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SIMULATION OF HYBRID CONTROL FOR AN EXPERIMENTAL AIRCRAFT TWO-RATE MODEL

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Abstract: *This paper describes a two-rate stochastic control system as state-space (SS) type decentralized models of multi-input/multi-output (MIMO) stochastic subsystems with the "fast" linear-quadratic-Gaussian (LQG) regulator, "slow" neural network (NN) regulator and fuzzy logic (FL) control structure.*

An illustrative example – LQG, NN and FL hybrid control of two-rate decentralized stochastic model of a tracking system for an experimental aircraft – was carried out using the proposed two-rate SS decentralization technique. This example demonstrates that this research technique results in simplified low-order MIMO autonomous subsystems with various discretization periods and with various speeds of actuation, and shows the quality of the proposed technique.

The simulation results with use of software package Simulink demonstrate that this research technique would work for real-time MIMO stochastic systems.

Keywords: *simulation, multirate, LQG control, neural networks, fuzzy logic.*

1. INTRODUCTION

Many physical and engineering problems are appropriately described by MIMO dynamical models. The computational efforts required for analysis, design and control of such models are quite excessive. It is therefore considered desirable to develop decentralized multirate reduced-order submodels that approximate the dynamic behaviour of MIMO systems.

Sandell et al. (1978) reviewed the previous research and divided it into four categories: Model Simplification, Interconnected Systems, Decentralized Control, and Hierarchical Control. In opinion of these authors, we do not believe that the existing mathematical tools are powerful enough to define a preferable structure for decentralized and/or hierarchical control. Sandell et al. (1978) claimed that with respect to designing decentralized controllers for many physical large-scale systems a good combination of engineering judgement and analysis can be used to define in a reasonable way a special structure for the dynamic system.

Kokotovic et al. (1972, 1976) showed that the singular perturbation theory for difference equations involves a list of ingredients-order reduction, separation of time scales, and boundary layer phenomena. Sufficient conditions are given under which the solution of the original problem tends to the solution of a low-order problem.

The lifting method (Li et al., 1999) is used to analyze the multirate system in the SS framework. It is clear that the fast sampled model can be easily identified from the input excitation of an open-loop process running under a multirate sampling scheme with slow output sampling and fast control.

Tornero et al. (1999) studied the multi-rate controller, which updates the controller output faster than the measurement sampling frequency by a factor of N. The controlled system is continuous, and the discrete time controller is obtained as its zero order hold equivalent. The simplest case of $N = 2$ has been examined in some details.

In Astrov et al. (2000), a method for describing a two-rate continuous-time stochastic control system in SS is presented. The block diagram showing functioning of all subsystems with the reference input vector and the optimal linear-quadratic regulators is offered. Simulation and animation results for a control system of a tracked vehicle with the help of MATLAB/Simulink show the quality of the proposed method.

This paper expands the basic ideas presented in (Astrov et al., 2000) and describes a two-rate stochastic control system as SS type discrete-time models of MIMO decentralized stochastic autonomous subsystems with LQG, NN and FL hybrid control.

The goal of this paper is to show the applicability of the proposed two-rate LQG, NN and FL hybrid control technique to the stochastic model of a tracking system for an experimental aircraft.

2. STATE EQUATIONS FOR TWO-RATE STOCHASTIC SYSTEMS

A SS description of stochastic discrete-time MIMO system is given by

$$x[(t+1)\Delta] = Fx(t\Delta) + Gu(t\Delta) + v(t\Delta) \qquad (1)$$

$$y(t\Delta) = Hx(t\Delta) + w(t\Delta), t = 0,1,\ldots \qquad (2)$$

where

$x(t\Delta) \in R^n$, $u(t\Delta) \in R^m$, $y(t\Delta) \in R^p$, $v(t\Delta) \in R^n$, $w(t\Delta) \in R^p$

are the state, control input, output, noise of excitation of state and noise of measurement vectors, respectively.

Suppose that we consider the linear transformation $z(t\Delta) = Qx(t\Delta)$, where Q is a nonsingular matrix. It is easy to see that (1)-(2) are transformed into the equations

$$z[(t+1)\Delta] = J_d z(t\Delta) + \tilde{G}u(t\Delta) + \tilde{T}v(t\Delta) \qquad (3)$$

$$y(t\Delta) = \tilde{H}z(t\Delta) + w(t\Delta) \qquad (4)$$

where

$J_d = QFQ^{-1}, \tilde{G} = QG, \tilde{T} = Q, \tilde{H} = HQ^{-1}$.

The SS equations (3)-(4) may be written in terms of partitioned submatrices as follows

$$z_1[(t+1)\Delta] = \Lambda_1 z_1(t\Delta) + G_1 u(t\Delta) + T_1 v(t\Delta) \qquad (5)$$

$$z_2[(t+1)\Delta] = \Lambda_2 z_2(t\Delta) + G_2 u(t\Delta) + T_2 v(t\Delta) \qquad (6)$$

$$y(t\Delta) = H_1 z_1(t\Delta) + H_2 z_2(t\Delta) + w(t\Delta), t = 0,1,\ldots \qquad (7)$$

where

eigenspectrum of matrix Λ_1 consists of small eigenvalues, and eigenspectrum of matrix Λ_2 consists of large eigenvalues,

$$J_d = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}, \ \tilde{G} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \ \tilde{T} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix},$$

$$\tilde{H} = \begin{bmatrix} H_1 & H_2 \end{bmatrix}, \ z(t\Delta) = \begin{bmatrix} z_1(t\Delta) \\ z_2(t\Delta) \end{bmatrix}.$$

Definition 1: Vectorial variables are said to be "slow" variables if these variables are slowly varied constants between the sampling periods. Otherwise, the vectorial variables are said to be "fast" variables if these variables are strongly varied variables.

Assume that the relation of discretization periods for the "fast" and "slow" subsystems is a real number N:

$$\frac{\Delta_s}{\Delta_f} = N \qquad (8)$$

Let us consider the first time interval $0 \langle \tau_f \langle \tau_{fs}$.

According to **Definition 1**, it is possible to consider the variable z_2 as a "slow" function of time on this interval. Hence, assuming that $z_2((t+1)\Delta_f) \approx z_2(t\Delta_f)$ on this interval, from (6), we find

$$z_2(t\Delta_f) = (I - \Lambda_2)^{-1} G_2 u(t\Delta_f) + \\ + (I - \Lambda_2)^{-1} T_2 v(t\Delta_f) \quad (9)$$

From (5), (7) and (9), we find that the SS equations for the "fast" subsystem may be written as

$$z_f[(t+1)\Delta_f] = F_f z_f(t\Delta_f) + \\ + G_f u_f(t\Delta_f) + T_f v_f(t\Delta_f) \quad (10)$$

$$y_f(t\Delta_f) = H_f z_f(t\Delta_f) + E_f u_f(t\Delta_f) + \\ + w_f(t\Delta_f), \quad t = 0,1,..., \quad (11)$$

where

$$F_f = \Lambda_1, G_f = G_1, T_f = T_1, H_f = H_1,$$
$$E_f = H_2(I - \Lambda_2)^{-1} G_2, \ y_f(t\Delta_f) = y(t\Delta_f),$$
$$V_f = H_2(I - \Lambda_2)^{-1} T_2, \ z_f(t\Delta_f) = z_1(t\Delta_f),$$
$$u_f(t\Delta_f) = u(t\Delta_f), \ v_f(t\Delta_f) = v(t\Delta_f),$$
$$w_f(t\Delta_f) = w(t\Delta_f) + V_f v(t\Delta_f)$$

We shall proceed now to consideration of the second time interval $\tau_s \geq \tau_{fs}$. According to **Definition 1**, it is possible to consider the variable z_1 as a "fast" function of time, achieving on this interval a steady meaning. Hence, assuming that $z_1((k+1)\Delta_s) \approx z_1(k\Delta_s)$ on this interval, from (5), we find

$$z_1(k\Delta_s) = (I - \Lambda_1)^{-1} G_1 u(k\Delta_s) + \\ + (I - \Lambda_1)^{-1} T_1 v(k\Delta_s) \quad (12)$$

From (6), (7) and (12), we find that the SS equations for the "slow" subsystem may be written as

$$z_s[(k+1)\Delta_s] = F_s z_s(k\Delta_s) + \\ + G_s u_s(k\Delta_s) + T_s v_s(k\Delta_s) \quad (13)$$

$$y_s(k\Delta_s) = H_s z_s(k\Delta_s) + E_s u_s(k\Delta_s) + \\ + w_s(k\Delta_s), \quad k = 0,1,\ldots \quad (14)$$

where

$$F_s = \Lambda_2, \ G_s = G_2, \ T_s = T_2, \ H_s = H_2,$$
$$E_s = H_1(I - \Lambda_1)^{-1} G_1, \ V_s = H_1(I - \Lambda_1)^{-1} T_1,$$
$$z_s(k\Delta_s) = z_2(k\Delta_s), \ u_s(k\Delta_s) = u(k\Delta_s),$$
$$v_s(k\Delta_s) = v(k\Delta_s), \ y_s(k\Delta_s) = y(k\Delta_s),$$
$$w_s(k\Delta_s) = w(k\Delta_s) + V_s v(k\Delta_s).$$

3. EXAMPLE

In this Section, the proposed two-rate decentralization technique is tested through various simulations.

The model of a multivariable tracking system for an experimental aircraft (Vaccaro, 1995) represents the longitudinal dynamics as perturbed from steady-state straight and level trim conditions, with a nominal velocity of 881 ft/sec.

The state variables are defined as follows:

$x_1 =$ roll angle,

$x_2 =$ perturbation velocity in x direction,

$x_3 =$ velocity in z direction,

$x_4 =$ roll rate.

The system inputs are:

$u_1 =$ canard angle,

$u_2 =$ flaperon angle,

$u_3 =$ thrust (pounds).

The system outputs are:

$y_1 =$ pitch angle,

$y_2 =$ flight-path angle,

$y_3 =$ aircraft velocity (ft/sec).

All angles are measured in degrees. For this example a sampling period $\Delta = 0.02$ seconds was chosen.

The matrices (F,G,H) of the SS model (1)-(2) are:

$$F = \begin{bmatrix} 1 & -6.3539\times10^{-7} & 5.9878\times10^{-6} & 0.02 \\ -0.6415 & 0.9984 & 0.0008 & -0.3914 \\ -0.0134 & -0.0017 & 0.9723 & 17.6489 \\ 1.6175\times10^{-5} & -0.0001 & 0.0006 & 1.0003 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.006 & -0.0007 & -5.6116\times10^{-8} \\ 0.2923 & -0.0413 & 0.0025 \\ 4.6425 & -2.9267 & -0.0001 \\ 0.6032 & -0.0751 & -5.6469\times10^{-6} \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2.45\times10^{-5} & -0.0011 & 0 \\ 0 & 1 & 0.0219 & 0 \end{bmatrix}$$

The "fast"-subsystem (10)-(11) matrices are

$$F_f = [0.8827],$$
$$G_f = [48.7861 \quad -4.7429 \quad -0.0003],$$

$$H_f = \begin{bmatrix} -0.0008 \\ 0.0003 \\ -0.0022 \end{bmatrix},$$

$$T_f = \begin{bmatrix} 0.1354 & 0.0388 & -0.5671 & 85.219 \end{bmatrix},$$

$$E_f = \begin{bmatrix} -17.642 & 3.4292 & 0.0041 \\ -17.6525 & 3.4665 & 0.0041 \\ 7123.2 & -1399.4 & -0.0974 \end{bmatrix},$$

$$V_f = \begin{bmatrix} 493.5965 & 1.5694 & -0.5275 & -30.8832 \\ 511.8909 & 1.5695 & -0.5428 & -30.9658 \\ -192820 & -4.2873 & 218.017 & 12061 \end{bmatrix}$$

The "slow" subsystem (13)-(14) is specified by the following SS matrices

$$F_s = \begin{bmatrix} 0.9958 & 0.0079 & 0 \\ -0.0079 & 0.9958 & 0 \\ 0 & 0 & 1.5251 \end{bmatrix},$$

$$G_s = \begin{bmatrix} -69.7103 & 13.7167 & 0.0074 \\ 4.4142 & -0.8322 & 0.0122 \\ 311.3267 & -44.6634 & -0.0041 \end{bmatrix}$$

$$T_s = \begin{bmatrix} 399.8225 & 0.5358 & -0.4392 & -24.5065 \\ 2.9812 & 1.0068 & 0.0211 & 0.7413 \\ 0.4286 & -0.067 & 0.4303 & 85.1273 \end{bmatrix},$$

$$H_s = \begin{bmatrix} 0.0025 & -0.0012 & 0.0015 \\ 0.0025 & -0.0013 & 0.0004 \\ -0.0074 & 0.9967 & -0.0086 \end{bmatrix},$$

$$E_s = \begin{bmatrix} -0.3385 & 0.0329 & 2.1354\times10^{-6} \\ 0.1276 & -0.0124 & -8.051\times10^{-7} \\ -0.9279 & 0.0902 & 5.8541\times10^{-6} \end{bmatrix},$$

$$V_s = \begin{bmatrix} -0.0002 & -0.0001 & 0.001 & -0.1516 \\ 0.0001 & 2.5995\times10^{-5} & -0.0004 & 0.0572 \\ -0.0007 & -0.0002 & 0.0028 & -0.4156 \end{bmatrix}$$

Simulation results for the case of the offered block scheme (see Fig. 1) are given in Figs. 5-7.

The LQG regulator for "fast" subsystem, and NN regulator with Many ADAptive LINear Elements (MADALINE) for "slow" subsystem with three neurons are shown in Figs. 2-3.

A FL control structure (see Fig. 4), which consists of the FL controller and switches, is designed to control the appropriate outputs of control subsystems to achieve the two-rate control goal. This FL controller uses Mamdani-style fuzzy inference with the nine rules base.

It can be seen that $N = 4.9$ from (8) (where $\Delta_f = 0.02, \Delta_s = 0.098$ seconds) is chosen as a relation of discretization periods for the "fast" and "slow" subsystems.

Note that good results were obtained using only simple LQG and NN regulators and single FL controller in the sense of a minimum rule base.

These results support the theoretical predictions well and demonstrate that this research technique would work for real systems.

4. CONCLUSIONS

In this paper a two-rate SS decentralization technique is proposed. The obtained subsystems not only have reduced dimensions of SS matrices and various discretization periods (small and large), but also various speeds of actuation (fast and long response times) and time sharing property. Further analysis of the decentralized control subsystems can be produced separately with the help of modern computer-aided control analysis software.

In example of this paper, the centralized stochastic system of fourth order (the SS type of MIMO stochastic model of the longitudinal dynamics for a certain experimental aircraft) is partitioned on the "fast" subsystem of first order and on the "slow" subsystem of third order. This example shows the computational procedure and the applicability of this decentralization technique for real-time systems.

The block diagram of functioning of all decentralized subsystems with the LQG, NN and FL control structures is offered. The discrete-time process simulations were carried out in a MATLAB/Simulink environment.

Although many of the details inevitably relate with this particular system, there is sufficient generality for this research technique to be applied to other MIMO discrete-time stochastic control systems.

The contribution of the paper is twofold: to develop new decentralization schemes appropriate for real-time multirate LQG, NN and FL hybrid control applications, and to present the results of two-rate LQG, NN and FL hybrid control for any chosen SS type of MIMO stochastic model in simulation form.

A significant advantage of this approach is the relative simplicity that may be easily used for more complex systems than the model of a tracking system for an experimental aircraft, and may be improved considerably with the use of more powerful computers to be ideally suited to real-time control applications.

Potential applications are evaluated by control exploration and by computer simulations, using SS type of MIMO discrete-time stochastic models with multirate phenomena.

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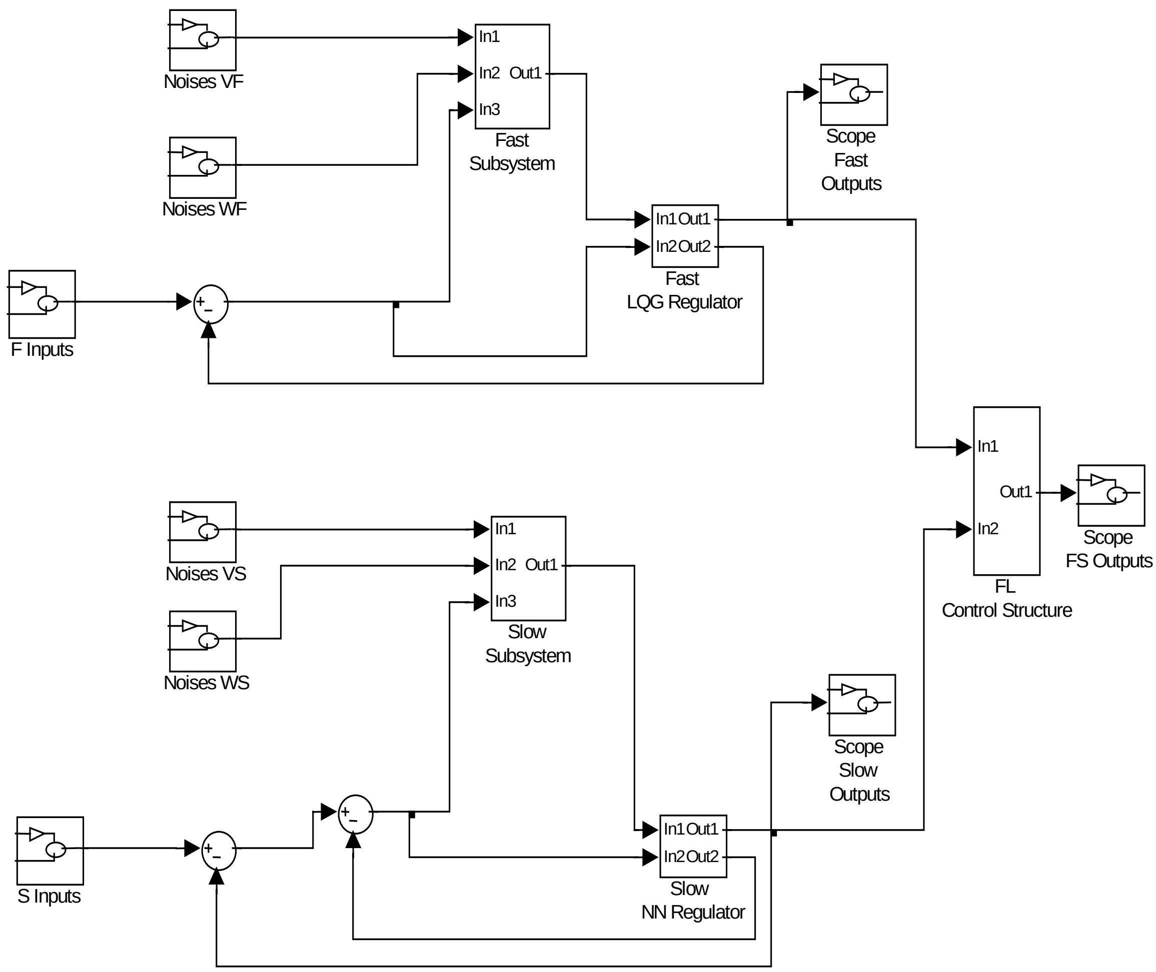

Fig. 1. The block scheme of a two-rate decentralized control system.

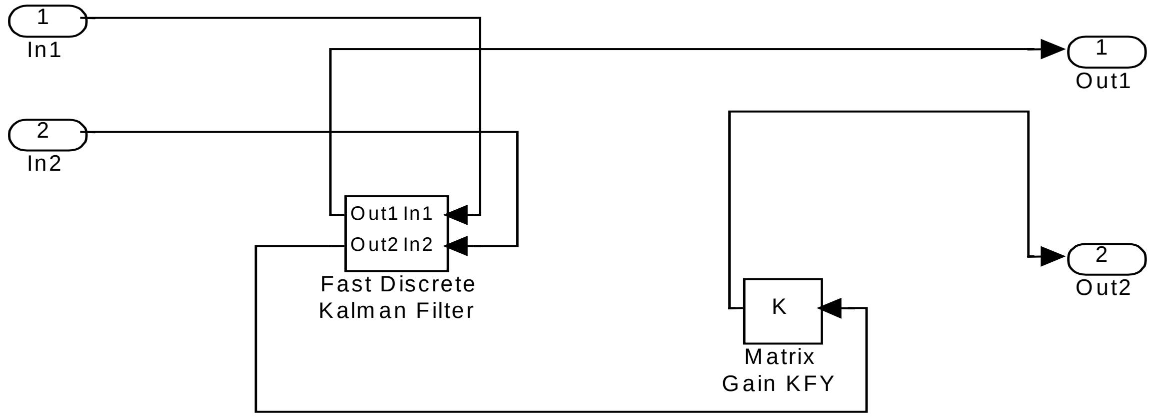

Fig. 2. The block scheme of the fast LQG regulator.

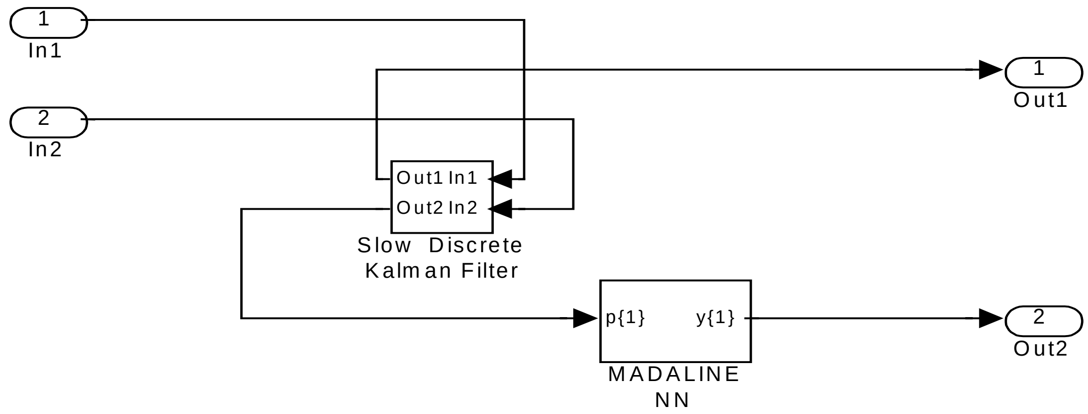

Fig. 3. The block scheme of the slow NN regulator.

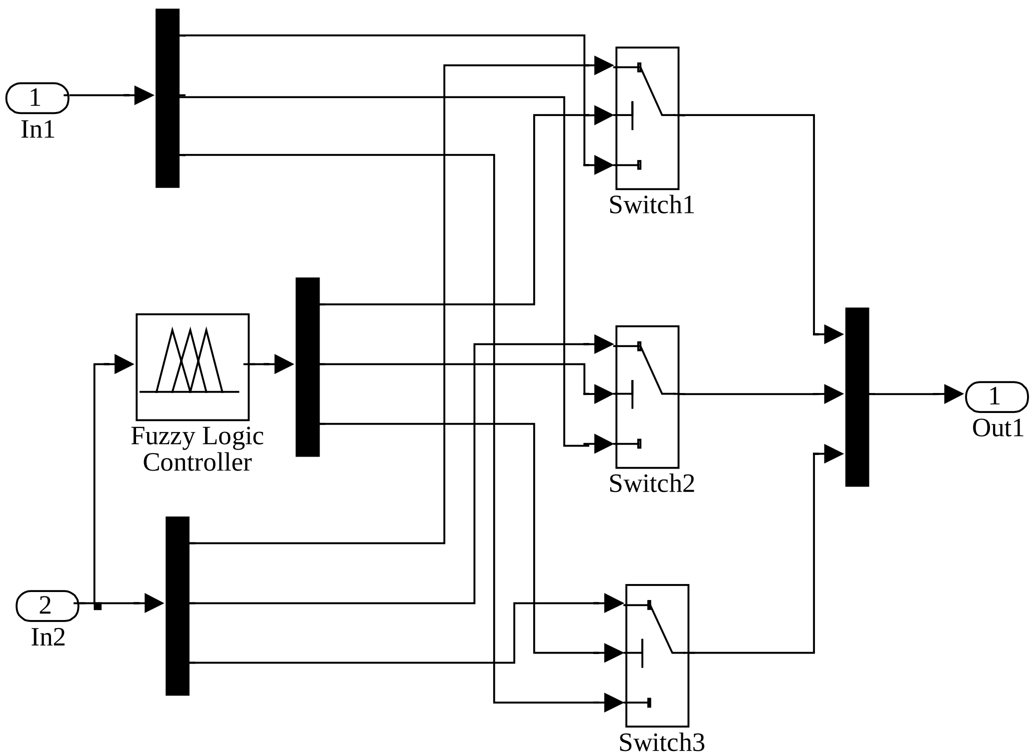

Fig. 4. The block scheme of the FL control structure.

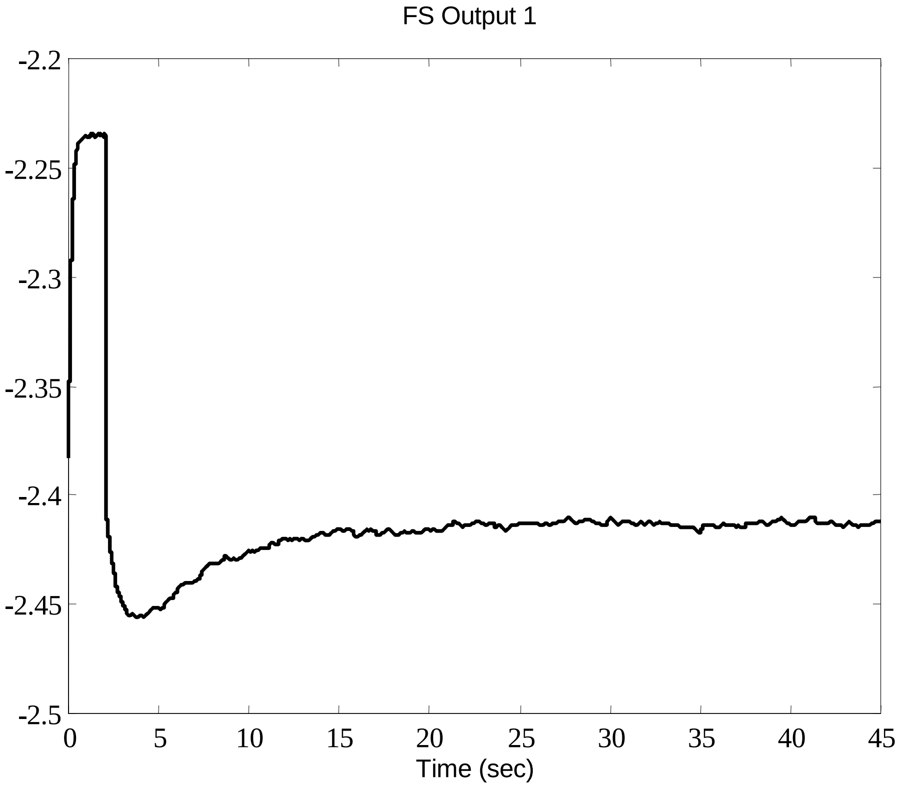

Fig. 5. Response to the output y_1.

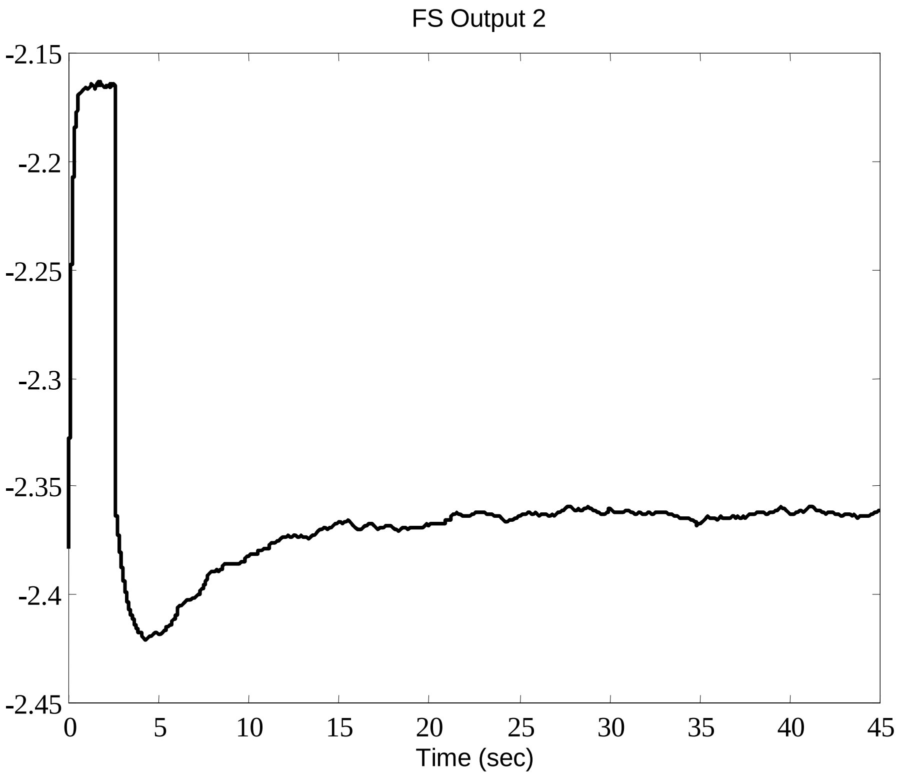

Fig. 6. Response to the output y_2.

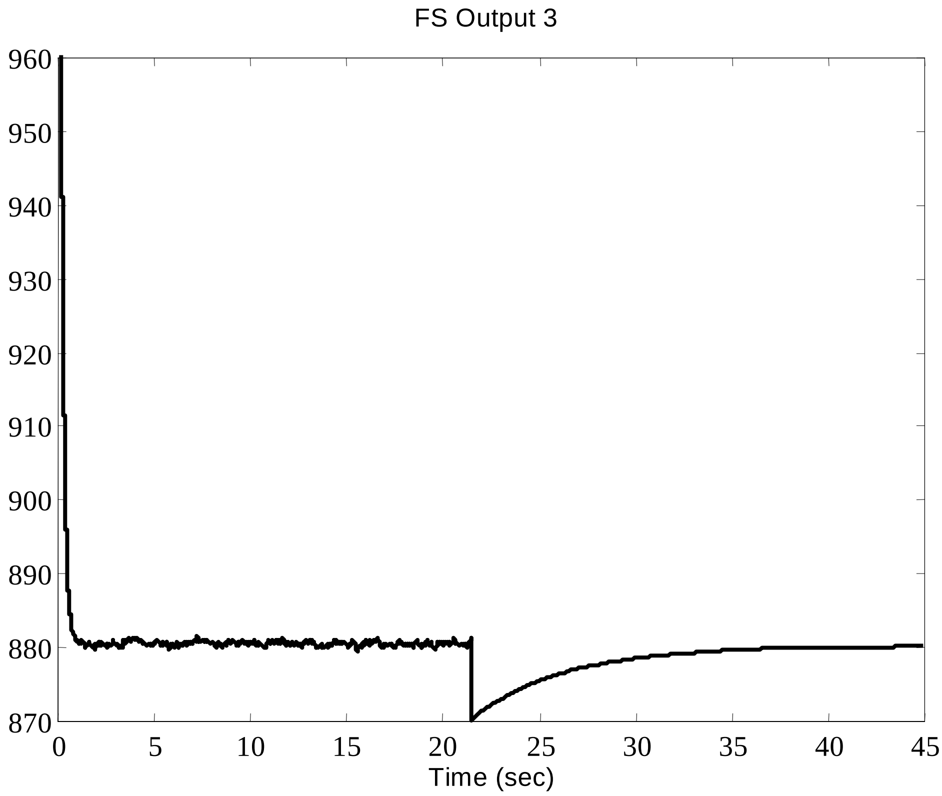

Fig. 7. Response to the output y_3.