# Adaptive Synchronization of Noise-Perturbed Complex Delayed Network Systems

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**Abstract:** Concerning the effect of internal noise and communication delay in the synchronized process, we introduce a noise-perturbed complex network dynamical model and design an adaptive feedback controller to synchronize the proposed network. Based on invariance principle of stochastic time-delay differential equations, a sufficient condition in terms of linear matrix inequality is derived to guarantee global synchronization of the network via the adaptive controller. The analytical results show that the adaptive control scheme is of robustness against the noise resulted from the internal synchronized errors. Finally, a numerical simulation is provided to verify the effectiveness of the adaptive strategy.

*Keywords*: synchronization, complex networks, noised-perturbed, time-delay

## 1. INTRODUCTION

Recently, synchronization of coupled chaotic oscillators has attracted a great number of researchers. This is partly due to that complex networks of coupled chaotic oscillators have been widely used to describe various complex systems in biophysics, neuroscience, and technology (Boccaletti, Kurths, Osipov, Valladares, and Zhou, 2002; Belykh, Belykh, and Hasler, 2004; Lü and Chen, 2005; Boccaletti, Wang, Chavez, Amann, and Pecora, 2006). Yet in practice, such synchronization is urgently expected. For example, Peskin (1975) reported that synchronous beats of the heart cells are regulated by the activity of the pacemaker cells situated at the sinoatrial node. Then it is necessary to design a controller to guarantee the synchronization, which is also called synchronization control. In this endeavor, much valuable work shows that the synchronization behavior can be boosted or eliminated by feedback control based on those complex dynamical network models with deterministic structures and coupling relationships. Therefore, it is potentially of great significance to investigate synchronization control problem of dynamical systems on complex networks.

During the past decade, distributed synchronization control of complex networks has attracted a great deal of attention. Sorrentino, Bernardo, Garofalo, and Chen (2007) suggested that the controllability of a coupled complex network via pinning can be assessed by means of a Master Stability Function approach. Wang and Sun (2010) studied the robustness problem of pinning a general complex dynamical network, particularly for some changes on network architecture. Chen, Liu, and Lu (2007) pointed out that a general complex network can be pinned by a single controller if the coupling strength is large enough. Wang and Chen (2002) investigated the control problem for a scale-free dynamical network by applying local feedback injections to a fraction of network nodes. Wu, Wei, Li, and Xiao (2009) investigated the pinning control strategy for stabilizing a complex network with uncertain couplings to a homogenous orbit based on the V-stability tool, which associates the dynamics of the nodes with passivity degrees.

Note that, in engineering practice, the dynamics of each node in complex networks can not be precisely observed due to external disturbance, so that the analysis of network synchronization becomes much more complex (Wu, Wei, Li, and Xiao, 2009; Zhang, Li, and Lin, 2008). The influence of noise on the behaviors of nonlinear network communication among the sensors is very diverse which might cause the multiple nodes system to diverge or oscillate. How to control the appearance of synchronized states in the dynamical networks is of great significance in theory and potential applications. Xiao and Xu (2009) designed an adaptive-feedback controller to synchronize a class of noise-perturbed two bi-directionally coupled chaotic systems with time-delay. Sun and Cao (2007) investigated lag synchronization for two coupled chaotic systems with noise perturbation and unknown DNNs.

However, most of these investigations have focused on two coupled chaotic systems, and few works have addressed the effects of external noise-perturbed for complex coupled network systems. To solve the synchronized problems for the multiple nodes of complex network systems is a challenging task, because the nonlinear coupling between nodes and external disturbances in many real-world systems will aggravate difficulties in analyzing the synchronized errors trajectory of the controlled network. Besides, once the topology with communication delay is involved, the analysis of synchronization for networks becomes much more complex. With this background, we study the synchronization control for a class of noise-perturbed networks with communication time-delay.

In this paper, we attempt to investigate the stability of synchronized error system with the effect of internal noise and communication delay in the synchronized process. In doing the analysis, we obtain the dynamical model of the synchronized errors for the complex networks. Then, based on invariance principle of stochastic time-delay differential equations, we investigate how to design the controller for synchronization of the complex network systems with time-delay and noise-perturbed. Finally, synchronization is achieved globally for the multiple nodes of network systems from effects of noise and time-delay. The analytical results show that the controller has a certain robustness against the noise resulted from the internal synchronized errors.

The paper is organized as follows. In Section 2, we present a model for a class of noise-perturbed complex network systems with time-delay and define the problem addressed. The main converging results of the noise-perturbed complex dynamical network with communication time-delay systems are brought forth in Section 3. Following that, Section 4 gives the simulation result. Finally, some conclusions are drawn in Section 5.

## 2. SYSTEM MODEL

Generally, a complex network consisting of $n$ identical nodes with communication time-delay in the presence of d-dimensional nonlinear vector can be formulated as

$$\dot{x}_i(t) = f(x_i) + \sum_{j\neq i} a_{ij}(x_j(t-\tau) - x_i(t-\tau)) \tag{1}$$

where $x_i \in R^d$ is the state vector, $i, j = 1, \cdots, n$. $\tau$ is the time delay of the network transmission. The initial condition of (1) are given by $x_i(t) = \varphi_i(t) \in C([-\tau, 0], R^d)(i = 1, 2, \cdots, n)$. Here, $C([-\tau, 0], R^d)$ denotes the set of all continuous functions from $[-\tau, 0]$ to $R^d$. $f = (f_1, \cdots, f_d)^T : R^d \to R^d$ is a smooth vector function. The weighted adjacency matrix $A = [a_{ij}]$ is the configuration coupling matrix representing the topological structure of the network, in which the coupling strengths $a_{ij}$ are defined as follows: if there is a connection between node $i$ and node $j(j \neq i)$, then $a_{ij} > 0$; otherwise $a_{ij} = 0$.

Assume the desired target trajectory $s(t)$ driven by the dynamical equation

$$\dot{s}(t) = f(s(t))$$

where, $s(t) \in R^d$ called synchronization state which is usually an equilibrium point, a periodic orbit, an aperiodic orbit or a chaotic orbit.

Define the synchronized error vector

$$e_i(t) = x_i(t) - s(t), i = 1, \cdots, n.$$

where $e_i = (e_{i1}, e_{i2}, \cdots, e_{id})^T \in R^d$.

In fact, the coupled systems are inevitably affected by different environment. Particularly, the noise is resulted from the internal synchronized errors. From the perspective of control theory, designing controllers is an effective method in synchronizing a complex network of coupled dynamical systems. The goal of control is to achieve complete synchronization.

Considering a controlled network consisting of $n$ identical nodes, we assume that the noisy non-linear dynamics of each synchronized error is

$$de_i(t) = (f(x_i(t) - f(s(t)))dt + \sum_{j\neq i} a_{ij}(e_j(t-\tau) - e_i(t-\tau))dt$$
$$+\sigma_i(t, e_i(t), e_i(t-\tau))dw_i(t) + u_i(t)dt, \quad i = 1, \cdots, n \tag{2}$$

where, $u_i(t) \in R^d$ is the control input, $w_i(t) = [w_{i1}(t), \cdots, w_{id}(t)]^T$ is d-dimensional Brownian motion defined on a complete probability space $(\Omega, F, \psi)$ with a natural filtration $\{F_t\}_{t\geq 0}$ generated by $\{w_i(s) : 0 \leq s \leq t\}$, where $\Omega$ is associated with the canonical space generated by $w_i(t)$, and $F$ is associated with $\sigma$-algebra generated by $\{w_i(t)\}$ with the probability measure $\psi$. Moreover, $dw(t) = [\eta_1(t), \cdots, \eta_{nd}(t)]^T \in R^{nd}$ is the white noise, in which every two elements are statistically independent, $E[\eta_i(t)\eta_j(t')] = \delta_{ij}\delta(t-t')(i, j = 1, \cdots, nd)$, the mathematical expectation $E[\eta_i] = 0$. It is noted that the noise intensity $\sigma_i = diag\{\sigma_{i1}(e_{i1}(t), e_{i1}(t-\tau)), \cdots, \sigma_{id}(e_{id}(t), e_{id}(t-\tau))\}$ for $0 \leq i \leq n$ is intrinsic to the dynamical system in the synchronized process (i.e. independent of the inputs), which is consistent with experimental findings by the synchronized error. $\sigma_i$ is continuous nonlinear matrix-valued function, which is usually called the noise coupling strength function, and $\sigma_i(t, 0, 0) \equiv 0$.

The initial condition associated with system (2) is given in the following form:

$$e_i(\theta) = \phi_i(\theta), \quad -\tau \leq \theta \leq 0, \quad i = 1, \cdots, n \tag{3}$$

where, the initial data $\phi_i \in C([-\tau, 0], R^d)$ is a continuous function satisfying that $\int_{-\tau}^{0} E\|\phi_i(\theta)\|^2 d\theta < \infty$.

Throughout this paper, the following Assumptions are need:

Assumption 1. Suppose that $f(\cdot)$ satisfies a Lipschitz condition. That is, there exists a Lipschitz constant $\alpha$ such that $\|f(x_i(t)) - f(s(t))\| \leq \alpha\|x_i(t) - s(t)\|$ for $0 \leq i \leq n$, and $f(0) \equiv 0$.

Assumption 2. Suppose that $\sigma_i(t, x, y)$ satisfies the Lipschitz condition for $\forall i = 1, \cdots, n$. Moreover, there exist constant matrices of appropriate dimensions $G_{1i}$, $G_{2i}$ such that

$trace[\sigma_i^T(t, x, y)\sigma_i(t, x, y)] \leq \|G_{1i}x\|^2 + \|G_{2i}y\|^2, \ \forall (t, x, y) \in R^+ \times R^d \times R^d$.

The assumption of $\sigma_i(t, 0, 0) \equiv 0$ if $e_i(t) \equiv 0$, and the Assumptions 1-2 make sure that is a trivial solution of the synchronized error vector $e_i(t)$ on $t \geq -\tau$.

Under Assumptions 1-2, this paper is devoted to design the controller to reach synchronization for networks consisting of $n$ identical nodes in the presence of d-dimensional noisy. Synchronization of the noise-perturbed network (2) is guaranteed if error vector $e_i(t)$ is asymptotically stable in mean square with the initial condition (3), in sense that

$$\lim_{t\to\infty} E\left\{\left\|e_i(t)\right\|^2\right\} = 0, \forall i = 1,\cdots,n$$

where, $\|\cdot\|$ denotes the Euclidean norm, $E\{\cdot\}$ denotes expectation.

## 3. ADAPTIVE CONTROLLER DESIGN

This section presents an adaptive controller to synchronize the noise-perturbed network systems with propagation delay. Given the dynamical system (2), Note that $\sigma_i(t)\equiv 0$ when $e_i(t)$ is equal to zero. However, if the feedback gains are fixed, the feedback energy must be the maximal when the synchronization is achieved. It further indicates that the traditional linear feedback control means a kind of waste in practice. To make economical use of energy, we choose an adaptive synchronization controller and a noise-perturbed update laws of parameters. We introduce the adaptive feedback gains controller $u_i(t)$

$$\begin{aligned} &u_i(t) = -\tilde{\varepsilon}_i e_i \\ &\dot{\tilde{\varepsilon}}_{ij} = \gamma_{ij} e_{ij}^2, i=1,\cdots,n.\ \ j=1,\cdots,d. \end{aligned} \tag{4}$$

where $\gamma_i > 0, (i=1,\cdots,n)$ are arbitrary constants. $\tilde{\varepsilon}_i = diag\{\tilde{\varepsilon}_{i1},\cdots,\tilde{\varepsilon}_{id}\}$ is the adaptive-feedback gain matrix with nonzero initial value $\varepsilon_i(0)$. We separate the adaptive-feedback gain $\varepsilon_i$ with zero initial value and the initial feedback gain $\varepsilon_i(0)$ from $\tilde{\varepsilon}_i$. It is worth pointing out that $\varepsilon_i$ will be automatically adjusted to a suitable constant in the process of network synchronization, which is different from the traditional linear feedback.

Then, the dynamical equation of $e_i(t)$ is

$$\begin{aligned} d e_i(t) = [&f(x_i(t)) - f(s(t)) + \sum_{j\neq i} a_{ji}(e_j(t-\tau) - e_i(t-\tau)) - \varepsilon_i e_i(t) \\ &-\varepsilon_i(0) e_i(t-\tau)]dt + \sigma_i(e_j(t)-e_i(t), e_j(t-\tau)-e_i(t-\tau))dw_i(t) \end{aligned} \tag{5}$$

It's easy to see that the dynamics (5) can also be described as

$$\begin{aligned} de(t) = [&Jf(t) - (L\otimes I_d)e(t-\tau) - \varepsilon_\wedge e(t) - \varepsilon_{\wedge 0} e(t-\tau)]dt \\ &+\sigma_\wedge(t, e(t), e(t-\tau))dw(t) \end{aligned} \tag{6}$$

where, $e=(e_1,\cdots,e_n)^T \in R^{nd}$ is the state error vector, $w=(w_1,\cdots,w_n)^T\in R^{nd}$, $Jf(t) = \left[f(x_1(t)) - f(s(t)),\cdots, f(x_n(t)) - f(s(t))\right]^T$, $\sigma_\wedge = diag\{\sigma_1,\cdots,\sigma_n\}$, $\varepsilon_\wedge = diag\{\varepsilon_1,\cdots,\varepsilon_n\}$, $\varepsilon_{\wedge 0} = diag\{\varepsilon_1(0),\cdots,\varepsilon_n(0)\}$, $\otimes$ is the Kronecker product notation, $L$ is the Laplacian matrix of the weighted digraph. Correspondingly, the Laplacian matrix is defined as $L=\left[l_{ij}\right]$, where $l_{ii}=d_i$ and $l_{ii} = -a_{ij}, i\neq j$. $d_i = \sum_{j=1}^n a_{ij}$ for $i,j=1,\ \cdots,\ n$.

By (6), we find that $\varepsilon_{\Lambda 0}$ could be considered as a linear fixed feedback diagonal matrix, whose elements are not all zero. It is evident that

$$\begin{aligned} de(t) = [&Jf(t) - (L\otimes I_d + \varepsilon_{\wedge 0})e(t-\tau) - \varepsilon_\wedge e(t)]dt \\ &+\sigma_\wedge(t, e(t), e(t-\tau))dw(t) \end{aligned} \tag{7}$$

Then, the following provides a sufficient condition to guarantee all nodes are synchronized to the desired noise-free system by designing the controller $\varepsilon_{\Lambda 0}$, and the error dynamical system (7) is globally stable with the initial condition $e(0)$.

**Theorem 1.** Under the Assumptions 1-2, the noise-perturbed response system (7) achieves global synchronization with $e(0)$, if there exist positive definite matrix $Q\in R^{nd\times nd}$ and constant diagonal matrix $B$ satisfying

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12}^T & H_{22} \end{bmatrix} < 0 \tag{8}$$

where

$$H_{11} = Q + I_{nd}\otimes\alpha + B + \frac{1}{2}G_1^T G_1$$

$$H_{12} = -\frac{1}{2}(L\otimes I_d + \varepsilon_{\wedge 0})$$

$$H_{22} = -Q + \frac{1}{2}G_2^T G_2$$

and $G_1 = diag\{G_{11},\cdots,G_{1n}\}$, $G_2 = diag\{G_{21},\cdots,G_{2n}\}$, $B = diag\{b_1,\cdots,b_n\}$, $b_i = diag\{b_{i1},\cdots,b_{id}\}$, $G_1$ and $G_2$ are the noise coupling constant matrices for $\sigma_i(t,x,y)$.

**Proof.** Based on the LaSalle-type invariance principle for stochastic differential equation with time delay (Mao, 2002), we introduce the continuous differential non-negative function:

$$\begin{aligned} V(t,e(t),e(t-\tau)) = &\frac{1}{2}e^T e + \int_{t-\tau}^t e^T(s)Qe(s)ds \\ &+\sum_{i=1}^n\sum_{j=1}^d \frac{1}{2\gamma_{ij}}(b_{ij}+\varepsilon_{ij})^2 \end{aligned} \tag{9}$$

where $Q$ is a positive definite matrix, and $b_{ij}$ $(i=1,\cdots,n. j=1,\cdots,d)$ are constants to be determined.

The stochastic derivative of $V$ along the trajectory of error system (7) can be expressed by

$$\begin{aligned} DV(t,e(t),e(t-\tau)) = &\partial V_t(t,e(t)) + \partial V_e(t,e(t)) \\ &\cdot\left[Jf(t) - (L\otimes I_d + \varepsilon_{\Lambda 0})e(t-\tau) - \varepsilon_\Lambda e(t)\right] \\ &+\frac{1}{2}trace(\sigma_\wedge^T(t,e(t),e(t-\tau))V_{ee}(t,e(t))\sigma_\wedge(t,e(t),e(t-\tau))) \end{aligned}$$

where

$$\partial V_e(t,e(t)) = \left(\frac{\partial V_e(t,e(t))}{\partial e_1},\cdots,\frac{\partial V_e(t,e(t))}{\partial e_n}\right)$$

$$V_{ee}(t,e(t)) = \left(\frac{\partial^2 V(t,e(t))}{\partial e_i \partial e_j}\right)_{n\times n}$$

From Eq. (9) and the dynamics of system (7), we have

$$\begin{aligned} \partial V_t(t,e(t)) = &e^T(t)Qe(t) - e^T(t-\tau)Qe(t-\tau) \\ &+\sum_{i=1}^n\sum_{j=1}^d (b_{ij}+\varepsilon_{ij})e_{ij}^2 \end{aligned}$$

By above analysis, we obtain that

$$\begin{aligned} &DV(t,e(t),e(t-\tau)) \\ &= e^T(t)Qe(t) - e^T(t-\tau)Qe(t-\tau) + \sum_{i=1}^n\sum_{j=1}^d (b_{ij}+\varepsilon_{ij})e_{ij}^2 \\ &+e^T(t)Jf(t) - e^T(t)(L\otimes I_d + \varepsilon_{\wedge 0})e(t-\tau) - e^T(t)\varepsilon_\wedge e(t) \\ &+\frac{1}{2}\sum_{i=1}^n trace(\sigma_\wedge^T(t,e(t),e(t-\tau))\sigma_\wedge(t,e(t),e(t-\tau))) \\ &= e^T(t)Qe(t) - e^T(t-\tau)Qe(t-\tau) + e^T(t)Be(t) + e^T(t)\varepsilon_\wedge e(t) \\ &+e^T(t)Jf(t) - e^T(t)(L\otimes I_d + \varepsilon_{\wedge 0})e(t-\tau) - e^T(t)\varepsilon_\wedge e(t) \end{aligned}$$

$$+\frac{1}{2}\sum_{i=1}^{n} \operatorname{trace}(\sigma_{\wedge}^{T}(t,e(t),e(t-\tau))\sigma_{\wedge}(t,e(t),e(t-\tau)))$$
$$= e^{T}(t)Qe(t) - e^{T}(t-\tau)Qe(t-\tau) + e^{T}(t)Be(t)$$
$$+e^{T}(t)Jf(t) - e^{T}(t)(L\otimes I_d + \varepsilon_{\wedge 0})e(t-\tau)$$
$$+\frac{1}{2}\sum_{i=1}^{n} \operatorname{trace}(\sigma_{\wedge}^{T}(t,e(t),e(t-\tau))\sigma_{\wedge}(t,e(t),e(t-\tau)))$$

Under Assumptions 1-2, then we have

$$DV(t,e(t),e(t-\tau))$$
$$\le e^{T}(t)Qe(t) - e^{T}(t-\tau)Qe(t-\tau) + e^{T}(t)Be(t) + e^{T}(t)$$
$$\cdot(I_{nd}\otimes\alpha)e(t) - e^{T}(t)(L\otimes I_d + \varepsilon_{\wedge 0})e(t-\tau)$$
$$+\frac{1}{2}\sum_{i=1}^{n}(e_i^{T}(t)G_{1i}^{T}G_{1i}e_i(t) + e_i^{T}(t-\tau)G_{2i}^{T}G_{2i}e_i(t-\tau))$$
$$= e^{T}(t)Qe(t) + e^{T}(t)(I_{nd}\otimes\alpha)e(t) + e^{T}(t)Be(t)$$
$$+\frac{1}{2}e^{T}(t)G_1^{T}G_1e(t) - e^{T}(t)(L\otimes I_d + \varepsilon_{\wedge 0})e(t-\tau)$$
$$+\frac{1}{2}e^{T}(t-\tau)G_2^{T}G_2e(t-\tau) \ -e^{T}(t-\tau)Qe(t-\tau)$$
$$=\begin{bmatrix} e(t) \\ e(t-\tau)\end{bmatrix}^{T}\begin{bmatrix} H_{11} & H_{12} \\ H_{12}^{T} & H_{22}\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau)\end{bmatrix}$$

(10)

Then, a sufficient condition for $DV<0$ holds if and only if $H<0$.

From (10) and It$\hat{o}$ formula, it is obvious to see that

$$E\{V(t)\} - E\{V(t_0)\} = E\left\{\int_{t_0}^{t} DV(s)ds\right\} \qquad (11)$$

By the definition of $V(t)$ in (9), there exits a positive constant $\lambda_1$ such that

$$\lambda_1 E\left\{\|e(t)\|^2\right\} \le E\{V(t)\} \le E\{V(t_0)\} + E\left\{\int_{t_0}^{t} V(s)ds\right\}$$
$$\le E\{V(t_0)\} + \lambda_{\max} E\left\{\int_{t_0}^{t}\|e(s)\|^2 ds\right\} \qquad (12)$$

where $\lambda_{\max}$ is the maximal eigenvalue of $H$ and it is negative.

Therefore, from (12) and the discussion in Ref. (Kushner, 1967), we know that $E\left\{\|e(t)\|^2\right\}\to 0$, the error dynamical system (7) is globally asymptotically stable in mean square to guarantee synchronization of the network. This completes the proof.

From Theorem 1, we can see that the adaptive-feedback controller can synchronize the complex networks with communication time-delay, which is related to the noise intensity matrix $G_{1i}$, $G_{2i}$. It fits well to the engineering practice. At the same time, the analytical results show that the adaptive-feedback controller has a certain robustness against the noise resulted from the internal synchronized errors. The theorem provides a sufficient condition to design the controller for the networks which guarantee that all nodes achieve synchronization.

It is obvious that $H$ is a real symmetric and linear matrix. Since every diagonal entry of $B$ is to be determined, according to the theory of linear matrix inequality, we can find a positive definite matrix $Q$ to ensure that $H$ is definitely negative if $B$ is negative and small enough, so that we obtain the feedback control parameters $\varepsilon_{\wedge 0} = diag\{\varepsilon_1(0),\cdots,\varepsilon_n(0)\}$.

## 4. A NUMERICAL SIMULATION

In this section, a simulation is given to illustrate the theoretical results obtained in the previous sections. Assume that the controlled network (1) consists of 4 identical Chua systems, where the node dynamics is given by

$$\begin{bmatrix}\dot{x}_{i1}\\ \dot{x}_{i2}\\ \dot{x}_{i3}\end{bmatrix} = \begin{bmatrix} c_1(x_{i2}-x_{i1}+q(x_{i1})) \\ c_2(x_{i1}-x_{i2})+c_3x_{i3} \\ -c_4x_{i2}\end{bmatrix}$$

where $q(\cdot)$ is a piecewise linear function of the form

$$q(x_{i1}) = \begin{cases} -c_6x_{i1}-c_5+c_6 & x_{i1}>1 \\ -c_5x_{i1} & |x_{i1}|\le 1 \\ -c_6x_{i1}+c_5-c_6 & x_{i1}<-1\end{cases}$$

in which $c_i>0$ with $i=1,2,3,4$ and $c_5<c_6<0$. If the system parameters are chosen to be

$$c_1=7, c_2=0.35, c_3=0.5, c_4=7, c_5=-5.714, c_6=-0.143$$

Obviously, function $f(\cdot)$ satisfies Assumption 1. Then Chua's oscillator has a chaotic attractor shown in Figure 1.

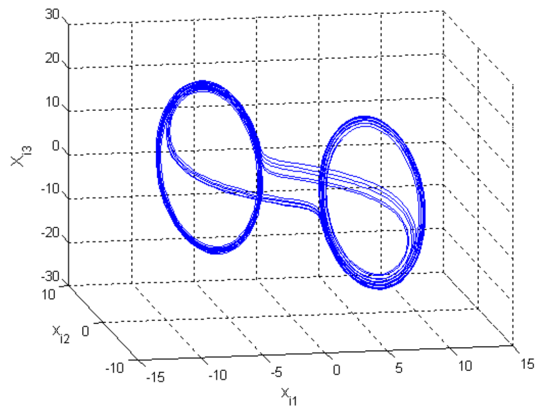


Fig. 1. The chaotic attractor of Chua's oscillator

Consider that the network obeys topological structure of the network model, where the graph Laplacian is defined as

$$L=\begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2\end{bmatrix}$$

According to the effect of the coupling, internal noise and parametric mismatch, we set the noise intensity

$$\sigma_{\wedge}(t,e(t),e_{\tau}(t))$$
$$=\begin{bmatrix} a_1I_3e_1(t)+b_1I_3e_{\tau1}(t) & 0 & 0 & 0 \\ 0 & a_2I_3e_1(t)+b_2I_3e_{\tau1}(t) & 0 & 0 \\ 0 & 0 & a_3I_3e_1(t)+b_3I_3e_{\tau1}(t) & 0 \\ 0 & 0 & 0 & a_4I_3e_1(t)+b_4I_3e_{\tau1}(t)\end{bmatrix}$$

and $w(t)$ is a 4-dimensional Brownian motion satisfying Assumption 2, it is easy to get

$$G_1 = \sqrt{4}diag(|a_1|,|a_2|,|a_3|,|a_4|) \otimes I_3 ,$$
$$G_2 = \sqrt{4}diag(|b_1|,|b_2|,|b_3|,|b_4|) \otimes I_3 .$$

The parameters in the complex network systems are taken as $a_1 = 0.1$ , $a_2 = 0.3$ , $a_3 = -0.3$ , $a_4 = -0.2$ , $b_1 = 0.2$ , $b_2 = -0.4$ , $b_3 = 0.5$, $b_4 = 0.4$, time-delay $\tau = 1$, $\varepsilon_{\wedge 0} = I$ , the white noise $w_i(t)$ is shown in Figure 2, which satisfy the condition in Theorem 1. Therefore the noise-perturbed complex networks with communication time-delay can be synchronized.

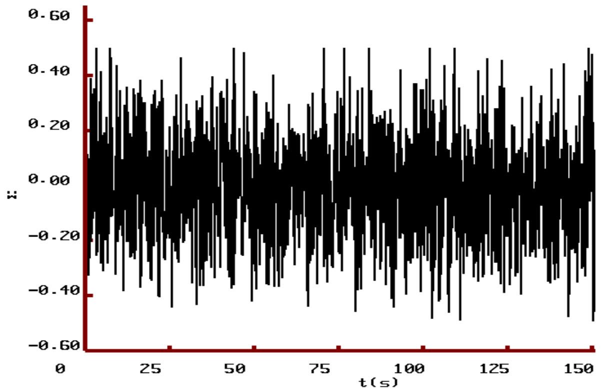


Fig. 2. The white noise $w_i(t)$ of the network

Now, we present the simulation result for synchronization problems of the complex networks with noise-perturbed and communication time-delay. Figure 3 shows the state error trajectories with noise in the controlled network, where synchronization index $E\{\|e_i(t)\|\} = E\{\|x_i(t) - s(t)\|\}$. Clearly, from Figure 3, synchronization is achieved. The numerical result shows that the approach of proposed adaptive synchronization is very effective.

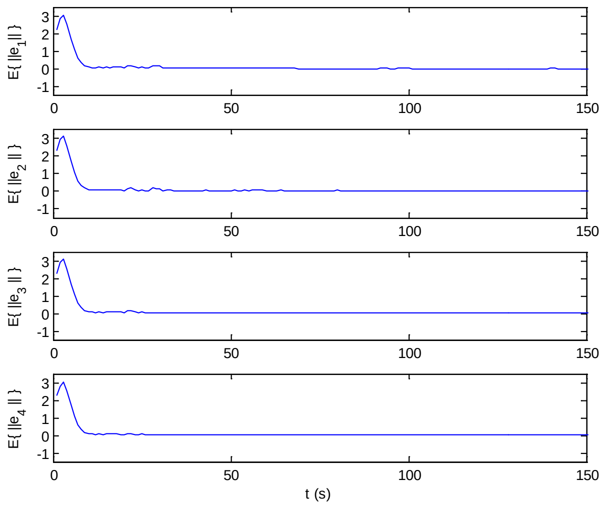


Fig. 3. The synchronization errors of the controlled network

## 5. CONCLUSIONS

Communication delay and noise are inevitable in modelling, controlling, and optimizing complex systems. However, the two factors are difficult to be handled by traditional techniques, especially for large-scale systems. In this paper, we investigate synchronization control for complex networks with the effect of internal noise and communication delay existing in many real-world systems. A noise-perturbed complex delayed network dynamical model is introduced. Based on invariance principle of stochastic time-delay differential equations, we investigate how to design the adaptive controller for synchronization of the networks with noise and communication delay. Moreover, a stability criterion in the terms of linear matrix inequalities is established for global synchronization of the controlled dynamical networks. Finally, we demonstrate its practicability through computer simulation. As has been shown by analytical and numerical result, the proposed controller has strong robustness against the effect of internal noise and communication delay, which could be commonly applied to realistic situations of engineering.

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