CEAI, Vol. 9, No. 1, pp. 3-10, 2007 Printed in Romania

STABILITY ANALYSIS APPROACH FOR FUZZY LOGIC CONTROL SYSTEMS WITH MAMDANI TYPE FUZZY LOGIC CONTROLLERS

Radu-Emil Precup[1], Marius L. Tomescu[2], Stefan Preitl[1], József K. Tar[3], Adrian Sebastian Paul[1]

[1] *"Politehnica" University of Timisoara, Department of Automation and Applied Informatics, Bd. V. Parvan 2, RO-300223 Timisoara, Romania, E-mail: {rprecup, spreitl}@aut.utt.ro, paul.adrian.sebastian@gmail.com*

[2] *"Aurel Vlaicu" University, Computer Science Faculty, Complex Universitar M, Str. Elena Dragoi 2, RO-310330 Arad, Romania, E-mail: tom uav@yahoo.com*

[3] *Budapest Tech Polytechnical Institution, Institute of Intelligent Engineering Systems, Bécsi út 96/B, H-1034 Budapest, Hungary, E-mail: tar.jozsef@nik.bmf.hu*

Abstract: *This paper presents a stability analysis approach for a class of nonlinear processes controlled by Mamdani type fuzzy logic controllers (FLCs). The stability analysis is performed in the sense of Lyapunov dedicated resulting in the derivation of an original stability analysis approach that can be used as design method for FLCs with guaranteed stability. Quadratic positive definite Lyapunov functions candidate are employed in this context. The approach is expressed in terms of sufficient stability conditions for fuzzy logic control systems with Mamdani type FLCs. An illustrative example validates the stability analysis approach by designing an FLC to control a nonlinear process.*

Keywords: *fuzzy logic controller, Lyapunov stability, nonlinear system, invariance principle, Mamdani fuzzy rule.*

1. INTRODUCTION

Fuzzy logic control is a convenient and easily understandable initial nonlinear approach to controlling complex, uncertain or even ill-defined processes. Stability is one of the structural properties of fuzzy logic control systems of particular importance to their applications.

A fuzzy logic controller (FLC) can be viewed as a real-time expert system that involves fuzzy logic to ensure the desired / imposed control system performance indices – with respect to several inputs including the reference one and the load type disturbance ones, and to parametric variations and even uncertainties as well – usually measured in the output. Indeed, they provide a means of converting a linguistic control strategy derived from expert knowledge into automatic control strategies and offer a means of interrogating the control system evolution and performance. In these conditions the necessity for systematic design methods of fuzzy logic control systems becomes more and more important [10]. The stability analysis methods enable the design providing conditions that enable the parameter setting for FLCs.

In principle, for the stability analysis of fuzzy logic control systems controlling nonlinear processes any method can be utilized which is suitable for the analysis of nonlinear dynamical systems [6]. Which method is the best one to use depends only on the prerequisites. There, the structure of the system, the type of information describing the process and the type of sufficient conditions for the stability are usually the key points. Current trends in the stability analysis in case of fuzzy logic control systems with Mamdani type FLCs include the Lyapunov's [11] and Krasovskii's [12] approaches, the describing function method [6], the algebraic approaches [1] or the use of Mamdani fuzzy dynamic models [13]. The new stability analysis approach proposed in the sequel is different to the application of Lyapunov's theorem [6, 15] in several important aspects and allows more applications. In particular, it is well-suited to controlling processes where the derivative of the Lyapunov function candidate is not negative definite, therefore applying LaSalle's invariance principle to nonlinear processes controlled by Mamadani type FLCs can be applied to a wide area of nonlinear dynamic systems. Another important difference is that the stability of the closed-loop system is guaranteed by the stability of each fuzzy subsystem.

This paper addresses the following topics. In section 2, a short review of fuzzy logic control systems with Mamdani type FLCs is given with some definitions and properties. In section 3 the new stability analysis approach is suggested and proved. Next, section 4 gives an illustrative example and section 5 validates the approach presenting simulation results that correspond to an example. The final part of the paper, section 6, contains some concluding remarks.

2. FUZZY LOGIC CONTROL SYSTEM STRUCTURE

A fuzzy logic control system consists of a process and a fuzzy logic controller (FLC) as shown in Fig. 1. Let $X \subset R^n$ be a universe of discourse. It is accepted the following class of single input nonlinear dynamical systems modelled by the state-space equations in (1):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u, \tag{1}$$

where: $\mathbf{x} \in X$, $\mathbf{x} = [x_1 \quad x_2 \quad \ldots \quad x_n]^T$ is the state vector, $n \in IN^*$, $\dot{\mathbf{x}} = [\dot{x}_1 \quad \dot{x}_2 \quad \ldots \quad \dot{x}_n]^T$ is the derivative of $\mathbf{x}$ with respect to the time variable t, $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}) \quad f_2(\mathbf{x}) \quad \ldots \quad f_n(\mathbf{x})]^T$ and $\mathbf{b}(\mathbf{x}) = [b_1(\mathbf{x}) \quad b_2(\mathbf{x}) \quad \ldots \quad b_n(\mathbf{x})]^T$ are functions describing the dynamics of the process, u is the control signal fed to the process, obtained by the Centre of Gravity (COG) defuzzification method for Mamdani type FLCs.

The i-th IF–THEN rule in the fuzzy rule base of the FLC, referred to as Mamdani fuzzy rule, is expressed in terms of the following form:

Rule i: IF x_i is $\widetilde{X}_{i,1}$ AND … AND x_n is $\widetilde{X}_{i,n}$ THEN u is $\widetilde{Y}_i$, $i = \overline{1,r}$, $r \in IN, r \geq 2$, (2)

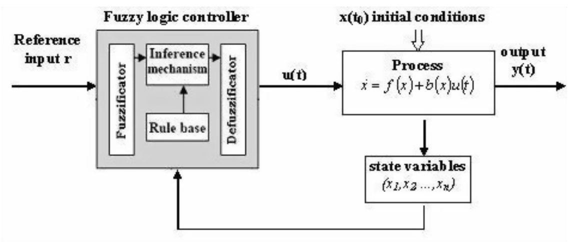

Fig.1. Fuzzy logic control system structure.

where $\widetilde{X}_{i1}, \widetilde{X}_{i2}, \ldots, \widetilde{X}_{i,n}$ are fuzzy sets describing the linguistics terms (LTs) of input variables, $\widetilde{Y}_i$ describes the LTs of output variables, and r is the total number of rules. Note that Y represents the output domain or control signal domain and $\mu_{Y_i} \neq 0, i = \overline{1,r}$.

The activation degree of the i-th Mamdani fuzzy rule is:

$$\alpha_i(\mathbf{x}) = \min(\mu_{\widetilde{X}_{i,1}}(x_1), \mu_{\widetilde{X}_{i,2}}(x_2), \ldots, \mu_{\widetilde{X}_{i,n}}(x_n)). \tag{3}$$

It is assumed that for any $\mathbf{x}$ belonging to the input universe of discourse, X, there exists at least one α_i among all rules that is not equal to zero.

The conclusion (control signal) of each rule is calculated using the COG defuzzification method applied for:
$c_i(\mathbf{x}, y) = \min(\alpha_i(\mathbf{x}), \mu_{\tilde{Y}_i}(y))$, that is:

$$u_i(\mathbf{x}) = \begin{cases} \dfrac{\int_Y y \cdot c_i(\mathbf{x}, y)\,\mathrm{d}y}{\int_Y c_i(\mathbf{x}, y)\,\mathrm{d}y} & \text{if } \alpha_i(\mathbf{x}) > 0 \\ 0 & \text{if } \alpha_i(\mathbf{x}) = 0 \end{cases}. \quad (4)$$

The aggregation of all rules is done in terms of applying (5):

$$c(\mathbf{x}, y) = c_1(\mathbf{x}, y) + c_2(\mathbf{x}, y) + \ldots + c_r(\mathbf{x}, y). \quad (5)$$

Consequently, the control signal fed to the process will be:

$$u = \frac{\int_Y y \cdot c\,\mathrm{d}y}{\int_Y c\,\mathrm{d}y} = \frac{\sum_{i=1}^{r} \int_Y y \cdot c_i\,\mathrm{d}y}{\sum_{i=1}^{r} \int_Y c\,\mathrm{d}y}. \quad (6)$$

This FLC with the structure described before is referred to as AND-SUM-COG fuzzy logic controller [4].

Property 1: For any AND-SUM-COG fuzzy logic controller the following relationships hold: $u_{\min}(\mathbf{x}) \le u(\mathbf{x}) \le u_{\max}(\mathbf{x})$ for any $\mathbf{x} \in X$, where $u_{\min}(\mathbf{x}) = \min\limits_{i=\overline{1,r}}(u_i(\mathbf{x}))$,.

Proof. Let $\mathbf{x}_0 \in X$ and $A_i = \int_Y c_i(\mathbf{x}_0, y)\,\mathrm{d}y$.

If $\alpha_i(\mathbf{x}_0) \ne 0$ then $u_i(\mathbf{x}_0)$ in (7) is bounded:

$$u_i(\mathbf{x}_0) = \frac{\int_Y y \cdot c_i(\mathbf{x}_0, y)\,\mathrm{d}y}{\int_Y c_i(\mathbf{x}_0, y)\,\mathrm{d}y} = \frac{\int_Y y \cdot c_i(\mathbf{x}_0, y)\,\mathrm{d}y}{A_i}$$

And

$$u_i(\mathbf{x}_0) \cdot A_i = \int_Y y \cdot u_i(\mathbf{x}_0, y)\,\mathrm{d}y. \quad (7)$$

Therefore:

$$u(\mathbf{x}_0) = \frac{\int_Y y \cdot c(\mathbf{x}_0, y)\,\mathrm{d}y}{\int_Y c(\mathbf{x}_0, y)\,\mathrm{d}y} =$$

$$= \frac{\int_Y y \cdot (c_1(\mathbf{x}_0, y) + c_2(\mathbf{x}_0, y) + \ldots + c_r(\mathbf{x}_0, y))\,\mathrm{d}y}{\int_Y (c_1(\mathbf{x}_0, y) + c_2(\mathbf{x}_0, y) + \ldots + c_r(\mathbf{x}_0, y))\,\mathrm{d}y} =$$

$$= \frac{u_1(\mathbf{x}_0)A_1 + u_2(\mathbf{x}_0)A_2 + \ldots + u_r(\mathbf{x}_0)A_r}{A_1 + A_2 + \ldots + A_r}. \quad (8)$$

If $u_{\min}(\mathbf{x}) = \min\limits_{i=\overline{1,r}}(u_i(\mathbf{x}))$ and

$u_{\max}(\mathbf{x}) = \max\limits_{i=\overline{1,r}}(u_i(\mathbf{x}))$, then it results that:

$$u(\mathbf{x}_0) = \frac{u_1(\mathbf{x}_0)A_1 + u_2(\mathbf{x}_0)A_2 + \ldots + u_r(\mathbf{x}_0)A_r}{A_1 + A_2 + \ldots + A_r} \le$$

$$\le \frac{u_{\max}(\mathbf{x}_0)A_1 + u_{\max}(\mathbf{x}_0)A_2 + \ldots + u_{\max}(\mathbf{x}_0)A_r}{A_1 + A_2 + \ldots + A_r} =$$

$$= \frac{u_{\max}(\mathbf{x}_0)(A_1 + A_2 + \ldots + A_r)}{A_1 + A_2 + \ldots + A_r} = u_{\max}(\mathbf{x}_0) \quad (9)$$

and

$$u(\mathbf{x}_0) = \frac{u_1(\mathbf{x}_0)A_1 + u_2(\mathbf{x}_0)A_2 + \ldots + u_r(\mathbf{x}_0)A_r}{A_1 + A_2 + \ldots + A_r} \ge$$

$$\ge \frac{u_{\min}(\mathbf{x}_0)A_1 + u_{\min}(\mathbf{x}_0)A_2 + \ldots + u_{\min}(\mathbf{x}_0)A_r}{A_1 + A_2 + \ldots + A_r} =$$

$$= \frac{u_{\min}(\mathbf{x}_0)(A_1 + A_2 + \ldots + A_r)}{A_1 + A_2 + \ldots + A_r} = u_{\min}(\mathbf{x}_0). \quad (10)$$

Concluding, $u_{\min}(\mathbf{x}) \le u(\mathbf{x}) \le u_{\max}(\mathbf{x})$ for any $\mathbf{x} \in X$.

3. STABILITY ANALYSIS APPROACH

To derive the stability theorem it is considered the fuzzy subsystem consisting of one Mamdani fuzzy rule and the process described in (1). In the following theorem, that expresses the stability analysis approach proposed here, it will be proved that if each subsystem is stable in the sense of Lyapunov, under a common Lyapunov function, the overall system will be also stable in the sense of Lyapunov.

Theorem 1: If $\mathbf{P}$ is a positive definite matrix and:

1. $V(\mathbf{x}) = \mathbf{x}^T \mathbf{P}\,\mathbf{x} \to \infty$ as $\|\mathbf{x}\| \to \infty$, $V(\mathbf{0}) = 0$,

2. $\dot{V}(\mathbf{x}) \le 0, \forall \mathbf{x} \in X$ for all fuzzy subsystems,

3. the set $\{\mathbf{x} \in X \mid \dot{V}(\mathbf{x}) = 0\}$ contains no trajectory of the system except the trivial trajectory $\mathbf{x}(t) = \mathbf{0}$ for $t \ge 0$,

then the fuzzy logic control system with the AND-SUM-COG fuzzy logic controller defined in section 2 and the process described in (1), is globally asymptotically stable in at the origin.

Proof. The Lyapunov function candidate $V(\mathbf{x}) = \mathbf{x}^T \mathbf{P}\, \mathbf{x}$ is set.

From $\dot{V}(\mathbf{x}) = \dot{\mathbf{x}}^T \mathbf{P}\, \mathbf{x} + \mathbf{x}^T \mathbf{P}\, \dot{\mathbf{x}}$ and (1), the result is:

$$\begin{aligned} \dot{V}(\mathbf{x}) &= (\mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u(\mathbf{x}))^T\, \mathbf{P}\,\mathbf{x} + \\ &+ \mathbf{x}^T\, \mathbf{P}(\mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u(\mathbf{x})) = \\ &= \mathbf{f}(\mathbf{x})^T\, \mathbf{P}\,\mathbf{x} + \mathbf{x}^T\, \mathbf{P}\,\mathbf{f}(\mathbf{x}) + \\ &+ (\mathbf{b}(\mathbf{x})^T\, \mathbf{P}\,\mathbf{x} + \mathbf{x}^T\, \mathbf{P}\,\mathbf{b}(\mathbf{x}))u(\mathbf{x}) = \\ &= F(\mathbf{x}) + B(\mathbf{x})u(\mathbf{x}), \end{aligned} \tag{11}$$

with:

$$\begin{aligned} F(\mathbf{x}) &= (f(\mathbf{x})^T\, \mathbf{P}\,\mathbf{x} + \mathbf{x}^T\, \mathbf{P}\,\mathbf{f}(\mathbf{x})), \\ B(\mathbf{x}) &= b(\mathbf{x})^T\, \mathbf{P}\,\mathbf{x} + x^T\, \mathbf{P}\,\mathbf{b}(\mathbf{x}). \end{aligned} \tag{12}$$

If $\mathbf{x}_0 = 0 \Rightarrow \dot{V}(\mathbf{x}_0) = 0$. What remains to be proved is that $\dot{V}$ is negative definite with respect to $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) + \mathbf{b}(\mathbf{x}(t))u(\mathbf{x}(t))$. Then, using condition 2 in theorem 1 the result will be (13):

$$\dot{V}(\mathbf{x}) = F(\mathbf{x}) + B(\mathbf{x})u_i < 0 \tag{13}$$

for any Mamdani fuzzy rule $i = \overline{1, r}$.

Now it is considered $\mathbf{x} \neq \mathbf{0}$. Three possible cases should be analyzed as follows.

Case 1: $B(\mathbf{x})$ is strictly positive. From property 1 it is obtained that:

$$\begin{aligned} &u_{\min}(\mathbf{x}) \le u(\mathbf{x}) \le u_{\max}(\mathbf{x}) \Rightarrow F(\mathbf{x}) + \\ &+ B(\mathbf{x})u_{\max}(\mathbf{x}) \le F(\mathbf{X}) + B(\mathbf{X})u(\mathbf{X}) \le \\ &\le F(\mathbf{X}) + B(\mathbf{X})u_{\max}(\mathbf{X}) < 0, \end{aligned} \tag{14}$$

therefore $\dot{V}(\mathbf{x}) < 0$.

Case 2: $B(\mathbf{x})$ is strictly negative. From property 1 the result is (15):

$$\begin{aligned} &u_{\min}(\mathbf{x}) \le u(\mathbf{x}) \le u_{\max}(\mathbf{x}) \Rightarrow 0 > F(\mathbf{x}) + \\ &+ B(\mathbf{x})u_{\max}(\mathbf{x}) \ge F(\mathbf{x}) + B(\mathbf{x})u(\mathbf{x}) \ge \\ &\ge F(\mathbf{x}) + B(\mathbf{x})u_{\min}(\mathbf{x}), \end{aligned} \tag{15}$$

hence once more $\dot{V}(\mathbf{x}) < 0$.

Case 3: $B(\mathbf{x}) = 0$. From (13) it results straightforward that $\dot{V}(\mathbf{x}) = F(\mathbf{x}) < 0$.

From the above three cases it may be concluded that whatever the value of $B(\mathbf{x})$ is, the expected results will be obtained, $\dot{V} \le 0$.

Condition 3 ensures the fulfilment of LaSalle's invariance principle from LaSalle's theorem referred in [5]. In these conditions, it is guaranteed that the equilibrium point at the origin is globally asymptotically stable.

4. ILLUSTRATIVE EXAMPLE

This section presents an example consisting of a mass-spring-damper system to be controlled by a Mamdani FLC. A scheme of principle of the controlled process is illustrated in Fig. 2.

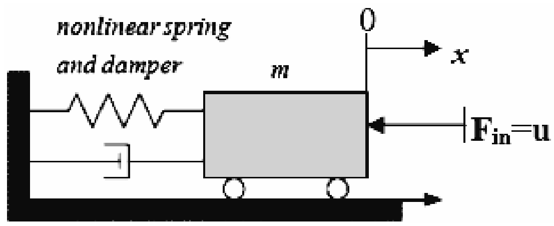

Fig. 2. Nonlinear mass-damper-spring system.

One mathematical model of this single input nonlinear process is written in terms of the differential equation (16):

$$m\ddot{x} + b\dot{x}|\dot{x}| + kx = u\,, \tag{16}$$

where: x – displacement of the body away from the rest (equilibrium) position, m – mass linked to the spring, $b\dot{x}|\dot{x}|$ – nonlinear dissipation or damping, $b > 0$, kx – spring term, $k > 0$, u – control signal representing an externally applied force (F_{in}) pushing on the mass.

A set of state variables sufficient to describe this system includes the position, x, and the velocity of the mass, $\dot{x}$. Therefore, we will define a set of state variables as $\{z_1, z_2\}$, with:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \in X = [-1,1] \times [-1,1],\ z_1(t) = x(t),$$

$$z_2(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t}, \tag{17}$$

In order to express (16) using the state variables, the defined state variables are substituted and it is obtained that:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) + \mathbf{b}(\mathbf{z})u \,, \tag{18}$$

so the controlled process has been transformed to the form (1) where:

$$\mathbf{f}(\mathbf{z}) = \begin{bmatrix} z_2 \\ \frac{1}{m}(-bz_2|z_2| - kz_1) \end{bmatrix}, \ \mathbf{b}(\mathbf{z}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{19}$$

The design of the Mamdani type AND-SUM-COG fuzzy logic controller will be presented as follows. It starts with setting the fuzzification module and Fig. 3 shows the membership functions that describe the LTs of the linguistic variables z_1 and z_2. The LTs representing Positive, Negative and Zero values are denoted by P, N and Z, respectively. Fig. 4 presents the membership functions that describe the LTs of the control signal.

The inference engine employs the fuzzy logic operators AND and OR implemented by the min and max functions, respectively. The inference engine is assisted by the complete set of fuzzy control rules illustrated in Table 1, and the COG defuzzification method is utilized.

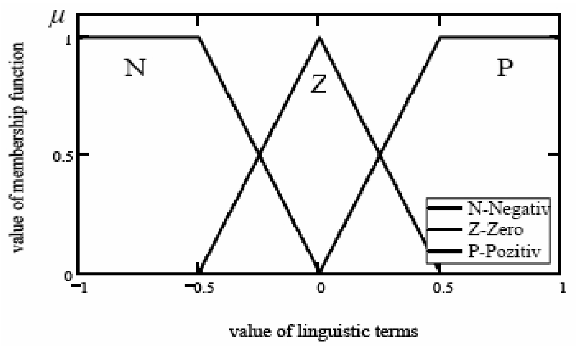

Fig. 3. Membership functions of z_1 and z_2.

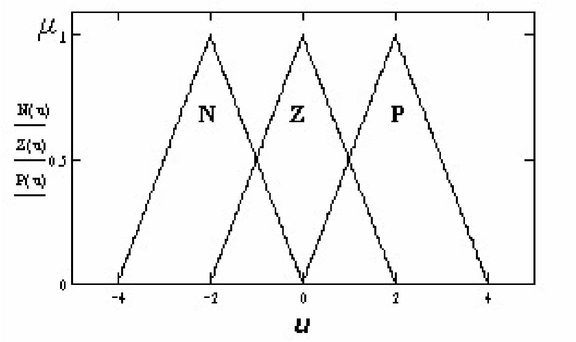

Fig. 4. Membership functions of u.

Table 1: Fuzzy control rule base.

Rule	Antecedent		Consequent
	z_1	z_2	u
1	**P**	**P**	**N**
2	**N**	**N**	**P**
3	**P**	**N**	**P**
4	**N**	**P**	**N**
5	**P**	**Z**	**Z**
6	**N**	**Z**	**Z**
7	**Z**	**P**	**N**
8	**Z**	**N**	**P**
9	**Z**	**Z**	**Z**

To prove that the system is stable by the proposed stability analysis, theorem 1 will be applied as follows to the suggested example. The Lyapunov function candidate in (20) is accepted:

$$V(\mathbf{z}) = \mathbf{z}^T \mathbf{P}\, \mathbf{z} = \frac{1}{2}(\frac{k}{m} z_1^2 + z_2^2)\,, \tag{20}$$

where:

$$\mathbf{P} = \begin{pmatrix} k/(2m) & 0 \\ 0 & 1/2 \end{pmatrix}. \tag{21}$$

Therefore V is positive and for $\|\mathbf{z}\| \to \infty$, then $V(\mathbf{z}) \to \infty$. It follows that $V(\mathbf{0}) > 0, \forall \mathbf{x} \neq \mathbf{0}$ and

$$\dot{V}(\mathbf{z}) = \frac{1}{m}(-bz_2^2|z_2| + z_2 u)\,. \tag{22}$$

From the fuzzy control rule base (Table 1) it may be observed that, if z_2 is Z, then u is Z too, otherwise z_2 and u are of opposite sign. So, taking (22) into account the derivative of V becomes $\dot{V}(\mathbf{z}) \leq 0$ for any fuzzy rule. Hence, $\dot{V}$ is negative semi-definite.

Next it will be proved that the condition 3 in theorem 1 holds. Assuming that there is a trajectory with:

$$z_2(t) = 0,\ z_1(t) \neq 0, \tag{23}$$

it results that:

$$\frac{\mathrm{d}z_2(t)}{\mathrm{d}t} = -\frac{k}{m} z_1(t) \neq 0\,, \tag{24}$$

which means that $z_2(t)$ cannot stay constant. So $\mathbf{z}(t) = \mathbf{0}$ is the only possible trajectory for which $\dot{V}(\mathbf{z}) = 0$. Hence the set $\{\mathbf{z} \in X \mid \dot{V}(\mathbf{z}) = 0\}$ contains no trajectory of the system except the trivial trajectory $\mathbf{z}(t) = \mathbf{0}$ for $t \geq 0$.

Thus, according to theorem 1, the fuzzy logic control system with the AND-SUM-COG fuzzy logic controller defined in section 2 and the process described in (1) is globally asymptotically stable at the origin.

5. SIMULATION RESULTS

The designed FLC is applied to the process described by equation (16), for *b*=0.1, *m*=6, *k*=6. Three simulation scenarios are accepted for the unforced system characterized by different initial states:

- the simulation scenario 1, with the initial state variables $z_1(0) = 1$ and $z_2(0) = -1$,
- the simulation scenario 2, with the initial state variables and $z_2(0) = 1$,
- the simulation scenario 3, with the initial state variables $z_1(0) = 1$ and $z_2(0) = 1$.

In case of the simulation scenario 1 the responses of z_1 and z_2 versus time (t) for the fuzzy logic control system are presented in Fig. 5. Next, accepting just the process described by (16), without FLC, keeping the parameters given above the response of z_1 versus time is shown in Fig. 6.

In case of the simulation scenario 2 the responses of z_1 and z_2 versus time for the fuzzy logic control system are illustrated in Fig. 7, and the response of z_1 versus time for the process is presented in Fig. 8.

Finally, in case of the simulation scenario 3 the behaviour of fuzzy logic control system is presented in Fig. 9. Fig. 10 illustrates the process behaviour observed in $z_1(t)$.

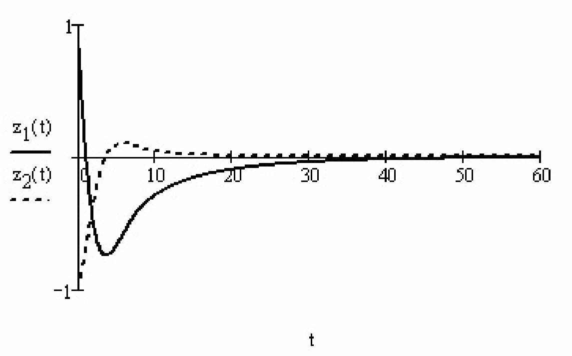

Fig. 5. Fuzzy logic control system behaviour in simulation scenario 1 for *r* = 0.

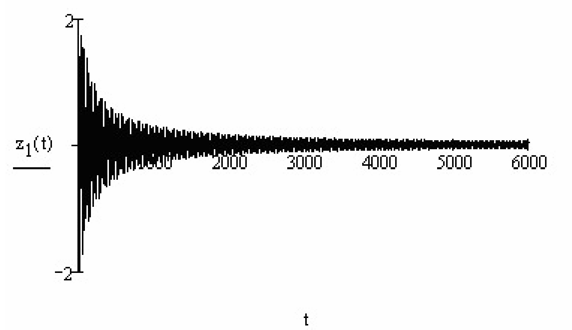

Fig. 6. Process behaviour in simulation scenario 1 for *u* = 0.

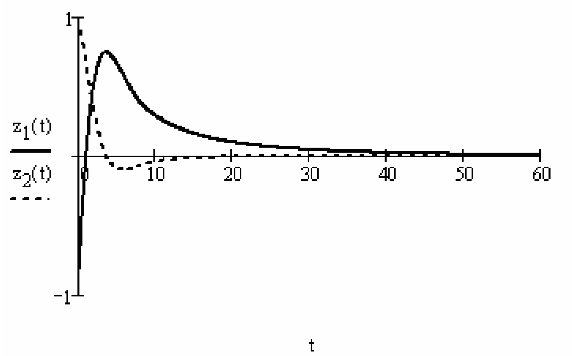

Fig. 7. Fuzzy logic control system behaviour in simulation scenario 2 for *r* = 0.

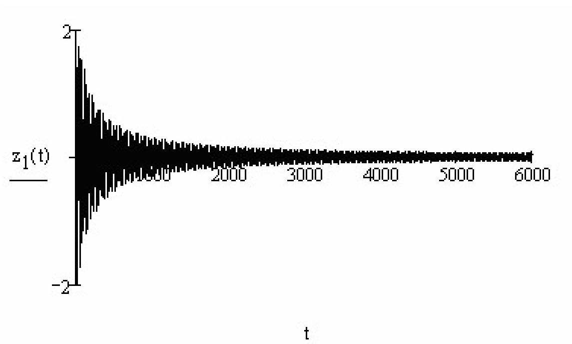

Fig. 8. Process behaviour in simulation scenario 2 for *u* = 0.

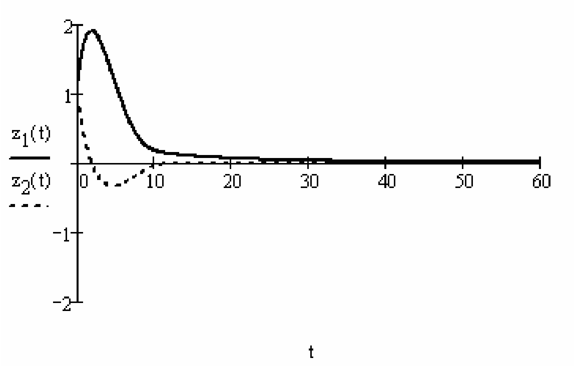

Fig. 9. Fuzzy logic control system behaviour in simulation scenario 3 for $r = 0$.

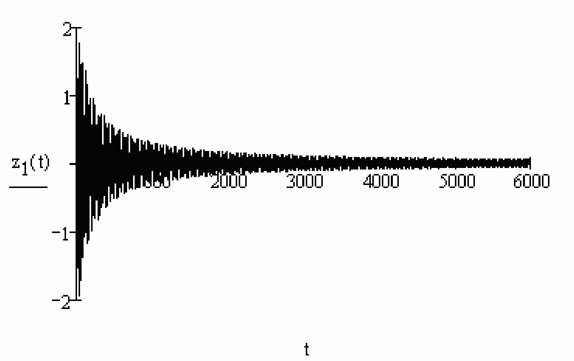

Fig. 10. Process behaviour in simulation scenario 1 for $u = 0$.

It can be observed that all dynamic behaviours of the unforced fuzzy logic control systems have been improved compared to the behaviours of controlled process. This improvement concerns alleviating the oscillations, reducing the overshoot and settling time. Summing up, the advantages of fuzzy logic control using the derived stability analysis approach to design the FLC can be emphasized clearly after examining Figs. 5-10.

6. CONCLUSIONS

An original approach to the globally asymptotically stability analysis of fuzzy logic control systems employing Mamdani type FLCs has been introduced. Using the proposed stability analysis approach, close to the approach in [14] dedicated to Takagi-Sugeno FLCs, makes the inserting of new fuzzy rules, the deleting of rules or the rule base pruning become very convenient due to the only need to fulfil the condition 2 in theorem 1.

This paper has shown, by an example, how the stability analysis suggested here by theorem 1, obtained by a reformulation of one result reported in [15] enabled by LaSalle's invariance principle, can be applied to a relatively general class of nonlinear processes constrained to (1). The stability analysis approach can be applied also in situations when the system has an equilibrium point different to the origin and / or the reference input is nonzero by an appropriately defined state-space transform [7].

Further research will be concentrated on the computer-aided design of the Mamdani type FLCs employing the stability analysis approach suggested in this paper. The state transforms necessary in dealing with non-autonomous systems will be part of the software that will be developed in this framework in order to increase its generality.

This stability analysis approach can be applied to other FLC structures that must just fulfil the property 1. Authors' intention is to cope with servo systems in manufacturing [8, 9], power electronics and digital audio signal processing [2, 3].

REFERENCES

[1] Andújar, J. M., Bravo, J. M., Peregrín, A., "Stability analysis and synthesis of multivariable fuzzy systems using interval arithmetic", Fuzzy Sets and Systems, vol. 148, pp. 337-353, Dec. 2004.

[2] Calvo, A., "Fuzzy control of chaos", in Integration of Fuzzy Logic and Chaos Theory (Z. Li, W. A. Halang, G. Chen, Eds.), Springer-Verlag, Berlin, Heidelberg, New York, pp. 99-125, 2006.

[3] Harb, A. M., Al-Smadi, I., "Chaos control using fuzzy controllers (Mamdani model)", in Integration of Fuzzy Logic and Chaos Theory (Z. Li, W. A. Halang, G. Chen, Eds.), Springer-Verlag, Berlin, Heidelberg, New York, pp. 127-155, 2006.

[4] Kandel, A., Langholz, G., Fuzzy Logic Control Systems, CRC Press, 1993.

[5] Khalil, H. K., Nonlinear Systems, 3rd Ed., Prentice Hall, Englewood Cliffs, NJ, 2002.

[6] Michels, K., Klawonn, F., Kruse, R., Nürnberger, A., Fuzzy Control: Fundamentals, Stability and Design of Fuzzy Controllers, Springer-Verlag, Berlin, Heidelberg, New York, 2006.

[7] Precup, R. -E., Preitl, S., Faur, G., "PI predictive fuzzy controllers for electrical drive speed control: methods and software

for stable development", Computers in Industry, vol. 52, pp. 253-270, Dec. 2003.
[8] Rudas, I. J., "Generalized T-operators", in Machine Intelligence Quo Vadis?" (P. Sincak, J. Vascak, K. Hirota, Eds.), World Scientific, pp. 179-198, 2004.
[9] Rudas, I. J., "Hybrid systems (integration of neural networks, fuzzy logic, expert systems, and genetic Algorithms)", in Encyclopedia of Information Systems, Academic Press, pp. 114-1-114-8, 2002.
[10] Sala, A., Guerra, T. M., Babuška, R., "Perspectives of fuzzy systems and control", Fuzzy Sets and Systems, vol. 156, pp. 432-444, Dec. 2005.
[11] Sugeno, M., Taniguchi, T., "On improvement of stability conditions for continuous Mamdani-like fuzzy systems", IEEE Trans. Systems, Man, Cybern. B, vol. 34, pp. 120-131, Jan. 2004.
[12] Tian, E., Peng, C., "Delay-dependent stability analysis and synthesis of uncertain T-S fuzzy systems with time-varying delay", Fuzzy Sets and Systems, vol. 157, pp. 544-559, Feb. 2006.
[13] Ying, H., "Structure and stability analysis of general Mamdani fuzzy dynamic models", International Journal of Intelligent Systems, vol. 20, pp. 103-125, Jan. 2005.
[14] Wong, L. K., Leung, F. H., Tam, P. K. S., "An improved Lyapunov function based stability analysis method for fuzzy logic control systems," Electronics Letters, vol. 36, pp. 1085-1086, June 2000.
[15] Wong, L. K., Leung, F. H. F., Tam, P. K. S., "A fuzzy sliding mode controller for nonlinear systems", IEEE Trans. Ind. Electron., vol. 48, pp. 32-37, Feb. 2001.