 0.1, G_k = I_6, Q_k = 0.001 I_6, S_i = 0$ and $R_{i,k} = \sqrt{i} P R_0 P^T$ with $R_0 = 0.5 \times \mathrm{diag}([1\ 4\ 7])$ is a permutation matrix that is randomly chosen for each node .

The $\sqrt{k}$ coefficient allows consideration of various different noise conditions for each node.

Initial state values are $x_0 = (10, 2, 8, 0.1, 0.1, 0.1)^T$ and $P_0 = I_6$.

The real vertical path (straight line) and the vertical position noise measurements at node 5 (dashed line) at node 5 are shown in Fig. 4. Moreover, Fig. 5 shows performance of state estimation while estimating vertical position for various different algorithms in the whole network. The remaining two curves are related to conventional fractional-order distributed Kalman filter (Ghanbari Firouzabadi et al., 2020) and improved fractional-order distributed Kalman filter.

It is seen that estimations performed by the improved fractional-order distributed Kalman filter are closer to the real path when compared with other conventional fractional-order distributed Kalman filters and the improved fractional-order distributed Kalman filter has been able to mitigate the effects of time delays in measurements quite well.

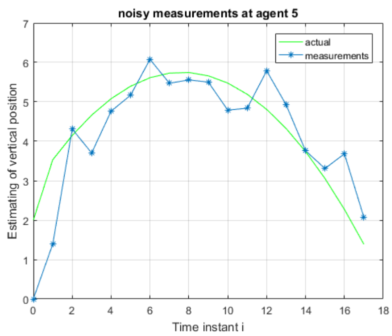

Fig. 4. Real vertical path and vertical position measurement noise at node 5.

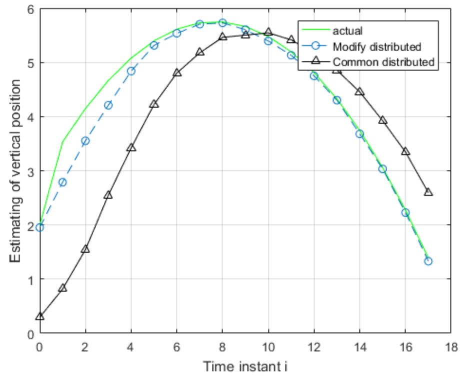

Fig. 5. Mean estimate of vertical position of all nodes by various different algorithms.

Remark 3. The mean squared deviation (MSD) metric is used for performance evaluation of fractional-order Kalman filters. It should be noted that the MSD metric is defined for all nodes by eq. (29):

$$\begin{aligned} & MSD_{i,k} = E\left\|x_i - \hat{x}_{i,k|k}\right\|^2 \\ & MSD_k^{avg} = \frac{1}{N} \sum_{i=1}^{N} MSD_{i,k} \end{aligned} \tag{29}$$

Where k is the time index and node i is where MSD is computed. There are different estimates produced by different distributed algorithms.

The MSD metric for the improved fractional-order Kalman filter is numerically compared with that of conventional fractional-order distributed Kalman filter. The value of MSD for the two algorithms is shown in Fig. 6. In this Figure, the x axis shows the number of iterations and the y axis is the value of MSD.

The error related to conventional fractional-order distributed Kalman filter is high, since the nodes do not have access to three-dimensional measurements of projectile motion and the pair $\{F, H_i^{loc}\}$ is unrecognizable as seen in the Figure.

We can conclude that the estimation obtained from the improved fractional-order distributed Kalman filter has considerable improvement over that of the conventional distributed Kalman filter by comparison and analysis of simulation.

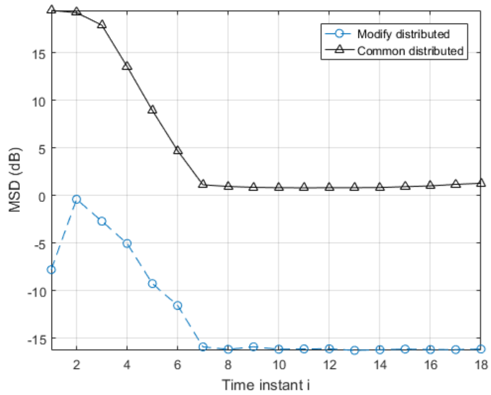

Fig. 6. The MSD metric for improved fractional-order distributed Kalman filter and conventional fractional-order distributed Kalman filter.

Table 3. MSD performance.

Algorithm	MSD performance(dB)	Computing time (s)
fractional-order distributed Kalman filter	5	2.2
fractional-order distributed Kalman filter with delay-time	-11	2.6

We may conclude from Table 3 that in the implemented approach, we increase estimation precision, the time needed for calculations increases and the value of MSD decreases. This points to the fact that error has lessened and precision has improved greatly although computational time has increased.

7. CONCLUSIONS

In this paper, a new improved fractional-order distributed Kalman filter algorithm is proposed for state estimation of time-delayed sensor network measurement. Then, performance of the mentioned algorithm is compared with that of conventional fractional-order distributed Kalman filter algorithm using MSD and mean to investigate its feasibility. Simulations results show that the improved fractional-order distributed Kalman filter has been able to mitigate the effects of time delays in measurements quite well. the accuracy of the estimation obtained by the improved fractional-order distributed Kalman filter has considerable improvement over that of the conventional distributed Kalman filter by comparison and analysis of simulation. Also, the implemented approach reflects that by increasing estimation precision, the time needed for calculations increases and the value of MSD decreases. This points to the fact that error has lessened and precision has improved greatly although computational time has increased.

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