

# RLS Estimation of Uncertainties in the Model Parameters and Decentralized Adaptive PI controller Using PSO for Chemical Multivariable Coupled System

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**Abstract:** This paper proposes a decentralized adaptive PSO-PI (particle swarm optimization-Proportional Integral) control strategy for an uncertain coupled multivariable system using input/output data. The example used to be examined is that of a distillation column, in which the model is considered unknown as well as these uncertainties. First, we have developed a recursive least squares (RLS) with exponential forgetting factor to estimate the parameters of the nominal system and these uncertainties. Then, the design of the decentralized adaptive PSO-PI controller is developed by the inverted decoupling network combined with the PSO-PI controllers, and all updated through the estimated parameters using RLS to get the adaptive controllers. To automatically adjust the parameters of the PSO-PI controllers with robustness within the closed-loop, a tuning formula is developed based on the fitness function of the PSO technique. The obtained results in comparative form with IMC-PI controller demonstrate that the proposed approach can find better performance for estimating and controlling multivariable process with a nominal model and variations of gains for different setpoint changes and disturbance rejection.

**Keywords:** Multivariable systems, recursive least squares (RLS), Decentralized control, particle swarm optimization (PSO), identification.

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## 1. INTRODUCTION

Implementations of control algorithms model-based are frequently under the assumption of prior identification of the dynamic characteristics of process and changes in operating point. According to reference (Xu et al., 2014), data-based controller design requires knowledge of I/O (input-output) measurement data and not a modeling based on assumptions.

The mathematical modeling often fails, because of uncertainties (Lee and Park, 2000), this is the case with dynamics that are strongly nonlinear (Wu et al., 2008; Su et al., 2012). System Identification is the theory and technique for system thematic design (Yuz et al., 2011; Herrera et al., 2011; Wang, 2011; Xie et al., 2010). Identification system will be useful for estimate the uncertainties which can be defined as a variation of model parameters (Zhong, 2009), making them a challenge to be tuned and control. Among the identification methods used: the stochastic gradient algorithm (SG) (Ding et al., 2008) and the RLS algorithm (Lennart, 1999; Ding et al., 2012).

In the contrast to the SG Algorithm, RLS algorithm achieves quicker convergence (Ding, 2013). RLS identification algorithm has been used by several processes, estimating unknown parameters (Tajdari, 2021) to automatically change the controller parameters in closed loop (Bai and Zhang, 2007; Li et al., 2014; Sundari and Nachiappan, 2014; Martinek et al., 2019; Sun et al., 2019).

The problem of building a mathematical model of a multivariable system is more complicated than a single-input, single-output system.

To illustrate multivariable processes, various mathematical models have been used, such as the state-space model and the transfer function matrix model (Lennart, 1999). Much work to identify multivariable systems based on the state-space model can be found in (Guidorzi and Diversi, 2003; Wigren, 2006; Li, 2013).

The PSO technique is widely applied to calculate the parameters of the PID controller; it is used in various engineering problems due to the high performance obtained (Banu and Uma, 2008; El-Gammal and El-Samahy, 2009; Allaoua et al., 2009; Ramli et al., 2009; Lin et al., 2010; Chen and Wang, 2011; Latha et al., 2013; Kumar and Gupta, 2013; Sultaniya and Gupta, 2014; Nithyarani et al., 2014; Kherici and Ali, 2014; Sungthong and Assawinchaichote, 2016; Illias et al., 2016; Chiou et al., 2016; Zhaosheng, 2016; Hamoudi, 2017; Yazgan et al., 2019).

In (Auxillia, 2015) has suggested an adaptive PSO focused on system identification and internal model sliding mode controller for the mass flow system. In (Yadav et al., 2019) have presented the estimate of the parameter and provided an extended predictive-based tuning approach for a Lab-Scale distillation column.

Different control strategies have been applied to the distillation column as can be found in (Murad et al., 1996; Benaskeur and Desbiens, 2002; Morilla et al., 2007; Regina and Mija, 2010; Arvani et al., 2009; Garrido et al., 2010; Mekki et al., 2013; Mekki, 2014; Mekki et al., 2016; Mekki et al., 2018). These scholars have not given adaptive control laws based on estimated model in any of their studies.

The approach of estimating the outputs of the WB model proposed in (Yadav et al., 2019) failed to find at the end exactly the parameters of the nominal model. In addition, we have not seen simulation figures that show the results obtained with uncertainties of the installation parameters of 30% in  $K$ ,  $\tau$  and  $\theta$  apart from the comparative tables and the responses of the nominal system with setpoint changes different from the nominal operating conditions of the column such as those presented by (Mekki et al., 2018).

For this, the first contribution is to obtain exactly the nominal model. It was considered that the uncertainties of the installation parameters of 20% in  $K$  are not known in advance. Each time, we will test the efficiency of RLS in estimating the parameters of the model. The second contribution is to realize a controller adapted to the variation of the parameters of the model.

Controlling the distillation process requires the development of a mathematical model that reflects the dynamics of the system. In this paper, identification is used to estimate all the parameters of the transfer matrix of the linearized transfer distillation process. It is assumed that the process is initially unknown during normal and uncertain operation and each parameter of the transfer functions of the transfer matrix can be estimated by applying the RLS method (this is conditional on the parameters being written in linear form).

The distillation column is a coupled multivariable system, and for control as (two independent) subsystems using the PSO-PI controllers to maintain the desired values, a decoupling network will be mandatory to minimize the effects of interactions existing between the control loops.

The calculations of the parameters of the PSO-PI controllers and the transfer functions of the reverse decoupling are adaptive and updated by the estimated parameters.

The linear regression method presents estimation errors of the system in (Yadav et al., 2019), it is for this reason, that we opted in this article to develop an RLS estimation algorithm to estimate the parameters nominal and uncertain value for a multivariable distillation system. This study is made only in simulation because of the lack of access to an experimental bench. The proposed controller is a decentralized PI using the PSO technique which helps to obtain the optimal gains while guaranteeing the best performance while respecting the operating conditions of the column.

## 2. DESCRIPTION OF DISTILLATION COLUMN SYSTEM AND MATHEMATICAL MODELING

The distillation column model is assumed to be structured as a first order system plus dead time (FOPDT). In a certain operational area, it is known as linear-time invariant (Yadav

et al., 2019). For the binary distillation process, the global equation of the system is given as a transfer function matrix (Mekki et al., 2018). This system has two manipulated variables are reflux  $L$  and steam flow rate  $V$  and two outputs are methanol composition in distillate (top)  $y_D$  and residue (bottom)  $x_B$ .  $F$  is the feed flow rate, it's considered as disturbance and  $s$  is the Laplace operator.

$$\begin{bmatrix} y_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} \frac{[K_P]_{11}}{[\tau_p]_{11}s+1} e^{-[\theta_p]_{11}s} & \frac{[K_P]_{12}}{[\tau_p]_{12}s+1} e^{-[\theta_p]_{12}s} \\ \frac{[K_P]_{21}}{[\tau_p]_{21}s+1} e^{-[\theta_p]_{21}s} & \frac{[K_P]_{22}}{[\tau_p]_{22}s+1} e^{-[\theta_p]_{22}s} \end{bmatrix} \cdot \begin{bmatrix} L(s) \\ V(s) \end{bmatrix} + \begin{bmatrix} \frac{[K_d]_{13}}{[\tau_d]_{13}s+1} e^{-[\theta_d]_{13}s} \\ \frac{[K_d]_{23}}{[\tau_d]_{23}s+1} e^{-[\theta_d]_{23}s} \end{bmatrix} \cdot F(s) \quad (1)$$

We can identify the parameters of this model by the gains  $([K_P]_{ij}, [K_d]_{ij})$ , the time constants  $([\tau_p]_{ij}, [\tau_d]_{ij})$  and delays  $([\theta_p]_{ij}, [\theta_d]_{ij})$ .

The last two parameters of the system are expressed in minutes (Mekki et al., 2018).

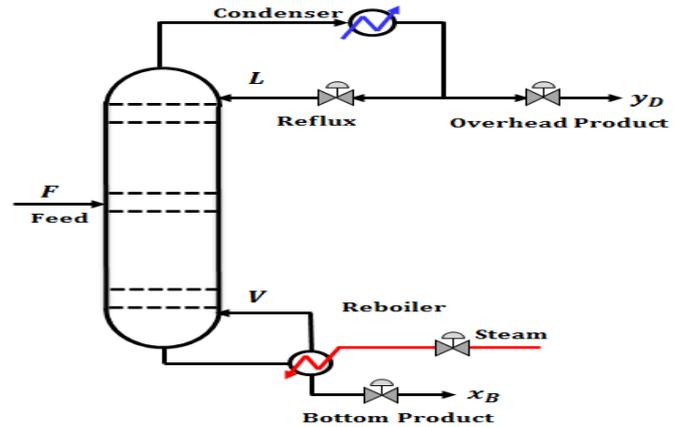


Fig. 1. Basic diagram of a continuous distillation column (Mekki et al., 2013).

## 3. DESIGN OF DECENTRALIZED ADAPTIVE PSO-PI CONTROLLER USING RLS IDENTIFICATION SYSTEM

### 3.1 RLS system identification

System identification is an operation for obtaining a mathematical model of a process from input and output signals. The identified transfer functions that represent the dynamics of the distillation column system are four linear FOPDTs. The form of these functions can be written as following:

$$[G_P]_{ij}(s) = \frac{[K_P]_{ij}}{[\tau_p]_{ij}s+1} e^{-[\theta_p]_{ij}s}, \quad i, j = 1, 2 \quad (2)$$

The variations of  $i, j = 1, 2$  are used in equations (3-17).

This identification must reproduce with great precision the dynamic characteristics of the process, taking into account in some cases the disturbances. In this paper, RLS algorithm is used for the identification and estimation of model parameters by simulations only.

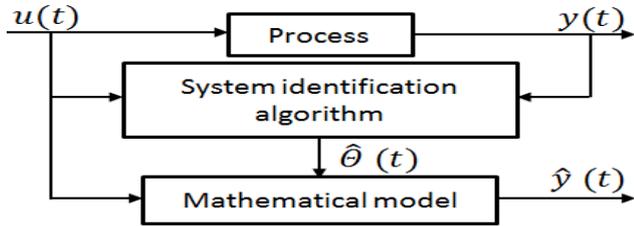


Fig. 2. Identification system of one transfer function (Sundari and Nachiappan, 2014).

This algorithm is based on input and output data from the distillation system. Assuming  $T$  is the sampling time and  $d_{ij}$  is the delay of the input signal. The  $z$  transform model of the distillation system is derived:

$$\begin{aligned} [G_P]_{ij}(z) &= z \left( \frac{1 - e^{-Ts}}{s} \cdot \frac{[K_P]_{ij}}{[\tau_P]_{ij} \cdot s + 1} e^{-[\theta_P]_{ij} s} \right) \\ &= \frac{B_{ij}(z^{-1})}{A_{ij}(z^{-1})} = \frac{b_{ij} z^{-1}}{1 - a_{ij} z^{-1}} \cdot z^{-d_{ij}} \end{aligned} \quad (3)$$

Where:

$$\begin{aligned} a_{ij} &= e^{-T/[\tau_P]_{ij}} \\ b_{ij} &= [K_P]_{ij} (1 - e^{-T/[\tau_P]_{ij}}) \end{aligned} \quad (4)$$

We started by estimating the  $a_{ij}$  and  $b_{ij}$  parameters using RLS.

However,  $d_{ij}$  parameters cannot be directly estimated, so, we designed RLS with exponential forgetting combined with model matching of zero frequency method for estimating all parameters once and for all (Bai and Zhang, 2007). Subsequently the  $[K_P]_{ij}$ ,  $[\tau_P]_{ij}$  and  $[\theta_P]_{ij}$  parameters of Eq. (2) can be computed.

In order to estimate  $d_{ij}$ , we display  $[G_P]_{ij}(z)$ 's numerator in Eq. (3),  $B_{ij}(z^{-1}) \cdot z^{-d_{ij}}$ , as  $[B_m]_{ij}(z^{-1})$ :

$$[B_m]_{ij}(z^{-1}) = [b_1]_{ij} z^{-1} + [b_2]_{ij} z^{-2} + \dots + [b_m]_{ij} z^{-m} \quad (5)$$

The  $m-1$ : is the largest potential delay time of the system, in the above equation. Then the estimation model of the distillation process is (Bai and Zhang, 2007).

$$\begin{aligned} [G_m]_{ij}(z) &= \frac{B_{ij}(z^{-1})}{A_{ij}(z^{-1})} \cdot z^{-d_{ij}} = \frac{[B_m]_{ij}(z^{-1})}{[A]_{ij}(z^{-1})} \\ &= \frac{[b_1]_{ij} z^{-1} + [b_2]_{ij} z^{-2} + \dots + [b_m]_{ij} z^{-m}}{1 - [\hat{a}_1]_{ij} z^{-1}} \end{aligned} \quad (6)$$

The vector that describes the parameter estimates are defined as follows:

$$[\hat{\Theta}]_{ij} = \left[ [\hat{a}]_{ij}, [\hat{b}_1]_{ij}, [\hat{b}_2]_{ij}, \dots, [\hat{b}_m]_{ij} \right]^T \quad (7)$$

Where, the model estimate output takes the form of:

$$[\hat{y}]_{ij}(k) = [\hat{\Theta}]_{ij}^T [h]_{ij}(k) \quad (8)$$

$$[h]_{ij}(k) = \begin{bmatrix} -[y]_{ij}(k-1), [u]_{ij}(k-1), \\ [u]_{ij}(k-2), \dots, [u]_{ij}(k-m) \end{bmatrix}^T \quad (9)$$

At every sample time, RLS algorithm is executed to update. The part of the output which cannot be predicted is called the error in the model prediction. It is described as follows:

$$[\varepsilon]_{ij}(k) = [y]_{ij}(k) - [\hat{\Theta}]_{ij}^T(k-1) [h]_{ij}(k) \quad (10)$$

Parameter estimates are updated with this error and we will write the expression below as:

$$[\hat{\Theta}]_{ij} = [\hat{\Theta}]_{ij}(k-1) + [G]_{ij}(k) [\varepsilon]_{ij}(k) \quad (11)$$

The gain matrix of the estimator is calculated by:

$$[K]_{ij}(k) = \frac{[P]_{ij}(k-1) [h]_{ij}(k)}{\rho + [h]_{ij}^T(k) [P]_{ij}(k-1) [h]_{ij}(k)} \quad (12)$$

Where: the forgetting factor is  $\rho \in [0, 1]$ .

For every new sample time, in Eq. (11), only the covariance matrix  $P$  is updated using the following equation:

$$[P]_{ij}(k) = \frac{1}{\rho} \left[ I - [K]_{ij}(k) [h]_{ij}^T(k) \right] [P]_{ij}(k-1) \quad (13)$$

If the value of  $\rho$  is smaller, the previous data will be forgotten quickly and the RLS algorithm with forgetting factor becomes faster. Therefore, the choice of  $\rho$  controls the capacity of the algorithm to follow the changes parameters (Pardoen et al., 2005; Abd-Elrady et al., 2008). We will choose  $\rho < 1$  (typically between 0.98 and 0.995). The value depends on the dynamics of the evolution of the system parameters (Lennart, 1999). The recursive equations (11) and (13) are initialized with given vector  $\hat{\Theta}_{ij}(0) = 0$  (if we have a division by one of the parameters, we choose it non-zero) and covariance matrix  $P(0) = P(0)' > 0$  is chosen large enough. After a few iterations, this matrix converges to zero, which can cause identification problems later. For this, there are approaches that offer to keep constant trace ( $P(0) = \alpha \cdot I$ , where  $I$ : is the identity matrix and  $\alpha$  is a large scalar).

Once  $\hat{\theta}_{ij}$  is estimated, at that moment it is feasible to compute the  $b_{ij}$  and  $d_{ij}$  parameters  $[G_P]_{ij}(z)$ 's numerator  $B_{ij}(z^{-1}) \cdot z^{-d_{ij}}$  by the following model matching of zero frequency method. Assuming that the frequency is denoted  $\omega = 0$ , let us assume the following relation by : (Bai and Zhang, 2007)

$$\begin{aligned} [B]_{ij}(z^{-1}) \cdot z^{-d_{ij}} \Big|_{z=e^{j\omega}} &= [\hat{B}_m]_{ij}(z^{-1}) \Big|_{z=e^{j\omega}} \\ \frac{[dB]_{ij}(z^{-1}) \cdot z^{-d_{ij}} \Big|_{z=e^{j\omega}}}{d\omega} &= \frac{d[\hat{B}_m]_{ij}(z^{-1}) \Big|_{z=e^{j\omega}}}{d\omega} \end{aligned} \quad (14)$$

In this way, we estimate  $b_{ij}$  and  $d_{ij}$  by the following equation:(Bai and Zhang, 2007)

$$[\hat{b}]_{ij} = \sum_{r=1}^l [\hat{b}_r]_{ij} \quad (15)$$

$$[\hat{d}]_{ij} = \left[ \frac{\left( \sum_{r=1}^l r \times [\hat{b}_r]_{ij} \right)}{\sum_{r=1}^l [\hat{b}_r]_{ij}} \right] - 1 \quad (16)$$

The entire value of  $d_{ij}$  parameter is defined by:

$$[\hat{d}]_{ij} = INT([\hat{d}]_{ij} + 0.5) \quad (17)$$

### 3.2 Decentralized adaptive controller

The design of a decentralized adaptive controller for a multivariable system such as the highly coupled distillation column is done in two parts:

The first step is to decompose the multivariable system of two inputs and two outputs into two subsystems of the form:

$$\begin{cases} y_D(s) = G_{11}(s) \cdot U_1(s) \\ x_B(s) = G_{22}(s) \cdot U_2(s) \end{cases} \quad (18)$$

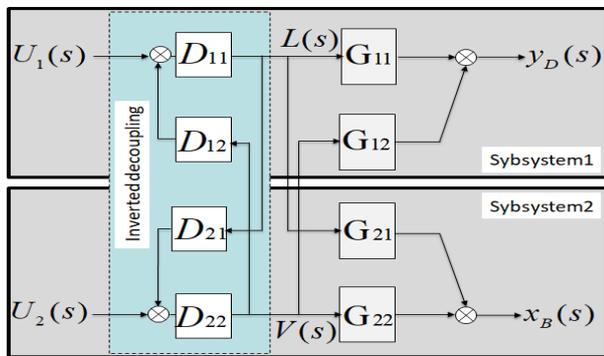


Fig. 3. Inverted decoupling for 2-input/2-output system (Mekki et al., 2013; Mekki et al., 2016), where  $D_{11} = D_{22} = 1$

Interactions between loops are eliminated by the inverted decoupling used in (Mekki et al., 2013; Mekki, 2014; Mekki

et al., 2016), where the expression of  $y_D$  must be independent from  $V$ , and  $x_B$  independent from  $L$ .

The inverted decoupling functions are given in the following (Mekki et al., 2013; Mekki, 2014; Mekki et al., 2016):

$$\begin{cases} D_{12} = \frac{[K_P]_{12} \cdot (\tau_{11}s + 1)}{[K_P]_{11} \cdot (\tau_{12}s + 1)} e^{-(\theta_{12} - \theta_{11}) \cdot s} \\ D_{21} = \frac{[K_P]_{21} \cdot (\tau_{22}s + 1)}{[K_P]_{22} \cdot (\tau_{21}s + 1)} e^{-(\theta_{21} - \theta_{22}) \cdot s} \end{cases} \quad (19)$$

The second step is the adaptation and updating of the expression of the inverted decoupling transfer functions as a function of the estimated values of the nominal and uncertain model. The three parameters  $([K_P]_{ij}, [\tau_P]_{ij}$  and  $[\theta_P]_{ij})$  of each transfer functions ( $G_{11}, G_{12}, G_{21}$  and  $G_{22}$ ) of the distillation process is considered unknown are estimated according to the RLS approach given in equations (10) to (13).

### 3.3 Particle Swarm Optimization-Proportional and Integral (PSO-PI) Controller

#### A. Optimization on Particle Swarm:

The PSO is an optimization technique supported by evolutionary computation (Zhaosheng, 2016); it had been developed by researching on swarm-based works to observe the social behavior of moving organisms like birds flock or fish school (Chiou et al., 2016; Malik et al., 2014; El-Gammal and El-Samahy, 2009). The PSO's essential variant is based on a population known as a swarm, and every individual within the group is claimed to be a particle. In our paper, at iteration  $t$ , each particle comprises two P and I parameters that have two dimensions in the searching space and particles be required to move in a two-dimensional space.

Every single particle has two vectors: position and velocity.

The position of the  $i^{th}$  particle is represented in the  $D$ -dimensional space ( $D$  is the number of search space) and defined as: (Latha et al., 2013; Allaoua et al., 2009)

$$X_{i,j}(t) = (X_{i,1}; X_{i,2}, \dots, X_{i,D}) \quad (20)$$

Where:  $i, j$  are particle index and dimension number.

Every particle gives minimum fitness value, it is considered the best position; it is recorded and represented as:

$$Pbest_{i,j} = (Pbest_{i,1}; Pbest_{i,2}, \dots, Pbest_{i,D}) \quad (21)$$

The index of the best particle among all the particles in the swarm is represented by  $gbest_D$ .

The  $i^{th}$  particle velocity is represented as:

$$V_{i,j}(t) = (V_{i,1}; V_{i,2}, \dots, V_{i,D}) \quad (22)$$

The changed velocity and position of each particle which be determined using the current velocity and the distance from  $Pbest_{i,D}$  to  $gbest_D$  as seen in the formulas below (Latha et al., 2013; Allaoua et al., 2009; Chiou et al., 2016):

$$V_{i,j}(t+1) = \begin{cases} W \cdot V_{i,j}(t) \\ + C_1 \cdot rand(\cdot) \cdot (Pbest_{i,j} - X_{i,j}(t)) \\ + C_2 \cdot rand(\cdot) \cdot (gbest_D - X_{i,j}(t)) \end{cases} \quad (23)$$

$$X_{i,j}(t+1) = X_{i,j}(t) + V_{i,j}(t+1) \quad (24)$$

$$i = 1, 2, \dots, N \quad j = 1, 2, \dots, D$$

Where:  $t$  the iterations (generations).  $C_1, C_2$ : are two positive constants, they represent the cognition and social components respectively.  $w$  corresponds to the inertia weight. It is used to keep balance in search ability between the index of the best particle in the group ( $Pbest_{i,D}$ ) and the best position of every particle ( $gbest_D$ ).  $N$  is the number of particles in the group. All these results have established for enhanced tuning to obtain the optimal controller parameters. Based on (Nithyarani et al., 2014; Hamoudi, 2017; Zhaosheng, 2016; Chiou et al., 2016), the random numbers of  $rand(\cdot)$  are uniformly between 0 and 1.

#### B. Tuning of PI controller based on PSO technique:

In order to find optimum tuning parameters for the two PI controllers, the PSO algorithm will be extended to the 2-D solution space for this study.

PSO has several advantages like ease in implementation, all the particles converge to the optimum solution rapidly and only few parameters are there to be adjusted.

The PI controller tuning parameters using the PSO algorithm can be found in (Hannachi et al., 2021; Premkumar et al., 2021; Sultana et al., 2021).

#### C. Fitness function:

The general form of the equation of two PI controllers is:

$$U_b(t) = KP_b \cdot \left( e_b(t) + \frac{1}{T_{i_b}} \int e_b(t) dt \right), \quad b = 1, 2 \quad (25)$$

Where,  $b$  is the number of the loop.  $U_b$  is the control signal based on the error signal  $e_b$  for each loop.  $KP_b, T_{i_b}$  are respectively the proportional gain and integral time.

In this article, PSO-PI controllers are applied to each transfer functions estimated by RLS. The optimum parameters  $\Phi_b$  of controllers can be achieved by minimizing the fitness function given by the equation below:

$$\begin{cases} \min J_b(\Phi_b) = Tr + Ts + Mp \\ \Phi_b = [KP_b, KI_b], \quad KI_b = KP_b / T_{i_b} \end{cases} \quad b = 1, 2 \quad (26)$$

We evaluated the objective function using the following three criteria: the rise time ( $Tr$ ), settling time ( $Ts$ ) and overshoot ( $Mp$ ).  $KI_b$  is known as the integral gain.

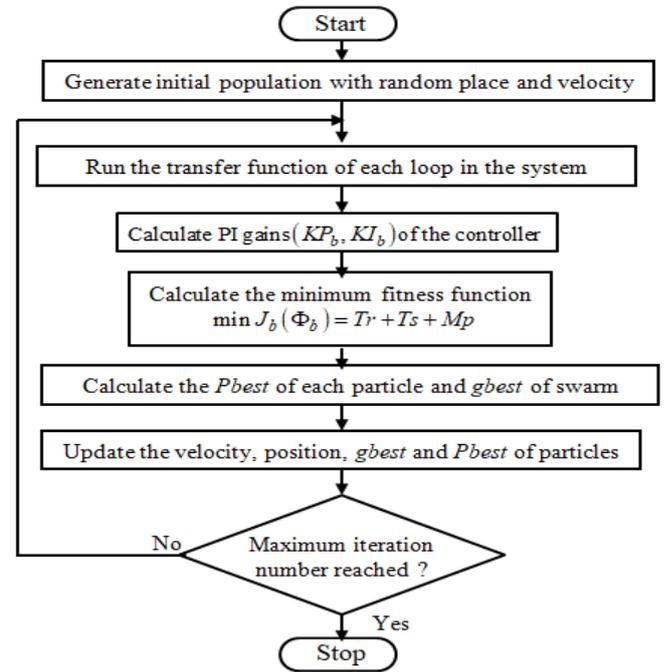


Fig. 4. Diagram for optimizing the parameters of PI controllers based on PSO technique.

## 4. CASE STUDY

### 4.1 Wood-Berry distillation column

The binary distillation column is used for separating a mixed feed stream into two parts: methanol and water. The mathematical representation given by Wood and Berry (WB) is the following: (Wood and Berry, 1973)

$$\begin{bmatrix} y_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8}{16.7s+1} e^{-s} & \frac{-18.9}{21s+1} e^{-3s} \\ \frac{6.6}{10.9s+1} e^{-7s} & \frac{-19.4}{14.4s+1} e^{-3s} \end{bmatrix} \cdot \begin{bmatrix} L(s) \\ V(s) \end{bmatrix} \quad (27)$$

$$+ \begin{bmatrix} \frac{3.8}{14.9s+1} e^{-8s} \\ \frac{4.9}{13.2s+1} e^{-3s} \end{bmatrix} \cdot F(s)$$

The time constants of equation (27) are in minutes.

### 4.2 Performance specifications

The same performance specifications cited in the paper (Mekki et al., 2018) have been taken into account except that the variability in the model will be expressed as differences in gains  $[K_p]_{ij}$  only as defined in Table 1.

By applying RLS algorithms to each individual transfer function (see Fig. 5) elements of an unknown two inputs two outputs (TITO) process despite the variation in gains are estimated perfectly, following the equations in Section 3.

**Table 1. The gains of the distillation column and their uncertainties.**

Parameter unknown	$ij$	Maximum (Max) +20%	Nominal	Minimum (Min) -20%
$[K_p]_{ij}$	11	15.36	12.8	10.24
	12	-15.12	-18.9	-22.68
	21	7.92	6.6	5.28
	22	-15.52	-19.4	-23.28

#### 4.3 Application of RLS and PSO-PI to WB distillation column

An analysis of TITO processes was used to carry out parameter estimation and validation of the control tuning system in this section (see Fig. 5). The suggested approach was put to the test against the IMC-PI controller. Several performance indices are used among them: IAE (Integral of Absolute value of Error), ISE (Integral of Square Error), ITAE (Integral of Time multiplied by the Absolute value of Error) and ITSE (Integral of Time multiplied by the Square Error) to compare and know which of the methods is the most appropriate to minimize the error signal and have the best performance.

The weight inertia is a crucial factor in the convergence of the PSO, according to (Latha et al., 2013). It is used to

manage how the previous velocities influence the new velocity at the current time step.

The PSO parameter values that provide the best results are described in Table 2. The estimated inverted parameters will be able to reduce the undesirable cross-couplings.

**Table 2. Parameters of PSO algorithms.**

Population Size	50
Number of iterations	50
$w$	0.9
$C_1 = C_2$	1.8
$N$	49

The optimum tuning parameters for both PI controllers are obtained after estimation, as seen in Table 3.

**Table 3. Comparison of PSO-PI and IMC-PI first and second loop parameters.**

Controllers	Parameters of two PI controllers			
	$PI_1$ of the top composition		$PI_2$ of the bottom composition	
	$KP_1$	$Ti_1$	$KP_2$	$Ti_2$
IMC-PI (Mekki, 2014)	0.3054	17.25	-0.0863	15.98
PSO-PI	0.6116	0.0475	-0.1442	0.0312

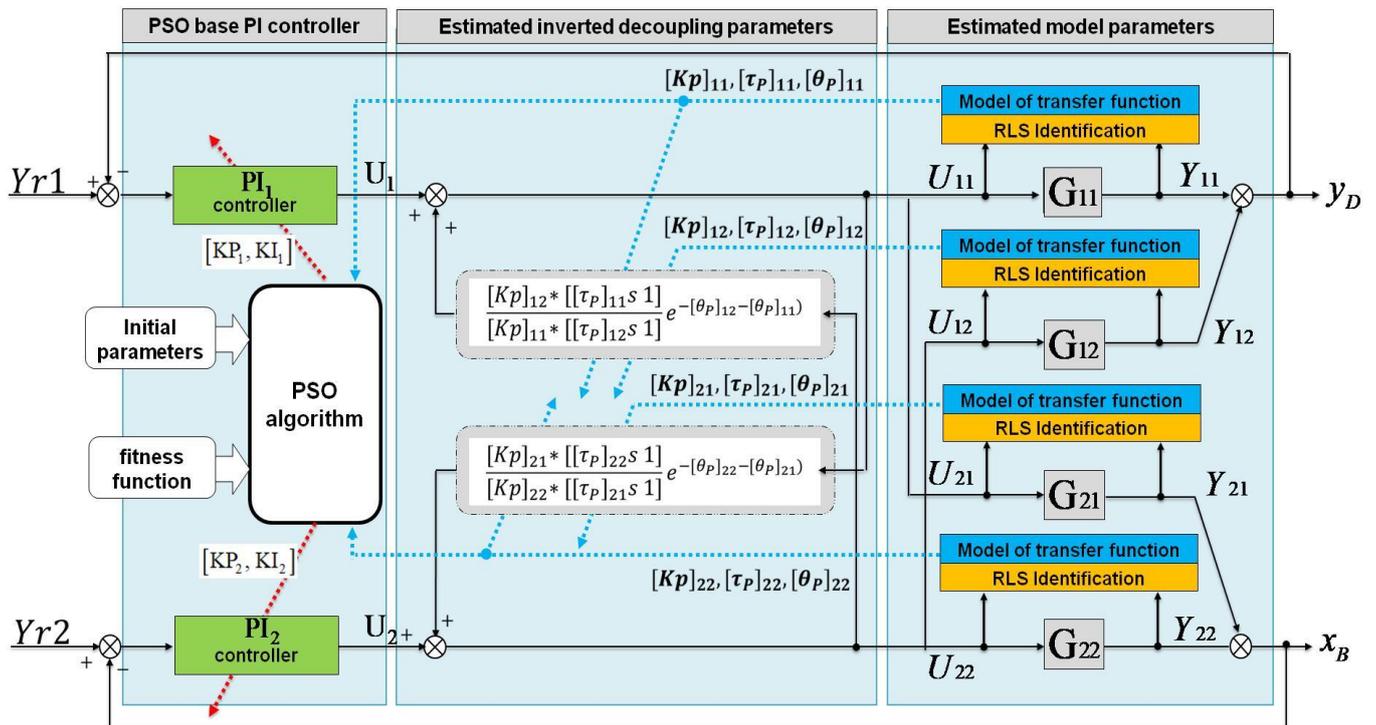


Fig. 5. Principle schema for estimation of model parameters and the decentralized adaptive PSO-PI controller.

#### 4.4 Simulation results and discussion

In this part, we used a WB model in order to design and validation of an adaptive decentralized PSO-PI controller for functions considered unknown.

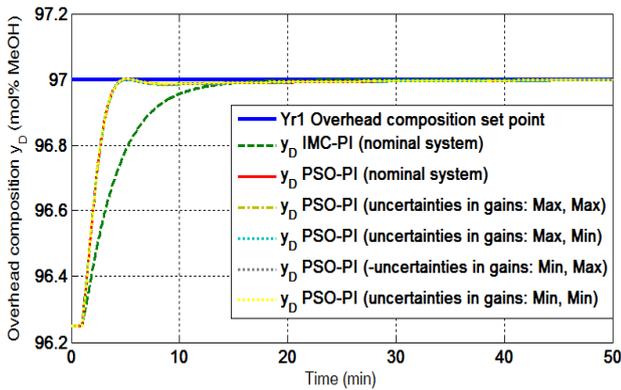
After estimation of the process parameters, two cases are studied in the presence of uncertainty modeled in gains only:

### A. Simulations with different set point changes:

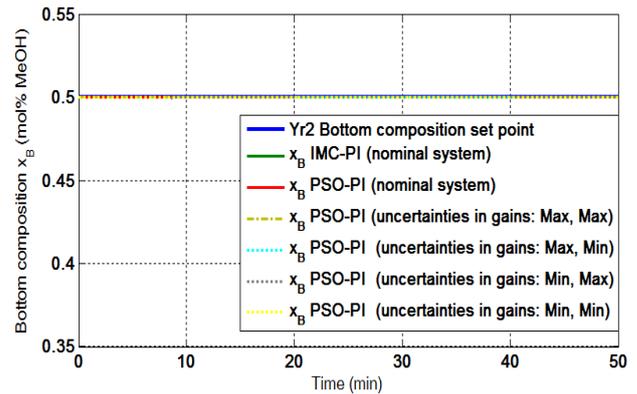
In Fig. 6(a), the set point of the composition in the top ( $Yr1$ ) was rising from 96.25 to 97 mol% MeOH but, as seen in Fig. 6(b) the bottom composition ( $Yr2$ ) is set to 0.5 mol% MeOH.

In Fig. 7(b),  $Yr2$  was rising from 0.5 to 0.6 mol% MeOH but, as seen in the Fig. 7(a) the top composition  $Yr1$  is set to 96.25 mol% MeOH.

The simulation results from the Fig. 6(a), Fig. 7(b) are detailed in Table 4, when changing the set point.



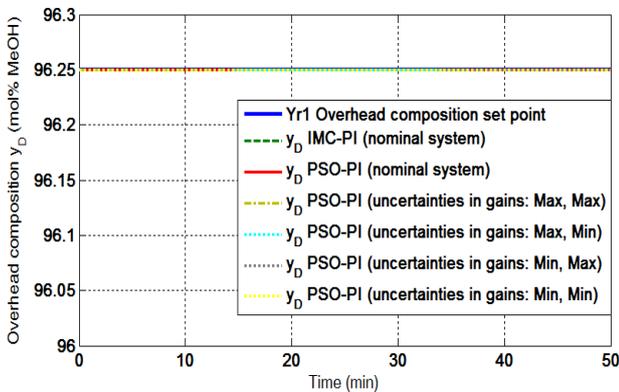
(a)



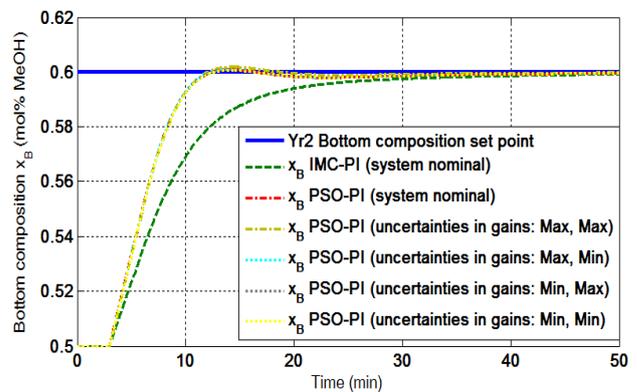
(b)

Figs. 6. Simulation for a set point change in top composition where only gains modified:

(a) top composition, (b) bottom composition.



(a)



(b)

Figs. 7. Simulation for a set point change in bottom composition when only gains modified:

(a) top composition, (b) bottom composition.

### B. Simulation with disturbance rejection:

At time zero min, we will apply a continuous disturbance  $F = 0.20$  (lb/min) which is the rise in the feed flow rate to see if the controller is robust to reject it, while keeping the concentration set points ( $Yr1 = 96.25$ ,  $Yr2 = 0.5$ ) mol% MeOH on the top and the bottom of the column of their operating points.

The feed rate of the mixture to be separated will descend to the bottom of the column. An increase in the latter will cause a rapid rise in the level at the bottom. To avoid flooding the

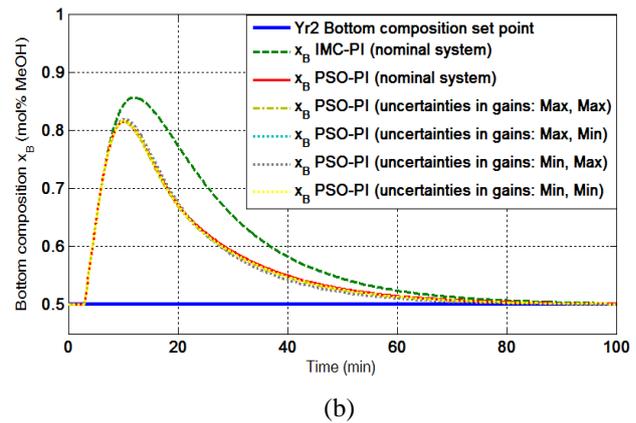
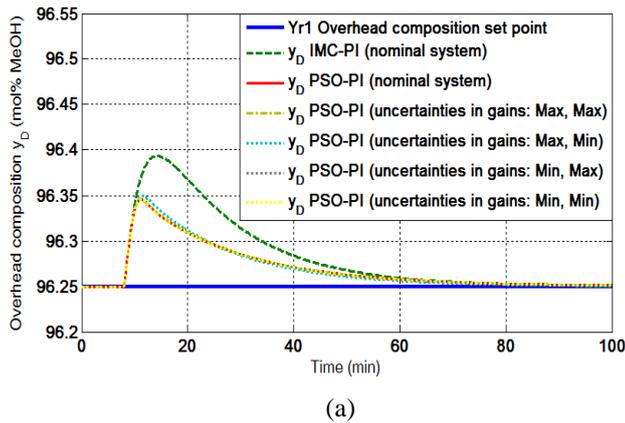
The analysis and simulation observations of the figures (Fig. 6(a), Fig. 7(b)) show that the control laws applied to the concentrations of the top and bottom have succeeded in perfectly imitating the exceedances of the setpoints, the influence of variations in the operating points and the uncertainties on the system with the best possible operating performance.

For the purpose of checking the approach suggested, we carried out the same tests in setpoints changes with the variations in gains like in (Mekki et al., 2018). By varying the gains of the system, it can be seen that we have obtained smooth responses with really negligible deviations.

lower trays of the column, an increase in the residue flow rate with a  $x_B$  concentration will be applied.

As seen in Fig. 8(b), the increase in the feed rate has a strong effect on the composition of the bottom product ( $x_B$ ) and drifts far enough from 63.82% of the desired final value with a slow action on disturbance rejection.

On the other hand, Fig. 8 (a) demonstrates that the excess on the top concentration ( $y_D$ ) is less than 0.1% of the desired final value and this despite the variations in the gains of the system imposed during the simulation.



Figs. 8. Simulation for disturbance rejection in top composition when only gains modified: (a) top composition, (b) bottom composition.

Table 4. Set point change of the outputs ( $y_D, x_B$ )

Outputs efficiency with decentralized IMC-PI and PSO-PI						
Setpoint change	Controlled variable, with		IAE	ITAE	ISE	ITSE
Yr1	$y_D$	IMC-PI	3.247	11.85	1.514	2.743
		PSO-PI	1.961	8.571	0.983	0.990
Yr2	$x_B$	IMC-PI	0.941	6.904	0.062	0.238
		PSO-PI	0.672	3.358	0.050	0.142

Table 5. Rejecting Disturbance of the outputs ( $y_D, x_B$ )

Outputs efficiency with decentralized IMC-PI and PSO-PI						
Fixed setpoint	Controlled variable, with		IAE	ITAE	ISE	ITSE
Yr1	$y_D$	IMC-PI	3.287	89.74	0.2934	5.854
		PSO-PI	2.015	56.03	0.1021	1.953
Yr2	$x_B$	IMC-PI	9.317	223.2	2.113	37.17
		PSO-PI	6.243	124.5	1.207	17.18

Figs. 8 shows that the responses ( $y_D, x_B$ ) of the nominal system are almost superimposed on the results of simulations of the changes in the gain values during the response to the disturbance rejection.

The two proposed regulators were compared with the four performance indices. The performances of the disturbances rejections are detailed in Table 5. The PSO-PI controller has the lowest errors between set points change and disturbance rejections.

### 5. CONCLUSIONS

We have proposed an RLS for estimation of model parameters and a decentralized adaptive PI controller using PSO to control the compositions of the top and bottom into a coupled binary distillation column.

All linearized first order plus dead time (FOPDT) transfer functions model parameters are correctly estimated in the nominal model and variations in gains uncertainties of continuous binary distillation column.

The advantage of the decentralized controller with only a nominal system has been developed to take into consideration

the uncertainties of the system under the name of an adaptive decentralized controller with used an inverted decoupling.

A perfect performance, even for variations in gains model was elaborated using the PSO-PI controller. The estimation of the RLS and the decentralized adaptive PSO-PI control presented in this paper for a nominal and uncertain system has obtained a great success in the different variations of setpoints and disturbance rejection.

The recommended strategy is based on the correct choice of weighting criteria, depending on the RLS and PSO techniques. The results obtained by simulations can serve as an excellent starting point for implementing online.

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