MARKOV PARAMETERS EVALUATION BY INTEGRAL METHODS

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Abstract: *This paper deals with a method for Markov parameters evaluation by integral methods for proper dynamical systems. The step response of the system is processed and a set of weighted integrals, performed on finite time intervals, allows creating an algebraic system of equations whose unknowns are the Markov parameters. Theoretically this is an infinite dimensional system but it is truncated to a finite order one. The convergence conditions and truncation error effects are analyzed. Because the Markov parameters are coefficients of Laurent series it is proposed a techniques for series convergence domains evaluation.*

Keywords: *system theory; identification, Markov parameters, weighted moments.*

1. INTRODUCTION

It is well known that in dynamical system identification one of the most difficult step is related to the order of the system appreciation. Usually in parameter identification approach a structure of the model is given and it follows that, based on different methods, the parameters of that given model to be determined in a such a way that some quality criteria to be optimized. It is a fundamental method the order evaluation of a dynamical system based on Hankel matrices built with Markov parameters, (Ho et al., 1965), (Kailath, 1980), both for single-input single-output systems as well as for multi-input multi-output systems (Chen et al., 1972), (Rozsa et al., 1975). The Markov coefficients are the coefficients of the transfer function power series development with respect to the variable s^{-1}. They correspond to the different order right side derivatives of the impulse response in the initial time moment.

If the transfer function expression is given it is not difficult to evaluate precisely its Markov parameters Vice versa, in identification problems, one starts from step response or impulse response and has to evaluate the initial time moments derivatives of different orders. This is a very difficult task, error sensitive especially if the response is measurement noise contaminated.

Markov parameters, as the values of the derivatives in the initial time moment of impulse or step response, univocally determine the values of that response for any time moment, that means their effect is time distributed. Starting from this observation, the paper proposes a method for Markov parameters determination by evaluating some integrals performed on finite time intervals similar to weighted moments shapes. Using integrals, which are global-based values or distributed-based values, instead of point-based values (values in some time instants), has the advantage of noise robustness but it asks for infinite dimensional algebraic equation solving.

However, through point-based values techniques, that means the evaluation of different order derivatives in the initial time instant, each Markov parameter can be individually determined. Unfortunately, it is not possible directly to evaluate derivatives having higher order than three. It is not recommended to approximate derivatives by finite differences.

Instead, using global-based techniques applied by evaluating some time integrals of the weighted moments type, each integral embeds the effects of all the coefficients, so information about Markov parameters are combined and, as a natural consequence, the final result appears as a system of algebraic equations, theoretically infinity dimensional one.

Many other problems, different of the Markov parameters evaluation, require study and solving of infinite dimensional algebraic equations.

Taking all these into consideration, some aspects regarding the computing errors committed by the infinite dimensional algebraic system truncation to a finite dimension one are analyzed. This approach is based on fundamental notions of complex variable functions as referred in (Sabac, 1965) and (Corduneanu, 1981).

2. MARKOV PARAMETERS

Let us consider a linear time invariant dynamical system expressed by the transfer function $H(s)$, with convergence abscissa σ_0, asymptotically stable, so $\sigma_0 < 0$. This is a single-input single-output system; the multivariable case could be analyzed by extension. Because $H(s)$ is a Laplace transform, it is defined and holomorphic, as a complex variable function, in the domain $\mathrm{Re}(s) < \sigma_0$ only.

In the following we shall consider by $H(s)$ the analytical extension of the transfer function in all the points $s \in \mathbb{C}$ of the complex plain, less its singular points, considered to by poles only. In such a way, we can look at $H(s)$ as being a meromorphic complex variable function, and we denote by $\sigma(H)$ the set of its poles. Shall we withdraw now from the expression of $H(s)$ its strictly proper part $\widetilde{H}(s)$,

$$H(s) = \gamma_0 + \widetilde{H}(s) \tag{1}$$

where

$$\lim_{\mathrm{Re}(s)\to\infty} \widetilde{H}(s) = 0 \tag{2}$$

The condition (2) will assure the existence of a neighborhood of the points at the infinity in the complex plane of $s \in \mathbb{C}$ where $\widetilde{H}(s)$ is holomorphic so the point $s = \infty$ is an ordinary point for $\widetilde{H}(s)$. Considering the poles placed in a bounded domain we can define

$$R = \max_{\lambda\in\sigma(H)} |\lambda|, \tag{3}$$

the radius of a circle having its center in the origin of the complex plane which contains all the poles of the function $\widetilde{H}(s)$ or the bounded poles of the $H(s)$.

Into such conditions, the Laurent series (4) of the function $\widetilde{H}(s)$ developed outward the disk with a radius R, $\Delta(R)$, is uniformly convergent,

$$\widetilde{H}(s) = \sum_{k=1}^{\infty} \gamma_k \cdot s^{-k} \quad , \quad |s| > R . \tag{4}$$

The coefficients γ_k, k=1, 2, ... are called the Markov parameters of the function $\widetilde{H}(s)$. They can be evaluated by an integral (5) around a circle $\Gamma(R_1)$ of the radius $R_1 > R$ such that the whole circle $\Gamma(R_1)$ is included in the holomorphy domain of the function $\widetilde{H}(s)$,

$$\gamma_k = \frac{1}{2\pi j} \cdot \int_{|s|=R_1} \widetilde{H}(s) \cdot s^{k-1} \cdot ds \quad k=1, 2, ... \tag{5}$$

If $\widetilde{H}(s)$ is a rational function, one can utilize the residuum theorem under the form,

$$\gamma_k = \sum_{\text{poliifc.}\widetilde{H}(s)s^{k-1}} \mathrm{Rez}\{\widetilde{H}(s) \cdot s^{k-1}\} \quad k=1, 2, ... \tag{6}$$

It can be observed, if we analyze the shape of series (4), that relations (5) and (6) are similar to the inverse z transform formula.

Considering (A, b, c^T, d, x) as being a state equation realization of the transfer function $H(s)$, or $(A, b, c^T, d=0, x)$ as being the state realization of its strictly proper part, $\widetilde{H}(s)$, the Markov parameters are given by relations,

$$\gamma_k = c^T \cdot A^{k-1} \cdot b, \; k=1, 2, ... ; \gamma_0 = 0 \tag{7}$$

Denoting by $\widetilde{h}(t) = \mathbf{L}^{-1}\{\widetilde{H}(s)\}$, the weighting function (impulse response) of the strictly proper part, and by applying the initial value theorem, one obtains

$$\gamma_k = \widetilde{h}^{(k-1)}(0^+) = \lim_{\substack{t\to 0\\ t>0}} \frac{d^{k-1}\widetilde{h}(t)}{dt^{(k-1)}}, \quad k=1, 2, ... \tag{8}$$

It is well known that the Markov parameters allow obtaining completely controllable and observable minimal order realization by state equations (Kailath, 1980), (Ionescu, 1985). The advantages of the Markov parameters are more relevant for multivariable systems (Chen et al., 1972), (Rozsa et al., 1975).
If the expression of a strictly proper transfer function is knownn, then Markov parameters can be precisely determined by using a series development of the form (4) or by using the relations (7). If an impulse response or a step response obtained by an experimental test it is available only then it is inadequate to use directly for numerical calculations the relation (8), when indexes exceeds three.

In this paper the information available for Markov parameters determination are represented by a string of numbers obtained by performing some integrals on the step response generated by an experimental test.

3. MARKOV PARAMETERS EVALUATION BY INTEGRALS

Let $y(t)$ be the step response, experimentally obtained, of a linear stable system with the transfer function $H(s)$, with unknown set of poles $\sigma(H)$ placed inside a circle $\Gamma(R)$, with radius R and centered in the complex plane origin. The circle $\Gamma(R)$ evaluation is equivalent with the estimation of the least time constant possible in that physical system. The Laplace transform of the step response $y(t)$,

$$Y(s)=H(s)\cdot\frac{1}{s} \tag{9}$$

is well-defined holomorphic function for $Re(s)>\sigma_0=0$ and we consider to be analytically extended in

$$\mathbb{C}\setminus\sigma(Y),\sigma(Y)=\sigma(H)\cup\{0\}\subset\Delta(R), \tag{10}$$

$\sigma(h)\subset\Delta(R)$ where $\Delta(R)$ being the disk bounded by the circle $\Gamma(R)$.
Under these hypothesis, the complex variable function

$$Y(s):\mathbb{C}\setminus\sigma(Y)\rightarrow\mathbb{C}, \tag{11}$$

the analytical extension of the Laplace transform, is holomorphic one in the domain $|s|>R$ even for $H(s)$ proper function because

$$\lim_{Re(s)\to\infty} Y(s)=0 \tag{12}$$

so that the point at the infinity is an ordinary point. As a result the series

$$Y(s)=\sum_{k=1}^{\infty}c_k s^{-k}, \quad |s|>R \tag{13}$$

is uniformly convergent for any value s such that $|s|>R$. Taking into consideration (1), (4), (9), (13), one obtains after identification

$$c_k=\gamma_{k-1}, \quad k\ge 1 \tag{14}$$

or, by applying the initial value theorem,

$$c_k=y^{(k-1)}(0^+)=\lim_{\substack{t\to 0\\ t>0}}\left(\frac{d^{k-1}y(t)}{dt^{k-1}}\right), k\ge 1 \tag{15}$$

It can be taken into consideration that

$$y^{(k)}(t)=\tilde{h}^{(k-1)}(t), \quad \forall t>0, \quad k\ge 1 \tag{16}$$

The coefficients c_k from the series (13) can be evaluated, equivalently as for the Z transform, by an integral on a circle $\Gamma(R_1)$ with the radius $R_1>R$, the integral value being irrespective of the radius value $R_1>R$

$$c_k=\frac{1}{2\pi j}\cdot\int_{|s|=R_1>R} Y(s)\cdot s^{k-1}\cdot ds \tag{17}$$

Because only stable systems are considered, the step response $y(t)$ is a bounded function, so it exists M, so that

$$M=\max_{t\ge 0}|y(t)|, \tag{18}$$

and one can use the condition of an original function

$$|y(t)|\le M\cdot e^{\sigma_0 t}, \ \sigma_0=0 \tag{19}$$

that means

$$|Y(s)|\le\frac{M}{\sigma-\sigma_0}, \quad Re(s)=\sigma>\sigma_0. \tag{20}$$

Taking into consideration (17), which means the Cauchy inequality is true (Sabac, 1965) ,

$$|c_k|\le M\cdot R^{k-1}, \quad k\ge 1 \tag{21}$$

where M may be considered having covering values as in (18) to express (19). In the series (13), the series rest $T_n(s)$ can be put into evidence,

$$Y(s)=S_n(s)+T_n(s)$$

where,

$$S_n(s)=\sum_{k=1}^{n}c_k\cdot s^{-k}. \tag{22}$$

The absolute value of the series rest, accomplishes the condition

$$\left|T_n(s)\right| \le \frac{M}{R} \cdot \frac{(R/|s|)^{n+1}}{1 - R/|s|}, \quad |s| > R \qquad (23)$$

but, for $s = \alpha > R$, $\alpha \in \mathbb{R}$, for which we denote $\xi = \frac{R}{\alpha} < 1$, and we have

$$\left|T_n(\alpha)\right| \le \frac{M}{R} \cdot \frac{(\xi)^{n+1}}{1-\xi}, \quad \xi = \frac{R}{\alpha} < 1. \qquad (24)$$

It is well known (Sabac, 1965), (Corduneanu, 1981), that the power series (25)

$$f(t) = \sum_{k=1}^{\infty} c_k \cdot \frac{t^{(k-1)}}{(k-1)!} \qquad (25)$$

has the properties:
It is convergent for any $t \ge 0$.
Uniformly converges on any finite time interval $t \in [0, \theta]$, θ as much as large one.
The series sum $f(t)$ is an original function with the convergence index R if the coefficients c_k satisfy the condition (21).
The Laplace transforms $Y(s)$ and $F(s)$ for $y(t)$ and $f(t)$ respectively, coincide and they are holomorphic in the domain $\mathrm{Re}(s) > R > \sigma_0$; $R > 0$.
The coefficients c_k from the series (13) and (25) coincide. In fact they represent the Markov parameters according to (14). Into the above conditions one can write,

$$y(t) = \sum_{k=1}^{\infty} c_k \cdot \frac{t^{k-1}}{(k-1)!}; \quad t \ge 0 \qquad (26)$$

Because the uniform convergent series (26) has the convergence radius of $+\infty$, then the series obtained by multiplying each term by a bounded factor $e^{-\alpha t}$, $\alpha > 0$

$$e^{-\alpha t} \cdot y(t) = \sum_{k=1}^{\infty} c_k \cdot \frac{e^{-\alpha t} \cdot t^{(k-1)}}{(k-1)!} \qquad (27)$$

is uniformly convergent for any finite time interval $t \in [0, \theta]$, and it is possible to perform term by term integration on this interval (Corduneanu, 1981), for any value of θ,

$$\int_0^{\theta} e^{-\alpha t} y(t) dt = \sum_{k=1}^{\infty} c_k \int_0^{\theta} \frac{e^{-\alpha t} t^{k-1}}{(k-1)!} dt \qquad (28)$$

Denoting

$$q_k = q_k(\theta, \alpha) = \int_0^{\theta} \frac{e^{-\alpha t} \cdot t^{k-1}}{(k-1)!} dt$$

$$m(\theta) = m(\theta, \alpha) = \int_0^{\theta} e^{-\alpha t} \cdot y(t) \cdot dt \qquad (30)$$

one obtains

$$\sum_{k=1}^{\infty} q_k(\theta, \alpha) \cdot c_k = m(\theta, \alpha) \qquad (31)$$

Relation (26) can be directly obtained from (13), but some detailed descriptions were necessary to precise convergence and existence domains, which for an expression as $Y(s)$ can be different if we understand by $Y(s)$ a Laplace transform, a sum of a convergent series or an analytical extension. The coefficients in equation (31) are irrespective of the response $y(t)$. They can be evaluated by using the relations,

$$q_1(\theta, \alpha) = \frac{1 - e^{-\alpha\theta}}{\alpha} \qquad (32)$$

$$q_{k+1}(\theta, \alpha) = -\frac{1}{\alpha} \cdot \frac{\theta^k}{k!} \cdot e^{-\alpha\theta} + \frac{1}{\alpha} \cdot q_k(\theta, \alpha), k \ge 1 \qquad (33)$$

These parameters can be interpreted as being weighted moments on finite time intervals of the unitary step function $\mathbf{1}(t)$,

$$q_k(\theta, \alpha) = \int_0^{\theta} \Psi_k(t, \alpha) \cdot \mathbf{1}(t) \cdot dt, \quad k \ge 1 \qquad (34)$$

with a weighting factor of $\psi_k(t, \alpha)$, where

$$\psi_k(t, \alpha) = e^{-\alpha t} \cdot \frac{t^{k-1}}{(k-1)!}, \quad k \ge 1 \qquad (35)$$

They have the shapes as in Fig. 1.

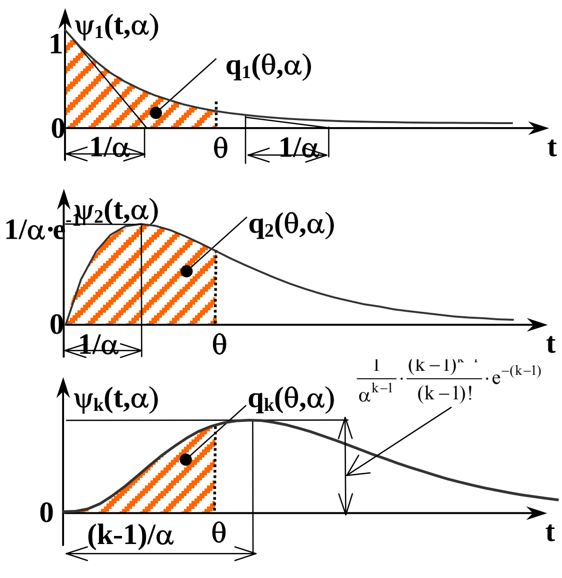

Fig 1. Shapes of weighting functions.

It can be observed that $\psi_1(t, \alpha)$ is an decreasing exponential function with a time constant of $1/\alpha$ but,

$$\text{For } k = 2 \qquad \dot{\psi}_2(t, \alpha) = (1 - \alpha t) \cdot e^{-\alpha t};$$

$$\dot{\psi}_2(0) = 1; \quad t_{max} = \frac{1}{\alpha} \quad \psi_{2max} = \frac{1}{\alpha} \cdot e^{-1} \qquad (36)$$

For $k \geq 3$ $\quad \dot{\psi}_k(t,\alpha) = (1 - \alpha\frac{t}{k-1}) \cdot \frac{t^{k-2}}{(k-2)!} \cdot e^{-\alpha t}$

$\dot{\psi}_k(0) = 0$; $t_{max} = \frac{k-1}{\alpha}$;

$$\psi_{k\,max} = \frac{1}{\alpha^{k-1}} \cdot \frac{(k-1)^{k-1}}{(k-1)!} \cdot e^{-(k-1)} \tag{37}$$

they have a maximum point $(t_{k\,max}, \psi_{k\,max})$.

The weighted moments of a function $y(t)$ have been defined and utilised in (Marin, 1992), (Marin et al., 1993),

$$m_k(\alpha) = \lim_{\theta\to\infty} m_k(\theta,\alpha) \tag{38}$$

where

$$m_k(\theta,\alpha) = \int_0^\theta \frac{(-1)^k}{k!} \cdot e^{-\alpha t} \cdot y(t) \cdot dt \tag{39}$$

It can be observed that the values $m(\theta)$ from the relation (30) represent

$$m(\theta) = m_0(\theta,\alpha) = \int_0^\theta e^{-\alpha t} \cdot y(t) \cdot dt \tag{40}$$

The unknown variable in equation (31) is a string of numbers. To get this it is necessary to build an algebraic system with an infinity number of equations. Such systems can be generated from (31) as follows:

1. Keep α constant and choose a string of distinct values $\{\theta_i\}_{i\geq 1}$ for the parameter θ.
2. Keep constant θ and choose a string of distinct values $\{\alpha_j\}_{j\geq 1}$ for the parameter α.
3. Use combinations of pairs $\{\theta_i, \alpha_j\}_{i\geq 1,\, j\geq 1}$.

The above infinite dimensional system of equations is approximated by a finite dimensional one, imposing to determine the first n values of the Markov parameters which constitutes the column vector,

$$w=[c_1 \ c_2 \ \dots \ c_n]^T \tag{41}$$

using n values for the parameter θ gathered in the column vector

$$\theta=[\theta_1 \ \theta_2 \ \dots \ \theta_n]^T \tag{42}$$

The finite system of equations is

$$A(\alpha,\theta) \cdot w = m(\theta,\alpha) + e(\theta,\alpha) \tag{43}$$

where

$$A(\alpha,\theta) = [a_1 \ a_2 \dots a_i \dots a_n]^T \tag{44}$$

$$a_i^T = [q_1(\theta_i,\alpha) \ \ q_2(\theta_i,\alpha) \dots q_n(\theta_i,\alpha)] \tag{45}$$

$$m^T(\theta,\alpha) = [m(\theta_1,\alpha) \dots m(\theta_n,\alpha)] \tag{46}$$

$$e^T(\theta,\alpha) = [e(\theta_1,\alpha) \dots e(\theta_n,\alpha)] \tag{47}$$

$$q_k(\theta_i,\alpha) = \int_0^{\theta_i} e^{-\alpha t} \cdot \frac{t^{k-1}}{(k-1)!} \cdot dt \tag{48}$$

$$m(\theta_i,\alpha) = \int_0^{\theta_i} e^{-\alpha t} \cdot y(t) \cdot dt \tag{49}$$

$$e(\theta_i,\alpha) = -\sum_{k=n+1}^{\infty} q_k(\theta_i,\alpha) \cdot c_k \tag{50}$$

The component $e(\theta_i,\alpha)$ of the vector $e(\theta,\alpha)$ expresses the error committed by truncating the series (31) to a finite number of terms, performing the evaluation with the parameters (θ_i,α). The absolute value of this error is bounded by

$$|e(\theta_i,\alpha)| \leq \sum_{k=n+1}^{\infty} |q_k(\theta_i,\alpha)| \cdot |c_k| \ ,$$

$$|e(\theta_i,\alpha)| \leq \sum_{k=n+1}^{\infty} |c_k| \cdot \int_0^{\theta_i} \psi_k(t,\alpha) dt \tag{51}$$

because the weighting functions $\psi_k(t,\alpha)$ are positive ones. In addition,

$$\int_0^{\theta_i} \Psi_k(t,\alpha) dt < \lim_{\theta_i\to\infty} \int_0^{\theta_i} \Psi_k(t,\alpha) \cdot dt =$$

$$= \int_0^{\infty} \frac{t^{k-1}}{(k-1)!} \cdot e^{-\alpha t} \cdot dt = \frac{1}{\alpha^k}, \forall \alpha > 0 \tag{52}$$

So $\quad |e(\theta_i,\alpha)| < \sum_{k=n+1}^{\infty} |c_k| \cdot \frac{1}{\alpha^k}, \quad \alpha > 0 \qquad (53)$

It can be observed that the right side term in the above inequality can be interpreted as coming from (22) for which $s = \alpha$, and according to (23)

$$|e(\theta_i,\alpha)| < \frac{M}{R} \cdot \frac{(R/\alpha)^{n+1}}{1-(R/\alpha)}, \quad \alpha > R > 0 \tag{54}$$

It exists a finite limit of the truncation error, the same for each equation, which is independent of the vector θ, but it depends on α and n, denoted

$$\varepsilon(\alpha,n) = \frac{M}{R} \frac{(R/\alpha)^{n+1}}{1-(R/\alpha)}, \quad \alpha > R > 0, \tag{55}$$

The parameters M and R are unknown. In addition the parameter α has to be selected from the convergence condition of the series rest (22), $\alpha > R$.

The string θ_i, of the values which constitute the vector θ, are selected only from the condition that, for an already chosen α, the matrix $A(\alpha,\theta)$ to be well conditioned.So, the n values of the Markov parameters are approximated by,

$$\hat{w} = [\hat{c}_1 \dots \hat{c}_n]^T, \ \hat{w} = A^{-1}(\alpha,\theta) \cdot m(\theta,\alpha) \tag{56}$$

with an error, determined by the series truncation,

$$\varepsilon_w = w - \hat{w} = A^{-1}(\alpha,\theta)\cdot e(\theta,\alpha)$$

Denoting

$B(\alpha,\theta) = A^{-1}(\alpha,\theta) = \{b_{ki}\},\ k,i \in [1,\ldots,n]$,

$b_{ki} = b_{ki}(\alpha,\theta)$ then the computed absolute value of a specific Markov parameter $c_k = w(k)$ accomplishes the condition

$$|c_k - \hat{c}_k| < b_k(\alpha,\theta)\cdot\varepsilon(\alpha,n) = \varepsilon_{k\,max} \tag{57}$$

where it has been denotes

$$b_k(\alpha,\theta) = \left|\sum_{i=1}^{n} b_{ki}(\alpha,\theta)\right| \tag{58}$$

4. PROCEDURE FOR THE CONVERGENCE DOMAIN EVALUATION

Several techniques can be imagined to appreciate the convergence radius and for selecting the factor α in suach away that, for a given value for n (that means the number of Markov parameters to be determined is imposed) to assure a minimum computing error.

The main idea is based on using the a priori information about the first Markov parameters, for example, $c_1 = y^{(0)}(0^+)$, $c_2 = y^{(1)}(0^+)$,which can be measured or numerically evaluated. If for a selected value of α, the ratio $\xi = \frac{R}{\alpha}$ (unknown R) has been assured, this means that the convergence conditions are accomplished and the inequality (24) takes place. In this case, the computed value $\hat{c}_1$ is finite, near the known value $c_1 = y^{(0)}(0^+)$. Otherwise, the value $\hat{c}_1$ is far away different of c_1 and it has to select for α a much greater value. If the convergence conditions, above mentioned, are accomplished, then the inequality (24) takes place and for $n=1, n=2$, we obtain,

$$g_1(\theta_i,\alpha) = |T_1(\alpha)| = |q_1(\theta_i,\alpha)\cdot c_1 - m(\theta_i,\alpha)| =$$
$$= |q_1(\theta_i,\alpha) y(0^+) - m(\theta_i,\alpha)| \tag{59}$$

$$g_2(\theta_i,\alpha) = |T_2(\alpha)| =$$
$$= |q_1(\theta_i,\alpha)\cdot c_1 + q_2(\theta_i,\alpha)\cdot c_2 - m(\theta_i,\alpha)|$$
$$= |q_1(\theta_i,\alpha)\cdot y(0^+) + q_2(\theta_i,\alpha)\cdot \dot{y}(0^+) - m(\theta_i,\alpha)| \tag{60}$$

So, $\forall\theta_i$,

$$g_1(\theta_i,\alpha) < \frac{M}{R}\cdot\frac{(R/\alpha)^2}{1-(R/\alpha)} = \frac{M}{\alpha}\cdot\frac{\xi^2}{1-\xi}$$

$$\text{if } \xi = (R/\alpha) < 1 \tag{61}$$

$$g_2(\theta_i,\alpha) < \frac{M}{R}\cdot\frac{(R/\alpha)^3}{1-(R/\alpha)} = \frac{M}{\alpha}\cdot\frac{\xi^3}{1-\xi}$$

$$\text{if } \xi = (R/\alpha) < 1 \tag{62}$$

According to the relation (18), the value of M, can be appreciated as,

$$M = \max_{t\geq 0}|y(t)| \tag{63}$$

Let we denote.

$$g_1(\alpha) = \max_{\theta_i}\{g_1(\theta_i,\alpha)\}$$
$$g_2(\alpha) = \max_{\theta_i}\{g_2(\theta_i,\alpha)\} \tag{64}$$

$$\eta = \frac{\xi^2}{1-\xi} \tag{65}$$

From (62) it results that

$$\eta > \frac{\alpha\cdot g_2(\alpha)}{M} = \eta_0 \Rightarrow \xi > \xi_0 \tag{66}$$

as it is illustrated in Fig. 2.

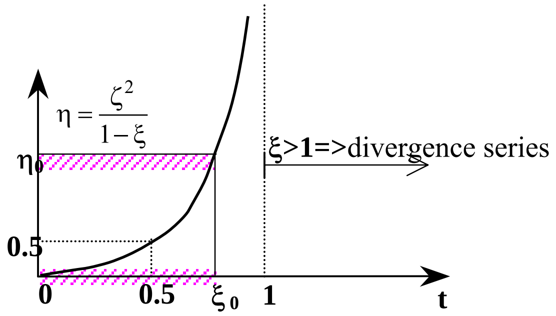

Fig. 2. Convergence domain.

In the convergence domain there is a self-equilibration process of the η_0 value that means the product $\alpha\cdot\xi_0 = R_0$ is changing around some values upper bounded by a value R, which expresses the radius of the series evaluation circle in the holomorphy domain. This circle contains all the poles of the expression $Y(s)$, but with no guaranty that R is just the convergence radius, that means the least value R for which the series is convergent. If for the ratio $\xi = R/\alpha$ it is selected a value $\xi > \xi_0$, this means that it is accepted an evaluation outside a circle having the radius R, where

$$R \geq \alpha\xi_0 = R_0. \tag{67}$$

From (61) it results that considering a ratio ξ_0 then the radius of the evaluation circle from the holomorphy domain will satisfy the inequality,

$$R_0 < \frac{M}{g_1(\alpha)}\cdot\frac{\xi_0^2}{1-\xi_0^2} = \frac{M}{g_1(\alpha)}\cdot\eta_0 = \alpha\cdot\frac{g_2(\alpha)}{g_1(\alpha)}. \tag{68}$$

It can be selected, as radius of convergence, the radius of a circle that contains all the poles of the Laplace transform $Y(s)$, having the value of

$$R_0 = \alpha \cdot \frac{g_2(\alpha)}{g_1(\alpha)} \tag{69}$$

Having already evaluated the convergence radius R_0, accepted as being the value R from relation (55), one can now appreciate the computing error of Markov parameters for $n \geq 3$, by using the relation (57). An open problem is related to the selection procedure of a division for θ and a value for α in such a way that this error to have minimum value and the matrix $A(\alpha,\theta)$ to be well conditioned. Intuitively can be observed that increasing the factor α will determine a decreasing of the truncation error $\varepsilon(\alpha,n)$ but the same time a decreasing of the conditioning factor of the matrix $A(\alpha,\theta)$. A large value for α in fact means a strong weighting of the response $y(t)$, will determine the integral evaluations to be concentrated toward the initial moment $t = 0$. This has unfavorable consequences in the effect reduction of the noise which has affected the useful signal $y(t)$ and which is filtered by integrating process.

5. CONCLUSIONS

The main goal of this paper is to analyze the possibility of Markov parameters evaluation for a linear proper system, based on equations built with information obtained by processing the experimental step response. These pieces of information are obtained performing weighted integrals on the step response over finite time intervals. The fundamental problems appeared in this study are related to the computing structures in the convergence domain of the Laurent series throughout the Markov parameters are defined. Remain open, for future researches, the problems related to the numerical calculation precision in this process.

6. REFERENCES

Chen C.T., Mital D. (1972). A simplified, Irreducible Realization Algorithm", IEEE, *Trans. on Aut. Constr*. Aug., 535-537.

Corduneanu A. (1981). *Differential equations.* Editura Facla, Iasi.

Ho B. L., Kalman R. E. (1965). Effective construction of linear state variable models from input-output data. *Proc3rdAllerton Conf.*, 449-459.

Ionescu V. (1985). *System Theory*. E.D.P. Bucureşti.

Kailath T. (1980). *Linear Systems.* Prentice-Hall, London.

Marin C. (1992). Process Control Algorithm. *UK Patent, Serial No* 9313*534.1/1992.*

Marin C., Wood J. (1993). Apparatus for and Method of Controlling a Process. *USA Patent No 08/108622/1993*.

Rozsa P., Sirha N. (1975). Minimal realization of a transfer function matrix in canonical forms. *Int. J. Control*. vol. 21, No. 2, 273-278.

Sabac I. (1965). *Special mathematics* (in Romanian), E.D.P. Bucureşti.