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BER IMPROVEMENTS WITH MULTILEVEL CODED MODULATION

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Abstract: *The paper compares some new methods for coded modulation constructions using multilevel codes obtained by combining channel coding and digital modulation. As reference examples are presented schemes based on phase shift keying. The comparison parameter is the bit error rate and consequently structures for optimal receivers are discussed. In particular it is shown how multistage decoding can be improved by using interleaving blocks between the three levels of the decoding structure and which are the advantages obtained - the increase in reliability, additional coding gains, a modest complexity of the structure, but with an undesired increased decoding delay. Finally the role of time diversity encoding in improving transmission performances on nongaussian channels is presented.*

Keywords: *coded modulation, multilevel codes, optimal rejection, gaussian noise, error rate performance.*

1. INTRODUCTION

It was realized that in order to approach the Shannon capacity limit, digital modulation and channel coding must be designed jointly for maximizing the data rate per unit bandwidth at a certain channel signal-to-noise ratio. By coded modulation we mean combined coding and modulation schemes that are jointly optimized. Trellis coded modulation (TCM) is a special case, an alternate strategy to block code modulation (BCM).

Both these methods may be considered as time diversity encoding methods. Traditionally L-fold time diversity is obtained by means of repeating a symbol in L different time slots. In terms of channel coding, that appears as a repetition code with minimum Hamming distance d_H=L. For conventional channel coding with phase shift keying (PSK) modulation it is known that a maximum Hamming distance yields the largest free Euclidean distance for the system and consequently such system has also the largest built-in degree of time diversity and so an optimal system for Gaussian channels is also optimal for a Rayleigh (fading) system. Unfortunately, as we will see, such a dual optimality does not exist for coded modulation schemes.

Trellis coded modulation has fond interest and immediate application since Ungerboeck's paper (Ungerboeck, 1982). With M = 2^m different phases one can transmit m bits per M-PSK symbol without coding. The key idea of coded phase modulation is to transmit only (m-1) bits and to use a rate (m-1)/m trellis code to generate sequences with high Euclidean distances in the complex plane. Normally bandwidth is not increased, because one phase symbol is transmitted per signaling interval in both cases.

In Ungerboeck's approach the (m-1) information bits are jointly encoded by one encoder and mapped into 2^m phase symbol using the principle of set-partitioning. Another approach to coded M-PSK with $M = 2^m$ is the concept of multilevel codes i.e. short binary block codes of length 2 to 8 binary symbols to construct 8-PSK codes of length 2 to 8 (Imai and Hirakawa, 1988). In this paper, we introduce additional concepts for improved decoding for multilevel codes and compare their decoding complexity and performance with a classical decoding concept and with a TCM code.

2. CODED MODULATION CONSTRUCTIONS

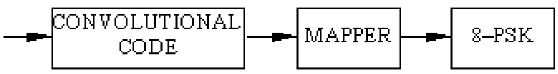

Fig. 1a.

Figure 1a shows a classical trellis coded 8-PSK system with a single rate 2/3 convolutional code (2,4) as is depicted in fig 1b, where the labels 0...8 represent the output 8-PSK symbols disposed in a symmetric constellation in counterclockwise order. This scheme has a 3 dB gain over QPSK in bit energy over noise density E_b/N_0, but there are error events of length L=1 due to the two parallel transitions between states. So, the achievable time-diversity is less than the Hamming distance, but quite this code is not the best one for a Gaussian channel, it is the one with the best time diversity at the same complexity level.

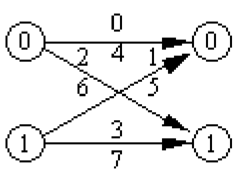

Fig. 1b.

Another solution to improve time diversity is the multiple trellis code modulation (MTCM) where more than one 8-PSK symbol is mapped onto each path of the trellis.

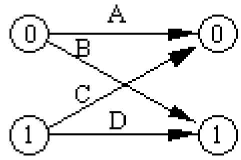

Fig. 2a.

Figure 2a shows the trellis section for a MTCM two states, 4/6 rate and blocks of two symbols per trellis path. The sets A, B, C, D of branches are given in figure 2b. Now all error events of length 1 or 2 correspond to Hamming distance 2 8-PSK symbols and so we have succeeded to improve the inherent time diversity to L=2 branches gaining asymptotically over uncoded QPSK, but only for fading channels.

$$A=\begin{bmatrix}0&0\\1&5\\2&2\\3&7\\4&4\\5&1\\6&6\\7&3\end{bmatrix}\quad B=\begin{bmatrix}0&4\\1&1\\2&6\\3&3\\4&0\\5&5\\6&2\\7&7\end{bmatrix}\quad C=\begin{bmatrix}0&2\\1&7\\2&4\\3&1\\4&6\\5&3\\6&0\\7&5\end{bmatrix}\quad D=\begin{bmatrix}2&0\\3&5\\4&2\\5&7\\6&4\\7&1\\0&6\\1&3\end{bmatrix}$$

Fig. 2b.

Let consider now the 8-PSK signal set with labeling of each point in the signal space just like Ungerboeck codes, like is shown in figure 3. It corresponds to a signal set S_0 with $M=2^m$ elements in the two-dimensional Euclidean space, where m is a positive integer [3 in our case]. Multilevel codes can be used to code such a signal; their construction is realized in fact by concatenating simple codes, associated with a single element from the signal set, defined through a partition chain.

So, for the chain in fig.3, the signal set is partitioned first in two 4-PSK sets and further into four 2-PSK set, such as finally we obtain eight subsets each containing one signal point.

Every level i, i=1,...,m of the partition chain is protected by a designated binary code $C^{(i)}$, which can be a block or convolutional code. The m outer codes $C^{(i)}$ and the partition chain $S_0/S_1.../S_m$ build up a multilevel code C, with the structure expressed by the following matrix:

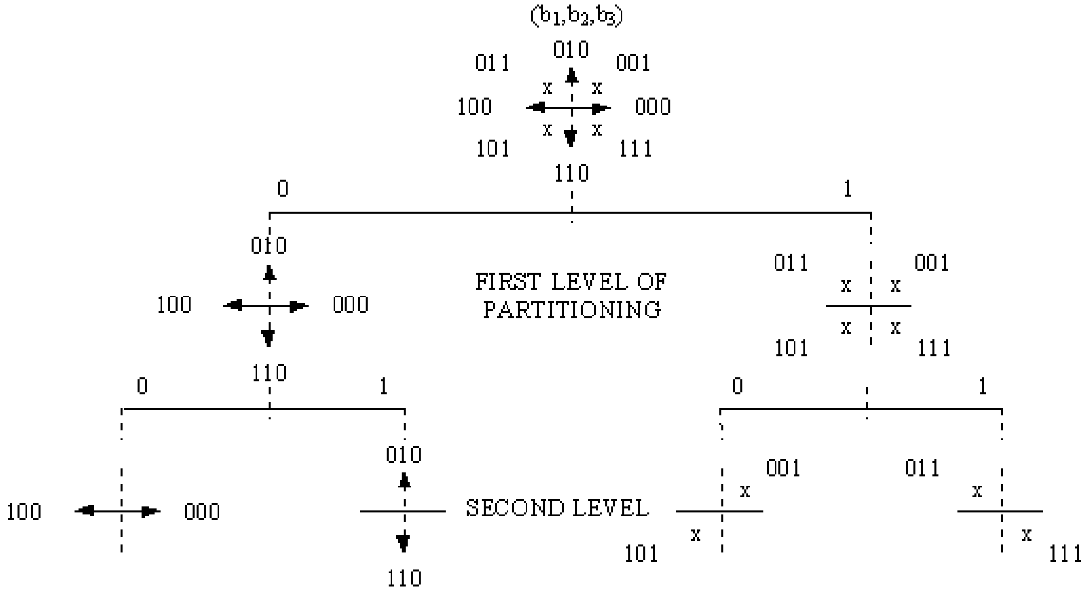

Fig. 3.

$$C = \begin{bmatrix} C^{(1)} \\ C^{(2)} \\ \vdots \\ C^{(m)} \end{bmatrix} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \cdots \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \cdots \end{bmatrix} \quad (1)$$

Every row $x_j^{(i)}$, j=1,2... contains a codeword of the corresponding outer code $C^{(i)}$. If this code is a block code, the index j runs until its length n_C. If $C_{(i)}$ is a convolutional code, we have no defined block length unless we terminate by tail bits. Every column $x_j^{(i)}$, i=1,...m, determines one point from the signal set S_0, which is then transmitted over the channel.

As consequence of this construction we get codes with minimum squared Euclidean distances between two different codewords. That means that we have in the Hamming space minimum Hamming distance in case of block codes or free Hamming distance in case of convolutional codes. In addition we obtain also a minimum asymptotic gain E_b/N_0 compared with uncoded QPSK.

In particular for 8-PSk we have m=3. Let the minimum Hamming distance (free distance) for each of the components binary codes be d_{Hi}, i=1,2,3. Then the minimum Hamming distance between any two 8-PSK sequences is $d_H = \min(d_{H1}, d_{H2}, d_{H3})$. In addition, a certain symbol rate should be achieved. If the total number of information bits into each encoder is K_i, the overall code length is n and the code rate is R bits/symbol, then $R = \Sigma R_i$ with $R_i = K_i/n$. Thus the code design problem is to achieve a diversity as large as possible at a given rate of transmission and decoding complexity.

Optimal decoding of multilevel codes can be performed by a maximum likelihood decoder that finds the better input sequence that maximize the probability of receiving the observed sequence, but in practice suboptimal decoding techniques are needed.

3. ENCODING AND DECODING MULTILEVEL SCHEMES

Figure 4 shows the classical encoder structure for a multilevel code with three levels. The incoming serial information bit stream u_K is fed into a demultiplexer, which divides it into three substreams $u_K^{(i)}$, i=1,2,3. Every substream is encoded by a designated code $C^{(i)}$ with the rate $R^{(i)}$. Then a multiplexer takes one coded bit $x_N^{(i)}$ from each level i to form a vector with three elements. This vector selects the finally transmitted 8-PSK symbol x_N according to the set partitioning principle from fig.3.

Several kinds of codes can be used, but in this paper we are mainly interested in combining convolutional codes. We have investigated punctured convolutional codes with memory v=3, having a total rate of about 2 bits/8-PSK-symbol, so that they can be compared with uncoded QPSK and simple TCM codes. Such a code can be defined by a vector having three components that correspond to the bit rates, i.e. (R_1, R_2, R_3).

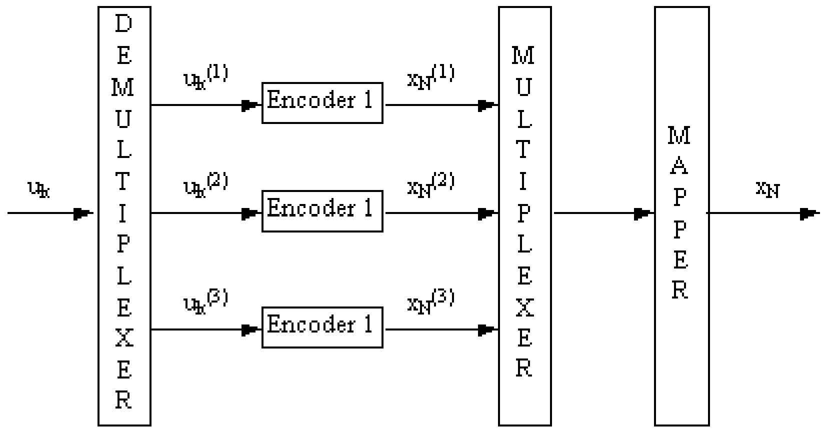

Fig. 4.

Table 1 gives a list of the codes further investigated in this paper, denoted with A, B, C, D including the rates of the three levels R_i, the corresponding Hamming distances dH_i, and the total rate R [bits/8-PSK-symbol]. SP means single parity check (dH=2), 1/1 means uncoded (dH=1).

Referring to the decoding principle, we mention first that in order to compare decoding performances we use the concept of decoding complexity, i.e. if we describe both block and convolutional codes by a trellis diagram, decoding complexity is defined by the number of trellis branches that have to be processed by a Viterbi algorithm per transmitted information bit.

The classical principle of decoding, called multistage decoding, breaks the decoding process in three stages. Fig. 5 shows the classical way to do multistage decoding as introduced in [2]. The samples y_N comings from the channel are fed into the decoder of the first level. An estimation of the transmitted information bit sequence $u_K^{(1)}$ is determined and forwarded to a multiplexer. Furthermore, estimates of the corresponding coded bits $x_N^{(1)}$ are fed to the decoder of level 2. This decoder obtains the same samples from the channel and determines the information bit sequence $u_K^{(2)}$ based on the decision $x_N^{(1)}$. The same procedure is applied to the third level.

Table 1. Comparison between codes.

	R_1	R_2	R_3	dH_1	dH_2	dH_3	Gain [dB]	R
A	1/4	3/4	1/1	13	3	1	3.0	2.00
B	1/4	3/4	15/16SP	13	3	2	4.6	1.94
C	2/3	2/3	2/3	10	6	4	4.7	2.00
D	1/2	2/3	2/3	12	8	7	4.8	1.83

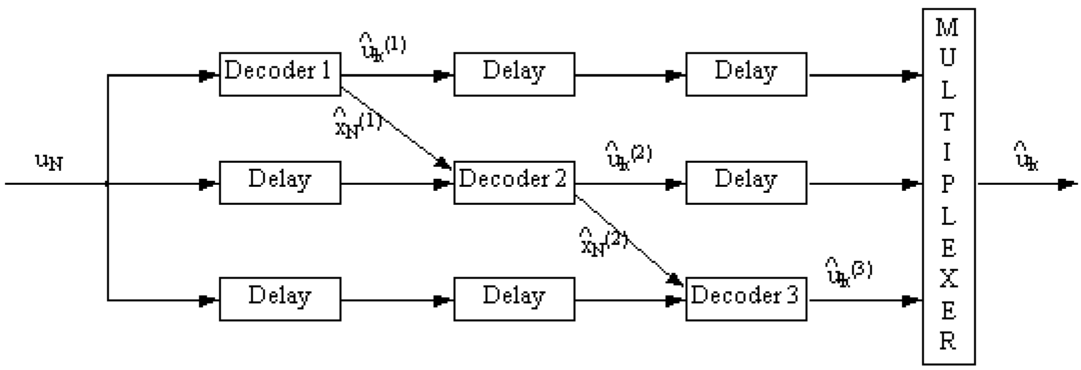

Fig. 5.

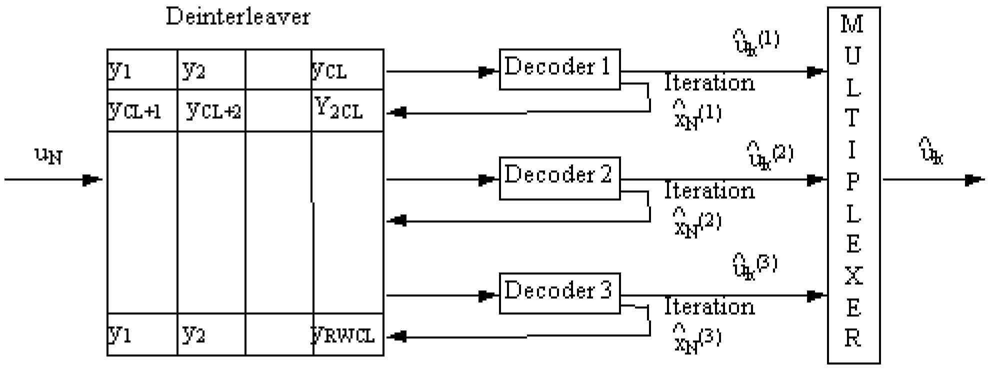

Fig. 6.

Finally, a multiplexer combines the three information bit sequences $u_K^{(i)}$ corresponding to the demultiplexing at the encoder and outputs u_K.

All multilevel codes using classical multistage decoding have about the same performances as the TCM code with 16 states (a gain of about 4.0 dB at a BER of 10^{-5}) and superior to the uncoded QPSK (2.6 dB asymptotic gain at the same BER). The code (1/4,3/4,1/1) has a gain of about 3 dB, but the others are far away [the best ensure 4.8 dB]. But there are also some inconvenient in the classical decoding procedure, using the Viterbi algorithm for decoding, such as error propagation from one level to subsequent levels and an increased number of nearest neighbors for the lower levels (Kasami *et al.*, 1991). Consequently, we propose in the next section a new improved decoding procedure.

4. USING INTERLEAVING TO IMPROVE THE DECODING PROCEDURE

Fig. 6 shows a block diagram corresponding to the improved multistage decoding procedure.

In order to avoid the error propagation effect, we introduce interleaving between the coded bit streams of each level (Woerz and Hagenauer, 1990). An interleaver matrix is added to the coder block diagram between the coders of each level and the mapping unit. At the decoder, the received symbols are first written into a deinterleaver matrix and are then processed. The interleaving has to be done in such a way that the reencoded bit streams of any two decoders are spread for the third decoder. To implement such an interleaver we used a two-dimensional block interleaver with RW rows and CL columns. Every cell of the interleaver contains one coded bit $x_N^{(i)}$, i=1,2,3 from each level. The c_N are mapped in to channel symbols x_N according to set partitioning. Fig.7 describes the interleaver cell enumeration (c_N are enumerated row-wise). The following rules are used in order to write the output $x_N^{(i)}$ of coder i into a cell $c_N^{(i)}$ of the interleaver: the coded bits of the first level $x_N^{(1)}$ are written row-wise the coded bits of the second level $x_N^{(2)}$ are written column-wise - the coded bits of the third level $x_N^{(3)}$ are written diagonally.

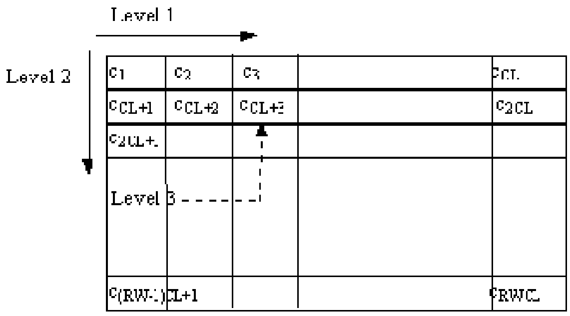

Fig. 7.

The decoders of levels 2 and 3 need the reencoded information bit sequence of the preceding levels to estimate their own information bit sequence. In the classical decoding, the decoders of levels 1 and 2 deliver this information in the form of +1 or -1. Because some of these decisions are likely to be erroneous, it is important to know their reliability and to compare it with the reliability obtained in the new structure. In this aim we use the Symbol-by-Symbol MAP (Maximum a-posteriori Algorithm) proposed by (Forney, 1973) which allows to extract the reliability information ($P(x_N^{(1)} = +1 \text{ or } -1)$ and consequently to compute the log-likelihood ratio:

$$L(x_N^{(1)}) = \log [P(x_N^{(1)}=+1) / P(x_N^{(1)}=-1) \quad (2)$$

Suppose we use a code in the i-th level that can be described by the trellis diagram shown in fig.8. The states are denoted by $s_{l,k}$, l=1...L, k=1...K, where L is the number of states and K the sequence length. A transition from $s_{l,k}$ to $s_{l',k+1}$ is denote by $\xi_{ll',k}$.

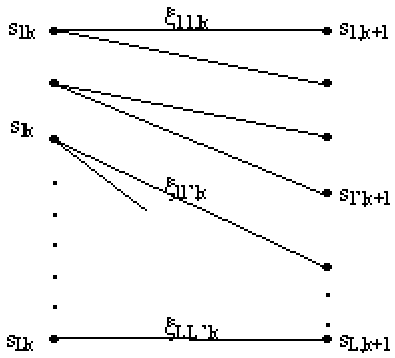

Fig. 8.

The MAP algorithm is a recursive procedure to compute the probability $P(\xi_{ll'}/y)$ of each possible transition $\xi_{ll'}$ having received a vector of samples y from the channel. Every transition is labeled with information bits u_K and the corresponding coded bits x_N. Let $S(u_K)$ be the set of all transitions $\xi_{ll'}$ which are labeled with information bit u_K=1. Then:

$$P(u_K=+1) = \Sigma\, P(\xi_{ll',k}/y)\ ;\xi_{ll',k}\varepsilon S(u_K) \qquad (3)$$

In the same way we can define a set $S(x_N)$ which includes all transitions $\xi_{ll',k}$ labeled with x_N=+1 and also the corresponding probability $P(x_N=+1/y)$.

Now we will present the method to use $P(x_N=+1/y)$ in subsequent stages. Contrary to the Viterbi algorithm, which work with the logarithm of probabilities, the MAP uses probabilities for decoding computations. The essential point in the method is that we sum up weighted a-posteriori probabilities $P(y_N/x_N^{(i)},x_N^{(j)})$ to obtain the total a-posteriori probability $P(y_N/x_N^{(i)})$.

5. SIMULATED PERFORMANCES

5.1. *BER Improvements with Convolutional Codes*

Let consider the two combined convolutional codes presented in the rows 3 and 4 in Table 1. If the encoder memory is 4, the free distance is 4, respectively 7, with only a slightly reduced rate. In both cases a 256 block interleaver of size 16 x 16 is used on code C_1. The decoder is a multi-stage decoder with iterative decoding. Figure 9 plots the performances obtained after two iterations for each code, denoted with b_i, respectively $b_{i'}$. We can see that the use of the R=1/2 code on the least significant bit reduces substantially the error rate, with only a loss of rate of 0.17 bits/symbol. The diagram shows clearly the superiority of the multilevel codes when compare with Ungerboeck code and uncoded 4 PSK.

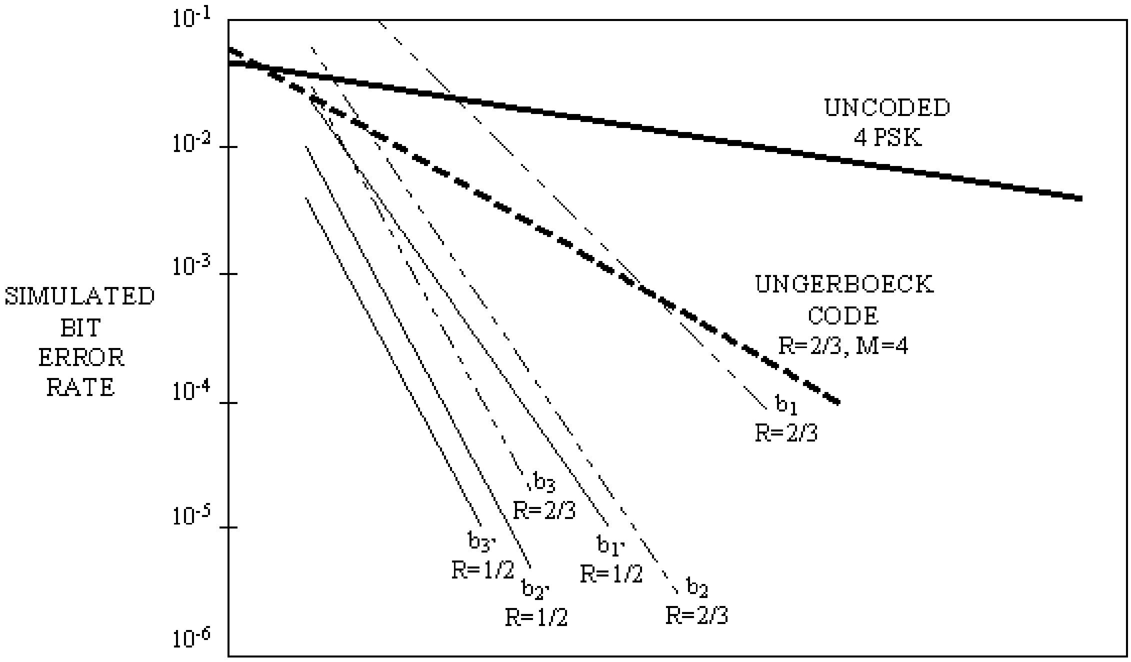

Fig. 9.

5.2. *Unequal Error Protection with Block Codes*

Unequal error protection (UEP) is obtained by providing higher time diversity through code selection for the most important data and by using a non-uniform signal constellation that provides for these data a larger Euclidean distance. Low transmission delay is obtained by matching the code rate and interleaver to the channel conditions. One of the typical application, where by using block coded modulation we can obtain good time diversity, low delay and UEP is the transmission of digital speech. The need of UEP arises because only a fraction of digitized speech data is extremely sensitive to channel errors (important data). Otherwise, in order to obtain a error rate that varies inversely as the SNR raised to a power that is determined by the minimum Hamming distance of the code, it is important that for the significant disturbance that is fading to be independent from symbol to symbol. This can be achieved to through the process of interleaving, by using rectangular arrays with the number of rows at least as large as the average fade duration and the number of columns should be equal to the decoding depth. For a fixed interleaver depth, short block codes have small decoding depth and hence reduce the end-to-end delay which is highly desirable in a speech communication system.

We have simulated the performances of two multilevel block codes. In the first case a 8-dimensional block code of rate 1,75 bits/symbol formed by using a repetition code C_1 of rate 1/4 bits/symbol and two even parity check C_2 and C_3, both of rate 3/4 bits/symbol, which ensure a minimum Hamming distance of 4 in C_1; thus bits encoded by C_1 are subject to a lower error probability than those of the other two codes. In the second case a 16-dimensional block code of rate 1,875 bits/symbol formed by choosing C_1 and C_2 to be the (8,4,4) Hamming code and C_3 to be a (8,7,1) parity check code. Interleaving in both examples is performed over 200 coded symbols. At a BER of 10^{-3}, in the first case we obtain a coding gain over uncoded differential QPSK of about 16 dB for the important data and of about 13 dB for the less important data; in the second case, 3 levels of UEP are obtained , with gains of about 16 dB for class 1, 14 dB for class 2 and 11 dB for class 3.

6. CONCLUSIONS

Coded modulation techniques are under intensive development. In this paper we have demonstrated how multilevels codes with large built in time diversity can be constructed and we have proposed a new method for multistage decoding with some advantages in comparison with the classical decoding. The new method proposes three additional concepts: two decoding steps (reiterated decoding), passing reliability information and interleaving. Only when all these three concepts are applied, a significant improvement in gain (of minimum 1.5 dB) is obtaining. Without two decoding steps, the improvement is very small, confirming the poor performances of multilevel codes, if only one decoding step is performed. At BER values in the range of 10^{-4} - 10^{-5} the best results are obtained with the codes (1/4,4/5,19/20SP) and (1/4,3/4,11/12SP) which ensure an additional gain of about 2 dB compared with 16 states TCM an of about 4 dB compared with uncoded QPSK, with only about 30% increase of decoding complexity. In fact, we propose a kind of "soft-outputs" decoder, which by interleaving ensures those BER values on poor channel too. Unfortunately, the additionally imposed decoding delay might be intolerable for certain applications, like voice transmissions. In this case other combination of block codes can be successfully applied. Another advantage of multilevel coding and multistage decoding is that the modulation rate can be easily varied for a given signal constellation. In fact, although most of the discussions above have centered on 8-PSK constellations, similar results can be obtained for other PSK or QAN signal sets. Additionally the proposed method offers another flexibility, because when using a convolutional code, we can periodically insert training bits on the information bit level. This allows from time to time to transmit a subset of the signal space, i.e. 4-PSK instead of 8-PSK, in order to obtain a lower BER in the neighboring bits and a faster synchronization scheme, with almost no complexity increase.

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