Structural Analysis for Fault Detection and Isolation Using the Matching Rank Algorithm for Residual Generation: Application on an Industrial Water Heating System

Alem Saïd, Benazzouz Djamel, Ichalalen Boudjema

Solid Mechanics & Systems Laboratory (LMSS)
University M'Hamed Bougara Boumerdes, Algeria;
E-mails: alem-said@umbb.dz, dbenazzouz@umbb.dz

Abstract: In this paper an innovative way of dealing with the generation of residuals for fault detection and isolation based on structural information is presented. The developed technique considers implementation issues therefore; it has a more realistic point of view compared to classical structural approaches. First, theoretical aspects of structural analysis are considered and introduced. Then the method of introducing them to test the structural proprieties is presented. Finally, we show how the matching rank algorithm can be applied in order to choose the most suited matching that leads to residual computational sequences. The approach is applied and demonstrated through an industrial application.

Keywords: Structural analysis, bipartite graph, analytic redundancy relations, fault detection and isolation, water heating system.

1. INTRODUCTION

In complex systems having a great number of sensors, the obtained information starting from the various parameters and signals must be controlled and offers a large possibility to detect any faults. This became a rigorous task with the increase of the number of subsystems (actuators/sensors).

The structural approach constitutes a general framework to provide information when the system becomes complex. The main aim of the structural approach application is to identify the subsystems which present redundancy.

This paper analyses the structural model based on the dynamical structural properties of the system. The system structural model is an abstraction of its behavior in a sense that on the structure of the constraints only the existence of links between variables and parameters is considered and not the constraints themselves. The links are represented by a bipartite graph, which is independent of the nature of the constraint (quantitative, qualitative, equations, rules, etc.), variables and parameters values.

Indeed, this approach offers a qualitative, a simple and an easy way to obtain the system behavioral model.

The structural analysis is concerned by the structural model properties, which are similar to the bi-partite graph analysis. As this graph is independent of the system parameters values, structural properties are true almost everywhere in the system parameter space.

The analysis of system structure, originally developed for the decomposition of large systems of equations for their hierarchical resolution, was adopted by the Fault Detection and Isolation (FDI) community (Staroswiecki *et al.,* 1999), and structural concepts were used for the analysis of system monitorability using complete matching on a graph.

A recent overview of structural analysis was presented by (Blanke *et al.,* 2006). Essential references of the research field on properties of residual generators for linear differential algebraic systems were treated in (Nyberg and Frisk, 2006), and combined structural and polynomial methods were pursued in (Krysander and Nyberg, 2005; Leitold and Hangos, 2001). Techniques for active fault isolation were treated in (Nyberg and Frisk, 2006) and suggested in a structural context in (Blanke *et al.,* 2006). Structural isolability properties were investigated and applied to through examples in (Düstegör, 2005; Dulmage and Mendelsohn, 1959).

Regarding to their simplicity, structural models can be applied in many systems to extract useful information in fault diagnosis and fault-tolerant control design domains. Therefore, structural analysis allows the local redundancy analysis of the system. It also allows highlighting the possibilities for monitoring through the structural properties the structural monitorability and the determination of the calculation sequences, where the result is a residue. The purpose of structure analysis of residues is to assess the faults detectability and isolability.

This paper considers the fault detection and isolation problem in the structured context where an overview of the structural analysis and the residual generation is given using an industrial application to show the efficiency of our approach.

Our new approach is resumed in the modified rank algorithm inspired from applied mathematics for the generation of new structural properties allowing the easiest and the maximum faults detectability and isolability using a maximal matching on the structural bipartite graph. Moreover, the proposed approach gives significant results between modelling process and simulations applied on an industrial application.

2. STRUCTURAL MODEL

This section provides a brief overview of the main concepts in structural analysis. It provides the bases for further extensions to enhanced structural faults detection and isolation.

The main idea in analytic fault diagnosis is to establish relations to test whether measured and other known variables satisfy all relations that describe the system's normal behavior. If this is not the case, some violation of the normal behaviour has occurred, i.e., one or more faults are present in the system. Relations that can be used for such testing are referred to as redundancy relations.

Let's consider a system be described by the set X of unknown variables, the set K of known variables and the set C of constraints related to these variables, thus there may exist a set Cm $\subseteq$ C from which all variables in X can be determined. The system that has this property is said to have a complete matching of unknown variables. If any existing constraints which were not used to obtain such matching, the set of unmatched constraints Cum $\subset$ C may be used to test the consistency between known variables and the system's normal behavior. Hence, redundancy relations are obtained from the unmatched constraints.

Solving unknown variables in a nonlinear system can be rather complex if using directly constrains in analytical form. Structural analysis offers a better solution in this case. It is a method which determines possible ways to solve the set of constraints. Based on the relations between constraints and unknown variables we construct the graph which will allow us to determine the unknown variables of the system.

The result of structural analysis is a receipt that, in a symbolic form, describes how unknown variables could be calculated from known variables, using the system constraints. Analytical expressions are not used until a complete structural solution is found. This reduces drastically the complexity of finding parity equations for fault diagnosis.

The main feature of the structural analysis approach is that graph theory exists and can be employed to find all possible ways of the system constraints sets that can be matched to unknown variables (Dulmage and Mendelsohn, 1963). As sets of unmatched constraints, in general, differ from matching to matching, structural analysis can determine the entire set of possible parity relations.

Being very useful as a first step of the analysis, the results of structural analysis are, however, only indicative of the existence of the associated analytical results. The existence of a structural parity relation do not guarantee the existence of an analytic counterpart but the non-existence in the structural domain do, however, it imply a non-existence also in the analytical domain.

Structural concepts were studied early by the applied mathematics community, and various theoretical algorithms were developed in (Hopcroft and Karp, 1973; Izadi-Zamanabadi *et al.,* 2003). Structural analysis was used intensively in chemical engineering for solving large sets of equations (Niemann, 2006; Unger *et al.,* 1995). The structural approach and their features are well adapted for analysing monitoring and diagnosis problems. They were first introduced in (Staroswiecki *et al.,* 1999). Extensions to the analysis of reconfigurability and fault-tolerance emerged were presented in (Staroswiecki and Declerck, 1989; Staroswiecki and Gehin, 2000). The structural analysis approach was presented in a digested form in (Blanke *et al.,* 2006). Structural analysis has hence evolved during several decades. However, the prominent features of the theory and its possibilities that offers have only become apparent to a larger community in the field of automation and automatic control over the last few years (Åström *et al.,* 2001; Krysander, 2006) with applications reported in, e.g., (Düstegör *et al.,* 2004; Düstegör 2005; Izadi-Zamanabadi and Starosweiki, 2000).

2.1 Structure as a bi-bipartite graph

The structural model represents the links between a set of variables and a set of constraints. It is an abstraction of the behavior model because it describes simply which variables are connected to which constraints. Hence, the structural model presents the basic features and properties of a system which are independent of its parameters. The behaviour model of a system is defined by a pair (C, Z) where Z is a set of variables and parameters and C is a set of constraints; see (Blanke *et al.,* 2006).

This structure can be represented by a bipartite graph. A graph is bipartite if its vertices can be separated in two disjoint sets C and Z in such a way that every edge has one endpoint in C and the other one in Z.

The undirected bipartite graph can be interpreted as follows:

- All the variables and parameters connected with a given constraint vertex have to satisfy the equation or rule of this constraint vertex.
- This graph allows representing the structure of general models including both differential and algebraic constraints.
- In the bipartite graph the Z vertices are represented by circles and C constraint vertices by bars.

Example 1:

Consider the following example of a single tank system. This system contains a tank, pump and controller. The flow is controlled via the level sensor.

The system consists of the components {tank, input valve, output pipe, level sensor, control algorithm). A continuous model with variable in continuous time is given by the following constraints:

- Tank c1: $\dot{h}(t) = qi(t) - q0(t)$
- Input valve $c2 : qi(t) = \alpha u(t)$
- Output pipe $c3 : q0(t) = k\sqrt{h(t)}$

- Level sensor $c4: y(t) = h(t)$
- Control algorithm c5: (1)

$$u(t) = \begin{cases} 1 \text{ if } y(t) \le h_0 - r \\ 0 \text{ if } y(t) \ge h_0 + r \end{cases}$$

- Derivative constraint c6 :

$$\dot{h}(t) = \frac{d}{dt} h(t)$$

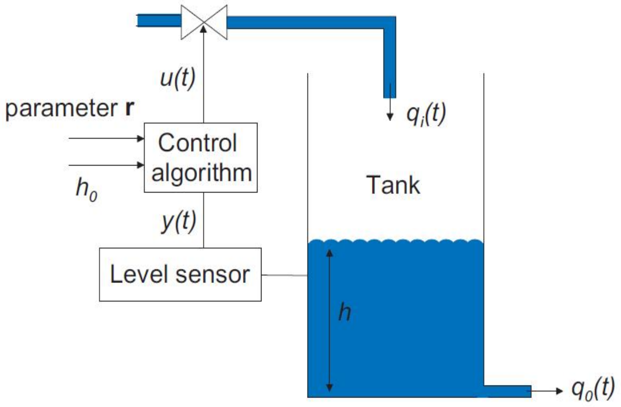

Fig. 1. Single Tank System.

Where: u (control variable), y (sensor output), h_0 (reference point constraint), r and k (given parameters), h (liquid level), q_i and q_0 (liquid inside and outside the tank), α (valve constraint).

The corresponding bipartite graph of example 1 is represented as follows:

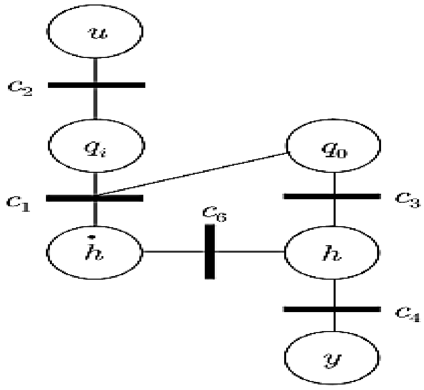

Fig. 2. Structure Graph of the Single Tank System.

This system can be represented by in the form of incidence matrix where every matrix columns correspond to a circle-vertex and every rows to a bar-vertex and the variables that are connected to constraints are marked by 1 in their corresponding boxes. The obtained matrix is given in Table 1.

The system variables and parameters can be decomposed into known and unknown variables. System input and output are examples of variables which are usually identified as known variables. In the same way, the entering parameters of example 1 are identified as known parameters. Notice that the known variables are available in real-time and can directly be employed in the diagnosis systems.

Table 1. Incidence Matrix of the System.

	Input/output		*internal variables*			
	u(t)	***y(t)***	***h(t)***	$\dot{h}(t)$	$q_i(t)$	$q_o(t)$
c_1				1	1	1
c_2	1				1	
c_3			1			1
c_4		1	1			
c_6			1	1		

2.2 Known and Unknown Variables

Unknown variables are not directly measurable, although there might exist some way to compute their value from the values of known variables. For the considered example 1, the last four columns of the incidence matrix $\{h, \dot{h}, q_i, q_0\}$ correspond to unknown variables, while the first two columns correspond to known variables {y, u}.

The variable set is divided into;

$$Z = K \cup X \tag{2}$$

Where K is known variables and X unknown variables.
And;

$$C = C_K \cup C_X \tag{3}$$

C_K is constraint which links only the known variables and C_X include those constraints in which at least one unknown variable appears.

3. MATCHING ON A BIPARTITE GRAPH

The basic tool for the structural analysis is the concept of matching on a bipartite graph. In other words, a matching is a causal assignment which associates unknown variables of the system with the system constraints from which they can be calculated. The unknown variables which cannot be coupled cannot be calculated. The variables which can be coupled several ways can be determined by various ways, which provide means for fault detection and a possibility for reconfiguration (Blanke *et al.,* 2006).

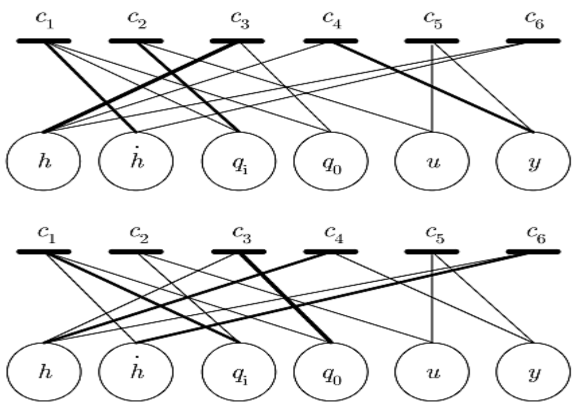

Fig. 3. Two Possible Matching for the Tank System.

In a bipartite graph the coupling is a set of arc edge such that any two edges have no common node (neither in C nor in Z), and it is represented by a bolt arc.

In the incidence matrix, the matching is represented by 1, and only one 1 is authorized in same column and line.

3.1 Graph Matching Algorithm

There exist many algorithms which are used to find the maximal matching, like the classical maximal matching algorithm, the maximal flow matching, Hungarian method, see (Hopcroft and Karp, 1973; Izadi-Zamanabadi *et al*., 2003).

The former ranking algorithm (Blanke *et al.,* 2006) can be used to achieve a complete matching on the unknown variables. The input of this algorithm is the incidence matrix or structure graph. In the first step of the algorithm, all known variables are marked and all unknown variables remain unmarked. Then every constraint that contains at most one unmarked variable is assigned by a rank 1. The constraint is matched with the unmarked variable and thus this variable will be marked. This step is repeated with an increasing rank number until no new variable can be matched. Every matched variable has associated a number or a rank which can be interpreted as the number of steps needed to calculate an unknown variable from the known ones.

Based on this algorithm, our new idea was integrated under a developed algorithm called modified algorithm. The modification starts from the 6th step, where we use the obtained non matched constraints to get more unmatched constraints by testing all possible matching on each constraint. This is done for each iteration. It is useful to get a complete set of unmatched constraints that represent the analytic redundancy relations (ARR). It is also useful information in monitoring such a system. An example of two possible matching is given in Fig.3 for the single tank system.

The former ranking algorithm may not find the complete matching, even if it exists. This can be seen in step 2 of the algorithm when it cannot found any unmarked variable.

By adding the measured constraint in step 6, we can improve this result by eliminating the possibility to get any conflict when they are unmarked variables, no variables will be available in step 2.

In addition, this step will recover the sensor placement. It will increase diagnostic performance when adding new sensor placements (Alem and Benazzouz, 2013).

An application of the matching or constraint propagation algorithm on the single tank example is given as follows:

As $\boldsymbol{u}$, $\boldsymbol{y}$ are known, only the variable set $\{\boldsymbol{h}, \dot{\mathbf{h}}, \boldsymbol{q_i}, \boldsymbol{q_0}\}$ has to be matched.

- **Starting set (rank 0):** $\{\boldsymbol{u}, \boldsymbol{y}\}$
- **First step (rank 1):** match $\boldsymbol{q_i}$ with $\boldsymbol{c_2}$, and match h with $\boldsymbol{c_4}$.
- **Second step (rank 2):** match $\boldsymbol{q_0}$ with $\boldsymbol{c_3}$, and match $\dot{\mathbf{h}}$ with $\boldsymbol{c_6}$.

Notice that all unknown variables were matched.

Modified Ranking Algorithm

Given: incidence matrix or structure graph

1. Mark all known variables
   i = 0
2. Find all constraints in the current table with exactly one unmarked variable. Associate rank i with these constraints and mark these constraints as well as its corresponding variable.
3. Set **i = i + 1**
4. If there are unmarked constraints whose all variables are marked, associate them with rank i, mark them and make the link with the pseudo-variable **ZERO**
5. If there are unmarked variables or constraints, continue with step 2.
6. Mark the constraint connected to pseudo-variable ZERO which has higher rank, then mark one unknown variable and add the measurement constraint to make it as known variable.
7. If all constraints and variables are marked or no novel unmarked constraints with all variables remains, all of them are marked, then **Break**.
8. Else, **Return** to step1

Return Matching and Ranking of the constraints

4. SYSTEM CANONICAL DECOMPOSITION

In graph theory, a classical result from bi-partite graph (Dulmage and Mendelsohn, 1963), states that any finite-dimensional graph can be decomposed into three sub graphs with specific properties associated with an over-constrained, a just-constrained and an under-constrained subsystem respectively.

This decomposition is canonical, i.e. for a given system, it is unique. The three subsystems play a major role in the analysis of the system structural properties: Observability, Controllability, and Monitorability.

A graph (C, Z) is called

- **Over-constrained** if there is a complete matching on the variables **Z** but not on the constraint **C**,
- **Just-constrained** if there is a complete matching on the variables **Z** and on the constraints **C**,
- **Under-constrained** if there is a complete matching on the constraints **C** but not on the variables **Z**.

5. PROPERTY OF STRUCTURAL ANALYSIS

5.1 Observability

Observability informs us about the unknown variables of our system which can be checked and observed.
Structural Observability condition:

- All the unknown variables are reachable from the known ones,
- The over-constrained and the just-constraint subsystems are causal,

- The under-constrained subsystem is empty.

The single tank example system is observable when the system is over-constrained or just-constrained. More over the system should have all its unknown variables reachable C, i.e. they can be calculated by the known variables.

5.2 Monitorability

A system is said to be monitorable if it can be determined, using only known variables whether the system constraints are satisfied or not and also by the design of fault detection and isolation algorithm based on ARR.

The ARR based fault diagnosis tries to identify faults by comparing the actual system behavior, with the theoretical behavior described by the system constraints. ARRs are the constraints that express this redundancy.

Notice that the ARR should have the following properties:

- Robust, i.e. insensitive to unknown inputs and unknown parameters;
- Sensitive to faults: this insures that they are not satisfied when faults are present, so that there is no missed detection;
- Structured: this insures that in presence of a given fault; only subsets of the ARR are not satisfied, thus allowing recognition of fault which has occurred.

The following two necessary conditions for fault monitorability are equivalent:

- $\mathbf{X}_{\varphi}$ is structurally observable in the system;
- $\boldsymbol{\varphi}$ belong to the structurally observable over-constrained part of the system (c, z)

i.e. unknown variables must be observable in the overstrained subset.

5.3 Development of the Analytic Redundancy Relations

Redundancy relations are sub-graphs of the structure graph, which are associated with the complete causal matching of the unknown variables associated with the over-constrained subsystem of the reduced bi-partite graph.

Redundancy relations are composed of alternated chains, which start with known variables and end with non-matched constraints, the chosen output is labelled ZERO.

Developing the set of residuals starts from building maximal matching of the given structural graph, under derivative causality, and identifying the redundancy relations as the non-matched constraints in which all the unknowns have been matched. This is performed based on the presented modified algorithms which find the maximal matching.

The obtained incidence matrix after matching of the single tank system is given in table 2.

Notice that the constraint c1 is the non-matched constraint for all matched variables.

Thus, the constraint c1 is used for developing the ARR. This is performed using Eq.4 which replaces all the variables by coupling them with the known variables Eq.5.

Table 2. Incidence Matrix after Matching.

	Unknown				***Ranking***	
	h	**$\dot{h}$**	***q_i***	***q_0***	**zero**	rank
c_1		1	1	1	1	2
c_2			1			0
c_3	1			1		1
c_4	1					0
c_6	x	1				1

$$\mathbf{c1}: 0 \;=\; \dot{\mathbf{h}}\,(t) \;+\; \boldsymbol{q}0(t) - \boldsymbol{q}\boldsymbol{i}(t)$$

$$\mathbf{c1}(\dot{\mathbf{h}}, \boldsymbol{q}0, \boldsymbol{q}\boldsymbol{i}) \rightarrow ZERO \tag{4}$$

By replacing the known variables we get:

$$\mathbf{c1}: 0 \;=\; \frac{d}{dt} y(t) \;+\; k\sqrt{y(t)} \;-\; \boldsymbol{\alpha}\,\boldsymbol{u}(t) \;=\; r(t) \tag{5}$$

r(t): residual obtained by the ARR.

The operations order of constraints is:

$$\mathbf{c1}\ (\boldsymbol{c6}\ (\boldsymbol{c4}\ (\boldsymbol{y})), \boldsymbol{c2}\ (\boldsymbol{h}), \boldsymbol{c3}\ (\boldsymbol{c4}\ (\boldsymbol{y}))) \;\rightarrow\; ZERO \tag{6}$$

5.4 Structural Detectability and Isolability

1- Structural Detectability

A violation of the constraint c is structurally detectable if and only if it has a nonzero Boolean signature in some residual r

$$\boldsymbol{c} \in \boldsymbol{C}_{\mathbf{Detectable}} \Leftrightarrow \exists \boldsymbol{r} : \boldsymbol{c} \neq 0 \;\Rightarrow \boldsymbol{r} \neq 0 \tag{7}$$

2- Structural Isolability

A violation of the constraint c_i is structurally isolable if and only if it has a unique signature in the residual vector.

In the given single-tank example, it contains one unmatched constraint c1. Back-tracking through the matching we obtain.

$$\mathbf{c1}\ (\boldsymbol{c6}\ (\boldsymbol{c4}\ (\boldsymbol{y})), \boldsymbol{c2}\ (\boldsymbol{h}), \boldsymbol{c3}\ (\boldsymbol{c4}\ (\boldsymbol{y}))) \;\rightarrow\; ZERO \tag{8}$$

We enter constraints to residual as follows;

$$\begin{array}{cc} & \begin{array}{ccccc} c1 & c2 & c3 & c4 & c6 \end{array} \\ r1 \leftarrow & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{9}$$

Hence, the violation in any of the constraint is structurally detectable but not structurally isolable see (Alem and Benazzouz, 2013).

Suppose that we have an incidence matrix given in Eq.10, which contains several residues. Thus only c1, c2, c5 are isolable and they have single signature residual, but c3, c4 are only observable.

$$\begin{array}{cc} & \begin{array}{ccccc} c1 & c2 & c3 & c4 & c5 \end{array} \\ \begin{array}{c} r1 \leftarrow \\ r2 \leftarrow \\ r3 \leftarrow \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{10}$$

Application

To test our approach we have chosen a water heat system given in Fig.4. It consists of a tank, a boiler, the thermal resistance and the water pump each one are controlled by a regulator on-off, a valve and three sensors, flow, thermal resistance and pressure sensors. Level and temperature are controlled by the two controllers u1 and u2 respectively.

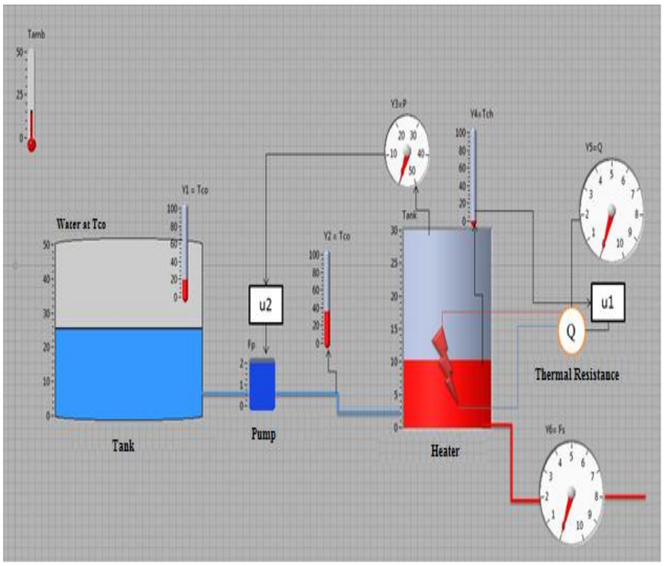

Fig. 4. Water Heating System.

The ARR principal starts by finding the analytical description of various system constraints. Then, we match unknown variables by using the ranking algorithm or any other matching or constraint propagation algorithm (Blanke *et al.*, 2006).

The ARRs are obtained from the non-matched constraints. Various ARRs can be obtained using different maximal matching. In addition, by combining some ARRs we can obtain new redundant relations and more residual generators (this is the main idea of our proposed algorithm).

Next, the system constraints are prearranged in three subsets as function of their structural representation, such as behavioural, measurement and control constraints. These subsets are explained as follows;

A. Behavioural Constraints

In order to characterise the system command behaviour we use the enthalpy change, which represents the heat exchange in the thermo-fluid system.

The balance energy equation is given by the Eq. 11.

$$\dot{H}g = \dot{Q}th + \dot{H}co + \dot{E}w - \dot{H}v - \dot{Q}p \tag{11}$$

$\dot{Q}_{th}$ (j/s) : Thermal power provided by the thermal resistance.
$\dot{H}_{co}$ (j/s) : enthalpy flows in the pipe.
$\dot{E}_{w}$ (j/s) : work furnished by the pressure forces.
$\dot{H}v$(j/s) : enthalpy variations of the generated steam.
$\dot{Q}_{p}$ (j/s) : thermal power loss.

$$\dot{Q}th = 1000 * Q \tag{12}$$

Q : measured by sensor Y_5.

$$\dot{H}co = Fco * h0 \tag{13}$$

h_0 (j/kg) : water specific enthalpy.

$$h0 = Cpe * Tco \tag{14}$$

C_{pe}: water specific heat, it depends slightly on the temperature variations. When the work varies from 4180 J to 4200 J, the temperature varies from 15°C to 100°C. The boiler temperature of the application varies between 50°C to 60°C, we can take C_{pe} = 4200 J to cover largely the temperature.

The flow pump F_p, and the inlet flow rate of the boiler are constant and almost equal. Thus we can consider that $F_{co} = F_p$.

By replacing Eq. (13) in Eq. (14) and we obtain:

$$\dot{H}co = Fp * Tco * 4200 \tag{15}$$

$$\dot{Q}p = \left(\frac{2\pi H}{\ln\frac{R}{r}} + \frac{S}{e}\right) * \lambda * (Tamb - Tch) \tag{16}$$

H (meter) is the boiler height,
R and r (meter) are the boiler outside and inside radius respectively.
T_{ch} (°) is the boiler temperature and T_{amb}(°) is the ambient temperature.
S (m^2) is the boiler area.
e (m) is the boiler surface plane thickness.
λ is the materials thermal conductivity.

- $\dot{H}v$ = 0 because in the water heat application the temperature varies between 50°C and 60°C. Thus at this level of temperature there is no generation of steam.
- $\dot{E}_w$ = 0 because the work produced by the pressure forces is based on pressure variations, for the given application the work is at constant atmospheric pressure.

Thus Eq. (11) is simplified and written as follows:

$$\dot{H}g = \dot{Q}th + \dot{H}co - \dot{Q}p \tag{17}$$

C1: $\dot{H}co = Fco * Tco * 4200$

C2: $\dot{Q}p = \left(\frac{2\pi H}{\ln\frac{R}{r}} + \frac{S}{e}\right) * \lambda * (Tamb - Tch)$

C3: $\dot{Q}th = 1000 * Q$

C4: $\dot{H}g = \dot{Q}th + \dot{H}co - \dot{Q}p$

More over we determine the other parameters of the system and given as follows;

C5 : $\dot{m}(t) = (Fp - Fs)$

Fs : outlet flow measured by the sensor $\mathbf{Y_6}$.
and ;

C6 : $Fs = Cd\sqrt{m(t)}$

C_d : valve discharge coefficient.
m(t) : tank water quantity.

$C7: Fp = f\ (u2)$

F_p : pump flow rate (which is constant for the case study).
u_2 : control variable.

$C8: \dot{Q}th = \partial f\ (u1)$

u_1 : control variable.

$C9: \dot{m}(t) = \frac{dm(t)}{dt}$

B. Measurements constraints

$M1: Y1 = Tco$

$M2: Y2 = Tco$

$M3: Y3 = \frac{m(t) * g}{s}$

$M4: Y4 = Tch$

$M5: Y5 = Q$

$M_6 : Y_6 = F_s$

C. Control Constraints

$CT: u1 = on-off\ (Tch)$

The control algorithm of u_1 is:
u_1= 0 if T_{ch} is between T=50°C and T=60°C.

u_1= 1 if T< 50°C.

$Cp: u2 = on-off\ (m\ (t))$

The control algorithm of u_2 is:
u_2= 0 if m= 20kg

u_2= 1 if m= 2kg

After studying the complete water heating system and determining various parameters needed for evaluating all the constraints, we apply the developed modified ranking algorithm for matching unknown variables. The obtained result of the incidence matrix is given in Table 3.

The matching is performed using the modified ranking algorithm; so we can say that it is the maximal possible matching.

The constraints $\{C1, C2, C3, C4, C9, M1, M3, M4, M5, M6\}$ are respectively matched with

$\{\dot{H}co,\ \dot{Q}p,,\ \dot{Q}th, \dot{H}g,\ \dot{m}(t), Tco, m(t), Tch, Q, Fs\}$

We have:

- $\dot{H}co = Fp * Y1 * 4200$
- $\dot{Q}p = \left(\frac{2\pi H}{\ln\frac{R}{r}} + \frac{S}{e}\right) * \lambda * (Tamb - Y4)$
- $\dot{Q} = 1000 * Y4$
- $m(t) = \frac{S}{g} * \frac{d}{dt} Y3$
- $Y1 = Tco$
- $m(t) = \frac{Y_3 * s}{g}$
- $Y4 = Tch$
- $Y5 = Q$
- $Y6 = Fs$

Table 3. Incidence Matrix with Matching on the Water Heating System.

	Unknown Variables										known Variables										
	$\dot{H}_{co}$	T_{co}	$\dot{Q}_p$	T_{ch}	$\dot{Q}_{th}$	Q	$\dot{H}_g$	$\dot{m}(t)$	m(t)	Fs	Fp	u_1	u_2	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Rank
C_1	1	1									1									1
C_2			1	1																1
C_3					1	1														1
C_4	1		1		1		1													2
C_5								1		1	1									Zero
C_6									1	1										Zero
C_7											1		1							Zero
C_8					1							1								Zero
C_9								1	×											1
M_1		1												1						0
M_2		1													1					Zero
M_3									1							1				0
M_4				1													1			0
M_5						1												1		0
M_6										1									1	0
CT												1		1						Zero
CP													1			1				Zero

The constraints $\{C_5, C_6, C_7, C_8, M_2)$ are non-matched constraints, which can represent the analytic redundancy relations.

They are matched to the pseudo-variable Zero on the incidence matrix given in Table 3. Their evaluations are obtained by using numerical values of the included variables, which will be used to generate the residues.

$$C5: \dot{m}(t) = (Fp - Fs) \Rightarrow \dot{m}(t) - (Fp - Fs) = 0$$

$$\Rightarrow r1 = \dot{m}(t) - (Fp - Fs)$$

$$\Rightarrow r1 = \frac{S}{g} * \frac{d}{dt} Y3 - Fp - Y6$$

$$C6: Fs = Cd\sqrt{m(t)} \Rightarrow C6: Fs - Cd\sqrt{m(t)} = 0$$

$$\Rightarrow r2 = Y6 - Cd\sqrt{\frac{Y_3 * s}{g}}$$

$$C7: Fp = f\ (u2) \Rightarrow Fp - f\ (u2) = 0$$

$$\Rightarrow r3 = Fp - f\ (u2)$$

$$with\ u2 = ON - OFF\ (Y3).$$

$$C8: \dot{Q}th = \partial f\ (u1) \Rightarrow \dot{Q}th - \partial f\ (u1) = 0$$

$$\Rightarrow r4 = 1000 * Y5 - \partial f\ (u1)$$

$$with\ u1 = ON - OFF(Y4).$$

$$M2: Y2 = Tco \Rightarrow Y2 - Tco = 0$$

$$\Rightarrow r5 = Y2 - Y1$$

$\mathbf{r_5}$ is obtained from the ARR.

$$CT: u1 = ON - OFF\ (Tch)$$

$$\Rightarrow r6 = u1 - ON - OFF\ (Y4)$$

$$CP: u2 = ON - OFF\ (m(t))$$

$$\Rightarrow r7 = u1 - ON - OFF\ (Y3)$$

As an example, we can observe that the leakage of the boiler can clearly be detected by analysing the residue $\mathbf{r_1}$.

On the other hand, we found that seven (7) non-matched constraints that can be used for the design of the analytic redundancy relations. Applying our modified ranking algorithm we end up with seven (7) residues which enable us to establish the fault signature matrix given in Table 4.

Table 4. Fault Signature Matrix.

	r_1	r_2	r_3	r_4	r_5	r_6	r_7	*Observ*	*Isolab*
Leakage	1							1	1
Y_1					1			1	0
Y_2					1			1	0
Y_3	1	1	1				1	1	1
Y_4				1		1		1	1
Y_5				1				1	1
Y_6	1	1						1	1
F_p	1		1					1	1

Through the fault signature matrix, we can remark that Y_1 and Y_2 have the same fault signature.

In this case, it is not possible to isolate the fault between them; they represent a material (hardware) redundancy.

Simulations

The simulation is performed under the following parameter values given in Table 5.

Table 5. Parameters Values used in the Simulations.

Parameter	Value	parameter	value
Fp (kg/s)	2	λ (W/m.K)	26
H (m)	1	T_{amb} (c°)	21
e (m)	0.08	Cd	0.16
R (m)	0.6	G (m/s^{-2})	9.81
r (m)	0.55		

In the simulation, we tested our algorithm by introducing some faults in the system, such as the boiler water leak failure, the pump failure and the pressure sensor failure Y3. Our objective of this testing is to evaluate the sensitivity of the obtained residues.

Through Fig.5 we observe that only residue r1 is sensitive to the boiler water leak. This has been provided by the theoretical aspect obtained by the fault signature matrix given in Table 5.

1. Residue sensitivity of the boiler leak.

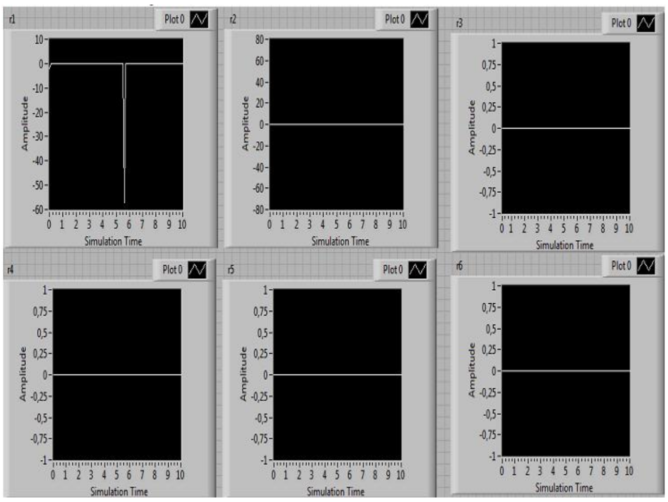

Fig. 5. Reaction of r1 of the water leak failure.

2. Residue sensitivity of the pump (actuator)

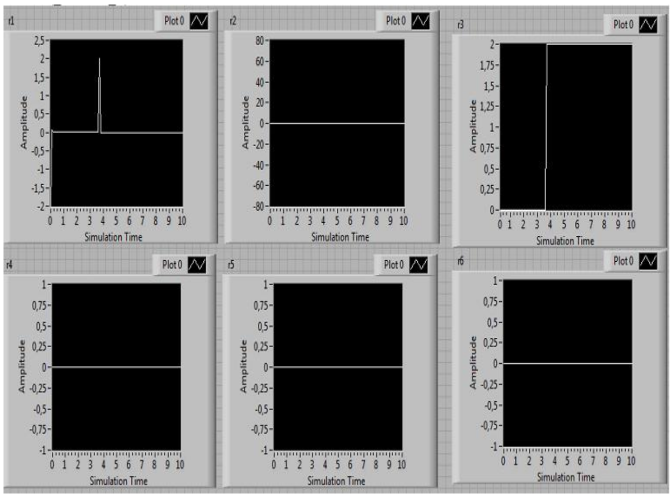

Fig.6. Reaction of r1 and r3 of pump (actuator) failure.

Considering the pump flow as constant, thus for any variation of flow or any other variations even if flow stopping during operation will be detected as a failure. This is observed through our obtained graph given in Fig.6. This result is also in conformity with the obtained failure signature matrix.

Through Fig. 7, we observe clearly the change on the obtained graph which shows how the pressure sensor Y3 is detected. This has been also provided in the obtained fault signature matrix given in Table 5.

Finally, by this approach we can take any other component of the water heat application, and introduce voluntarily any fault, it will be detected by our proposed modified ranking algorithm. Since there is similarity between the theoretical developed concept and the obtained simulation results we can say that the proposed matching algorithm (residual generation) by the structural analysis is efficient and thus the system can be easily monitorable.

3. Residue sensitivity of the pressure sensor Y3.

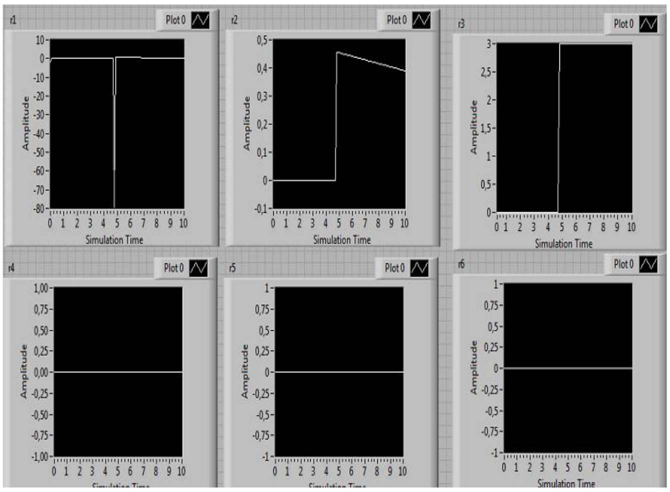

Fig. 7. Reaction of r1, r2 and r3 to the pressure sensor Y3 failure.

6. CONCLUSION

The structural analysis allows the local redundancy analysis of the system. It enhances the system monitoring through the structural properties. These proprieties are the monitorability, calculation sequences determination of unknown variables, ARR generations. The ARR evaluation deduces the desired residues.

The residual structure analysis allows the identification of detectable and isolable faults. Therefore, it offers the possibilities to implement other sensors in order to change the status of the system components from non-observable to observable (Alem and Benazzouz, 2013).

The water heating application specifies the consistency of the obtained results, where these are in conformity with theoretical and simulation aspects. The proposed algorithm based on ARR generation, evaluation and residual structure analysis is well adapted to design monitoring system.

The computational complexity of the proposed algorithm is polynomial in the number of unknowns making these condition suitable for real systems with a large number of constraints and unknown variables.

In our future work, we intend to evaluate the efficiency of the developed approach compared to the application of derivative and integral causalities for constraints matching and determination of calculation sequences.

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