Self-similarity Tests for Internet Traffic

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Abstract: The paper aims to demonstrate the fractal nature of the Internet traffic using self-similarity tests. First, it presents the self-similarity of the Internet traffic and proposes a fractal model of this traffic. Secondly, it offers an analysis of experimental results that prove the similarity in real traffic and a comparison with similar tests performed by simulation on a network model with scale-free topology. Furthermore, the authors demonstrate that the fractal structure of the topology influences the fractal behaviour of the Internet traffic.

Keywords: fractal analysis, self similarity, traffic model, long range dependency, scale-free topology

1. INTRODUCTION

In the first models proposed for the simulation of traffic in large communication networks only Poisson or renewal processes were considered. In either case arrivals were memory less in the Poisson case, or memory less at renewal points, and interarrival intervals were exponentially distributed. The Poisson arrival model and exponentially distributed holding time model allowed analytically and computationally simple Markov chains to be used for modeling.

Internet traffic behaves very differently from such simple Markovian models. Traffic measurements made at the Local Area Networks (LAN) and Wide Area Networks (WAN) suggest that traffic exhibits variability (traditionally called 'burstiness') over multiple time scales (Leland, *et al.,* 1994). The second order properties of the counting process of the observed traffic displayed behaviour that is associated with self-similarity, multi-fractals and/or long range dependence (LRD). This indicates that there is a certain level of dependence in the arrival process. Near-range and long-range dependencies often manifest themselves in a network by causing frequent and irremediable packet losses.

Dependencies and burstiness in traffic hence brought in an enormous amount of attention from researchers. They attempted to develop mathematically-based models that would help explain the nature of the systems exhibiting such phenomena and provide critical insight into the actual mechanisms that led to this behaviour. Models like fractional Brownian motion, chaotic maps etc. were suited to capture the second order self-similar behaviour of traffic (Ribeiro, *et al.,* 2000). Their results were difficult to get and harder to apply, and such models did not provide insight into the actual mechanism of traffic generation. A simpler, more accurate and analytically tractable model that provides more physical insight into why they are meaningful on physical grounds would help the designers produce more effective and efficient solutions.

Some ideas generated in the last decade offer promise towards crafting the model. Anderson and Nielsen (1998) illustrated that continuous parameter Markov chains (cpMc) can model the dependencies in network traffic over multiple time scales. Their model matched the second order properties of the self-similar process closely, but it was not sufficient for accurate prediction of queuing behaviour. Grossglauser and Bolot (1999) discussed both the importance of limiting the view to the finite range of time scales of interest, and the influence of marginal distributions in performance evaluation and prediction problems. Salvador et al. (2003) achieved some degree of success by using a fitting procedure that matched both the marginal distribution and auto covariance of the counting process, but a formal solution was still missing.

The Internet is a prime example of a self-organizing complex system, having grown mostly in the absence of centralized control or direction. In Internet, information is transferred in the form of packets from the sender to the receiver via routers, computers which are specialized to transfer packets to another router "closer" to the receiver. A router decides the route of the packet using only local information obtained from its interaction with neighboring routers, not by following instructions from a centralized server. A router stores packets in its finite queue and processes them sequentially. However, if the queue overflows due to excess demand, the router will discard incoming packets, a situation corresponding to congestion. A number of studies have probed the topology of the Internet and its implications for traffic dynamics (Coates, *et al.,* 2002; Dobrescu, *et al.,* 2004).

To efficiently control and route the traffic on an exponentially expanding Internet, one must not only capture the structure of current Internet, but allow for long-term network design. Until recently all Internet topology generators provided versions of random graphs but the discovery of Faloutsos (1999) that Internet is a scale-free network with a power-law degree distribution, changed this perception. Several contributors found that the Internet flow is strongly localized: most of the traffic takes place on a spanning network connecting a small number of routers which can be classified either as "active centers," which are gathering information, or "databases," which provide information. Experimental evidence for self-similarity in various types of data network traffic is already overwhelming and continues to grow. So far, simulations and analytical studies have shown that it may have a considerable impact on network performance that could not be predicted by the traditional short-range-dependent models. The most serious consequence of self-similar traffic concerns the size of bursts. Within a wide range of time-scales, the burst size is unpredictable, at least with traditional modeling methods.

This is the point from which the authors of this paper assume that the traffic behaviour is strong influenced and depends of the network free-scale structure. We have also demonstrated that the self-similarity confers a fractal aspect for both traffic and topology.

2. MODELING TRAFFIC SELF-SIMILARITY

2.1 General considerations

Using a number of experiments, two main results towards characterizing and quantifying the network traffic processes have been achieved:

First, self-similarity is an adaptability of traffic in networks. Many factors are involved in creating this characteristic. A new view of this self-similar traffic structure is provided. This view is an improvement over the theory used in most current literature, which assumes that the traffic self-similarity is solely based on the heavy-tailed file-size distribution.

Second, the scaling region for traffic self-similarity is divided into two timescale regimes: short-range dependence (SRD) and long-range dependence (LRD). Experimental results show that the network transmission delay (RTT time) separates the two scaling regions. This gives us a physical source of the periodicity in the observed traffic. Also, bandwidth, TCP window size, and packet size have impacts on SRD. The statistical heavy-tailedness (Pareto shape parameter) affects the structure of LRD. In addition, a formula to quantify traffic burstiness is derived from the self-similarity property.

Furthermore, studies of fractal traffic with multifractal analysis have given more interesting and applicable results: 1) At large timescales, increasing bandwidth does not improve throughput (or network performance). The two factors affecting traffic throughput are network delay and TCP window size. On the other hand, more simultaneous connections smooth traffic, which could result in an improvement of network efficiency. 2) At small timescales, traffic burstiness varies. In order to improve network efficiency, we need to control bandwidth, TCP window size, and network delay to reduce traffic burstiness. There are the tradeoffs from each other, but the effect is nonlinear. 3) In general, network traffic processes have a Hölder exponent α ranging between 0.7 and 1.3. Their statistics differ from Poisson processes. To apply this prior knowledge from traffic analysis and to improve network efficiency, a notion of the *efficient bandwidth*, EB, is derived to represent the fractal concentration set. Above that bandwidth, traffic appears bursty and cannot be reduced by multiplexing. But, below it, traffic is congested. An important finding is that the relationship between the bandwidth and the transfer delay is nonlinear.

The past few decades have seen an exponential growth in the amount of data being carried across packet switched networks, and particularly the Internet. During that time, a number of models for the traffic carried across them have also been proposed. Early attempts at modelling network traffic focussed on Markovian models, such as the Markov-Modulated Poisson Process (Zukerman and Rubin, 1994).

In recent analyses of traffic measurements, evidence of non-Markovian effects, such as burstiness across multiple time scales, long range dependence and self similarity; have been observed in a wide variety of traffic sources. As is clearly shown in many references (Erramilli, *et al*., 2000; Norros, 1995; Zwart, *et al*., 2001), the performance of processes exhibiting these properties is radically different from that of the traditional models. Given the evidence of long range dependence and self-similarity in such a wide variety of sources, it is clear that any general model for data traffic must account for these properties. This has led to the development of a number of new models.

2.2 A fractal traffic model

Mandelbrot and Van Ness (1968) introduced an analogy between *self-similar* (SS) processes and *fractal* processes. Referring directly to the incremental process $X_{s,t} = X_t - X_s$, they define stochastic self-similarity as:

$$X_{t0,t0+rt} = r^H X_{t0,\, t0+t}, \qquad \forall t_0,\, t,\, \forall r>0 \qquad (1)$$

Mandelbrot constructs his SS process (*fractional Brownian motion*, fBm) starting with two properties of the Brownian motion (Bm): it has independent increments and it is self-similar with Hurst parameter H= 0.5. Denoting Bm as B(t) and fBm as $B_H(t)$, here is a simplified version of Mandelbrot's definition of the fBm: $B_H(0) = 0$, $H \in [0,1]$ and

$$B_H(t) = \frac{1}{\Gamma(H+0.5)}\left\{\int_{-\infty}^{0}\left[(t-s)^{H-1/2} - (-s)^{H-1/2}\right]dB(s) + \int_{0}^{t}(t-s)^{H-1/2}dB(s)\right\} \qquad (2)$$

An SS process is called a *long-range dependence* (LRD) process if there are constants $\alpha \in (0,1)$ and $C>0$ such that

$$\lim_{k\to\infty}\frac{\rho(k)}{Ck^{-\alpha}} = 1 \qquad (3)$$

where $\rho(k)$ is the autocorrelation of lag k.

When represented in logarithmic coordinates, eq.(3) is called the *correlogram* of the process, and has an asymptote of slope $-\alpha$. It is to note that there are SS processes which are not LRD and, conversely, there are LRD processes which are not SS. However, the fBm with H>0.5 is both SS and LRD type.

In his landmark paper, Leland *et al.* report the discovery of self-similarity in local area network (LAN) traffic, more precisely Ethernet traffic. To be precise, we note that all methods used in the cited paper (and in numerous papers that followed) detect and estimate LRD rather than SS. Indeed, the only "proof" offered for SS *per se* is the visual inspection of the time series at different time-scales. "Self-similarity" (actually LRD) has since been reported in various types of data traffic: LAN, WAN, Variable-Bit-Rate video, SS7 control, HTTP etc. Lack of access to high-speed, high-aggregation links, and lack of devices capable of measuring such links have until recently prevented similar studies from being performed on Internet *backbone* links. In principle, traffic on the backbone could be qualitatively different from the types enumerated above, due to factors such as much higher level of aggregation, traffic conditioning (policing and shaping) performed at the edge, and much larger round-trip-time (RTT) for TCP sessions. Actually, some researches have even claimed that aggregating Internet traffic causes convergence to a Poisson limit. For reasons presented in the next section and based on the remarks that on *shorter time scales*, effects due to the network transport protocols are believed to dominate traffic correlations and on *longer time scales*, non-stationary effects such as diurnal traffic load patterns become significant, we disagree.

3. ESTIMATORS FOR SELF-SIMILARITY

As was shown before, the level of self-similarity in time series is usually expessed by the Hurst parameter H. The range of H is $0.5 \le H \le 1$; a strong self-similarity coresponds to values of H closed to 1.0. Several methods are used to estimate self-similarity in time series (Kelly, 2005); the widespread are: the statistical analysis based on the Rescaled adjusted range R/S, the Variance-Time analysis (or Dispersion for Counts analysis), the analysis based on periodogram (or correlogram) and the Whittle (or Gaussian semiparametric) estimator.

3.1 Rescaled range method

The rescaled range *(R/S)* is a normalized, nondimensional measure proposed by Hurst itself to characterise data variability. For a given set of experimental observations $X = \{X_n, n \in Z^+\}$ with the samples average $\overline{X}(n)$, samples dispersion $S^2(n)$ and samples range $R(n)$, the rescaled adjusted range is (or R/S statistic) is:

$$\frac{R(n)}{S(n)} = \frac{\max(0, \Delta_1, \Delta_2, ..., \Delta_n) - \min(0, \Delta_1, \Delta_2, ..., \Delta_n)}{S(n)} \qquad (4)$$

where $\Delta_k = \sum_{i=1}^{k} X_i - k\overline{X}$ for $k=1,2,..n$ (5)

In the case of many natural phenomena, when $n \to \infty$, then

$$E\left[\frac{R(n)}{S(n)}\right] \sim cn^H \qquad (6)$$

with c a constant positive integer value. Applying logarithms:

$$\log\left\{E\left[\frac{R(n)}{S(n)}\right]\right\} \sim H\log(n) + \log(c) \qquad (7)$$

So, one can estimate the value of H by the slope of a straight line that approximate the graphical plot of $\log\left\{E\left[\frac{R(n)}{S(n)}\right]\right\}$ as a function of $\log(n)$.

3.2 Variance-time analysis

The variance-time analysis is based on the property of slowly decrease of the dispersion of a self-similar aggregate process:

$$Var[X^{(m)}] = \frac{Var[X]}{m^\alpha} \qquad (8)$$

where $\alpha=2-2H$.

Equ. (8) van be written as:

$$\log\{Var[X^{(m)}]\} \sim \log[Var(X)] - \alpha \log(m) \qquad (9)$$

Because $\log\{Var[X^{(m)}]\}$ ia a constant that not depends of m, one can represent $Var(X)$ as a function of m in logarithmic axes. The line that approximate the resulting points has the slope α. The values between (-1; 0) of α represent self similarity .

Like the R/S method, the variance-time analysis is an euristhic method. Both methods can be affected by poor statistics (few realisations of the self similar process). They offers only a raw estimation of H.

3.3 Periodogram method

The estimation based on periodogram is amore accurated method than those based on aggregation, but it necessite to know *a priori* the mathematical parametrised model of the process. The periodogram is known also as the function of intensity $I_N(\omega)$ and represents the estimated spectral density of the stocastic process $X(t)$ (defined at discrete time intervals) and given, or the time period N by the equation:

$$I_N(\varpi) = \frac{1}{2\pi N}\left|\sum_{k=1}^{N} X_k e^{jk\varpi}\right|^2 \qquad (10)$$

where ω is the frecquency and X_k is the time series. When $\varpi \to 0$ the periodogram should be:

$$I_N(\varpi) \sim |\varpi|^{1-2H} \qquad (11)$$

The main drawback of this method is the need of a high processing capacity.

3.4 Whittle estimator

Whittle estimator derives from the periodogram, but uses a non graphic representation of the probability. Considering a self similar process with a *fBm* type model and its spectral density *S(ω,H)*, the value of the parameter *H* minimize the so named Whittle expression:

$$\int_{-\pi}^{\pi} \frac{I_N(\varpi)}{S(\varpi,H)} d\varpi \qquad (12)$$

Both periodogram and Whittle estimator are derived from the *Maximum likelihood estimation (MLE)* and offer good statistical properties, but can give errors if the model of the spectral density is false.

4. EXPERIMENTAL RESULTS

4.1 Analysis of real traffic

Using We present a case study for the analysis of self similarity by computing the Hurst exponent. In the experiments were calculated three graphical estimators: the rescaled adjusted range (R/S), the variance-time diagram (V-T) and the periodogram (Pg.). For the representation we have used the toolbox of MATLAB, but also a lot of original predefined functions optimized for numerical computing.

The data from the real traffic were captured with the utilitary *tcpdump*, for an interval of about 5 hours in an active period of the day (between 11:38 – 16:47 în October 12, 2005 on a AccessNet network in Bucharest (Ulrich, 2008). The network has about 20 active stations interconnected through a router. The captured traffic was splitted in 5 sequences of one hour each. Each sequence contains 36000 observations, and each observation represents the number of packets send in the network at every 100 miliseconds. For the capture of packets the application *tcpdump* is attached în the network socket in the Linux kernel. The application allows to note the moment of packet arrival (in kernel), the addresses and port numbers of the source and of the destination, the dimension of the frame and the type of the traffic (tcp, udp, icmp etc). In table 1 are presented the obtained results.

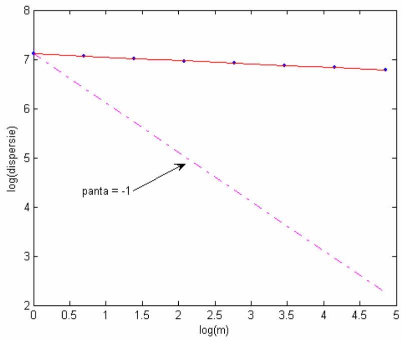
Table 1. Results of self-similarity tests.

Data set	Number of packets	Estimated Hurst parameter		
		R/S	V-T	Pg.
Trace1	775.077	0,98	0,97	0,99
Trace2	547.373	0,98	0,92	0,90
Trace3	197.795	0,83	0,77	0,74
Trace4	230.468	0,82	0,82	0,84
Trace5	319.905	0,97	0,93	0,99

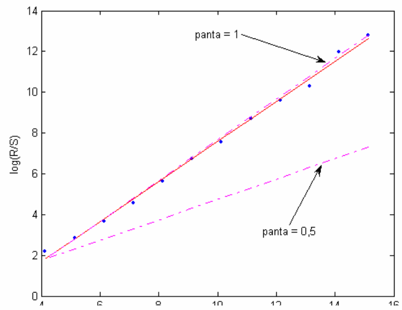
One can observe that for the first and the last sets, which correspond to busy intervals (11,40-12,40 and 15,40-16,40) the estimated Hurst parameter is very close to 1, while for the intervals when usually is a lunch pause, the values are significant little, but not under 0.74. The natural conclusion is that the self-similarity of the traffic increases when the degree of activity in the network is greater.

We present in the following some graphical results. We have choose only 6 diagrams, coresponding to data sets *Trace1* şi *Trace4* (3 diagrams for each set – fig.1 a, b, c for *Trace1* and fig.2 a, b, c for *Trace4*). Each diagram is represented in logarithmic coordinates and has as reference a line with slope -1 representing the value 0,5 of the Hurst parameter.

a.



b.



c.

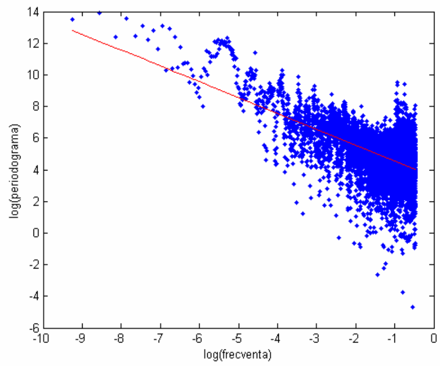


Fig. 1. Plots of graphical estimators for data set *Trace1*: a)R/S; b)V-T; c) Pg.

It is to note that the estimation of the Hurst parameter was made only for the time series realised with packets arrived in a time unit of 100 ms. Quite we have not presented the case when the times series are formed from the number of bytes arrived in a time unit, we can affirm that the results are similar.

Based on these experimental results, one can conclude that TCP maintains the long-range dependance from the Application layer to the Link layer. TCP uses an algorithm to control the end-to-end congestiopn by adapting the transmission rate at the network current status. If the network presentsw fluctuations at large time scales TCP will react in the same manner. TCP can acquire the self-similarity from a basic flow with self-similar traffic and maintain it on the path from source to destination. Combined TCP flows can spread the self-similarity on the whole network, which is a contribution to the global scaling, but maintaining in the same time the local scaling when this is already strong.

a.

b.

c.

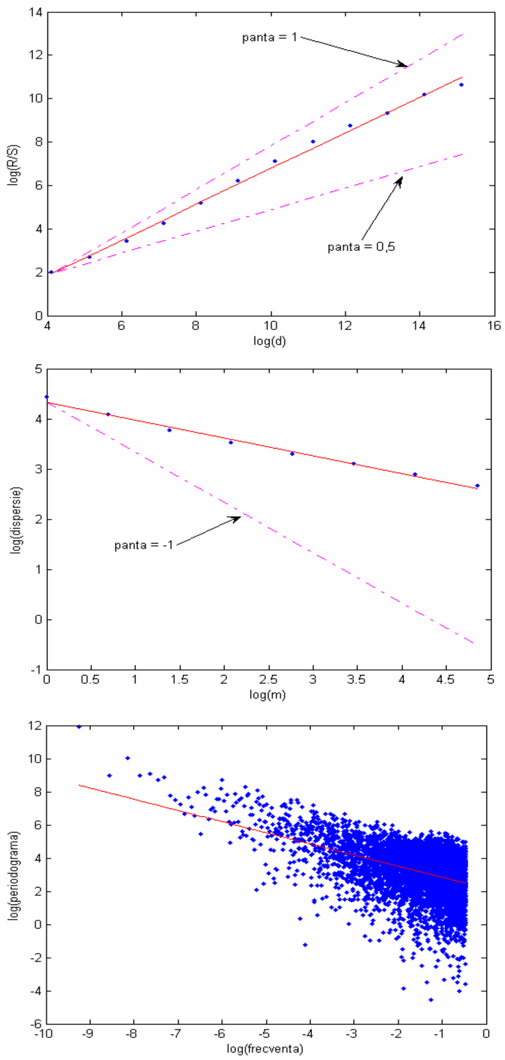


Fig. 2. Plots of graphical estimators for data set *Trace4*: a)R/S; b)V-T; c) Pg.

4.2 Analysis of the traffic model

a. Generation of the scale free topology

All real-life networks are finite so can be characterized by the degree of connectivity of the associated graph. But the degree distribution does not characterize the graph or ensemble in full. There are other quantities, such as the degree-degree correlation (between connected sites), the spatial correlations, etc. Several models have been presented for the evolution of scale-free networks, each of which may lead to a different ensemble. The first suggestion was the *preferential attachment* model by Barabasi and Albert, which came to be known as the “Barabasi-Albert (BA)” model. Several variants have been suggested to this model. One of them known as the “Molloy-Reed construction”, which ignores the evolution and assumes only the degree distribution and no correlations between nodes, will be considered in the following. Thus, the site reached by following a link is independent of the origin. This means that a new node will more probably attach to those nodes that are already very well connected, i.e. they have a large number of connections with other nodes from the network. Poor connected nodes, on the other hand, have smaller chances of getting new connections.

Besides following the repartition law mentioned above, some other restrictions (for example those related to cycles and long chains) had to be applied in order to make the generated model more realistic and similar to the Internet. Another obvious restriction is the lack of isolated components. A more subtle restriction is related to the TTL (Time-to-living) which is a way to avoid routing loops in a real Internet. This translates in a restriction for our topology – there can be no more that 30 nodes to get from any node to any other node. Another subtle restriction is that the generated network will also have redundant paths, multiple possible routes between nodes. In other words, the Internet model topology should not "look" like a tree, but should rather have numerous cycles. One more restriction is that we try to avoid long-line type of scale-free networks – a succession of several interconnected nodes – structure that does not have a real-life Internet equivalent, so our algorithm makes sure such a model is not generated.

The fractal nature of both traffic and topology of an Internet network and their reciprocal influence was tested considering simulated web traffic on an scale-free network model introduced by Dobrescu *et al.* (2008). The algorithm used for the generation of the scale-free network topology is generating networks with a cyclical degree that can be controlled; in our case, approximately 4% of the added nodes form a cycle. The generated topology consists of three types of nodes:

Routers, defined as nodes with one or several links. Routers do not initiate traffic and do not accept connections. Routers can be one of the following types: routers that connect primarily customers, routers that connect primarily servers and routers that connect primarily other routers. Routers that connect primarily customers have hundreds or thousands of type one connections (leaf nodes) and a reduced number of connections to other routers. Routers that connect primarily servers have a reduced number of connections to servers in the order of tenths and reduced number (2 or 3) connections to other routers. Routers that connect primarily routers have a number in the order of tenths of connections to other routers and do not have connections to neither servers nor customers.

Servers are defined as nodes with one connection but sometimes could have two or even three connections. Servers only accept traffic connections but do not initiate traffic.

Customers (end-users) defined as nodes that have only one connection, very seldom two connections. Customers initiate traffic connections towards servers at random moments but usually in a time succession. We chose a 20:80 customers to servers ratio.

We designed and implemented an algorithm that generates those subsets of the scale-free networks that are close to a real computer network such as the Internet. Our application is able to handle very large collections of nodes, to control the generation of network cycles, and the number of isolated nodes. The algorithm starts with a manually created network of several nodes, then using preferential attachment and growth algorithms, new nodes are added. We introduced an original component, the computation in advance of the number of nodes on each degree-level. The preferential attachment rule is followed by obeying to the restriction of having the optimal number of nodes per degree. In the simulation the whole network was splitted in subnetworks with at most 40 nodes. The parameter for simulation of such a subset were the total number of nodes/subnetwork is N=40, the number of the initial nodes is $m_0=5$ and a value $m=2$ (i.e. at each incremental step one add two links in order to maintain a non-zero grouping coefficient). Figure 3 depicts the network topology.

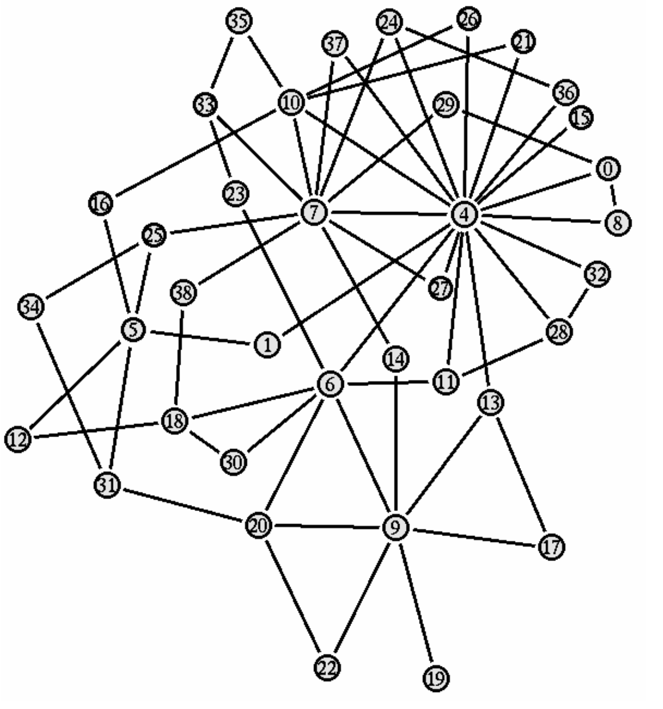


Fig. 3. Scale-free topology used for self-similar traffic simulation

b. Simulation of self-similar traffic

For the simulation of self-similar traffic it was used a superposition of ON-OFF sources after a Pareto distribution, with $1 \leq \alpha \leq 2$. The Pareto distribution has two parameters, the parameter of shape α and the low-cutting parameter β. The Cumulate Distribution Function (CDF) Pareto is $F(x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha}$, and the function of the probability density is $f(x) = \frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1}$ for $x>\beta$ and $\alpha>0$. Moreover, the parameter α is relationated with the Hurst parameter H as $H = \frac{3-\alpha}{2}$. In experiments we have used 32 associated traffic sources randomly associated to TCP traffic agents. The value of the shape coefficient α was 1,4 which lead to an expected value of $H=0,8$. In fig.4 are shown the diagrams of the aggregate number of packets on three time units: 1 second, 100 milliseconds and 10 milliseconds. The red color represents the zoom. The strong burstiness of the traffic in all three di... elf-similarity phenom...

a.

b.

c.

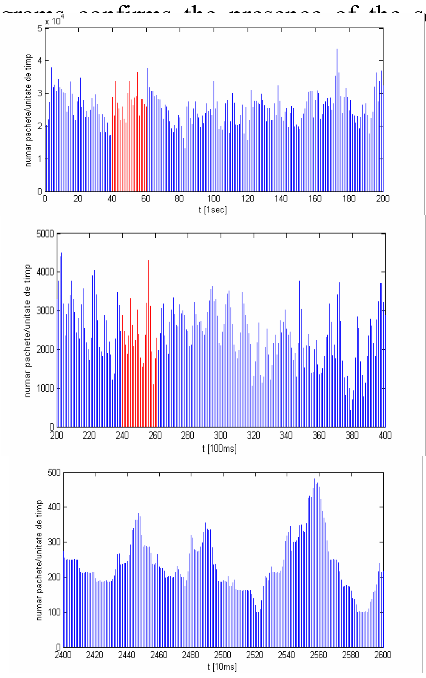


Fig. 4. Number of packets/s for three time scales: 1, 0.1 and 0.01 second.

The Hurst parameter for a given sequence was calculated using a number of three different estimation methods: the diagram of the rescaled domain, the diagram variance-time and the periodogram. In theory, the expected value of Hurst is 0.8, but the real results are, in the order in which the methods were presented: 0.8115, 0.8761 and 0.8825. Quite the coefficients for the last two methods are over evaluated, one can conclude that the tested model presents statistical self-similarity.

5. CONCLUSIONS

The advantage of the model proposed here is its flexibility: it offers an universally acceptable skeleton for potential Internet models, on which one can build features that could lead to further improvements. The model offers a realistic starting point for a general class of network topologies that combine the scale-free structure with a precise spatial layout.

Although the traffic processes in high-speed Internet links exhibit *asymptotic* self-similarity, their correlation structure at short time-scales makes their modeling as *exact* self-similar processes (like the fractional Brownian motion) inaccurate. Based on simulations made on the SFN based Internet model we conclude that Internet traffic retains its self-similar properties even under high aggregation.

The experiments have let to the following results: 1) self-similarity is an adaptability of traffic in the network and is not based only on the heavy-tailed file-size distribution; 2) the scaling region on traffic self-similarity is divided into two timescale regimes: short range dependencies (SRD), determined by bandwidth, TCP window size and packet size and long range dependencies (LRD), determined by the statistical heavy-tails; 3) in LRD, increasing the bandwidth does not improve throughput (or network performance; 4) there is a significant advantage in using fractal analysis methods to solve the problem of anomaly detection.

Because an accurate estimation of the Hurst parameter offers also a valuable abnormality indicator obtained for the bursty variables, for further work we expect to obtain a procedure to predict network traffic anomalies. Thus, by improving the capability of predicting impending network failures, it is possible to reduce network downtime and increase network reliability.

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