Voltage Control of DC Generators based on PWA hybrid systems

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Abstract: The main objective of this paper is the control of the voltage of a DC generator. To this aim, firstly, we identify the system using PieceWise Auto-Regressive eXogenous (PWARX) models due to their ability to approximate any nonlinear system with arbitrary precision. Secondly, we propose to design the one-step-ahead predictive control based on the identified models. The proposed control strategy exploit the same principle of the multimodel adaptive control with switching. The major advantage of the proposed control strategy is that the selection of the best controller is automatically achieved since at any time, the region closest to the current regression vector is used to assign the corresponding controller. The results of the proposed identification and control approaches are satisfying.

Keywords: PWARX models, Identification, Nonlinear systems, predictive Control.

1. INTRODUCTION

The representation of nonlinear dynamic systems by hybrid models of the class PieceWise AutoRegressive eXogenous (PWARX) has received great attention during the recent years within the international community of automation engineers Goudjil et al. (2016), Bako and Yahya (2019), Paoletti et al. (2019), van den Boom et al. (2016), Kim (2020). This is justified by two main reasons: fundamental and structural. Fundamentally, PWARX models are characterized by the property of universal approximation *i.e.* a PWARX model can be used to approximate with arbitrary precision the behavior of any nonlinear dynamic system. Structurally, the operating domain of a nonlinear system can be decomposed into a finite set of disjoint regions where a linear or affine sub-model is then associated to each region. Thus, the considered nonlinear system becomes by modeling like a PWARX system which switches between the different linear or affine submodels. These properties represent important advantages for the nonlinear systems represented by PWARX models because several control techniques from the theory of linear systems can be applied to these systems. Model predictive control has been the most successfully used strategy since it represent an efficient solution for the synthesis of controllers in the time domain. It is a technique of optimal control methodology, since the computed control sequence is calculated from the minimization of a criterion in order to maintain the system output close to the desired reference trajectory. The receding and the anticipatory action features distinguish it from other optimal control strategies Camacho et al. (2010). However, the performance of this control approach in the case of PWARX models, namely Hybrid Model predictive control (HMPC), remains sensitive to the choice of its tuning parameters due essentially to the switching phenomenon that characterized the PWARX models. Indeed, the switching from one sub-model to another implies the readjustment of the tuning parameters in order to maintain the desired performance. In the literature, some methods have been proposed for the parameters adjustement of generalized predictive control such as the parametric identification algorithms using fuzzy logic Chen and Narendra (2001), the multi-objective optimization algorithms Bempora and Penab (2009), the fuzzy supervisor Yahya et al. (2019) .

In this paper, we propose an alternative solution inspired by the principles of adaptive open loop control or switching multimodel control. The principle of this approach consists in synthetizing a controller for each sub-model ensuring the desired performances and then in developing a supervisor which allows selecting the best controller at every instant based on the minimization of a performance index. The precision of the control depends on the efficiency of the performance index. However, the synthesis parameters used by this index, such as the forgetting factor and the weightings for instantaneous and past errors, are set empirically. Consequently, a bad choice of these parameters leads to a degradation of the control law even to an instability of the control system. The use of the PWARX models allows to overcome this problem because the select of the best controller is done automatically based on the actual regressor which determines the well-partition of the regression space and then generates the active sub-model.

In this paper, we suggest the use of the PWARX models for the control of nonlinear systems by linear techniques and more particularly predictive control.

Predictive control is a control strategy widely used in research as well as in industry. This control strategy makes it possible to respond to the problems of regulating systems that may be subjected to constraints while solving an optimization problem. The principle of this control strategy involves knowing a mathematical model of the system in order to anticipate the future behavior of the process. The main idea of the predictive control can be summarized as follows: " *use a model in order to predict the behavior of the system and to choose the best decision by minimizing a performance criterion while respecting the constraints*".

The elements of predictive control are therefore a model of the system for the prediction, a performance criterion (cost function), constraints to be imposed on the state, input or output variables, an optimization algorithm generating the control law. For each element, several options can be considered, resulting in a multitude of predictive control algorithms. We are interested in this work in one-step ahead predictive control. The choice of a one-step prediction horizon is justified by the fact that the switching dynamics of the sub-models is a priori unknown and therefore switching from the actual sub-model to another can occur at each sampling instant.

This paper is organized as follows. Section 2 adresses the clustering technique based on the DBSCAN (Density-Based Spatial Clustering of Applications with Noise) algorithm to identify PWARX systems. Section 3 describes the proposed control strategy. In section 4, the identification and control approaches are applied on the DC generator.

2. MATHEMATICAL MODELING AND PROBLEM FORMULATION

2.1 PWARX systems representation

Dynamic hybrid systems are heterogeneous dynamical systems which exhibit both continuous and event phenomena. Among these systems, we distinguish the PieceWise AutoRegressive eXogenous (PWARX) which are obtained by decomposing the operating domain into a finite number of non-overlapping convex polyhedral regions, and then associating a linear or affine model to each region.

A PWARX system is defined as:

$$y(k) = \begin{cases} \theta_1^T \bar{\varphi}(k) + e(k) & if \quad \varphi(k) \in H_1 \\ \vdots \\ \theta_s^T \bar{\varphi}(k) + e(k) & if \quad \varphi(k) \in H_s \end{cases} \tag{1}$$

$$\varphi(k) = \begin{bmatrix} y(k-1) \\ \vdots \\ y(k-n_a) \\ u(k-1) \\ \vdots \\ u(k-n_b) \end{bmatrix} \quad \theta_i = \begin{bmatrix} a_{i,1} \\ \vdots \\ a_{i,n_a} \\ b_{i,1} \\ \vdots \\ b_{i,n_b} \\ g_i \end{bmatrix} \tag{2}$$

$$\bar{\varphi} = \begin{bmatrix} \varphi^T & 1 \end{bmatrix}^T. \tag{3}$$

where

- $y(k) \in \mathbb{R}$, $u(k) \in \mathbb{R}$, $e(k) \in \mathbb{R}$, $s \in \mathbb{N}$ are respectively the output, the input, the additive noise and the number of sub-models.
- $\theta_i \in \mathbb{R}^{n_a+n_b+1}$ is the parameter vector of the i^{th} sub-model having n_a and n_b as orders
- $a_{i,j}$ and $b_{i,j}$ represent the coefficients of the i^{th} sub-model and g_i represents the independent affine coefficient of the i^{th} sub-model.
- $\varphi(k) \in \mathbb{R}^{n_a+n_b}$ is the regressor vector.
- $H_i \in \mathbb{R}^{n_a+n_b}$ is the polyhedral partition of the i^{th} sub-model. The polyhedral partitions $H_i, i = 1, .., s$ must verify the following assumptions:

$$\begin{cases} \bigcup_{i=1}^{s} H_i = H \\ H_i \bigcap H_j = \emptyset \quad \forall i \neq j \end{cases} \tag{4}$$

2.2 PWARX systems identification

System identification consists in building mathematical models from the input-output measurements of the system. This approach is the most used in the field of system control because it ensures a good compromise between simplicity and precision of the model and is distinguished by its easy implementation. Moreover, it is generally applicable to all physical systems Bedoui et al. (2011).

The problem of PWARX system identification can be formulated as follows:

From a set of input / output measurements, estimate the orders of submodels, the number of submodels, the vectors of the parameters of the submodels and the coefficients of the affine hyperplanes defining the partitions of the regression space.

It is easy to deduce that the PWARX identification problem is one of the most difficult problems. To reduce this complexity, all existing approaches assume that the orders of the sub-models are known a priori as well as the number of sub-models for certain methods. Despite the consideration of these hypotheses, the subject still remains difficult because it requires the resolution of two problems which are the identification of the parameters of the sub-models and the estimation of the coefficients of the hyperplanes defining the regression domain. Numerous methods have been presented in the literature for the identification of PWARX models which can be classified in different solutions: algebraic Tian et al. (2011), classification Ferrari-Trecate et al. (2003), greedy solution Bemporad et al. (2003), Bayesian Juloski et al. (2005), bounded error solution Bemporad et al. (2005), sparse optimization Mattsson et al. (2016), and so on. Only the classification approach is considered in this work because it is based on a simple and informative procedure Lassoued and Abderrahim (2013a) Camacho et al. (2010) Lassoued and Abderrahim (2013b).

2.3 Clustering-based approach for the identification of PWARX models

This approach is based on a three main steps: data classification, parameter estimation and region reconstruction. The parameter estimation and the region reconstruction are ensured by efficient solutions which are the least squares method and the Support Vector Machine (SVM) method. But, the data classification step is realized by the *k-means* algorithm which is sensitive to additive noise. It also does not deal with outliers. Moreover, it assumes that the number of classes is known a priori Ferrari-Trecate et al. (2003). These drawbacks lead to a degradation of the estimation quality of the parameters of the sub-models as well as of the coefficients of the hyperplanes. To overcome these problems, we have advocated the use of other clustering algorithms. Among them, we cite the DBSCAN approach Lassoued and Abderrahim (2019) Kumar and Reddy (2016). This algorithm allows assigning the data into distinct classes while being based on certain density conditions, *i.e.* the classes are considered as dense regions which are separated by regions of low density. This method is also able to eliminate outliers during the partitioning process. In addition, it can determine the number of classes. This algorithm was tested on simulated examples and on real measurements where it provided the best results by comparison with the *k-means* method and its variants. Indeed, it is able to overcome the initialization problem which can lead to convergence towards local minima or even to a divergence of the algorithm. In addition, it automatically generates the number of sub-models. It also gives the best results even if the output is contaminated with noise having a high level Lassoued and Abderrahim (2014a).

Constructing local data sets We construct a local set C_k to each pair of data $\{\varphi(k), y(k)\}_{k=1}^{N}$. Every set C_k contain $\{\varphi(k), y(k)\}$ and their $(n_\rho - 1)$ nearest neighbors Ferrari-Trecate et al. (2001) verifying:

$$\forall(\breve{\varphi}, \breve{y}) \in C_k, \left\| \varphi(k) - \breve{\varphi} \right\|^2 \leq \|\varphi(k) - \hat{\varphi}\|^2, \forall(\hat{\varphi}, \hat{y}) \notin C_k. \quad (5)$$

The number of neighbors n_ρ is a random parameter. It has an important role in the algorithm. So, it must be properly chosen in order to ameliorate the identification result. For the obtained local sets $\{C_k\}_{k=1}^{N}$, we identify the parameters vectors $\{\theta_k\}_{k=1}^{N}$ using least square method or any standard linear regression techniques Ferrari-Trecate et al. (2002).

$$\theta_k = (\phi_k^T \phi_k)^{-1} \phi_k^T Y_k. \quad (6)$$

where

$$\phi_k = \left[\bar{\varphi}(t_k^1) ... \bar{\varphi}(t_k^{n_\rho})\right]^T,$$
$$Y_k = \left[y(t_k^1) ... y(t_k^{n_\rho})\right]^T$$

and $\left\{t_k^1, \cdots, t_k^{n_\rho}\right\}$ are the indices of data points belonging to the sets C_k.

Data clustering and parameters' estimation The objective of this step is to classify the parameters vectors $\{\theta_k\}_{k=1}^{N}$ into s clusters and determine the sub-models parameters vectors $\{\theta_i\}_{i=1}^{s}$.

We consider in this paper the DBSCAN approach. The main principle of this method is that for each data object of a cluster, the neighborhood of a given radius ϵ must hold in at least a minimum number *MinPts* of objects.
ϵ and $(MinPts)$ represent the input parameters of the algorithm. Therefore, they must be properly chosen in order to guarantee a good classification of the data.
Given ϵ and *MinPts* as input and a data set $S = \{\theta_k\}_{k=1}^{N}$, the ϵ-neighborhood of a point θ_k is defined as:

$$N_\varepsilon(\theta_k) = \{\theta_j \in S;\ \|\theta_k - \theta_j\| \leq \varepsilon\} \quad (7)$$

The DBSCAN constructs clusters by checking the ϵ-neighborhood of each object in the data set. So, a step of differentiation of points' nature is necessary. Then, the point is considered as:

- *core*, if the number of neighbors is higher or equal to *MinPts*.
- *border*, if the number of neighbors is less than *MinPts* and the considered point is within the neighborhood of any *core* point.
- *noise*, for the remaining points that are not *core* nor *border*.

The first cluster is formed by the first *core* point and its neighbors. Thereafter, all the other *core* points are recursively evaluated. If the considered point is not previously associated to a cluster, a new cluster will be created.
A detailed algorithm of the DBSCAN method is described in Lassoued and Abderrahim (2014b).

Region reconstruction The region reconstruction problem consists in determining a complete polyhedral partition $\{H_i\}_{i=1}^{s}$. Since the polyhedral regions are defined by hyperplanes, estimating the regions amounts to separating s sets of points by means of linear classifiers (hyperplanes) Richhariya and Tanveer (2018).
Separating the points in H_i from H_j, $i \neq j$ with an hyperplane without errors is a fabulous task because the sets H_i and H_j have intersecting convex hulls. Therefore, we just have to find the hyperplane that minimizes some misclassification index. For the s sets H_i,...,H_s, two types of linear separation can be handled:

- Binary classification: for each pair (H_i, H_j), with $i \neq j$, a linear classifier is constructed.
- Multi-class classification: a piecewise linear classifier is constructed having as object the discrimination of s classes.

The separation task can be accomplished by resorting to the support-vector machines SVMs which are considered among the best supervised learning algorithms Hsu and Lin (2002).

3. PROPOSED ONE-STEP-AHEAD PREDICTIVE CONTROL STRATEGY FOR PWARX SYSTEMS

After describing the nonlinear behaviour of the system by a PWARX model using the clustering based procedure of identification, we can proceed with the design of a system control law.

Since results of linear system control can be applied to the PWA model, we propose a simple control approach that would emphasize the accuracy of the model which is the predictive control. This method always attracts the attention of researchers as it is broadly used in the field of industry as well as in the field of research.

In this work, we are interested in the one-step ahead predictive control because switching from one submodel to another can occur at each sampling instant and therefore the switching dynamics of the submodels is not known a priori.
Obviously, this control strategy requires a mathematical model that permit the prediction of the behaviour of the

system and then it proceeds to minimize a performance criterion that can be constrained.

Indeed, in order to design the predictive control for PWARX systems, we propose a solution that exploit the same idea of multimodel adaptive control with switching Landau et al. (2011), Islam et al. (2016). This approach consists in selecting the suitable sub-model that represent the process at every sampling time by solving a performance criterion. Then, the output of the corresponding controller is applied to the process Anderson et al. (2001), Wang et al. (2005), Chen and Narendra (2001).

The main advantage o the proposed method is that the selection of the sub-model is based only on the regression vector which is constructed by the input and the output of the system. The convenient controller is then determined by the SVM approach which is already applied in determining the regions of the sub-model.
Therefore, since the nonlinear system is represented by a PWA model described by the parameters vectors of s sub-models with their corresponding regions, the control structure is then simplified as shown in Figure 1.

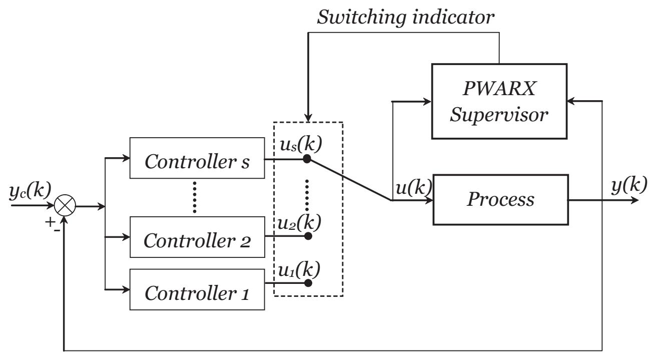

Fig. 1. Proposed PWARX control strategy.

The control law is calculated through the minimization of a quadratic criterion which penalizes the differences between the predicted outputs and the reference trajectory.
For each sampling time k, we dispose of the current output $y(k)$ and the precedent inputs and outputs $y(k), y(k-1), ..., y(k-n_a)$ and $u(k), u(k-1), ..., u(k-n_b)$.

The objectif of the one-step predictive control is to find the control low $u(k)$ that coincide the output $y(k)$ with the reference trajectory $y_c(k)$ at the instant $k+1$. More precisely, this control is obtained by minimizing the following criterion:

$$J = \frac{1}{2}\left\|y(k+1) - y_c(k+1)\right\|_Q^2 + \frac{1}{2}\left\|u(k)\right\|_R^2 \tag{8}$$

$Q \in \mathbb{R}^{m\times m}$ is a positive definite matrix and $R \in \mathbb{R}^{n\times n}$ is a semi definite positive matrix and m and n are respectively the number of outputs and inputs.

PWA control problem formulation
For PWA systems, the quadratic form of the criterion is given by:

$$\begin{aligned} J = [y(k+1) - y_c(k+1)]^T Q[y(k+1)-\\ y_c(k+1)] + u(k)^T R u(k) \end{aligned} \tag{9}$$

The output $y(k)$ of a PWA system is given by:

$$\begin{cases} a_{1,1}y(k-1) + .. + a_{n_a,1}y(k-n_a) + \\ b_{1,1}u(k-1) + .. + b_{n_b,1}u(k-n_b) + g_1 \\ \quad if\ \varphi(k) \in H_1 \\ \quad\vdots \\ a_{1,s}y(k-1) + .. + a_{n_a,s}y(k-n_a) + \\ b_{1,s}u(k-1) + .. + b_{n_b,s}u(k-n_b) + g_s \\ \quad if \varphi(k) \in H_s \end{cases} \tag{10}$$

The system output can be defined by the following equation:

$$\begin{aligned} y(k) &= \theta_{\sigma(k)}^T \bar{\varphi}(k) \\ &= a_{1,\sigma(k)}y(k-1) + \cdots + a_{n_a,\sigma(k)}y(k-n_a) \\ &\quad + b_{1,\sigma(k)}u(k-1) + \cdots + b_{n_b,\sigma(k)}u(k-n_b) \\ &\quad + g_{\sigma(k)} \end{aligned} \tag{11}$$

where $a_{i,\sigma(k)}$, $b_{i,\sigma(k)}$ and $g_{\sigma(k)}$ designate the parameters of the active sub-model and $\sigma(k)$ represents the active sub-model.

The criterion of the one-step predictive control allowing a coincidence between the output $y(k+1)$ and the reference $y_c(k+1)$ for a PWARX system described by the relation (11) can be written as:

$$J = Q.\Psi^2 + Ru^2(k) \tag{12}$$

where

$$\begin{aligned} \Psi = y_{ref}(k+1) - a_{1,\sigma(k)}y(k) - \cdots \\ -a_{n_a,\sigma(k)}y(k-n_a+1) - b_{1,\sigma(k)}u(k) - \cdots \\ -b_{n_b,\sigma(k)}u(k-n_b+1) - g_{\sigma(k)} \end{aligned}$$

An explicit solution of the control law $u(k)$ is obtained by minimising the criterion (12).

$$u(k) = \frac{Q.b_{1,\sigma(k)}.\Delta}{Q.b_{1,\sigma(k)}^2 + R} \tag{13}$$

where

$$\begin{aligned} \Delta = y_{ref}(k+1) - a_{1,\sigma(k)}y(k) - \cdots \\ -a_{n_a,\sigma(k)}y(k-n_a+1) - b_{2,\sigma(k)}u(k-1) - \cdots \\ -b_{n_b,\sigma(k)}u(k-n_b+1) - g_{\sigma(k)} \end{aligned}$$

A convenient choice of the weights Q and R can improve the stability of the one-step-ahead controller.
R can be considered as a tuning parameter allowing to obtain a balance between control magnitude and tracking accuracy.

4. EXPERIMENTAL VALIDATION: A DC VOLTAGE GENERATOR

4.1 System description

DC generator or direct current generator is one kind of electrical machine, and the main function of this machine is to convert mechanical energy into DC (direct current) electricity. The energy alteration process uses the principle of energetically induced electromotive force.

Two most important categories of DC generator exist namely separately excited and self-excited. In separately excited type, the field coils are strengthened from an autonomous exterior DC source.

In the self-excited type, the field coils are strengthened from the generated current with the generator. The generation of the first electromotive force will occur because of its outstanding magnetism within field poles.

The applications of different types of DC generators include the following.

* The separately excited type DC generator is used for boosting as well as electroplating. It is used for power and lighting purpose using a field regulator.

* The self-excited DC generator or shunt DC generator is used for power as well as ordinary lighting using the regulator. It can be used for battery lighting.

* The series DC generator is used in arc lamps for lighting, stable current generator, and booster.

* A compound DC generator is used to provide the power supply for DC welding machines.

In this paper, the studied system is a DC voltage generator which consists of a DC motor mechanically coupled with an induction motor. The speed of the induction motor is controlled by a variable-frequency drive. An acquisition card is used to adjust the frequency reference of the variable-frequency drive and acquire measurement of the DC voltage generated by the DC motor. Figure 2 is a real photo of the system.

Fig. 2. DC voltage generator.

The Induction motor is the most used electric motor worldwide in industrial facilities and large buildings. However, asynchronous motors do not inherently have the capability of variable speed operation. Therefore, using variable frequency drives is necessary.
By changing the supply frequency, the motor speed can be altered and thus the speed of a three-phase induction motor can be controlled.

The variable frequency drive used in this application is a DIGIDRIVE SK from Leroy Somer. Its control input is the frequency reference in $0 - 10VDC$ range.

The DC voltage generator is a DC motor with a shunt-excitation. It does not require an independent electrical supply. It is characterised by an unidirectional output and thus it may be considered as a source to supply its own field current.
The electrical characteristics of each component of the studied system are presented in Tables 1, 2 and 3.

Table 1. Electrical characteristics of the induction motor

Parameters	Value
nominal voltage	$400V$
frequency	$50Hz$
nominal speed	$1500rpm$
nominal power	$0.9kW$
nominal current	$1.75A$

Table 2. Electrical characteristics of the variable frequency drive

Parameters	Value
connection to a 3-phase supply	$380V - 480V$ $48Hz - 62Hz$
power for motor 4 poles	$1.5kW$
nominal current	$3.8A$
maximum current	$5.2A$

Table 3. Electrical characteristics of the generator

Parameters	Value
nominal voltage	$220V$
nominal speed	$1500rpm$
nominal power	$1.5kW$
nominal current	$6.8A$

4.2 System analysis

The frequency reference input of the variable-frequency drive represents the control input of the system $u(k)$.
The output of the system $y(k)$ is a linear transformation of the voltage U generated by the DC generator such that $y = U/22$.
According to the experimental studies, we can note that the real process is a Single-Input Single-Output (SISO) system. Indeed, we have used the static characteristic for the determination of the nature of the system and for the construction of the identification signal. Therefore, the process can be considered as non linear as shown in Figure 3. In addition, we have used the system step responses for different levels to derive the structure of the sub-models which as in Figure 4. This figure shows that it is a second order system.

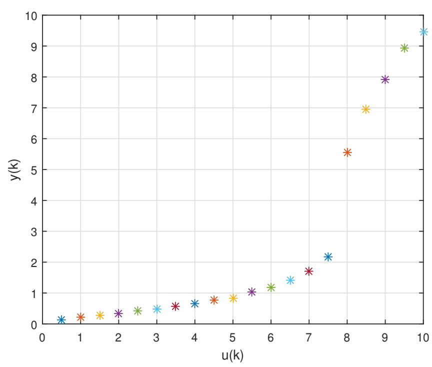

Fig. 3. Static characteristic of the real system.

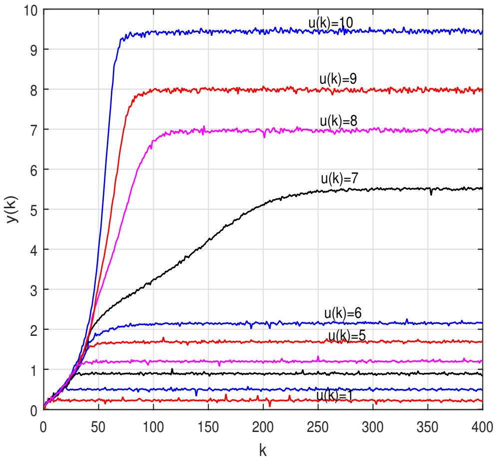

Fig. 4. Step responses of the real system.

4.3 Identification results

In this paragraph, the DC voltage generator is approximated by a PWARX model using the proposed identification method.
Indeed, the determination of a model to this process starts with the acquisition of some input-output measurements. We have picked out one measurement file for the identification and another one for the validation.

In fact, for nonlinear dynamic systems, the input must be designed to excite both all the bandwidth frequencies of the system and all the amplitude rang of the system. For these reasons, we have considered as input a multisine sequence (10 sinusoids) characterized by a frequency band [0.01 0.02] and an amplitude-range [1 10]. The sampling time of this signal is equal to 0.05s.
Figure 5 shows the input-output measurement files used for the identification of the real process.

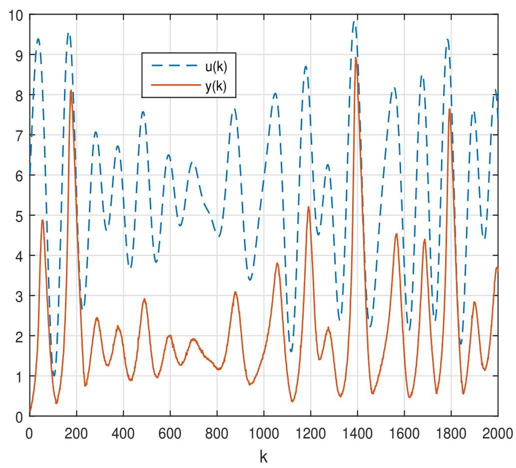

Fig. 5. Input-output data of the real system.

The proposed identification method is applied with the following synthesis parameters:
$n_\rho = 72$, $\epsilon = 0.23$ and $MinPts = 28$.
The orders of each sub-model n_a and n_b are fixed to the value 2 since this system can be considered as a second-order system around each operating point. The following structure is then adopted:

$$y(k) = \begin{cases} \begin{array}{l} -a_{1,1}y(k-1) - a_{1,2}y(k-2) \\ +b_{1,1}u(k-1) + b_{1,2}u(k-2) \end{array} & if\varphi(k) \in H_1 \\ \vdots & \\ \begin{array}{l} -a_{s,1}y(k-1) - a_{s,2}y(k-2) \\ +b_{s,1}u(k-1) + b_{s,2}u(k-2) \end{array} & if\varphi(k) \in H_s \end{cases} \tag{14}$$

where the regressor vector and the parameter vectors are as follows:

$$\varphi(k) = \begin{bmatrix} -y(k-1) \\ -y(k-2) \\ u(k-1) \\ u(k-2) \end{bmatrix} \qquad \theta_i(k) = \begin{bmatrix} a_{i,1} \\ a_{i,2} \\ b_{i,1} \\ b_{i,2} \end{bmatrix}$$

The obtained results are as follows:

- The number of sub-models s is equal to 4.
- The estimated parameter vectors θ_i(k), i = 1,..,4 are:

$$\theta_1^T = [0.5411 \quad 0.4397 \quad 0.6980 - 0.6920]$$

$$\theta_2^T = [0.3963 \quad 0.3100 \quad 0.1537 - 0.0856]$$

$$\theta_3^T = [0.3817 \quad 0.3562 \quad 0.2309 - 0.1751]$$

$$\theta_4^T = [0.4200 \quad 0.3675 \quad 0.2407 - 0.1958]$$

The obtained results of the estimated PWARX model and the real output are depicted in Figure 6 which prove the efficiency of the proposed approach. We can note that the four sub-models contribute to the estimated output evolution. In addition, the real and the estimated outputs have a similar evolution.

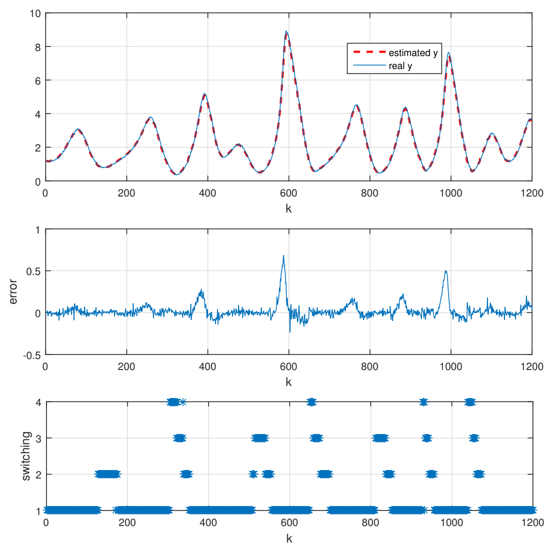

Fig. 6. Real and estimated outputs, estimation error and switching instances.

A new input-output measurement file is considered in order to validate the obtained PWARX model. Figure 7 prove the efficiency of the proposed method.

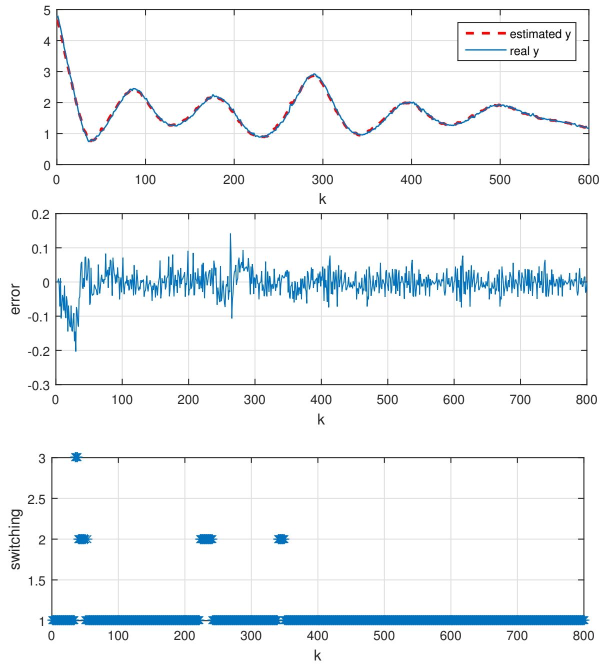

Fig. 7. Real and estimated outputs, estimation error and switching instances: validation.

4.4 Control results

The one-step ahead predictive strategy is applied using the obtained PWARX model using the reference and the disturbance trajectories as shown in Figure 8. The reference has a sinusoidal part to evaluate the tracking performance and includes two steps of different amplitudes to test the behavior in regulation.

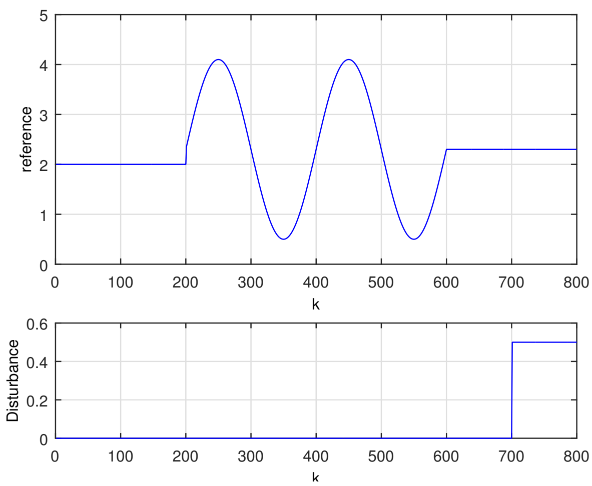

Fig. 8. Reference and disturbance evolutions.

Figure 9 shows the evolution of the output and the desired trajectory. We remark that the proposed control strategy provides good closed-loop performances. The evolution of the output is close to that of the reference trajectory. We remark also the convergence of the tracking error to zero.

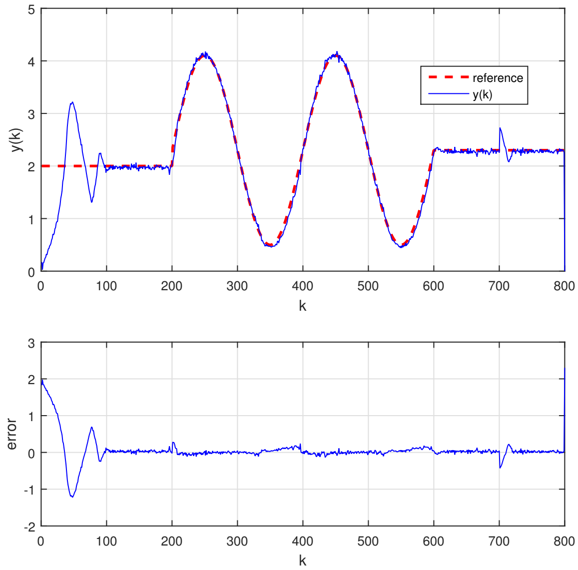

Fig. 9. Output and desired trajectory

As shown in Figures 10 and 11 the control law evolution is soft even when switching from one sub-model to another. It is noticeable that the control law allows to reject constant disturbances. Furthermore, there is switching of all sub-models with the reference trajectory evolution and one sub-model is activated if the trajectory remains unchanged.

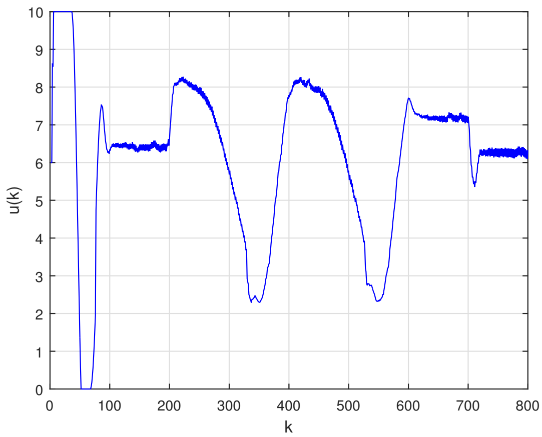

Fig. 10. Control evolution

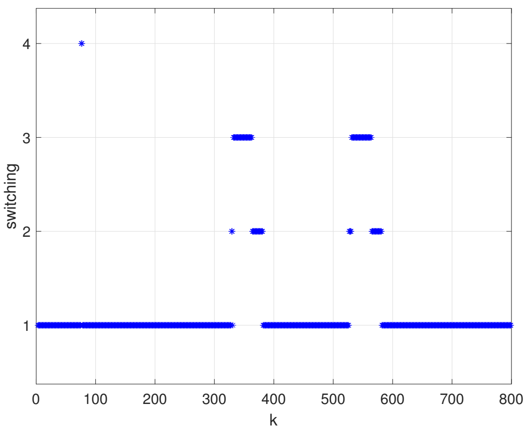

Fig. 11. Switching instances

From the obtained results, we can extract the following remarks:

- Good control and good tracking are performed.
- All sub-models are turned on by changing the reference signal.
- One sub-model is activated if the reference is unchanged.

5. CONCLUSION

In this paper, we have preconized the use of the PWARX models and the one-step predictive control to solve the problem of nonlinear system control. The suggested strategy is inspired by the switching multimodel control.

In fact, in the multimodel adaptive control with switching, the supervisor allows to select the best controller to be used at each sampling instant. The supervisor operation is based on the minimization of a criterion which relates the current and the past estimation errors weighted by some design parameters.

The major disadvantage of this supervisor lies in the fact that the choice of the design parameters is made empirically by the trial-and-error method. However, a bad choice of these parameters can lead to a degradation of the desired performances in a closed loop.

Our proposed approach suggests a control strategy where the selection of the best controller is done automatically by the SVM approach i.e., at any time, the region closest to the current regression vector is used to assign the corresponding controller.

The identification and the control approaches are successfully applied to the DC voltage genarator and the obtained results are satisfying. Indeed, it is important to point out that the implementation of the proposed approaches is generally difficult in the case of a real system. These difficulties arise at several levels such as the determination of the number of sub-models, the choice of the structure of the sub-models, the choice of the identification signal, the choice of the synthesis parameters, the stability of the control system, etc . To overcome these difficulties, we have suggested some heuristic solutions. Therefore, we used the static characteristic for the determination of the number of sub-models and for the construction of the excitation signal. In addition, the step responses can be used to deduce the structure of the sub-models. These choices are then refined both by a trial-and-error procedure. Indeed, the confirmation of these choices by fundamental concepts represents the core of our future contributions and therefore the problem is still open.

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