

Robust Fuzzy Backstepping Sliding Mode Controller For a Quadrotor Unmanned Aerial Vehicle

Hicham Khebbache, Mohamed Tadjine

*LCP, Automatic Control Department, National Polytechnic School,
10, Av. Hassen Badi, BP. 182, El-Harrach, Algiers, Algeria
(e-mail: khebbachehicham@yahoo.fr, tadjine@yahoo.fr).*

Abstract: The main purpose of this paper is to integrate fuzzy logic control and sliding mode control techniques based on backstepping approach to develop a robust fuzzy backstepping sliding mode controller (RFBSMC) for an under-actuated quadrotor UAV system under external disturbances and parameter uncertainties. First, a robust backstepping sliding mode control for quadrotor is introduced briefly. Moreover, a fuzzy logic inference mechanism is employed for implementing a fuzzy hitting control law to reduce the chattering phenomena on the conventional RBSMC. Lyapunov based stability analysis shows the main advantage of these control systems which are the trajectory tracking and the stability maintaining of the closed loop dynamics of quadrotor UAV even after occurrence of external disturbances and parameter uncertainties. The effectiveness of the complete system including the quadrotor, and the RFBSMC controller is demonstrated via some simulation results, and its advantages are indicated in comparison with the conventional RBSMC system.

Keywords: Backstepping approach, Fuzzy inference mechanism, Lyapunov stability, Quadrotor, Robust control, Sliding mode control.

1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have been developed for performing various missions in the military and civil areas. Quadrotors are one of UAVs which consist of two rods and four actuators as shown in the Fig. 1. Even though its structure is simple, the quadrotor is a VTOL (Vertical Take-off and Landing) and can perform most of missions that helicopters can do. In some aspects, the quadrotors have better maneuverability than helicopters because quadrotors have four rotors, which can increase the mobility and loadability.

Quadrotors are classified as rotorcraft, as opposed to fixed-wing aircraft, because their lift is derived from four rotors. The use of four rotors allows each individual rotor to have a smaller diameter than the equivalent single-rotor helicopter, allowing them to store less kinetic energy during flight and thus reduces the damage caused by the rotors hitting any objects. By enclosing the rotors within a frame, the rotors can be protected during collisions

The dynamic model of quadrotor UAV has six degree-of-freedom (DOF) with only four independent thrust forces generated by four rotors. It is difficult to control all these six outputs with only four control inputs. Moreover, uncertainties associate with dynamic model also bring more challenge for control design. To solve the quadrotor UAV tracking control problem, many techniques have been proposed. First of all, several backstepping and sliding mode controllers have been developed. (Bouabdallah *et al.*, 2005) presented a backstepping and sliding mode controllers in order to stabilize the

complete system (*i.e.* translation and orientation). However, a full-state backstepping technique based on Lyapunov stability theory and a backstepping sliding mode control are studied in (Madani *et al.*, 2006a, 2006b). Yet other Nonlinear control methods using backstepping and sliding mode approaches have been proposed in (Adigbli *et al.*, 2007). Moreover, a backstepping control method, which allowed the tracking of the various desired trajectories along (x, y, z) axis and yaw angle is proposed in (Bouadi *et al.*, 2007). Another backstepping controller, introducing the Frenet-Serret Theory (Backstepping+FST) is used for attitude stabilization (Colorado *et al.*, 2010), that includes estimation of the desired angular acceleration as a function of the aircraft velocity. Furthermore, a modified backstepping approach, which reduce the number of control parameters by half compared with the classical backstepping approach used in the literature is developed and applied to control the quadrotor UAV in (Saif *et al.*, 2012). A fuzzy integral sliding mode-backstepping controller for an autonomous quadrotor helicopter (X4-flyer) is proposed in (Meguenni *et al.*, 2012), in which the integral action is replaced by an inference fuzzy system to eliminate the static error. Moreover, a boundary layer ("sat" function) is used to reduce the chattering phenomenon. However, the fuzzy logic inference mechanism is not employed in the stability analysis.

There are also robust controllers designed for quadrotor systems. The authors in (Raffo *et al.*, 2008) proposed a nonlinear H_∞ controller in order to stabilize the rotational movements, whereas a control law based on backstepping approach was used to solve the path tracking problem of the quadrotor UAV. While in (Bouchoucha *et al.*, 2008, 2011), a robust nonlinear PI, a classical and second order sliding mode

control techniques for attitude stabilization and attitude tracking under external disturbances have been proposed and successfully validated in simulation and real time. Moreover, an integral backstepping and an integral sliding mode controllers for the same objectives of previous authors have been also implemented in real time on an embedded system Based on a dsPIC μC (Seghour *et al.*, 2011), a direct adaptive sliding mode controller for attitude stabilization and altitude trajectory tracking of the quadrotor in presence of parameter uncertainties is proposed in (Bouadi *et al.*, 2011). A robust control approach denoted sliding control based on the output feedback linearization is developed for quadrotor system to attenuate the parametric uncertainties (Khelfi *et al.*, 2012). Another nonlinear control algorithms based on the super-twisting algorithm (STA) which are able to ensure robustness with respect to bounded external disturbances, has been designed for attitude stabilization and attitude tracking with an experimental implementations on a real quadrotor (Derafa *et al.*, 2012), and combined with the block control (BC) technique for trajectory tracking of a quadrotor helicopter (Luque-Vega *et al.*, 2012a, 2012b, 2012c).

Yet other robust popular methods for handling unknown nonlinearities are to introduce neural networks tuned online, by using neural network observer with an output feedback controller (Dierks *et al.*, 2010), by using adaptive control techniques (Das *et al.*, 2009; Nicol *et al.*, 2011), and by using sliding-mode under-actuated control (SMUC) to design a hybrid neural-network-based sliding-mode under-actuated control (HNNMUC) (Hwang, 2012) for the under-actuated quadrotor system.

The aim of this paper is to design a robust control scheme for a quadrotor UAV under bounded uncertainties. To accomplish the mentioned motivation a RFBSMC system is developed for this system. First, a traditional RBSMC system is introduced, in which, its robustness toward uncertainties is demonstrated. However, the undesired chattering phenomenon may exist in inputs control of quadrotor. In order to remedy this phenomenon, a fuzzy hitting control laws are embedded into the RBSMC system, where the Lyapunov stability theorem is used to proofing the robustness of the RFBSMC system. This study is organized as follows. In Section 2, the nonlinear model of a quadrotor aircraft is presented. The proposed robust control algorithms are described in Section 3. Section 4 is devoted to the presentation and the discussion of simulation results, when the proposed scheme is applied to the quadrotor. Finally, conclusions and futures advances are provided in Section 5.

2. DYNAMICAL MODEL

2.1 Quadrotor dynamic model

The quadrotor is composed of four rotors. Two diagonal motors (1 and 3) are running in the same direction whereas the others (2 and 4) in the other direction to eliminate the anti-torque. On varying the rotor speeds altogether with the same quantity the lift forces will change affecting in this case the altitude z of the system and enabling vertical take-off/on landing. Yaw angle ψ is obtained by speeding up/slowing

down the diagonal motors depending on the desired direction. Roll angle ϕ allows the quadrotor to move towards y direction. Pitch angle θ allows the quadrotor to move towards x direction. The rotor is the primary source of control and propulsion for the UAV. The Euler angle orientation to the flow provides the forces and moments to control the altitude and position of the system (Fig. 1).

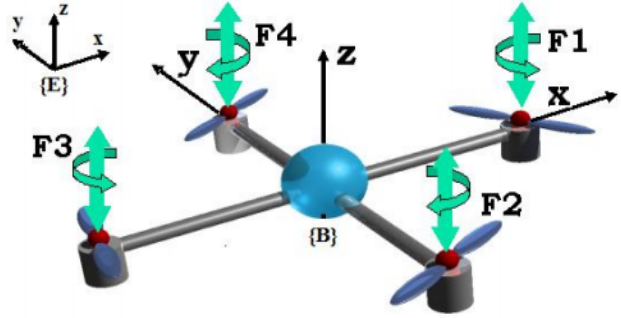


Fig. 1. Quadrotor configuration.

The absolute position is described by the three coordinates (x, y, z) and its attitude by the three Euler's angles (ϕ, θ, ψ) . under the conditions $(-\pi/2 < \phi < \pi/2)$ for roll, $(-\pi/2 < \theta < \pi/2)$ for pitch and $(-\pi \leq \psi < \pi)$ for yaw. The complete model (position and orientation dynamic) is given like in (Bouabdallah *et al.*, 2004, 2005) by:

$$\begin{cases} \ddot{\phi} = \frac{(I_y - I_z)}{I_x} \dot{\theta} \dot{\psi} - \frac{J_r}{I_x} \Omega_r \dot{\theta} + \frac{l}{I_x} u_2 \\ \ddot{\theta} = \frac{(I_z - I_x)}{I_y} \dot{\phi} \dot{\psi} + \frac{J_r}{I_y} \Omega_r \dot{\phi} + \frac{l}{I_y} u_3 \\ \ddot{\psi} = \frac{(I_x - I_y)}{I_z} \dot{\phi} \dot{\theta} + \frac{1}{I_z} u_4 \\ \ddot{z} = -g + \frac{\cos(\phi) \cos(\theta)}{m} u_1 \\ \ddot{x} = \frac{1}{m} (\cos \phi \cos \psi \sin \theta + \sin \phi \sin \psi) u_1 \\ \ddot{y} = \frac{1}{m} (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) u_1 \end{cases} \quad (1)$$

The system's inputs are posed u_1, u_2, u_3, u_4 and Ω_r a disturbance, obtaining:

$$\begin{cases} u_1 = b (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ u_2 = lb (\omega_4^2 - \omega_2^2) \\ u_3 = lb (\omega_3^2 - \omega_1^2) \\ u_4 = d (-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \\ \Omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4 \end{cases} \quad (2)$$

where, $I(x,y,z)$ is the body inertia, J_r is the rotor inertia, m is the total mass of the structure, g is the gravity constant, b is the thrust coefficient, d is the drag coefficient, l is the distance from the center of mass to the rotor shaft, and ω_i is the angular speed of the rotor i .

2.2 Rotors dynamic model

The rotors are driven by DC motors with the well known equations (Tayebi et al. 2006):

$$\begin{cases} J_r \dot{\omega}_i = \tau_i - Q_i \\ v_i = \frac{R_a}{k_m k_g} \tau_i + k_m k_g \omega_i, \quad i \in \{1, 2, 3, 4\} \end{cases} \quad (3)$$

where, $\tau_i, i \in \{1, \dots, 4\}$ are the control inputs of the system, which represent the torques produced by the rotors, Q_i is the reactive torque generated, in free air, by the rotor i due to rotor drag, which is given by $Q_i = d\omega_i^2$, R_a is the motor resistance, k_m is the motor torque constant, k_g is the gear ratio, and v_i is voltage of the motor i .

3. ROBUST CONTROLLERS DESIGN

The main object of the control algorithms developed in this paper is to design a robust output tracking controller which makes the output of the system $\{x(t), y(t), z(t), \psi(t)\}$ to track the desired output $\{x_d(t), y_d(t), z_d(t), \psi_d(t)\}$.

The uncertain model resulting by adding of unknown terms which represent external disturbances and parameter uncertainties in the model (1) can be written in a state-space form $\dot{X} = f(X) + g(X)U + d(X, U) = f(X, U) + d(X, U)$ with X state vector, U inputs vector, and $d(X, U)$ uncertain vector, such as:

$$\begin{aligned} X &= (x_1, \dots, x_{12})^T = (\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z, \dot{z}, x, \dot{x}, y, \dot{y})^T \\ U &= (u_1, u_2, u_3, u_4)^T \end{aligned} \quad (4)$$

Therefore:

$$f(X, U) = \begin{pmatrix} x_2 \\ a_1 x_6 + a_2 \Omega x_4 + b_1 u_2 \\ x_4 \\ a_3 x_6 + a_4 \Omega x_2 + b_2 u_3 \\ x_6 \\ a_5 x_2 + b_3 u_4 \\ x_8 \\ -g + \cos(x_1) \cos(x_3) u_1 / m \\ x_{10} \\ u_x u_1 / m \\ x_{12} \\ u_y u_1 / m \end{pmatrix}, \quad d(X, U) = \begin{pmatrix} 0 \\ A_\phi / I_x + \delta_\phi(x_4, x_6, u_2) \\ 0 \\ A_\theta / I_y + \delta_\theta(x_2, x_6, u_3) \\ 0 \\ A_\psi / I_z + \delta_\psi(x_2, x_4, u_4) \\ 0 \\ A_z / m + \delta_z(x_1, x_3, u_1) \\ 0 \\ A_x / m + \delta_x(u_1) \\ 0 \\ A_y / m + \delta_y(u_1) \end{pmatrix} \quad (5)$$

with

$$\begin{cases} a_1 = \frac{I_y - I_z}{I_x}, \quad a_2 = -\frac{J_r}{I_x}, \quad a_3 = \frac{I_z - I_x}{I_y}, \quad a_4 = \frac{J_r}{I_y}, \quad a_5 = \frac{I_x - I_y}{I_z} \\ b_1 = \frac{l}{I_x}, \quad b_2 = \frac{l}{I_y}, \quad b_3 = \frac{1}{I_z} \end{cases} \quad (6)$$

$$\begin{cases} u_x = (\cos \phi \cos \psi \sin \theta + \sin \phi \sin \psi) \\ u_y = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \end{cases} \quad (7)$$

Where $A_T = (A_\phi, A_\theta, A_\psi)^T$ and $A_F = (A_x, A_y, A_z)^T$ are the resulting aerodynamic forces and moments acting on the quadrotor, they are computed from the aerodynamic coefficients C_i as $A_i = (1/2) \rho_{air} C_i W^2$ Where ρ_{air} is the air density and W is the velocity of the UAV with respect to air (Gessow et al., 1967; Benallegue et al., 2008). Moreover, $(\delta_\phi, \delta_\theta, \delta_\psi)^T$ and $(\delta_x, \delta_y, \delta_z)^T$ are the parameter uncertainties related to the quadrotor motions.

Assumption 1: The model uncertainty is assumed to be bounded, as follows:

$$|d_j(X, U)| \leq d_j^+; \quad i \in [1, \dots, 6] \text{ and } j = 2i \quad (8)$$

Where $\{d_1^+, d_2^+, d_3^+, d_4^+, d_5^+, d_6^+\}$ are positive constants.

3.1 Adopted control strategy of quadrotor

It is worthwhile to note in the quadrotor system that the angles and their time derivatives do not depend on translation components. On the other hand, the translations depend on the angles. We can ideally imagine the overall system described by (5) as constituted of two subsystems, the angular rotations and the linear translations.

From (7) it easy to show that:

$$\begin{cases} \phi_d = \arcsin(u_x \sin(\psi_d) - u_y \cos(\psi_d)) \\ \theta_d = \arcsin\left(\frac{(u_x \cos(\psi_d) + u_y \sin(\psi_d))}{\cos(\phi_d)}\right) \end{cases} \quad (9)$$

The control scheme advocated for the overall system is then logically based on two loops (internal loop and external loop). The internal loop contains four control laws: control of roll, control of pitch, control of yaw and control of altitude. The external loop includes two control laws of positions x and y . The external control loop generates a desired reference of roll (ϕ_d) and pitch (θ_d) through the correction block. This block corrects the rotation of roll and pitch depending on the desired yaw (ψ_d) (illustrated by equation (9)). The synoptic scheme below shows this control strategy:

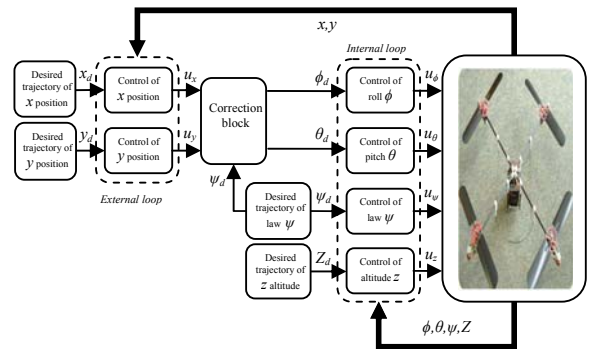


Fig. 2. Synoptic scheme of the proposed control strategy

3.2 Robust backstepping sliding mode control of Quadrotor

The recursive nature of the proposed control design is similar to the standard backstepping methodology. However, the proposed control design uses backstepping to design controllers with a zero-order sliding surface at each step (Zhou *et al.*, 2007). The benefit of this approach is that each actual controller can compensate the unknown bounded terms $di(t)$; $i \in [1, \dots, 6]$. The design proceeds as follows:

3.2.1 Attitude control

Three separate controllers are designed to track the desired roll, pitch, and yaw angles. For the first step we consider zero-order sliding surface:

$$s_1 = x_1 - \phi_d \quad (10)$$

Let the First Lyapunov function candidate:

$$V_{\phi 1}(s_1) = \frac{1}{2}s_1^2 \quad (11)$$

The time derivative of (11) is given by:

$$\dot{V}_{\phi 1}(s_1) = s_1 \dot{s}_1 = s_1(x_2 - \dot{\phi}_d) \quad (12)$$

The stabilization of s_1 can be obtained by introducing a new virtual control x_2 :

$$x_{2d} = \dot{\phi}_d - c_1 s_1; \quad c_1 > 0 \quad (13)$$

The equation (12) is then:

$$\dot{V}_{\phi 1}(s_1) = -c_1 s_1^2 \leq 0 \quad (14)$$

For the second step we consider the following zero-order sliding surface:

$$s_2 = x_2 - x_{2d} = x_2 - \dot{\phi}_d + c_1 s_1 \quad (15)$$

The augmented Lyapunov function is given by:

$$V_{\phi 2}(s_1, s_2) = \frac{1}{2}(s_1^2 + s_2^2) \quad (16)$$

Its time derivative is then:

$$\begin{aligned} \dot{V}_{\phi 2}(s_1, s_2) &= s_1 \dot{s}_1 + s_2 \dot{s}_2 \\ &= s_1(-c_1 s_1 + s_2) + s_2(\ddot{x}_2 - \ddot{\phi}_d + c_1 \dot{s}_1) \\ &= -c_1 s_1^2 + s_2(s_1 + a_1 x_4 x_6 + a_2 \Omega_r x_4 \\ &\quad + b_1 u_2 + d_1 - \ddot{\phi}_d + c_1(-c_1 s_1 + s_2)) \end{aligned} \quad (17)$$

In order to compensate the unknown term $d_1(t)$ related to roll motion, an auxiliary control effort is referred to as hitting control effort represented by $(u_{h,1})$, it is given as follows:

$$u_{h,1} = k_{h,1} \text{sign}(s_2) \quad (18)$$

The stabilization of (s_1, s_2) can be obtained by introducing the following sliding mode actual control:

$$\begin{aligned} u_2 &= \frac{1}{b_1}(u_{2n} - u_{h,1}) \\ &= \frac{1}{b_1}((\ddot{\phi}_d - c_1(-c_1 s_1 + s_2) - s_1 - c_2 s_2 - a_1 x_4 x_6 - a_2 \Omega_r x_4) - u_{h,1}) \end{aligned} \quad (19)$$

It result that:

$$\begin{aligned} \dot{V}_{\phi 2}(s_1, s_2) &= -c_1 s_1^2 - c_2 s_2^2 - s_2(k_{h,1} \text{sign}(s_2) - d_1) \\ &\leq -c_1 s_1^2 - c_2 s_2^2 - |s_2|(k_{h,1} - d_1^+) \leq 0 \end{aligned} \quad (20)$$

where $\{c_2, k_{h,1}\}$ are positive constants with $k_{h,1} > d_1^+$, $\text{sign}(\cdot)$ is the usual sign function.

Similarly, pitch and yaw controls are:

$$\begin{aligned} u_3 &= \frac{1}{b_2}(\ddot{\theta}_d - c_3(-c_3 s_3 + s_4) - s_3 - c_4 s_4 - a_3 x_2 x_6 \\ &\quad - a_4 \Omega_r x_2 - k_{h,2} \text{sign}(s_4)) \end{aligned} \quad (21)$$

$$\begin{aligned} u_4 &= \frac{1}{b_3}(\ddot{\psi}_d - c_5(-c_5 s_5 + s_6) - s_5 - c_6 s_6 - a_5 x_2 x_4 \\ &\quad - k_{h,3} \text{sign}(s_6)) \end{aligned}$$

with

$$\begin{cases} s_3 = x_3 - \theta_d \\ s_4 = x_4 - \dot{\theta}_d + c_3 s_3 \\ s_5 = x_5 - \psi_d \\ s_6 = x_6 - \dot{\psi}_d + c_5 s_5 \end{cases} \quad (22)$$

where $\{c_3, c_4, c_5, c_6, k_{h,2}, k_{h,3}\}$ are positive constants with $k_{h,2} > d_2^+, k_{h,3} > d_3^+$.

3.2.2 Altitude control

The altitude control u_1 is obtained using the same approach described in the previous section.

$$\begin{aligned} u_1 &= \frac{m}{(\cos(x_1) \cos(x_3))}(\ddot{z}_d - c_7(-c_7 s_7 + s_8) - s_7 - c_8 s_8 \\ &\quad + g - k_{h,4} \text{sign}(s_8)) \end{aligned} \quad (23)$$

with

$$\begin{cases} s_7 = x_7 - z_d \\ s_8 = x_8 - \dot{z}_d + c_7 s_7 \end{cases} \quad (24)$$

where $\{c_7, c_8, k_{h,4}\}$ are positive constants with $k_{h,4} > \delta_4^+$.

3.2.3 Position control

The zero-order sliding surfaces for x and y positions are defined as:

$$\begin{cases} s_9 = x_9 - x_d \\ s_{10} = x_{10} - \dot{x}_d + c_9 s_9 \\ s_{11} = x_{11} - y_d \\ s_{12} = x_{12} - \dot{y}_d + c_{11} s_{11} \end{cases} \quad (25)$$

The control laws are derived using RBSMC technique as:

$$\begin{aligned} u_x &= \frac{m}{u_1} (\ddot{x}_d - c_9(-c_9 s_9 + s_{10}) - s_9 - c_{10} s_{10} \\ &\quad - k_{h,5} \text{sign}(s_{10})); \quad u_1 \neq 0 \\ u_y &= \frac{m}{u_1} (\ddot{y}_d - c_{11}(-c_{11} s_{11} + s_{12}) - s_{11} - c_{12} s_{12} \\ &\quad - k_{h,6} \text{sign}(s_{12})); \quad u_1 \neq 0 \end{aligned} \quad (26)$$

where $\{c_9, c_{10}, c_{11}, c_{12}, k_{h,5}, k_{h,6}\}$ are positive constants with $k_{h,5} > d_5^+$, $k_{h,6} > d_6^+$.

3.3 Robust fuzzy backstepping sliding mode control of Quadrotor

The major advantage of control algorithms based on sliding mode techniques is its insensitivity to parameter variations and external load disturbances once on the switching surface. Unfortunately, such performances are obtained at price of extremely high control activity. As consequence, the *chattering* phenomenon always occurs in the sliding and steady state modes as high frequency oscillations about the desired equilibrium point of the system, and may excite unmodelled high frequency dynamics. Therefore, an RFBSMC system, in which a fuzzy logic inference mechanism is used to mimic the hitting control laws will be developed in this section.

Let the sliding surfaces $s_j, j \in [2, 4, 6, 8, 10, 12]$ be the input linguistic variables of the fuzzy logic system, and the hitting control laws $u_{f,j}, j \in [1, \dots, 6]$ be the output linguistic variables. According to the spirit of the hitting control laws, one can systematically build the fuzzy control rule base involved in the RFBSMC system as follows (Wai, 2007):

- Rule1: If s_j is N, then $u_{f,j}$ is NE
- Rule2: If s_j is Z, then $u_{f,j}$ is ZE
- Rule3: If s_j is P, then $u_{f,j}$ is PE

with the fuzzy labels for s_j and $u_{f,j}$; Z: Zero, N: Negative, P: Positive, ZE: Zero effort, NE: Negative effort, PE: Positive effort. Each fuzzy label of the input variables s_j is triangular membership function, fuzzy labels of the output variables $u_{f,j}$ are a singletons membership functions as shown in Fig. 3.

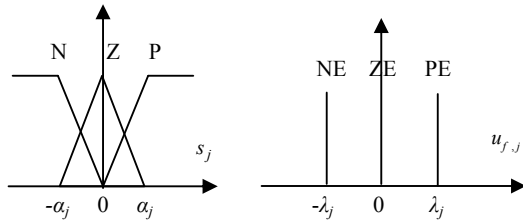


Fig. 3. Membership functions of input s_j and outputs $u_{f,j}$.

Then, the hitting control laws can be calculated by fuzzy logic inference mechanism as:

$$u_{f,j} = \frac{\sum_{i=1}^3 \mu_{j,i} \lambda_{j,i}}{\sum_{i=1}^3 \mu_{j,i}} = \mu_{j,1} \lambda_{j,1} + \mu_{j,2} \lambda_{j,2} + \mu_{j,3} \lambda_{j,3}; \quad j \in [1, \dots, 6] \quad (27)$$

where $0 \leq \mu_{j,1} \leq 1$, $0 \leq \mu_{j,2} \leq 1$ and $0 \leq \mu_{j,3} \leq 1$ are the firing strengths of rules 1, 2 and 3, respectively; $\lambda_{j,1} = \lambda_j$, $\lambda_{j,2} = 0$ and $\lambda_{j,3} = -\lambda_j$ are the center of output membership functions (ZE, NE and PE). The relation $\mu_{j,1} + \mu_{j,2} + \mu_{j,3} = 1$ is valid according to the triangular membership functions. Moreover, the fuzzy hitting control efforts can be further analyzed as the following four conditions, and only one of four conditions will occur for any value of s_j according to Fig. 3.

Condition 1: only rule 1 is triggered ($s_j > \alpha_j$, $\mu_{j,1} = 1$, $\mu_{j,2} = \mu_{j,3} = 0$)

$$u_{f,j} = \lambda_j; \quad j \in [1, \dots, 6] \quad (28)$$

Condition 2: rule 1 and 2 are triggered simultaneously ($0 < s_j \leq \alpha_j$, $0 < \mu_{j,1}, \mu_{j,2} \leq 1$, $\mu_{j,3} = 0$)

$$u_{f,j} = \lambda_{j,1} \mu_{j,1} = \lambda_j \mu_{j,1}; \quad j \in [1, \dots, 6] \quad (29)$$

Condition 3: rule 2 and 3 are triggered simultaneously ($-\alpha_j < s_j \leq 0$, $\mu_{j,1} = 0$, $0 \leq \mu_{j,2}, \mu_{j,3} < 1$)

$$u_{f,j} = \lambda_{j,3} \mu_{j,3} = -\lambda_j \mu_{j,3}; \quad j \in [1, \dots, 6] \quad (30)$$

Condition 4: only rule 3 is triggered ($s_j \leq -\alpha_j$, $\mu_{j,1} = \mu_{j,2} = 0$, $\mu_{j,3} = 1$)

$$u_{f,j} = -\lambda_j; \quad j \in [1, \dots, 6] \quad (31)$$

According to (28)-(31), it can see that $s_j (\mu_{j,1} - \mu_{j,3}) \geq 0$ (Wai, 2007). Totally, the RFBSMC laws can be represented as:

$$\begin{aligned} u_i &= \frac{1}{g_i} (u_{in} - u_{f,j}) & i \in [2, 3, 4, 1], j \in [1, \dots, 4] \\ &= \frac{1}{g_i} (u_{in} - \lambda_j (\mu_{j,1} - \mu_{j,3})) & \text{and } l = 2j \\ u_{\chi} &= \frac{m}{u_1} (u_{\chi^n} - u_{h,j}) & \chi \in [x, y] \text{ and } j \in [5, 6] \\ &= \frac{m}{u_1} (u_{\chi^n} - \lambda_j (\mu_{j,1} - \mu_{j,3})) \end{aligned} \quad (32)$$

Thus, define a global Lyapunov function candidate as:

$$V = \frac{1}{2} \sum_{i=1}^{12} s_i^2; \quad i \in [1, \dots, 12] \quad (33)$$

the time derivative of this Lyapunov function gives:

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^{12} c_i s_i^2 - \sum_{j=1}^6 s_l (\lambda_j (\mu_{j,1} - \mu_{j,3}) - d_j); \quad l = 2j \\ &\leq -\sum_{i=1}^{12} c_i s_i^2 - \sum_{j=1}^6 |s_l| (\lambda_j |\mu_{j,1} - \mu_{j,3}| - d_j^+); \quad l = 2j \end{aligned} \quad (34)$$

If the following inequality is satisfied:

$$\lambda_j > \frac{d_j^+}{|\mu_{j,1} - \mu_{j,3}|} ; j \in [1, \dots, 6] \quad (35)$$

then, $\dot{V} \leq 0$, it can be concluded that the RFBSMC controller given by (32) guarantees the stability of the controlled system under bounded uncertainties representing the external disturbances and the parameter uncertainties.

4. SIMULATION RESULTS

In order to verify the performance of the RFBSMC system developed in this paper, three simulations were made considering different flight conditions. First, an application without uncertainties. Second, a simulation with aerodynamic force and moment disturbances and finally, one considering a variation of model parameters (body inertia, rotor inertia and total mass of the system), with introducing the aerodynamic force and moment disturbances.

The physical parameters of the used quadrotor are: $m = 0.42$ kg, $g = 9.806$ m/s², $l = 20.5$ cm, $b = 2.9842 \times 10^{-5}$ N/rad/s, $d = 3.2320 \times 10^{-7}$ N.m/rad/s, $J_r = 2.8385 \times 10^{-5}$ kg.m², $I_x = I_y = 3.8278 \times 10^{-3}$ kg.m², $I_z = 7.1345 \times 10^{-3}$ kg.m² (Derafa *et al.*, 2006, 2012). The motors parameters are: $k_{m,i} = 4. \times 10^{-3}$ N.m/A, $k_{g,i} = 5.6$, $R_i = 0.67$ Ω , $i = \{1, \dots, 4\}$ (Taybi *et al.* 2006). The initial conditions for the helicopter are: $x_i(0) = \text{col}(0,0,0,0.5,0)$ with $i = \{1, \dots, 6\}$ for rotational subsystem and $x_i(0)=0$ with $i = \{7, \dots, 12\}$ for translational one.

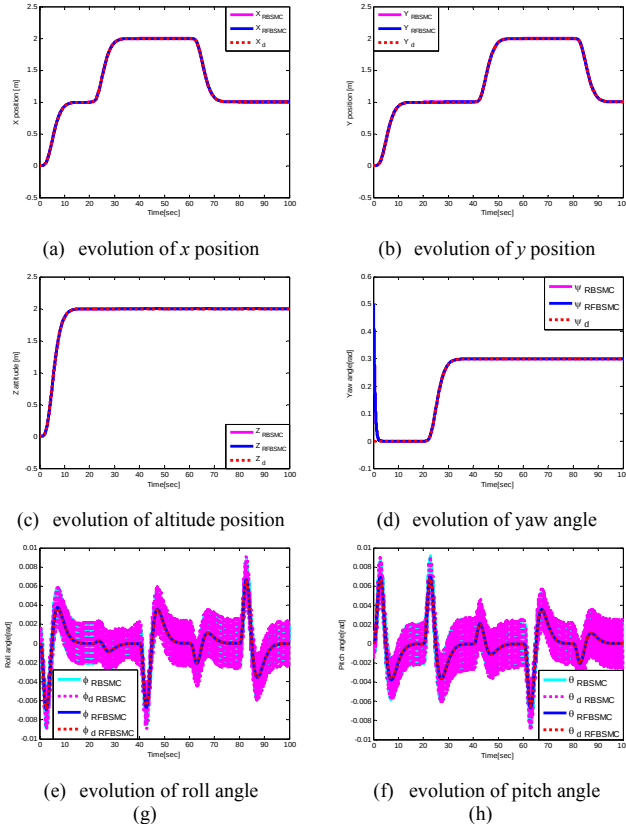


Fig. 4. Tracking simulation results of all trajectories, Case a.

The controllers parameters are:

For RBSMC system: $c_i = \text{col}(2,5,2,5,2,5,5,10,2,5,2,5)$ with $i = \{1, \dots, 12\}$, and $k_{h,j} = 2.5$ with $j = \{1, \dots, 6\}$. For RFBSMC system: $c_i = \text{col}(2,5,2,5,2,5,5,10,2,5,2,5)$ with $i = \{1, \dots, 12\}$, $\alpha_j = 0.5$, $\lambda_j = 4$, with $j = \{1, \dots, 6\}$.

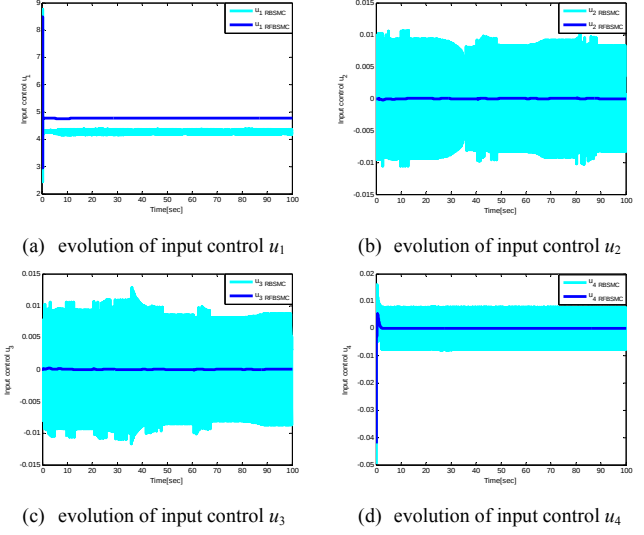


Fig. 5. Simulation results of all inputs control, Case a.

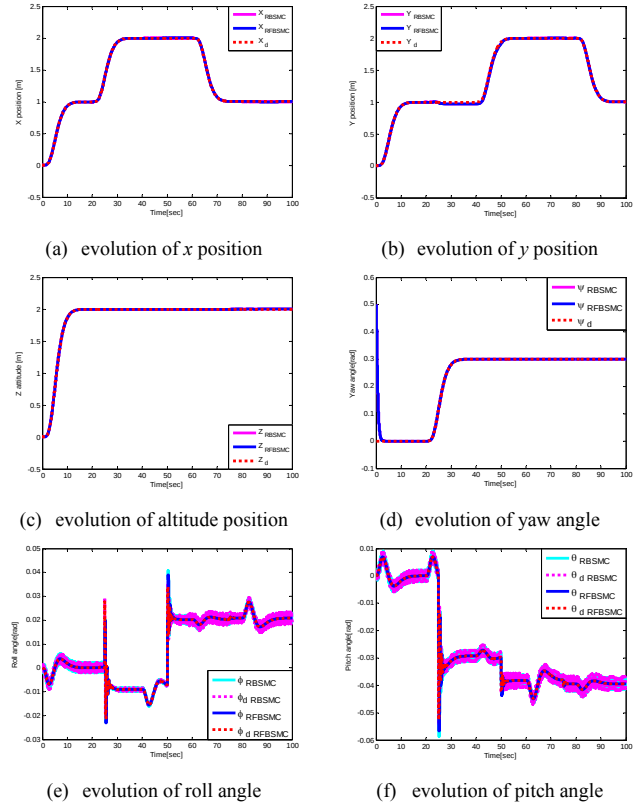


Fig. 6. Tracking simulation results of all trajectories, Case b.

For reason of derivations of the virtual controls, the desired trajectories are chosen in a manner to avoid initial conditions problem. So the reference trajectories chosen for $x_d(t)$, $y_d(t)$,

$z_d(t)$ and $\psi_d(t)$ are filtered with a six order filter defined by the transfer function $1/(s+1)^6$ where s is the *Laplace* variable to make it smooth in curve and zero initial conditions before exciting the system. The reference trajectories used in simulation are given like in (Luque-Vega *et al.*, 2012a).

Case a: Flight without disturbances

In the first case, we have a free flight without any uncertainty. The obtained results are shown in Figs. 4-5.

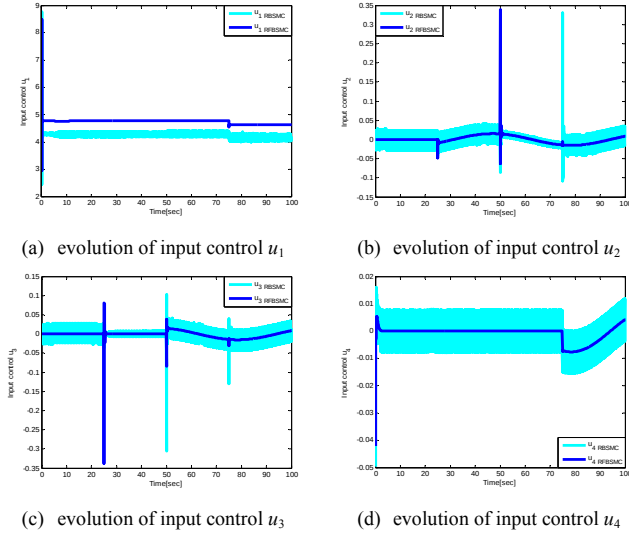


Fig. 7. Simulation results of all inputs control, Case b.

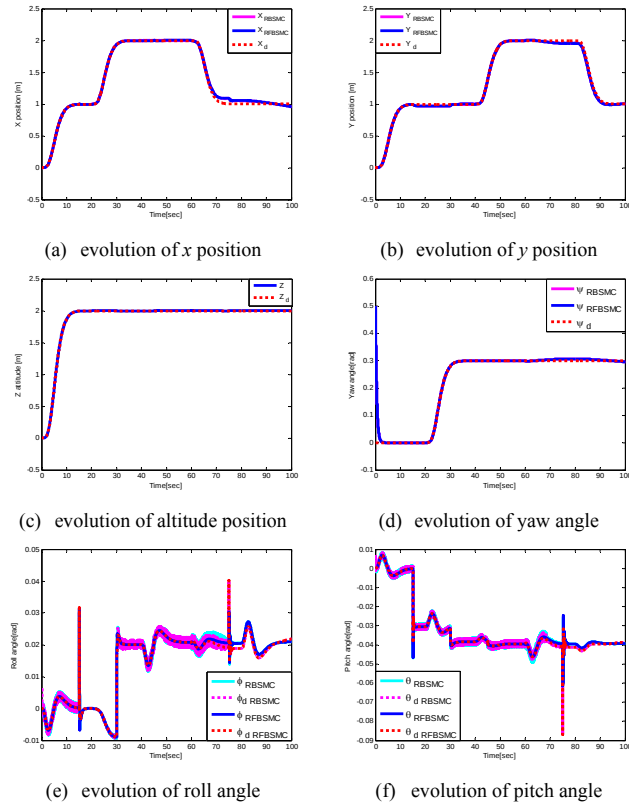


Fig. 8. Tracking simulation results of all trajectories, Case c.

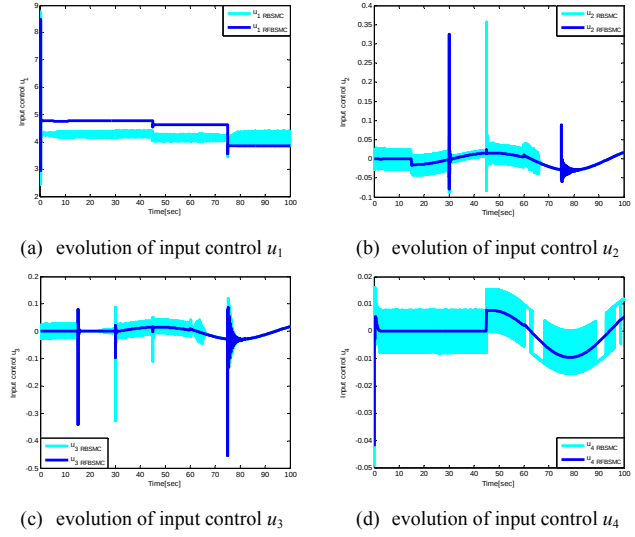


Fig. 9. Simulation results of all inputs control, Case c.

Case b: Flight with bounded external disturbances

Here, the aerodynamic force disturbances are $A_x = 0.126$, $A_y = 0.126$ and $A_z = 0.126$ occurring respectively at 25, 50 and 75s. Moreover, the aerodynamic moment disturbances are $A_\phi = 0.004 \times \sin(0.1t)$, $A_\theta = 0.004 \times \sin(0.1t)$, and $A_\psi = 0.008 \times \sin(0.1t)$ occurring also at 25, 50, and 75s respectively. The obtained results are depicted in Figs. 6-7.

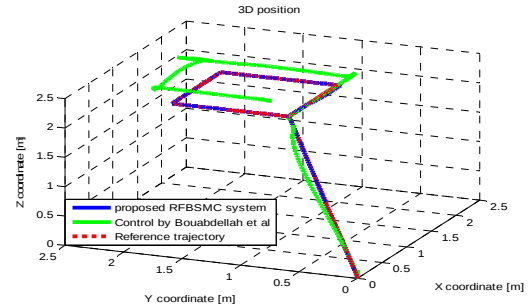


Fig. 10. Comparison between the RFBSMC system and the controller proposed by Bouabdallah et al. 2005, Case b.

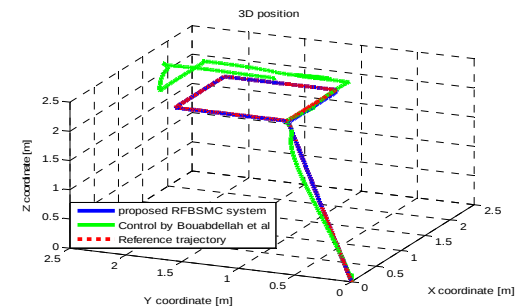


Fig. 11. Comparison between the RFBSMC system and the controller proposed by Bouabdallah et al. 2005, Case c.

Case c: Flight with bounded external disturbances and parameter uncertainties

For this latest case, the aerodynamic force and moment disturbances are ($A_x = 0.126$, $A_\phi = 0.004 \times \sin(0.1t)$), ($A_y = 0.126$, $A_\theta = 0.004 \times \sin(0.1t)$), ($A_z = 0.126$, $A_\psi = 0.008 \times \sin(0.1t)$) occurring at 15, 30 and 45s, respectively. Moreover, we consider an uncertainty of 50% on (a_i, b_i) with (i, j) = (1,..., 5; 1,..., 3) for rotational subsystem, and 50% on ($1/m$) for translational one occurring at 60 and 75s, respectively. The obtained results are shown in Figs. 8-9.

It is concluded from the simulations, made without disturbances (Case a), that the proposed controller systems gives satisfactory results of the quadrotor aircraft with regard to the desired trajectory, showing that the desired pitch and roll angles are bounded, with a strongly reducing of chattering phenomenon in RFBSMC system compared to the RBSMC system (see Figs. 4 (e,f) and 5). When aerodynamic force and moment disturbances are introduced (Case b), or even the appearance of parameter uncertainties with the presence of aerodynamic force and moment disturbances (Case c), the results in Figs. 6 and 8 reflect the robustness of the good performance of the proposed control algorithm. It is able to reject them thanks to the robustness properties of the fuzzy hitting control efforts in the RFBSMC system, without the need of any estimation procedure. It is worth noting that the obtained input control signals given by the proposed RFBSMC system are acceptable and physically realizable (see Figs. 7 and 9). On the other hand, Fig. 10 and Fig. 11 shows a comparison between the 3D position along (x, y, z) axis obtained using the RFBSMC system and the one proposed by (Bouabdallah et al., 2005), where the good performance of the proposed algorithm scheme is highlighted and the importance of considering the bounded uncertainties summarized by external disturbances and parameter uncertainties in the analysis is justified.

5. CONCLUSIONS AND FUTUR WORKS

To realize a performing and robust control of the quadrotor aircraft, a robust fuzzy backstepping sliding mode controller (RFBSMC) is developed. Compared with RBSMC system, the RFBSMC system results in robust control performance with strongly reduced chattering control efforts. It can be concluded from all simulation results above that the both control systems work effectively. In spite of the occurring external disturbances and parameter uncertainties, the dynamic behavior of quadrotor aircraft presents high performances in terms of stability, trajectory tracking and robustness with respect to external disturbances and parameter uncertainties. In the further work, the experimental implementation of the proposed control scheme will be addressed.

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