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SLIDING MODE FUZZY CONTROL FOR THE STEP MOTORS

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Abstract. *This paper proposed a sliding mode fuzzy control – SMFC, for the two-phase step motors used in high torque and low speed position control applications. The design procedure is based on a sliding mode controller with boundary layer (SMC-BL). The procedure presented concentrates only on the fuzzy part since the rest part can be adopted from the SMC. This structure of controller (SMFC) generates a non-linear transfer characteristic in contrast with that of SMC with BL. The controller drives the step motor phase windings using field-orientation control rather than stepping mode. This method drives the motor at maximum theoretical performance.*

Keywords: *step motor, field-oriented control, sliding mode fuzzy controller*

1. INTRODUCTION

Step motors used in industrial applications (robotic systems or industrial machinery) can exhibit undesirable behavior such as stepping resonance and skipped steps. This is due to the drive method and is not due to the motor itself. By field-oriented control (FOC), the motor can improve its dynamic performances. The motor becomes a dynamic AC drive properly for applications demanding high torque and low speed position control.

Sliding mode control with boundary layer (SMC-BL) can offer a number of attractive properties for industrial applications for the step motors (insensitivity to the parameter variations and external disturbances). Once the controlled system enter in the *sliding regime*, the dynamics of the system is determined by the choice of sliding hyper-planes and are independent of uncertainties and external disturbances. The control law consists of the following terms: *compensation term, filter term, feed-forward term and control term*. To improve the system response of the SMC-BL, the control term was replaced with a *fuzzy term*. This new *sliding mode fuzzy controller* offer a non-linear transfer characteristic in contrast to that of the SMC – BL.

$$\begin{cases} \dfrac{d\theta_m}{dt} = \omega_m. \\ \dfrac{d\omega_m}{dt} = \dfrac{1}{J_m} \cdot (k_m \cdot (\Psi_M \cdot i_{sq\theta} + (L_{sd} - L_{sq}) \cdot i_{sd\theta} \cdot i_{sq\theta}) - B_m \cdot \omega_m - m_r); \\ \dfrac{di_{sd\theta}}{dt} = \dfrac{1}{L_{sd}} (u_{sd\theta} - R \cdot i_{sd\theta} + z_r \cdot \omega_m \cdot L_{sq} \cdot i_{sq\theta}); \\ \dfrac{di_{sq\theta}}{dt} = \dfrac{1}{L_{sq}} (u_{sq\theta} - R \cdot i_{sq\theta} - \omega \cdot L_{sd} \cdot i_{sd\theta} + z_r \cdot \Psi_M \cdot \omega_m); \end{cases} \tag{1}$$

2. FIELD-ORIENTED MOTOR CONTROL

The hybrid step motors driven by the controller are two-phases (perpendicular to each other) motors. The stator magnetic field can be oriented in any direction by the current in the motor windings. By maintaining the stator magnetic field vector 90^0 (electrical) ahead of the magnetic field vector of the rotor (in the direction of rotation), then the motor is field-oriented and the torque will maximum for a given power supply voltage. If winding currents are sine waves shifted 90^0 one to each other, then the resulting stator magnetic field vector will rotate at the sinusoidal frequency. Mathematical model for the step motor, in the synchronous rotating *dq* reference frame is documented in [1], [4] represented by (1), where:

- $L_{sd} \approx L_{sq} \approx L_s$ are the inductance of the stator phases;
- $i_{sd\theta}$ and $i_{sq\theta}$ are the currents of $d\theta$ and $q\theta$ axis;
- Ψ_M is the permanent magnet flux;
- θ_m is the rotor position;
- ω_m is the rotor speed;
- $u_{sd\theta}$ and $u_{sq\theta}$ are the $d\theta$ - axis and $q\theta$ - axis voltages;
- z_r is number of rotor teeth;
- R is the phase resistance;
- J_m is the rotor inertia;
- B_m is the viscous friction;
- m_r is load torque.

The field-oriented control (FOC) imposes to keep the $d\theta$ axis current - $i_{sd\theta}$, always to zero. The torque equation becomes

$$m_e = k_m \cdot i^*_{sq\theta}, \tag{2}$$

where $k_m = z_r \Psi_M$, is the motor constant.

The order of the model (1) can be reduced if the *d-q* currents are considered as reference inputs for the system drive. Therefore, state space equations for the hybrid stepper motor with field-oriented driven by a current controlled PWM inverter are given by the following equations:

$$i^*_{sd\theta} = 0;$$

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{B_m}{J_m} & 0 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \\ + \begin{bmatrix} \dfrac{k_m}{J_m} \\ 0 \end{bmatrix} \cdot i^*_{sq\theta} + \begin{bmatrix} -\dfrac{1}{J_m} \\ 0 \end{bmatrix} \cdot m_r \tag{3}$$

where the new state vector is $\bar{x} = [x_1(t) \quad x_2(t)]^T = [\omega_m) \quad \theta_m]^T$.

3. SMFC CONTROL DESIGN

The configuration of the field-oriented hybrid stepper motor servo-drive with SMC is shown in Fig.1. The position control employs the *q*-axis current. We have defined the tracking error ***e*** as:

$$\boldsymbol{e} = [\theta_d - \theta_m \quad \omega_d - \omega_m]^T = [e \quad \dot{e}]^T, \tag{4}$$

where $\theta_d(t)$ specifies the references trajectory. The state equation (3) correspond to a second-order system of which general form is

$$\ddot{x} = f(\mathbf{x}, t) + b(\mathbf{x}, t) \cdot u + \tilde{d} \tag{5}$$

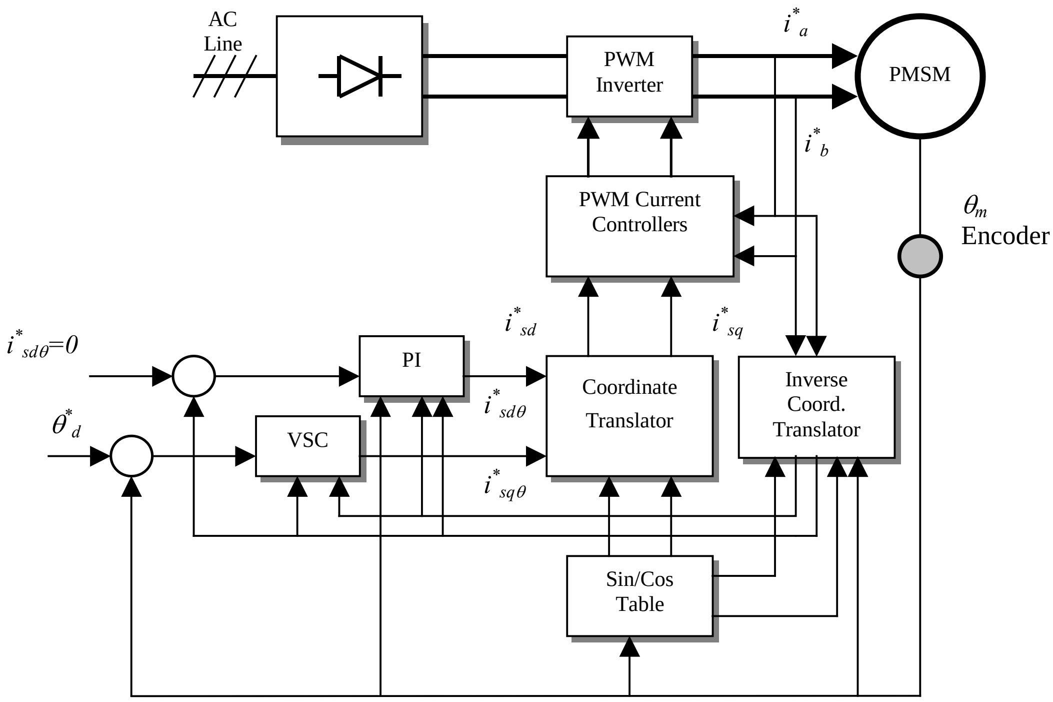

Fig. 1. SMC controller for field-oriented hybrid stepper motor drive.

Consequently, the sliding surface becomes a line,

$$s = \lambda \cdot e + \dot{e}. \quad (6)$$

The traditional sliding-mode control with boundary layer (SMC-BL) law takes the form

$$u = \hat{b}^{-1}(-\hat{f} + G \cdot (\ddot{x}^d - \lambda \dot{e}) - \\ - G \cdot \boldsymbol{K}(\mathbf{x}, t) \cdot sat(s/\Phi), \quad (7)$$

where Φ is called the thickness of the boundary layer (BL). The BL corresponds to substituting of the function $sgn(s)$ by a saturation function.

The control law consists of the following terms:

- *compensation term* - $u_{comp} = -\hat{b}^{-1}\hat{f}$;
- *filter term* - $u_{filt} = -\hat{b}^{-1} G \cdot \lambda \dot{e}$;
- *feed-forward term* - $u_{ff} = -\hat{b}^{-1} G \cdot \ddot{x}^d$;
- *control term* - $u_{comp} = -\hat{b}^{-1} G \boldsymbol{K}(\mathbf{x}, t) \cdot sat(s/\Phi)$.

In the control term, the part $u_{diag} = \boldsymbol{K}(\mathbf{x}, t) \cdot sat(s/\Phi)$ is of diagonal form with the "*diagonal*" $s = 0$. Considering that $\boldsymbol{K}(\mathbf{x}, t) = const$ and the state vector $\boldsymbol{e}$ located inside of BL we have that

$$u_{diag} = -\frac{\boldsymbol{K}(\mathbf{x}, t)}{\Phi} \cdot s = \frac{\boldsymbol{K}(\mathbf{x}, t) \cdot |s|}{\Phi} \operatorname{sgn}(s) \cdot \quad (8)$$

The analytical formulation of control law for the diagonal fuzzy logic controller – FLC, is

$$u_{fuzz} = -K_{fuzz}(e, \dot{e}, \lambda) \cdot \operatorname{sgn}(s). \quad (9)$$

A comparison of (8) and (9) shows the close relationship between an SMC with BL and a diagonal FLC [7]. The transfer characteristic $u_{fuzz} = f(s)$ of the diagonal form FLC is non-linear in contrast to that of the SMC-BL. Also, for the FLC the state vector $\boldsymbol{e}$ is restricted by bounds on the fuzzy state space. The diagonal form FLC changes the magnitude of u depending on the distance $|s|$ between the state vector $\boldsymbol{e}$ and the diagonal $s = 0$. Therefore, the corresponding control law is $u_{fuzz} = -\boldsymbol{K}(\mathbf{x}, t) \cdot |s| \cdot \operatorname{sgn}(s)$. The advantage of a Sliding Mode Fuzzy Controller (SMFC) over a FLC is that the number of fuzzy rules is reduced considerably. The above control law (7) is modified into a fuzzy control law

$$u = -\hat{b}^{-1}\hat{f} + \hat{b}^{-1} \cdot G \cdot (\ddot{x}^d - \lambda \dot{e}) - \hat{b}^{-1} \cdot G \cdot u_{fuzz}, \quad (10)$$

where u_{fuzz} was introduced instead of u_{diag}.

The diagonal is denoted by equation $s = \lambda \cdot e + \dot{e} = 0$, where λ represents the slope of the diagonal. The states located on a diagonal play an important role, since the controller output u changes its sign. An important step in design of SMFC is the choice of the number of fuzzy value for the controller inputs and outputs respectively, and also the form and location of the corresponding membership functions. For our controller we assume the same odd number for the fuzzy value of inputs and outputs respectively.

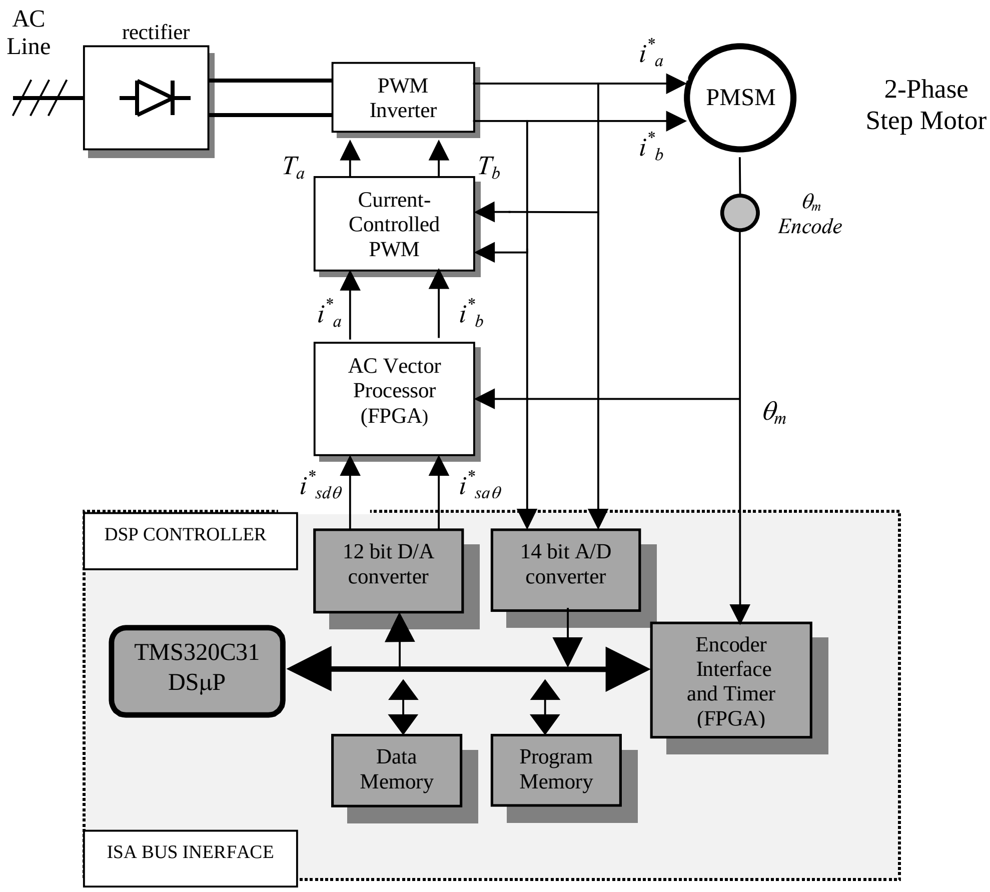

Fig. 2. DSP control of the hybrid stepper motor drive.

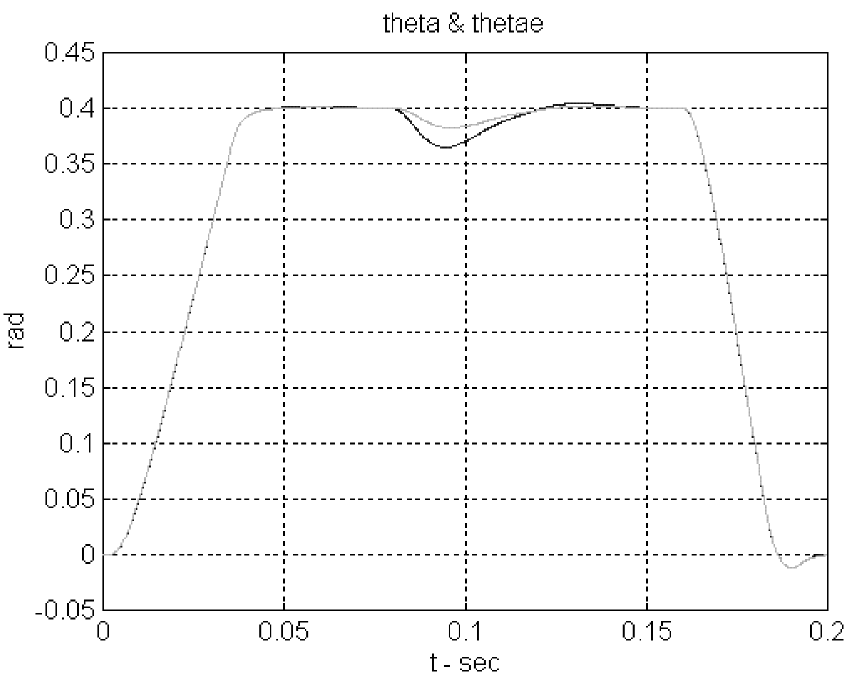

Fig. 3.a. rotor position - θ_m ;

4. HARDWARE DESIGN

The functional diagram of the hardware system, based on a DSP TMS320C31 controller structure, is shown in Fig. 2. The controller can be depicted in parts: DSP controller and a power converter unit provided with the current sensors. The current controllers work in the rotor frame. The DSP generates two sinusoidal waveforms with frequency and amplitude computed by the DSP firmware. The sine waves are applied to the motor windings through two full H bridge power drive The PWM output works like a current source inverter rather than voltage source inverter. Figures 4a and 4b present the stator phase currents - i_{sa} i_{sb} and, the filtered/non-filtered phase current, respectively.

The parameters for the step motor, are: 200 steps/rotation z_r=50 rotor teeth, i_N=2.0A, m_N=0.3Nm, R=3Ω, L_S=0.02mH, B_m = 3x10^{-3} Nm/s/rad, J_m= 1.25x10^{-4} kgm^2. In Fig. 3 the response of step motor at a positioning cycle for a step reference θ^*_m=0.4 rad is shown. These results are plotted the data acquisitions obtained by the hardware in loop simulation model.

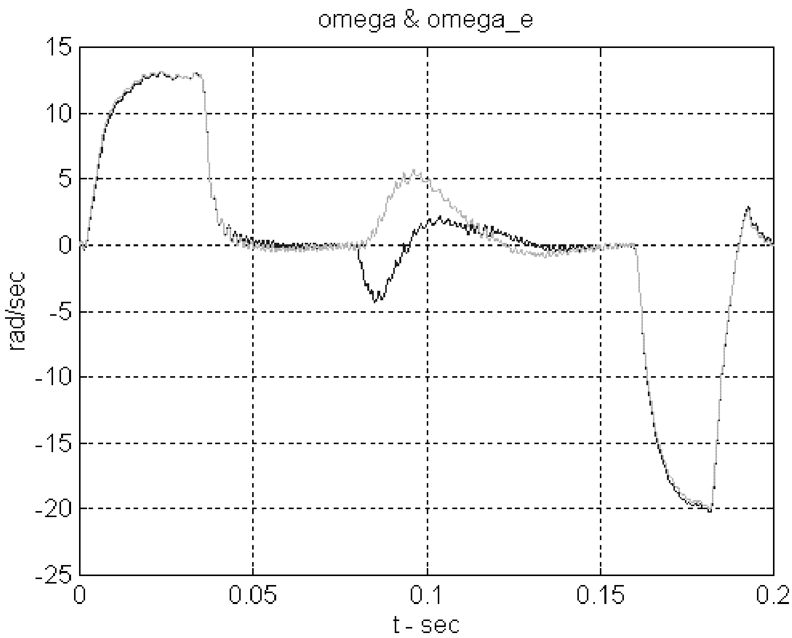

Fig. 3.b. speed ω_m and $\hat{\omega}_m$.

Fig. 3. Step response of the controller for a positioning cycle.

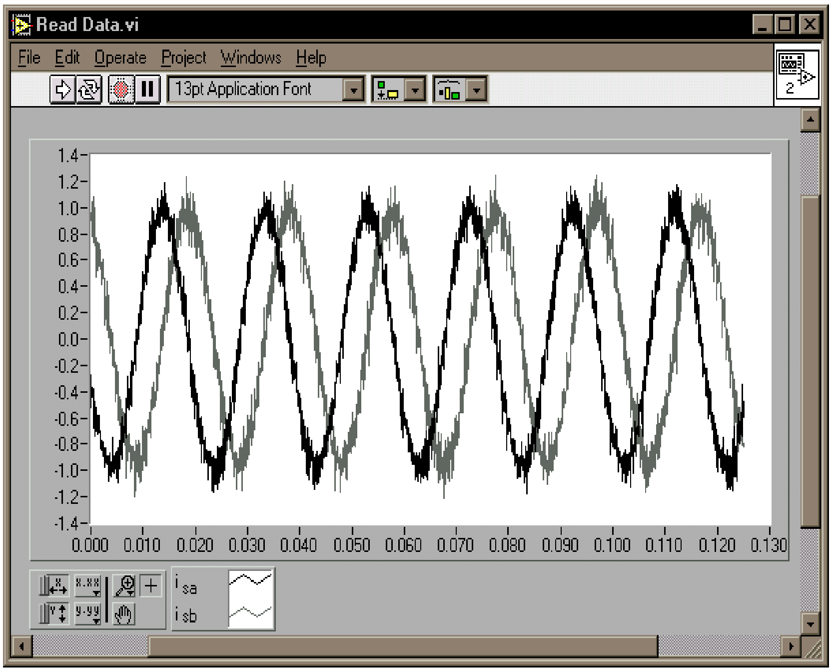

Fig. 4.a.. Stator phase currents - i_{sa} and i_{sb}.

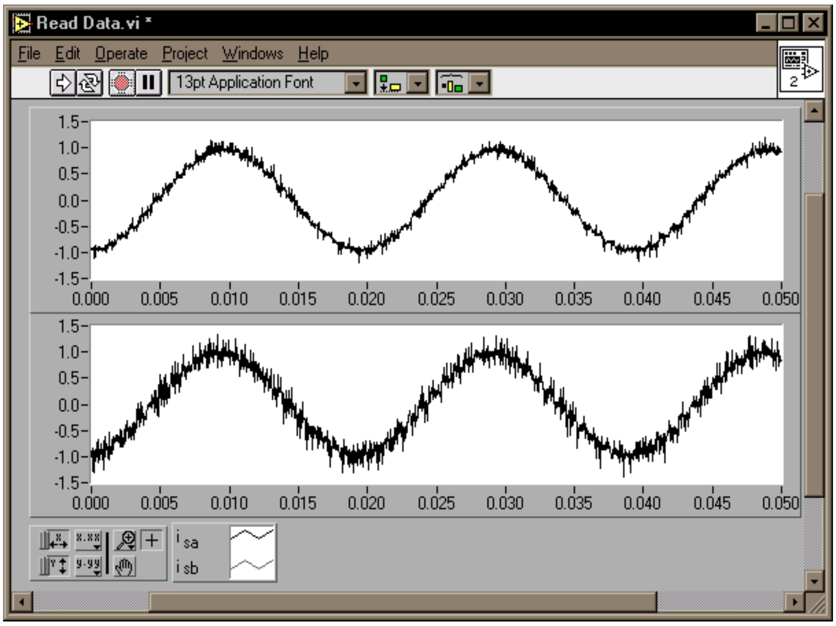

Fig. 4.b. Filtered and non-filtered current.

5. CONCLUSION

This paper has discussed the sliding mode fuzzy control (SMFC) for the two-phase step motors. The procedure presents only the fuzzy part since the rest part is based on a well-known sliding mode controller with boundary layer (SMC-BL).

This structure of controller (SMFC) generates a non-linear transfer characteristic in contrast with that of SMC with BL. The hardware controller, which is based on a TMS320C31, drives the step motor phase windings using field-orientation control rather than stepping mode operation. This method drives the motor at maximum theoretical performance. The PWM output works like a current source inverter rather than voltage source inverter. In this way the motor becomes very properly for industrial applications (robotic systems or industrial machinery) where high torque and low speed are required.

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