

Suppressing Axial-Torsional Coupled Vibrations in Drillstrings

Belem Saldivar^{*,**} Sabine Mondié^{*} Jean Jacques Loiseau^{**}
Vladimir Rasvan^{***}

^{*} *Departamento de Control Automático, CINVESTAV-IPN, Av. IPN
No. 2508, Col. San Pedro Zacatenco, C.P. 07360, México D.F.,
Mexico (e-mail: msaldivar,smondie@ctrl.cinvestav.mx).*

^{**} *Institut de Recherche en Communications et Cybernétique de
Nantes, 1 rue de la Noé, BP 92101, 44321 NANTES cedex 3, France
(e-mail: Jean-Jacques.Loiseau@irccyn.ec-nantes.fr)*

^{***} *Department of Automatic Control, University of Craiova, A.I. Cuza,
13 Craiova, RO-200585 Romania. (e-mail:
vrasvan@automation.ucv.ro)*

Abstract: In drilling operation, a wide variety of oscillations causing failures often arise. Torsional vibrations (stick-slip) are originated by the cutting device (bit) motion, these vibrations in turn excite axial oscillations causing a phenomenon known as bit-bouncing. This paper addresses two important challenges: the modeling of the coupled axial and torsional dynamics in a vertical oilwell drilling system and the design of an effective controller to reduce undesirable behaviors. Through the D'Alembert transformation, the distributed parameter model of the drillstring is reduced to a neutral-type time-delay equation which effectively describes the oscillatory behavior of the system. The Lyapunov theory allows to develop an efficient strategy for the control synthesis guaranteeing the elimination of the stick-slip and bit-bounce. This approach leads the "practical" stabilization of the closed loop system. All results can be easily generalized to any time-delay system subject to bounded disturbances. The effectiveness of the strategy is validated through simulations.

Keywords: stick-slip, bit-bouncing, neutral-type time-delay systems, practical stabilization.

1. INTRODUCTION

The presence of intense vibrations in a drill-string has been considered for many years as one of the most relevant causes of loss of performance in the drilling process. In the oil industry, the improvement of drilling performance is a matter of crucial economical interest. The main kinds of drilling oscillations caused by the impact of the bottom hole assembly (BHA) with the borehole, depicted on Fig. 1 are: torsional (stick-slip), axial (bit-bouncing) and lateral (whirling). These vibrations are generally quite complex in nature, they are intimately coupled and may occur simultaneously.

In order to reduce the costs of failures, extensive research efforts have been conducted in the last five decades to suppress drillstring vibrations, see for instance: Halsey et al. (1988), Sananikone (1993), Pavone and Desplans (1994), Jansen and van den Steen (1995), Serrarens et al. (1998), Navarro-López and Cortés (2007), Puebla and Álvarez (2008), Saldivar et al. (2009), Saldivar et al. (2011).

The literature available on the subject can largely be divided in publications focused on axial, torsional or lateral vibrations. In Leine (1997), the survey gives an overview on publications which are related to the stick-slip.

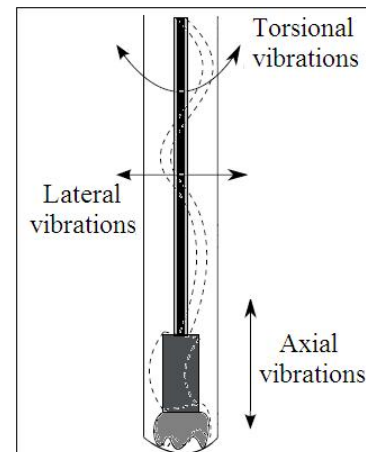


Fig. 1. Vibrations in oilwell drillstrings.

Drilling system complexity brings out a modelling and a control problem. The model must accurately describe the most important phenomena arising in real wells and it has to be simple enough for analysis and control purposes.

The wave equation is widely used to reproduce the oscillatory behavior of physical systems, this modeling strategy was adopted in Tucker and Wang (1999a), Tucker and Wang (1999b), Challamel (2000) and Tucker and Wang

(2003) to describe oilwell drilling system vibrations. By means of a direct transformation an input-output model described by a neutral-type time-delay equation is derived. It simplifies the analysis and simulations. The procedure allowing to transform the partial differential equation model to a delay system of neutral type was presented for the first time in Cooke and Krumme (1968), see also Mounier et al. (1995), Balanov et al. (2002), Blakely and Corron (2004), Barton et al. (2007), Saldivar et al. (2011).

The control methods found in the specialized literature include operational guidelines to overcome the stick-slip problem as well as active control methods using feedback (Sananikone (1993), Jansen and van den Steen (1995), Serrears et al. (1998)). Even though these control solutions succeed to suppress the stick-slip vibrations, their effect on the bit-bouncing phenomenon has not been analyzed.

In the real world, systems usually exhibit nonlinear characteristics and uncertainties for which the theoretical definitions of asymptotic or exponential stability can be too restrictive. Clearly, the state of a system may be mathematically unstable, but the response oscillates close enough to the equilibrium, to be considered as acceptable. In this situation, the notion of Lyapunov asymptotic or exponential stability is not appropriate. More suitable performance specifications, from an engineering point of view, are given by the definition of practical stability, also referred to as ultimate boundedness with a fixed bound (Hale and Verduyn (1993)). These definitions characterize the system transient behavior with estimates of the bounds on the system trajectories (Anabtawi (2011)).

This paper develops an effective strategy based on Lyapunov techniques and the descriptor approach introduced in Fridman (2001) to tackle the stabilization problem of neutral-type time-delay systems subject to nonlinear bounded perturbations. The methodology allows the feedback control synthesis for the suppression of the stick-slip and bit-bounce, furthermore it leads to a practical stable closed loop system.

This contribution is organized as follows: in Section 2, the drilling system modeling is presented; Section 3 gives conditions for a guaranteed evolution of the error which corresponds to the existence of a practical exponential estimate of the neutral-type system response; the main result is given in Section 4, the control design strategy for the system stabilization is developed, the practical stabilizability conditions are given in terms of a bilinear matrix inequality (BMI); in Section 5, simulation results of the axial-torsional coupled model are presented, the controller performance is evaluated; conclusions are given in the last section.

Notation. Throughout the paper we will use the Euclidean norm for vectors, it will be denoted by $|\cdot|$ and the induced norm for matrices with the notation $\|\cdot\|$. The notation $|\cdot|_h$ for any function ϕ from $\mathcal{C}^1([-h; 0], \mathcal{R}^n)$ is $\sup_{s \in [-h; 0]} (|\phi(s)|)$ and $\|\phi\|_h = \max \left(|\phi(s)|_h, \left| \dot{\phi}(s) \right|_h \right)$. $\mathcal{R}$ stands for the field of real numbers, $\mathcal{R}^n$ denotes n -dimensional Euclidean space, and $\mathcal{R}^{n \times m}$ is the set of $n \times m$ real matrices. $\mathcal{C}^1([-h, 0], \mathcal{R}^n)$ stands for the set of continuously differentiable functions on the interval $[-h, 0]$. We denote by $x(t, \phi)$ or $x(t)$ the solution of the system

with initial conditions ϕ and by x_t or $x_t(\phi) = \{x(t + \theta, \phi) \mid \theta \in [-h, 0]\}$ the state of the system. The superscript $'T'$ stands for matrix transposition, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the smallest and largest eigenvalues of a symmetric matrix A , respectively. The notation $P > 0$ for $P \in \mathcal{R}^{n \times n}$ means that P is a symmetric and positive definite matrix.

2. DRILLING SYSTEM MODELING

The main components of the drilling system are: drill-string, it is composed by pipes sections screwed end to end each other which are added as the bore hole depth increases; drill collars, they are heavy stiff steel tubulars used at the bottom of a BHA to provide weight on bit and rigidity; bit, it is the rock cutting device to create the borehole; drilling mud or fluid, it has the function of cleaning, cooling and lubricating the bit.

The purely torsional excitations of a drill-string of length L , described by the rotary angle $\theta(\xi, t)$ can be modeled by the wave equation (Tucker and Wang (1999a), Tucker and Wang (1999b), Challamel (2000), Tucker and Wang (2003), Fridman et al. (2010), Boussaada et al. (2012)):

$$\frac{\partial^2 \theta}{\partial \xi^2}(\xi, t) = p^2 \frac{\partial^2 \theta}{\partial t^2}(\xi, t), \quad p = \sqrt{\frac{I}{GJ}}, \quad \xi \in (0, L), \quad t > 0 \quad (1)$$

coupled to the mixed boundary conditions:

$$\begin{aligned} GJ \frac{\partial \theta}{\partial \xi}(0, t) &= c_a \left(\frac{\partial \theta}{\partial t}(0, t) - \Omega(t) \right); \\ GJ \frac{\partial \theta}{\partial \xi}(L, t) + I_B \frac{\partial^2 \theta}{\partial t^2}(L, t) &= -T \left(\frac{\partial \theta}{\partial t}(L, t) \right), \end{aligned} \quad (2)$$

where $\theta(\xi, t)$ is the angle of rotation, I is the inertia, G is the shear modulus, J is the geometrical moment of inertia and I_B is a lumped inertia to represent the assembly at the bottom hole.

The drillstring is rotated from the surface ($\xi = 0$) by an electrical motor, Ω is the angular velocity coming from the rotor that does not match the rotational speed of the load $\frac{\partial \theta}{\partial t}(0, t)$. The constant c_a is related to the local torsion of the drillstring. The undriven extremity ($\xi = L$) is subject to a torque T which is a function of the bit speed $\frac{\partial \theta}{\partial t}(L, t)$. The nature of $T \left(\frac{\partial \theta}{\partial t}(L, t) \right)$ is a critical aspect of the model and is discussed in details below.

Integration along characteristics of the hyperbolic partial differential equations (D'Alembert method) allows the association of certain system of functional differential equations to the mixed problem, more precisely, a one-to-one correspondence may be established and proved between the solutions of the mixed problem for hyperbolic partial differential equations and the initial value problem for the associated system of functional equations (Rasvan and Niculescu (2002)).

By reducing the boundary value problem (1-2) to a neutral-type time delay equation it is possible to exploit techniques from dynamic systems theory to gain insight into the complexity involved in the analysis and simulation of infinite dimensional systems.

The general solution of the one-dimensional wave equation that models vibrations of a string can be written as: $\theta(\xi, t) = \phi(t + p\xi) + \psi(t - p\xi)$, where the variables $\gamma = t +$

$p\xi$, and $\eta = t - p\xi$ result from the application of the method of characteristics to (1), ϕ and ψ are continuously differentiable real-valued functions of one variable.

The boundary conditions can be rewritten as:

$$\begin{aligned} p \left(\frac{\partial \phi}{\partial \gamma}(t) - \frac{\partial \psi}{\partial \eta}(t) \right) &= \frac{c_a}{GJ} \left(\frac{\partial \phi}{\partial \gamma}(t) + \frac{\partial \psi}{\partial \eta}(t) - \Omega(t) \right), \\ p \left(\frac{\partial \phi}{\partial \gamma}(t + \tau) - \frac{\partial \psi}{\partial \eta}(t - \tau) \right) &= -\frac{I_B}{GJ} \frac{\partial^2 \phi}{\partial \gamma^2}(t + \tau) \\ &\quad - \frac{I_B}{GJ} \frac{\partial^2 \psi}{\partial \eta^2}(t - \tau) - \frac{1}{GJ} T \left(\frac{\partial \phi}{\partial \gamma}(t + \tau) + \frac{\partial \psi}{\partial \eta}(t - \tau) \right), \end{aligned}$$

where $\tau = pL$. Let $z(t)$ the angular velocity at the bottom extremity of the drill string: $z(t) = \frac{\partial \theta(L, t)}{\partial t} = \dot{\phi}(t + \tau) + \dot{\psi}(t - \tau)$. Under appropriate manipulations, the following neutral-type time-delay equation which describes the input-output behavior of the system is obtained (Saldivar et al. (2011)):

$$\begin{aligned} \dot{z}(t) - \Upsilon \dot{z}(t - 2\tau) &= -\Psi z(t) - \Upsilon \Psi z(t - 2\tau) \\ &\quad - \frac{1}{I_B} T(z(t)) + \frac{1}{I_B} \Upsilon T(z(t - 2\tau)) + \Pi \Omega(t - \tau) + \omega, \end{aligned} \quad (3)$$

where $\Pi = \frac{2\Psi c_a}{c_a + \sqrt{IGJ}}$, $\Upsilon = \frac{c_a - \sqrt{IGJ}}{c_a + \sqrt{IGJ}}$, $\Psi = \frac{\sqrt{IGJ}}{I_B}$, and $\tau = \sqrt{\frac{I}{GJ}}L$. The additive bounded variable ω allows us to take into consideration uncertainties, disturbances and/or unmodeled dynamics in the system, we assume that ω is bounded in module. The time-delay τ is directly dependent on the length of the rod. It should be pointed out that the model transformation is exact, i.e., (3) is not an approximation of the wave equation model.

The stick-slip phenomenon constitutes the major source of failures in the oil extraction industry. It is identified by "stick stages" in which the bit rotation is completely stopped and "slip stages" in which the angular velocity of the bit is greatly increased regarding its nominal value, furthermore the torque at the bit level manifests periodic oscillations. Torsional vibrations occur when the friction between the bit and the rock is significant. The bit might eventually get stuck and then, after accumulating energy in terms of torsion, be suddenly released.

The following nonlinear expression introduced in Navarro-López and Cortés (2007) allows to approximate the physical phenomenon at the bit level:

$$T(z(t)) = c_b z(t) + W_{ob} R_b \mu_b(z(t)) \text{sign}(z(t)). \quad (4)$$

The first term (viscous damping torque) represents the influence of the drilling fluid and the second one (dry friction torque) approximates the bit-rock contact, R_b stands for the bit radius and W_{ob} is the weight on the bit. The bit dry friction coefficient $\mu_b(z(t))$ is described by:

$$\mu_b(z(t)) = \mu_{cb} + (\mu_{sb} - \mu_{cb}) e^{-\frac{\gamma_b}{v_f} |z(t)|}, \quad (5)$$

where $0 < \gamma_b < 1$ is a constant defining the velocity decrease rate and $\mu_{cb}, \mu_{sb} \in (0, 1)$ are the Coulomb and static friction coefficients. The constant velocity $v_f > 0$ is introduced in order to have appropriate units. The friction torque (4-5) leads to a decrease of the torque on the bit (TOB) with increasing bit angular velocity for low velocities which acts as a negative damping (Stribeck effect) and is the cause of stick-slip self-excited vibrations. The exponential decaying behavior of $T(z(t))$ coincides with experimental torque values. In Saldivar et al. (2011),

the model (3-5) is validated with simulation results in close qualitative agreement with field observations regarding stick-slip vibrations.

The angular velocity coming from the rotary table $\Omega(t)$ is usually taken as control input with the following structure:

$$\Omega(t) = u(t) + \Omega_0 \quad (6)$$

where $u(t)$ is a stabilizing controller which should eliminate the stick-slip phenomenon and Ω_0 is the constant angular velocity desired at the bit level. In order to shift the operating point, the new variable x is defined: $x(t) = z(t) - \Omega_0$. In view of the TOB model (4-5) and (6), the model (3) is rewritten as:

$$\begin{aligned} \dot{x}(t) + d\dot{x}(t - 2\tau) &= a_0 x(t) + a_1 x(t - 2\tau) + bu(t - \tau) \\ &\quad - \frac{1}{I_B} T_{nl}(x(t) + \Omega_0) + \frac{\Upsilon}{I_B} T_{nl}(x(t - 2\tau) + \Omega_0) + \omega, \end{aligned} \quad (7)$$

where T_{nl} stands for the nonlinear term of the TOB model (4-5):

$$T_{nl}(z(t)) = W_{ob} R_b \left(\mu_{cb} + (\mu_{sb} - \mu_{cb}) e^{-\frac{\gamma_b}{v_f} |z(t)|} \right) \text{sign}(z(t))$$

and $a_0 = -\Psi - \frac{c_b}{I_B}$, $a_1 = \frac{\Upsilon c_b}{I_B} - \Upsilon \Psi$, $d = -\Upsilon$, and $b = \Pi$.

The high bit speed level caused by stick-slip torsional motion can excite severe axial vibrations in the bottom hole assembly which may induce the bit to intermittently lose contact with the rock surface hitting the drill area with great strength. This phenomenon, known as bit-bounce, is identified by drastic variations of the weight on the bit.

The modeling strategy for the axial dynamic is inspired by the fact that any mass subject to a force in stable equilibrium acts as a harmonic oscillator for small vibrations (Challamel (2000)):

$$m_0 \ddot{Y}(t) + c_0 \dot{Y}(t) + k_0(Y(t) - \text{ROP}t) = -\mu_1 [T(z(t)) - T(z(t - 2\tau))] \quad (8)$$

the variables Y , $\dot{Y}$ and $\ddot{Y}$ stand for the axial variables: position, velocity and acceleration respectively, the rate of penetration (ROP) is an axial speed imposed at the surface, m_0 , c_0 , and k_0 represent the mass, damping and spring constant. The constant μ_1 depends on the bit geometry (bit radius, rake angle), it is easily obtained for a flat bit (Challamel (2000)). The change of variable $y(t) = Y(t) - \text{ROP}t$ yields the following equivalent model:

$$m_0 \ddot{y}(t) + c_0 (\dot{y}(t) + \text{ROP}) + k_0 y(t) = -\mu_1 [T(z(t)) - T(z(t - h))] \quad (9)$$

It should be pointed out that only the torsional model (7) will be employed in the design of the stabilizing controller. The obtained control law will be tested in the axial-torsional coupled model to evaluate, through simulations, its performance.

3. PRELIMINARY RESULT

Consider the following time-delay system of neutral-type:

$$\begin{aligned} \dot{x}(t) + D\dot{x}(t - h) &= A_0 x(t) + A_1 x(t - h) + n(t), \\ x(\theta) &= \phi(\theta), \quad \theta \in [-h, 0], \end{aligned} \quad (10)$$

where $h \geq 0$ is the time delay, $D \in \mathcal{R}^{n \times n}$ is Schur stable, $A_0, A_1 \in \mathcal{R}^{n \times n}$, $n(t)$ satisfies $\|n(t)\| \leq \zeta$, $\zeta > 0$, $t \geq 0$.

Definition (Practical stability or ultimate boundedness with a fixed bound). The system (10) is ε -practically stable if for some $\varepsilon > 0$ there exists $0 < t^*(\varepsilon, \phi) < \infty$ such that $\|x(t, \phi)\|^2 \leq \varepsilon$ for all $t \geq t^*(\varepsilon, \phi)$.

The next result follows the ideas introduced in Villafuerte et al. (2011).

Lemma 1. Let the neutral-type time-delay system given in (10). If there is a functional $V(x_t)$ such that the following conditions are satisfied:

$$\alpha_1 \|x(t)\|^2 \leq V(x_t) \leq \alpha_2 \|x_t\|_h^2, \quad \forall t \geq 0, \quad (11)$$

$$\frac{d}{dt} V(x_t) + \sigma V(x_t) \leq \beta, \quad \forall t \geq 0, \quad (12)$$

where σ, β are positive constants and $\alpha_2 > \alpha_1 > 0$. Then, the guaranteed evolution of the error is established:

$$\|x(t)\|^2 \leq e^{-\sigma t} \left(\frac{\alpha_2}{\alpha_1} \|\phi\|_h^2 - \frac{\beta}{\alpha_1 \sigma} \right) + \frac{\beta}{\alpha_1 \sigma}, \quad (13)$$

for any initial condition ϕ . The system (10) is μ -practically stable with $\mu \geq \frac{\beta}{\alpha_1 \sigma}$ for all $t \geq t^*(\mu, \phi)$, where

$$t^*(\mu, \phi) = \begin{cases} 0, & \text{if } \|\phi\|_h \leq \sqrt{\frac{\beta}{\alpha_2 \sigma}}; \\ \frac{1}{\sigma} \ln \left(\frac{\alpha_2 \sigma \|\phi\|_h^2 - \beta}{\alpha_1 \sigma \mu - \beta} \right), & \text{elsewhere.} \end{cases} \quad (14)$$

Proof. Multiplying (12) by $e^{\sigma \theta}$ gives

$$\frac{d}{d\theta} (e^{\sigma \theta} V(x_t)) \leq \beta e^{\sigma \theta}.$$

Integrating the above expression from 0 to t yields $V(x_t) \leq e^{-\sigma t} V(0, x_0) + \frac{\beta}{\sigma} (1 - e^{-\sigma t})$. It follows from (11) that

$$\begin{aligned} \alpha_1 \|x(t)\|^2 &\leq e^{-\sigma t} V(0, x_0) + \frac{\beta}{\sigma} (1 - e^{-\sigma t}) \\ &\leq \alpha_2^{-\sigma t} e \|\phi\|_h^2 + \frac{\beta}{\sigma} (1 - e^{-\sigma t}), \end{aligned}$$

then,

$$\|x(t)\|^2 \leq e^{-\sigma t} \left(\frac{\alpha_2}{\alpha_1} \|\phi\|_h^2 - \frac{\beta}{\alpha_1 \sigma} \right) + \frac{\beta}{\alpha_1 \sigma}. \quad (15)$$

Notice that for the initial conditions fulfilling $\frac{\alpha_2}{\alpha_1} \|\phi\|_h^2 \leq \frac{\beta}{\alpha_1 \sigma}$, the inequality $\|x(t)\|^2 \leq \frac{\beta}{\alpha_1 \sigma} \quad \forall t \geq 0$ is satisfied. Otherwise, if

$$\frac{\alpha_2}{\alpha_1} \|\phi\|_h^2 > \frac{\beta}{\alpha_1 \sigma}, \quad (16)$$

it follows that $\|x(t)\|^2 \leq \mu$, $\forall t \geq t^*(\mu, \phi)$, where

$$\mu > \frac{\beta}{\alpha_1 \sigma}. \quad (17)$$

In view of (15), the time $t^*(\mu, \phi)$ is derived from the inequality

$$0 < e^{-\sigma t} \left(\frac{\alpha_2}{\alpha_1} \|\phi\|_h^2 - \frac{\beta}{\alpha_1 \sigma} \right) \leq \mu - \frac{\beta}{\alpha_1 \sigma},$$

which implies $\left(\frac{\alpha_2 \sigma \|\phi\|_h^2 - \beta}{\alpha_1 \sigma} \right) \leq e^{\sigma t} \left(\frac{\alpha_1 \sigma \mu - \beta}{\alpha_1 \sigma} \right)$ and the result follows. Notice that conditions (16) and (17) imply that $\alpha_2 \sigma \|\phi\|_h^2 - \beta > 0$, $\alpha_1 \sigma \mu - \beta > 0$ and $\alpha_2 \sigma \|\phi\|_h^2 - \beta > \alpha_1 \sigma \mu - \beta$ hence t^* exists.

In the next section, the control design strategy for the "practical" stabilization of a class of nonlinear neutral-type time-delay systems leading to a guaranteed evolution of the error is developed.

4. MAIN RESULT

Theorem 2. Let a neutral-type time-delay system of the form:

$$\begin{aligned} \dot{x}(t) + D\dot{x}(t-2\tau) &= A_0 x(t) + A_1 x(t-2\tau) \\ &+ B u(t-\tau) + C_0 f(t) + C_1 f(t-2\tau) + \omega, \end{aligned} \quad (18)$$

where τ is a constant time delay, $D \in \mathcal{R}^{n \times n}$ is Schur stable, $A_0, A_1 \in \mathcal{R}^{n \times n}$, $B \in \mathcal{R}^{n \times m}$, f satisfies $\|f(t)\| \leq \zeta$, $\zeta > 0$, $t \geq 0$, and ω is such that:

$$\|\omega\|_{K_\omega} = \omega^T K_\omega \omega \leq 1, \quad t \geq 0. \quad (19)$$

Consider a control law of the form:

$$u(t-\tau) = K_0 \dot{x}(t-h) + K_1 x(t-h). \quad (20)$$

If there exist positive definite matrices $P, S, R \in \mathcal{R}^{n \times n}$, and $P_2, P_3, P_4, P_8 \in \mathcal{R}^{n \times n}$, $P_5, P_6, P_7 \in \mathcal{R}^{n \times m}$, $P_9 \in \mathcal{R}^{m \times m}$, $P_{10}, P_{11}, K_0, K_1 \in \mathcal{R}^{m \times n}$, and $\beta > 0$ such that the BMI $\Phi < 0$ is feasible for some $\sigma > 0$, where Φ is given in (21) (next page), then, for any initial condition ϕ , the system (18) in closed loop with the control law (20) satisfies the "practical" exponential estimate (13) with $\alpha_1 = \lambda_{\min}(P)$ and $\alpha_2 = \lambda_{\max}(P) + 2\tau\lambda_{\max}(S) + 4\tau^2\lambda_{\max}(R)$. In addition system (10) is μ -practically stable for all $t \geq t^*(\mu, \phi)$, where $t^*(\mu, \phi)$ is given in (14).

Proof. Consider the functional

$$\begin{aligned} V(x_t) &= x^T(t) P x(t) + \int_{t-2\tau}^t e^{\sigma(s-t)} x^T(s) S x(s) ds \\ &+ h \int_{-2\tau}^0 \int_{t+\theta}^t e^{\sigma(s-t)} \dot{x}^T(s) R \dot{x}(s) ds d\theta. \end{aligned} \quad (22)$$

Notice that condition (11) of Lemma 1 is satisfied with $\alpha_1 = \lambda_{\min}(P)$, and $\alpha_2 = \lambda_{\max}(P) + 2\tau\lambda_{\max}(S) + 4\tau^2\lambda_{\max}(R)$. The time derivative of (22) satisfies

$$\begin{aligned} \frac{dV(x_t)}{dt} + \sigma V(x_t) &\leq 2x^T(t) P \dot{x}(t) + \sigma x^T(t) P x(t) \\ &+ x^T(t) S x(t) - e^{-\sigma 2\tau} x^T(t-2\tau) S x(t-2\tau) \\ &- e^{-\sigma 2\tau} (x(t) - x(t-2\tau))^T R (x(t) - x(t-2\tau)) \\ &+ 4\tau^2 \dot{x}^T(t) R \dot{x}(t). \end{aligned} \quad (23)$$

Now, following the descriptor approach introduced in Fridman (2001), two null terms derived from the system dynamic and the controller structure (20) are added to the right hand side of the inequality (23):

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P - P_2^T + A_0^T P_3 & \Phi_{14} & \Phi_{15} & P_2^T C_0 + A_0^T P_6 & P_2^T C_1 + A_0^T P_7 & P_2^T + A_0^T P_8 \\ * & \Phi_{22} & A_1^T P_3 & \Phi_{24} & \Phi_{25} & A_1^T P_7 & A_1^T P_8 & A_1^T P_9 \\ * & * & 4\tau^2 R - P_3 - P_3^T & \Phi_{34} & \Phi_{35} & P_3^T C_0 - P_6 & P_3^T C_1 - P_7 & P_3^T - P_8 \\ * & * & * & \Phi_{44} & \Phi_{45} & P_4^T C_0 - D^T P_6 & P_4^T C_1 - D^T P_7 & P_4^T - D^T P_8 \\ * & * & * & * & \Phi_{55} & P_5^T C_0 + B_1^T P_6 & P_5^T C_1 + B_1^T P_7 & P_5^T + B_1^T P_8 \\ * & * & * & * & * & P_6^T C_0 + C_0^T P_6 & P_6^T C_1 + C_0^T P_7 & P_6^T + C_0^T P_8 \\ * & * & * & * & * & * & P_7^T C_1 + C_1^T P_7 & P_7^T + C_1^T P_8 \\ * & * & * & * & * & * & * & P_8^T + P_8 - \beta K_\omega \end{bmatrix}, \quad (21)$$

$$\begin{aligned} \Phi_{11} &= A_0^T P_2 + P_2^T A_0 + S - e^{-\sigma 2\tau} R + \sigma P, \\ \Phi_{12} &= e^{-\sigma 2\tau} R + P_2^T A_1, \quad \Phi_{14} = -P_2^T D + A_0^T P_4, \\ \Phi_{15} &= P_2^T B + A_0^T P_5, \\ \Phi_{22} &= -e^{-\sigma 2\tau} (S + R) + P_1^T K_1 + K_1^T P_{11}, \\ \Phi_{24} &= A_1^T P_4 + K_1^T P_{10} + P_{11}^T K_0 \end{aligned}$$

$$\begin{aligned} \Phi_{44} &= -P_4^T D - D^T P_4 + P_{10}^T K_0 + K_0^T P_{10}, \\ \Phi_{45} &= P_4^T B - D^T P_5 + K_0^T P_9 - P_{10}^T, \\ \Phi_{55} &= P_5^T B_1 + B_1^T P_5 - P_9^T - P_9, \\ \Phi_{25} &= A_1^T P_5 + K_1^T P_9 - P_{11}^T, \\ \Phi_{34} &= -P_3^T D - P_4, \quad \Phi_{35} = P_3^T B_1 - P_5, \end{aligned}$$

$$\begin{aligned} 0 &= 2[P_2 \dot{x}(t) + P_3 \dot{x}(t-2\tau) + P_4 \dot{x}(t-2\tau) + P_5 u(t-\tau) \\ &\quad + P_6 f(t) + P_7 f(t-2\tau) + P_8 \omega]^T [-\dot{x}(t) \\ &\quad - D \dot{x}(t-2\tau) + A_0 x(t) + A_1 x(t-2\tau) \\ &\quad + B u(t-\tau) + C_0 f(t) + C_1 f(t-2\tau) + \omega], \\ 0 &= 2[P_9 u(t-\tau) + P_{10} \dot{x}(t-2\tau) + P_{11} x(t-2\tau)]^T \\ &\quad [-u(t-\tau) + K_0 \dot{x}(t-2\tau) + K_1 x(t-2\tau)], \end{aligned} \quad (24)$$

where $P_2, P_3, P_4, P_8 \in \mathcal{R}^{n \times n}$, $P_5, P_6, P_7 \in \mathcal{R}^{n \times m}$, $P_9 \in \mathcal{R}^{m \times m}$, $P_{10}, P_{11}, K_0, K_1 \in \mathcal{R}^{m \times n}$. The condition (19) implies that $\omega^T K_\omega \omega - 1 \leq 0$, then for any $\beta > 0$, the inequality $-\beta(\omega^T K_\omega \omega - 1) \geq 0$ holds, thus, in order to take into account the perturbation restriction, the term:

$$-\beta(\omega^T K_\omega \omega - 1) \quad (25)$$

is added to (23). In view of (23-25), after symmetrization of cross terms, the following inequality holds,

$$\frac{dV(x_t)}{dt} + \sigma V(x_t) - \beta \leq \eta^T \Phi \eta$$

where

$$\eta = (x(t) \ x(t-2\tau) \ \dot{x}(t) \ \dot{x}(t-2\tau) \ u(t-\tau) \ f(t) \ f(t-2\tau) \ \omega)^T.$$

If $\Phi < 0$ the condition (12) is satisfied and the result follows from Lemma 1.

Remark 1. The synthesis of the controller (20) is directly obtained by solving the BMI $\Phi < 0$ with Φ given in (21).

5. NUMERICAL RESULT

In this section, simulation results of the torsional-axial coupled model introduced in Section 2 are presented. The strategy developed in Section 4 is applied to find the synthesis of a stabilizing controller ensuring the elimination of the stick-slip oscillations and consequently the reduction of the bit bouncing phenomenon.

Given that only the torsional model (7) is used to design the stabilizing controller, such methodology is reduced to the scalar case. Regarding the system of general form (18), the nonlinear function T_{nl} corresponds to $f(t)$. The obtained controller will be applied to the axial-torsional coupled model to evaluate its performance, through simulations.

The parameters used in the sequel are shown below, they represent a common case in practice (Challamel (2000)):

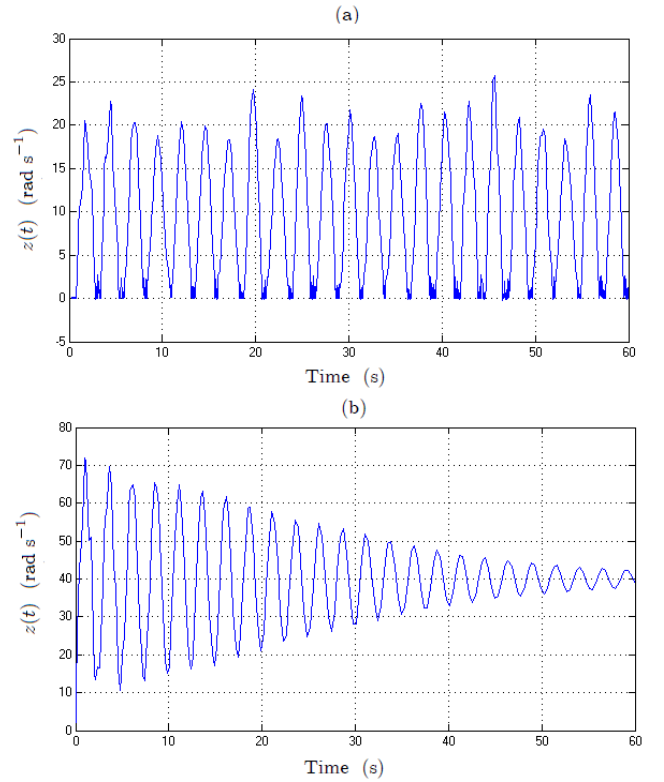


Fig. 2. Angular velocity at the bottom extremity $z(t)$ of the open loop drilling system for: (a) $\Omega_0 = 10 \text{ rad s}^{-1}$ (stick-slip), (b) $\Omega_0 = 40 \text{ rad s}^{-1}$.

$$\begin{aligned} L &= 1172 \text{m}, & I &= 0.095 \text{Kg m}, & J &= 1.19 \times 10^{-5} \text{m}^4, \\ \mu_1 &= 257 \text{m}^{-1}, & I_B &= 89 \text{Kg m}^2, & G &= 79.3 \times 10^9 \text{N m}^{-2}, \\ v_f &= 1, & R_b &= 0.155575 \text{m}, & c_b &= 0.03 \text{N m s rad}^{-1}, \\ \gamma_b &= 0.9, & W_{ob} &= 97347 \text{N}, & c_a &= 2000 \text{N m s}, \\ \mu_{sb} &= 0.8, & m_0 &= 37278 \text{Kg}, & k_0 &= 1.55 \times 10^6 \text{Kg s}^{-2}, \\ \mu_{cb} &= 0.5, & \text{ROP} &= 0.01 \text{m s}^{-1}, & c_0 &= 16100 \text{kg s}^{-1}. \end{aligned}$$

The simulations were developed through the variable step Matlab-Simulink solver ode45 (Dormand Prince Method). The constants of the torsional drilling model (7) take the following values: $a_0 = -3.3645$, $a_1 = -2.4878$, $d = -0.7396$, and $b = 5.8523$, the time delay is $\tau = 0.3719$, we consider that $\omega^2 \leq 0.8$.

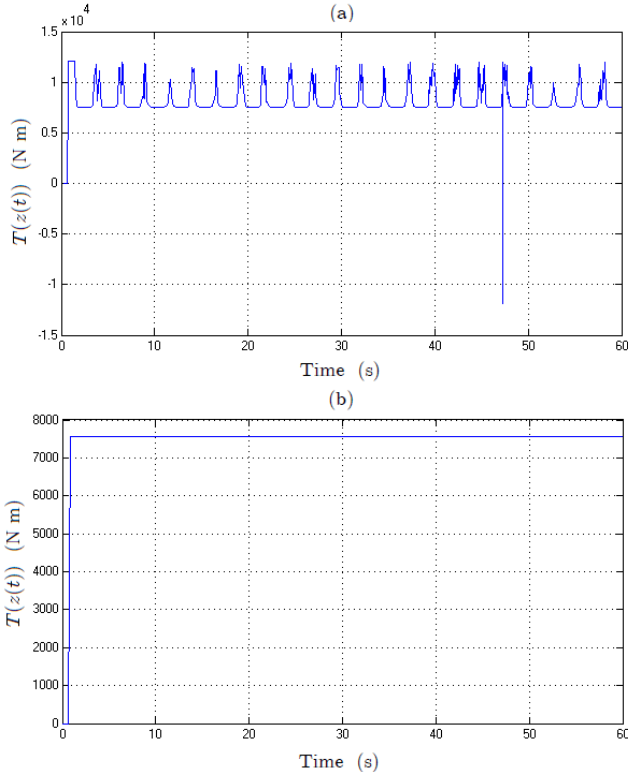


Fig. 3. Torque on the bit $T(z(t))$ for: (a) $\Omega_0 = 10 \text{ rad s}^{-1}$, (b) $\Omega_0 = 40 \text{ rad s}^{-1}$.

When the cutting device is off the formation, it is assumed that both extremities of the drillstring are rotating at the same velocity. When the rock-bit interaction begins, undesired oscillations may simultaneously appear.

Figs. 2-4 show the response of the open loop system. The stick-slip behavior can be seen in Fig. 2(a). As observed in practice, the increase of the surface angular velocity $\Omega(t)$ induces the reduction of the stick-slip. The qualitative affinity between the simulations presented and the field data observed in Halsey et al. (1988) is remarkable. During the "stick stage", the bit rotation stops, causing high values of TOB (Fig. 3). During the "slip stage", the torsional energy accumulated along the drillstring is liberated leading to extremely large torsional oscillations meanwhile, the bit does not lose contact with the rock. It seems that some of the accumulated energy in the "stick stage" is transferred into axial oscillations.

The bit-bouncing phenomenon can be seen in Fig. 4(a). This phenomenon takes place due to the axial resonance causing the bit to be off the formation. Whereas the bit is off bottom, the TOB is null, and consequently, the critical frequencies for lateral and axial vibrations change (Christoforou and Yigit (2003)). If control actions are not applied, these oscillations persist with energy exchange between different kinds of vibration. It is clear that this behavior will lead to system failure because the drilling rod is subject to intense stress. It is well known that the drilling oscillations are mostly driven by the rotation at the bit level. Then, an efficient controller for the suppression of drilling vibrations must include the measurement and feedback of motion at the ground level. Using Theorem 2, it is possible to obtain the synthesis of the controller

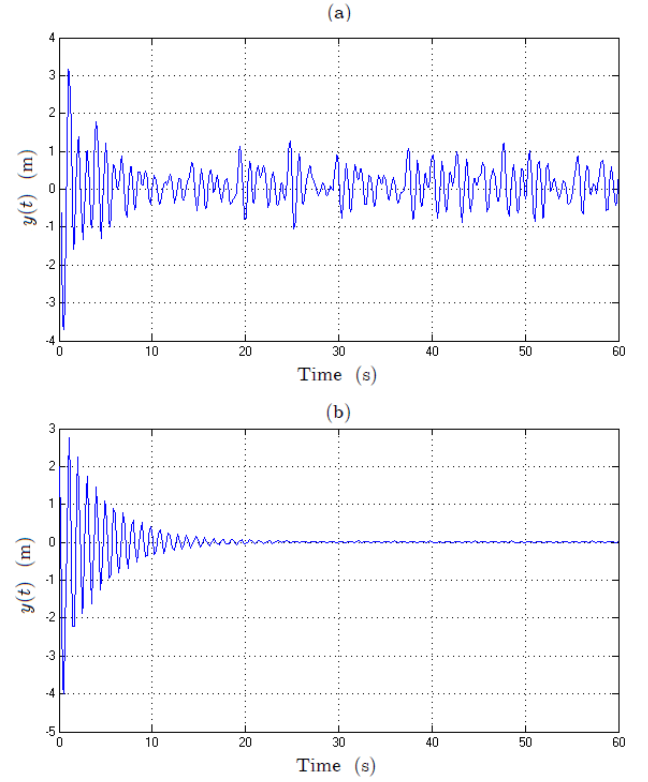


Fig. 4. Variable $y = Y - ROPt$ for: (a) $\Omega_0 = 10 \text{ rad s}^{-1}$ (bit-bouncing), (b) $\Omega_0 = 40 \text{ rad s}^{-1}$.

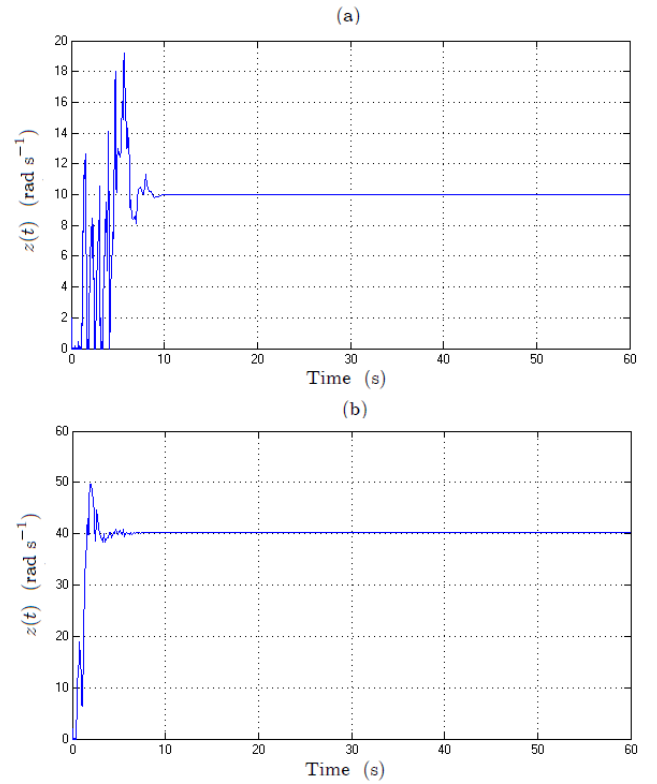


Fig. 5. Angular velocity at the bottom extremity $z(t)$ of the drilling system in closed loop with the control law (26) with: (a) $\Omega_0 = 10 \text{ rad s}^{-1}$ (reduction of stick-slip), (b) $\Omega_0 = 40 \text{ rad s}^{-1}$ (elimination of stick-slip).

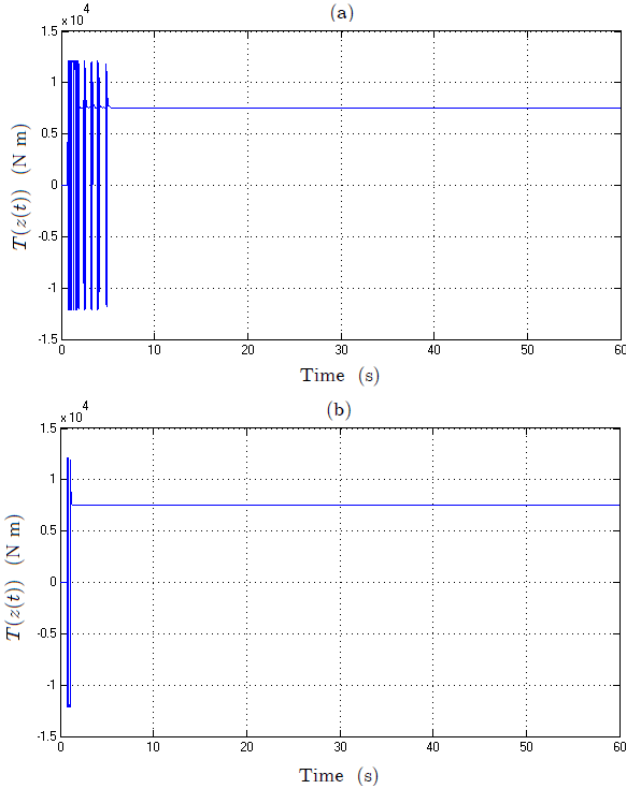


Fig. 6. Torque on the bit $T(z(t))$ of the drilling system in closed loop with the control law (26) with: (a) $\Omega_0 = 10 \text{ rad s}^{-1}$, (b) $\Omega_0 = 40 \text{ rad s}^{-1}$.

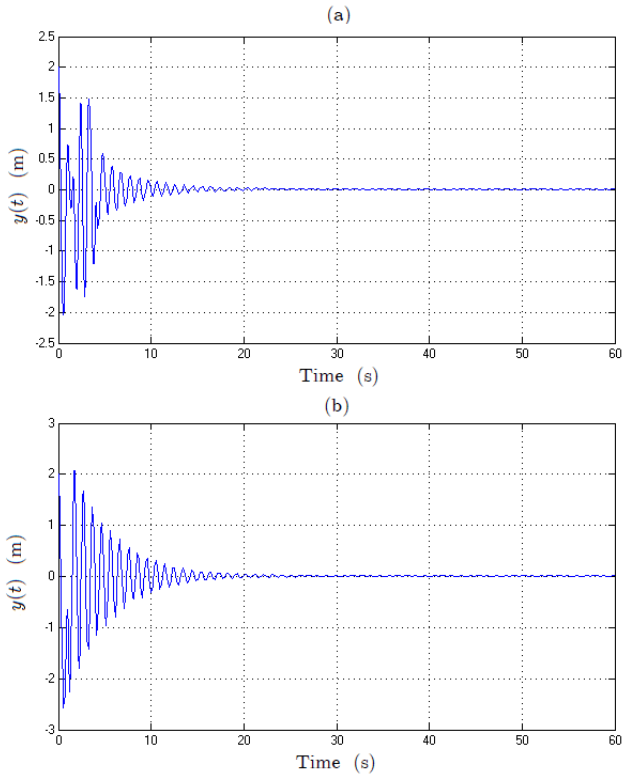


Fig. 7. Variable $y = Y - ROPt$ when the controller (26) is applied with: (a) $\Omega_0 = 10 \text{ rad s}^{-1}$, (b) $\Omega_0 = 40 \text{ rad s}^{-1}$.

(20) by means of the solution of the BMI $\Phi < 0$ using an appropriate computational tool such as the package "PENBMI" of MATLAB. For $\sigma = 1.5$, a feasible solution of the BMI $\Phi < 0$ is:

$$\begin{aligned} \beta &= 1.3627, & K_0 &= -0.1264, & K_1 &= 0.3984, \\ P &= 7.9283, & S &= 17.4167, & R &= 5.9585, \\ P_2 &= 336.8970, & P_6 &= 1.1003, & P_9 &= 1.2411 \times 10^5, \\ P_3 &= 100.1450, & P_7 &= -0.8138, & P_{10} &= 1.5684 \times 10^4, \\ P_4 &= 4.9788, & P_8 &= -97.8304, & P_{11} &= -4.9447 \times 10^4, \\ P_5 &= 39.3970, \end{aligned}$$

The controller $u(t) = -0.1264\dot{x}(t - \tau) + 0.3984x(t - \tau)$ "practically" stabilizes the system (7). Furthermore, it leads a guaranteed evolution of the error:

$$\|x(t)\|^2 \leq e^{-1.5t} \left(3.0497 \|\phi\|_{2\tau}^2 - 0.1146 \right) + 0.1146$$

is satisfied. The stabilizing controller can be written in terms of the bit speed $z(t)$ as:

$$u(t) = -0.1264\dot{z}(t - \tau) + 0.3984z(t - \tau) - 0.3984\Omega_0. \quad (26)$$

Figs. 5-7 show the response of the drilling system when the feedback controller (26) is applied. The stick-slip is eliminated. It is remarkable that, although only the torsional dynamic is directly controlled, the axial vibration amplitude is decreased to satisfactory levels too. It should be pointed out that the selection of the angular velocity at the rotary table level has influence on the drilling system response. The bit-bounce phenomenon is reduced because the source of excitation was eliminated.

Remark 2. Nowadays there are diverse tools which allows the drilling downhole measurements in real time like the Drill String Acceleration Tool (DSA) developed by the Borehole Research Group - Lamont-Doherty Earth Observatory (BRG-LDEO) or the Measurement While Drilling (MWD) tool developed by the Esso Research Company which allows to measure axial, torsional and bending forces and moments; axial, lateral and angular accelerations; internal and external pressure etc. Therefore the proposed controller given in (26) is suitable to be applied regardless of the use of a state observer or estimator.

6. CONCLUSION

A comprehensive analysis of the axial-torsional coupled oscillations occurring in the drilling system has been presented. The drilling model presented in this contribution reproduces the mutual coupling of the aforementioned vibration modes observed in real wells.

It has been shown that the stick-slip phenomenon is reduced by means of increasing the rotary table velocity, however, high rotation speeds give rise to lateral instabilities resulting in whirl motion. For that reason, the stabilizing controller design is essential in suppressing undesirable behaviors.

In this contribution, a control design strategy based on Lyapunov theory for a certain class of nonlinear neutral-type time-delay systems which leads to a "practical stable" closed loop system allowing a guaranteed evolution of the

error has been developed. This approach is applied to the torsional model of the drilling system to suppress the stick-slip phenomenon. Simulation results show that because of the intimate coupling between the torsional and axial vibration, the elimination of the stick-slip vibration also suppresses the bit-bounce.

The obtained controller performance also shows that the drilling operation can be done at lowest speed, which is impossible with empirical strategies.

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