

TRACKING CONTROL ALGORITHM FOR INDUSTRIAL ROBOTS WITH UNKNOWN FRICTION AND PAYLOAD

Lőrinc Márton

*Dept. of Electrical Engineering
Sapientia Hungarian University of Transylvania
540053 Tg. Mures, P-ta. Trandafirilor 61. Romania
martonl@ms.sapientia.ro*

Abstract: *The paper presents a path tracking control for robotic manipulators. The proposed control law uses the model of the robot but the payload mass and the frictional parameters are considered totally unknown. The unknown parameters are determined using on-line gradient parameter estimation techniques. Using Lyapunov method it was shown that the resulting controller achieves guaranteed final tracking accuracy. Simulation results are presented to illustrate the effectiveness and achievable control performance of the proposed scheme.*

Keywords: *Friction compensation, Tracking robot control, Parameter estimation*

1. INTRODUCTION

Friction is universally present in the motion of bodies in contact. It plays a role in the simplest actions of living, such as walking, grasping and stacking. In servo controlled machines and industrial robots friction has an impact in all regimes of operation. It can generate undesired effects such that steady state error or limit cycles. The friction force can be described with models which parameters are time varying, depend on external factors such as lubricating conditions, temperature, humidity and other factors which hardly can be controlled. These parameters cannot be determined a-priori.

At the other hand, there is a tendency in robotic industry to build robots with lightweight arms to avoid unnecessary energy consumption. For this reason in the robot model, the mass of the payload cannot be neglected related to the masses of the arms. The mass of the payload in

many application is unknown, varies according to the specific task of the robot.

These considerations suggest that the friction parameters and the mass of the payload should be estimated on-line. The other parameters of the robot arm (masses, inertias, length of the arms) are often catalog data, which are given by the manufacturer of the robotic system [2]. If only the friction forces and the payload are unknown in the model it is unnecessary to use adaptive control algorithms, which estimates all the parameters of the robot.

In the past years the adaptive control methods [3] became a wide spread method to control not only the linear but also the nonlinear systems with unknown parameters. Adaptive control of robotic manipulators has also been intensively studied in the control community. Early results can be found in the work of Slotine and Li [4]. The introduction of a neural network in the adaptive control law to handle uncertainties due

to payload uncertainty was proposed by Leahy et al. [5]. A decentralized adaptive control algorithm for trajectory tracking was proposed by Fu [6]. The controller can be implemented in a decentralized way i. e. a sub-controller is independently and locally equipped for each joint servo loop. Lewis et al. [7] combine the neural control technique with linear optimal control theory to control rigid manipulators with completely unknown models. Barambones and Etxebarria [8] propose a controller for generic manipulator with unknown parameters and sliding-mode control, which robustifies the design and compensates the neural approximation errors.

2. THE CONTROL ALGORITHM

2.1 Robot Modelling

The mathematical model of an open chain, rigid, n degree of freedom robot is given by:

$$H(\underline{q})\ddot{\underline{q}} + C(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} + D(\underline{q}) = \underline{\tau} - \underline{h}_F(\dot{\underline{q}}) \quad (1)$$

where $\underline{q}$ is the joint position vector of the robot and the vector $\underline{\tau}$ represents the control signal given by current controlled DC servos. The following notations is used: $H(\underline{q})$ is the generalized inertia matrix, $C(\underline{q}, \dot{\underline{q}})$ shows the effect of the centripetal and Coriolis forces, $D(\underline{q})$ is the gravity effect, $\underline{h}_F(\dot{\underline{q}})$ represents the effect of the friction force which acts on the joints of the robot.

The following *properties* of the robot model are well known:

- I. The matrix $H(\underline{q})$ is positive definite
- II. The matrix $\dot{H}(\underline{q}) - C(\underline{q}, \dot{\underline{q}})$ is skew-symmetric, namely for any $\underline{x} \in R^n$ we have:

$$\underline{x}^T (\dot{H}(\underline{q}) - 2C(\underline{q}, \dot{\underline{q}}))\underline{x} = 0$$

To develop a control law with friction and payload estimation, firstly let us consider the following assumptions:

Assumption I (Payload) Let us assume that the dimensions of the payload can be neglected compared with the dimensions of the robotic arm so its length and inertia is neglected. In this case, the terms of the robotic model can be written in the following form:

$$H(\underline{q}) = H_R(\underline{q}) + mH_L(\underline{q}) \quad (2)$$

$$C(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} + D(\underline{q}) = C_R(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} + D_R(\underline{q}) + m(C_L(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} + D_L(\underline{q})) \quad (3)$$

where m is the mass of the payload. H_R , C_R and D_R are terms of the robotic arm model without payload. H_L , C_L and D_L are known terms that can be determined from the robot model. The mass of the payload is bounded.

Assumption II (Friction Modelling) The friction force acts separately on each joint of the robot. It can be written in a linearly parameterized form:

$$h_{Fi}(\dot{q}_i) = \mu(\dot{q}_i)\theta_{PFi}^T \underline{\xi}_{PFi}(\dot{q}_i) + (1 - \mu(\dot{q}_i))\theta_{NFi}^T \underline{\xi}_{NFi}(\dot{q}_i) \quad (4)$$

where $\mu(\dot{q}_i) = 1$ if $\dot{q}_i \geq 0$ and 0 otherwise. The vectors θ_{Fi} contain the unknown parameters of the friction model and $\underline{\xi}_{Fi}(\dot{q}_i)$ are known regressor vectors. This relation means that two separate linearly parameterized models, one for the positive velocity regime (P) and the other for negative velocity regime (N) can describe the friction. The switching between these two models occurs at the zero velocity. If we incorporate the switching function in the regressor vector we have:

$$h_{Fi}(\dot{q}_i) = \theta_{Fi}^T \underline{\xi}_{Fi}(\dot{q}_i) \quad (5)$$

where $\theta_{Fi} = (\theta_{PFi} \ \theta_{NFi})$ and $\underline{\xi}_{Fi}(\dot{q}_i) = (\mu(\dot{q}_i)\underline{\xi}_{PFi}(\dot{q}_i) \ (1 - \mu(\dot{q}_i))\underline{\xi}_{NFi}(\dot{q}_i))$

2.2 The control problem

The problem is to design a control input $\underline{\tau} = (\tau_1 \ \tau_2 \ \dots \ \tau_n)^T$ such that the joint position $\underline{q} = (q_1 \ q_2 \ \dots \ q_n)^T$ track the desired trajectory $\underline{q}_d = (q_{d1} \ q_{d2} \ \dots \ q_{dn})^T$. The desired trajectories q_{di} are known bounded functions of time with bounded known derivatives.

We define the error metric S_i that describes the desired dynamics of the error system for the i 'th joint:

$$S_i(t) = \left(\frac{d}{dt} + \lambda_i\right)e_i \quad e_i = q_i - q_{di} \quad (6)$$

where λ_i are positive constants.

Using this metric the control problem can be reformulated as follows: *design a control law $\underline{\tau}$ such that the tracking error metrics $S_i(t)$ satisfy $|S_i(t)| \rightarrow 0$ for $t \rightarrow \infty$*

The error dynamics for the entire robot can be written in the form:

$$\underline{S} = \dot{\underline{e}} + \Lambda \underline{e} \quad (7)$$

Using the relation (1) and *Assumption I* we get:

$$\begin{aligned} H(\underline{q})\dot{\underline{S}} = & -C_R(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} - D_R(\underline{q}) - H_R(\underline{q})(-\ddot{\underline{q}}_d + \Lambda \dot{\underline{e}}) \\ & - m(C_L(\underline{q}, \dot{\underline{q}})\dot{\underline{q}} + D_L(\underline{q}) + H_L(\underline{q})(-\ddot{\underline{q}}_d + \Lambda \dot{\underline{e}})) \\ & - \underline{h}_F(\dot{\underline{q}}) + \underline{\tau} \end{aligned} \quad (8)$$

2.3 Control Law

The control law is formulated as follows:

$$\begin{aligned} \underline{\tau} = & H_R(\ddot{\underline{q}}_d - \Lambda \dot{\underline{e}}) + C_R(\dot{\underline{q}}_d - \Lambda \underline{e}) + D_R + \hat{m}\underline{\xi}_m \\ & + \hat{\underline{h}}_F(\dot{\underline{q}}) - K_S \underline{S} \end{aligned} \quad (9)$$

where $\underline{\xi}_F$ is defined in *Assumption II*, $\underline{\xi}_m = H_L(-\ddot{\underline{q}}_d + \Lambda \dot{\underline{e}}) + C_L(-\dot{\underline{q}}_d + \Lambda \underline{e}) + D_L$, $\hat{m}$ denotes the estimated value of the payload, $\hat{\underline{h}}_F(\dot{\underline{q}})$ denotes the estimated friction force.

The matrix K_S is defined as a diagonal matrix $K_S = \text{diag}(K_{Si})$ where $K_{Si} > 0$.

The adaptation laws for the friction parameters in the i 'th joint and for the mass of the payload are:

$$\dot{\hat{\theta}}_{ij} = \Gamma_{ij} S_i \zeta_{Fij} \quad \dot{\hat{m}} = \gamma_m \underline{S}^T \underline{\xi}_m \quad (10)$$

where $\Gamma_j = \text{diag}(\gamma_{ij})$ with $\gamma_{ij} > 0$ and $\gamma_m > 0$.

2.4 Lyapunov analysis

In this Subsection will be shown that the introduced control law solves the control problem presented in Subsection 2.2. To examine the proposed control law let us consider the following Lyapunov-like function:

$$V(t) = \frac{1}{2} \underline{S}^T H(\underline{q}) \underline{S} + \frac{\gamma_m^{-1}}{2} \tilde{m}^2 + \frac{1}{2} \sum_{i=1}^n \tilde{\theta}_{Fi}^T \Gamma_i^{-1} \tilde{\theta}_{Fi} \quad (11)$$

The time derivate of $V(t)$ is given by:

$$\begin{aligned} \dot{V}(t) = & \frac{1}{2} \underline{S}^T H(\underline{q}) \dot{\underline{S}} + \frac{1}{2} \underline{S}^T \dot{H}(\underline{q}) \underline{S} \\ & + \gamma_m^{-1} \tilde{m} \dot{\tilde{m}} + \sum_{i=1}^n \tilde{\theta}_{Fi}^T \Gamma_i^{-1} \dot{\tilde{\theta}}_{Fi} \end{aligned} \quad (12)$$

where $\tilde{m} = m - \hat{m}$ and $\tilde{\theta}_{Fi} = \underline{\theta}_{Fi} - \hat{\underline{\theta}}_{Fi}$.

By substituting the expression of the control law (9) in the equation of error dynamics (8) and adding

$$(C_R + mC_L)(-\dot{\underline{q}}_d + \Lambda \underline{e}) - (C_R + mC_L)(-\dot{\underline{q}}_d + \Lambda \underline{e})$$

on the right side of the obtained relation we have:

$$\begin{aligned} H(\underline{q})\dot{\underline{S}} = & -C \underline{S} - \tilde{m}(H_L(-\ddot{\underline{q}}_d + \Lambda \dot{\underline{e}}) + \\ & C_L(-\dot{\underline{q}}_d + \Lambda \underline{e}) + D_L) - \tilde{\underline{h}}_F(\dot{\underline{q}}) - K_S \underline{S} \end{aligned} \quad (13)$$

In the view of (13) and robot arm propriety 2 and introducing the expression of adaptation laws (10) the derivate of Lyapunov function (12) becomes:

$$\dot{V}(t) = -\underline{S}^T K_S \underline{S} \leq 0 \quad (14)$$

In the equilibrium we have $S(t) = 0$. According to Lyapunov stability theory the error metric $S(t)$ converge to equilibrium state 0 and the control which uses the control law (9) and the adaptation law (10) is globally and asymptotically stable.

3. SIMULATION RESULTS

We demonstrate the performance of the proposed sliding mode control algorithm using a dynamical model of a 3 DOF cylindrical manipulator. The first degree of freedom represents the rotation around the robot base. The second degree of freedom represents the vertical, and the third degree of freedom represents the horizontal translation of the payload. The dynamics of this robotic system is nonlinear with strong coupling between the first and third degrees of freedom.

The equations of motion in terms of generalized coordinates $\underline{q} = (q_1 \ q_2 \ q_3)^T$ representing the positions of the joints and the applied torques $\underline{\tau} = (\tau_1 \ \tau_2 \ \tau_3)^T$ at these joints is given by:

$$\begin{aligned}
& \begin{pmatrix} H_{11} & 0 & 0 \\ 0 & M_0 + M_1 + m & 0 \\ 0 & 0 & M_1 + m \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} + \\
& + \begin{pmatrix} (2M_1q_3 - M_1R + 2mq_3)\dot{q}_3\dot{q}_1 \\ (M_0 + M_1 + m)g \\ -\frac{1}{2}(2M_1q_3 - M_1R + 2mq_3)\dot{q}_1^2 \end{pmatrix} \quad (15) \\
& = \begin{pmatrix} \tau_1 - h_{F1}(\dot{q}_1) \\ \tau_2 - h_{F2}(\dot{q}_2) \\ \tau_3 - h_{F3}(\dot{q}_3) \end{pmatrix}
\end{aligned}$$

with $H_{11} = J_0 + M_1 \frac{R^2}{3} + M_1(q_3^2 - Rq_3) + mq_{3L}^2$,

J_0 and M_0 represents the inertia and the mass of the base of the robot, M_0 is the mass of the horizontal arm, m is the mass of the payload, R is the length of the horizontal arm and g is the gravitational acceleration.

In the friction model the viscous friction term, which is proportional with the velocity, and the constant Coulombic friction term, which depends only on the sign of velocity, was considered. For each joint it can be written in the following form:

$$\begin{aligned}
h_{Fi}(\dot{q}_i) &= \mu(\dot{q}_i)(a_{iP}\dot{q}_i + b_{iP}) \\
&+ (1 - \mu(\dot{q}_i))(a_{iN}\dot{q}_i + b_{iN}) \quad i = 1 \dots 3 \quad (16)
\end{aligned}$$

The switching function μ is defined in *Assumption II*. Note that the parameters a_{iP} and a_{iN} respectively b_{iP} and b_{iN} may differ in the positive and negative velocity regimes.

The expressions of the model matrices and vectors necessary for the control law (H_R , H_L , C_R , C_L , D_R , D_L) can be obtained easily from (15).

The numerical values chosen for simulation proposes were chosen as $J_0 = 0.01 \text{ kgm}^2$, $M_0 = 2 \text{ kg}$, $M_1 = 1 \text{ kg}$, $R = 1 \text{ m}$, $g = 9.81 \text{ m/s}^2$, $m = 0.5 \text{ kg}$, $a_{iN} = -a_{iP} = 2 \text{ N}$, $b_{iN} = b_{iP} = 0.2 \text{ Ns/m}$, $i = 1 \dots 3$. The mass of the payload and the friction parameters were considered totally unknown. For the adaptation laws the initial values for all these parameters were taken 0.

The reference trajectory was chosen in such way that all the joints to have acceleration, deceleration and constant speed regimes. The

third joint is tested both in positive and negative velocity regimes.

Simulations were performed with and without adaptation. As it can be seen the control guarantees precise tracking and tracking error metrics convergence (Fig. 1). The performance of the adaptation laws can be seen in the Fig. 3 – 6. Due to the switching function μ , the friction parameters are tuned separately in positive and negative velocities. It can be noticed in the case of the third joint which is used both for negative and positive velocities.

4. REFERENCES

- [1] T. ZODIAC, Theory of Robot Control. Springer-Verlag London Ltd, 1996.
- [2] P. Corke and B. Armstrong-Helouvry, "A search for consensus among model parameters reported for the Puma 560 robot", in Proc. IEEE Conf. Robotics and Automation, 1994, pp. 1608–1613.
- [3] J. -J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, New Jersey: Prentice Hall International, Inc., 1991.
- [4] J. -J. E. Slotine and W. Li, "Adaptive manipulator control: A case study", IEEE Trans. on Automatic Control, vol. 33, no. 11, pp. 995–1003, November 1988.
- [5] M. Leahy, M. Johnson and S. Rogers, "Neural network payload estimation for adaptive robot control", IEEE Trans. on Neural Networks, vol. 2, no. 1, pp. 93–100, January 1991.
- [6] L. -C. Fu, "Robust adaptive decentralized control of robot manipulators", IEEE Trans. on Automatic Control, vol. 37, no. 1, pp. 106–110, January 1992.
- [7] Y. Kim, F. Lewis and D. Dawson, "Intelligent optimal control of robotic manipulators using neural networks", Automatica, vol. 36, pp. 1355–1364, 2000.
- [8] O. Barambones and V. Etxebarria, "Robust neural control for robotic manipulators", Automatica, vol. 38, pp. 235–242, 2002.

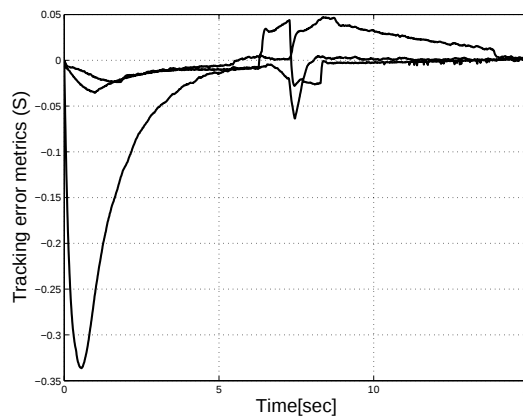


Fig. 1 Tracking Error Metrics

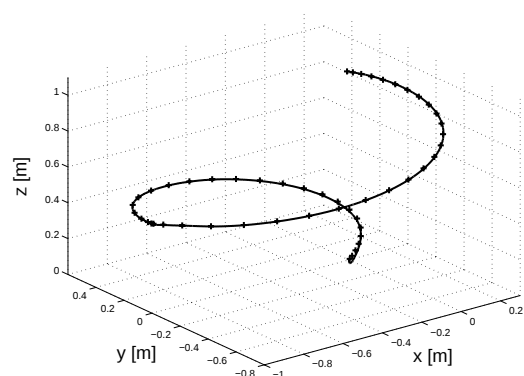


Fig. 2 Reference (+) and Real Trajectory

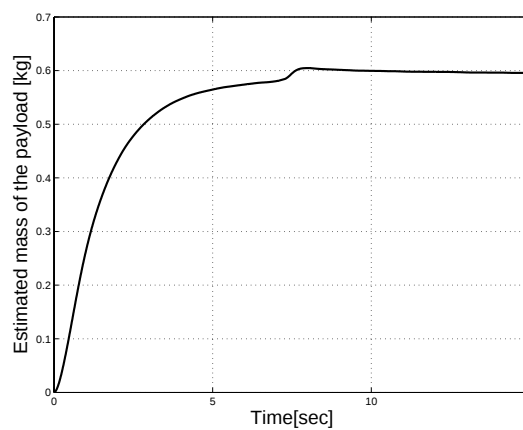


Fig. 3 Estimated payload mass

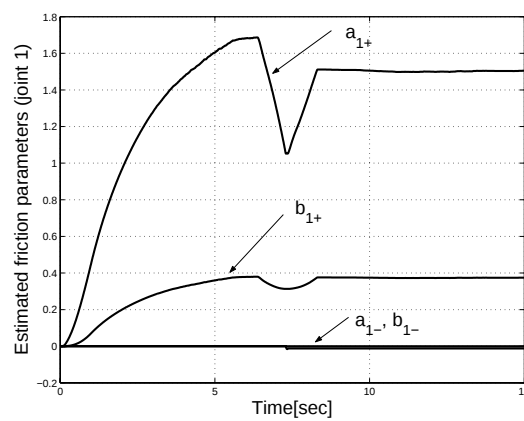


Fig. 4 Estimated Friction Parameters (joint 1)

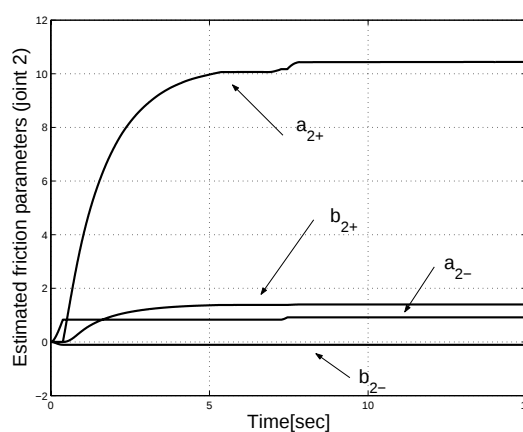


Fig. 5 Estimated Friction Parameters (joint 2)

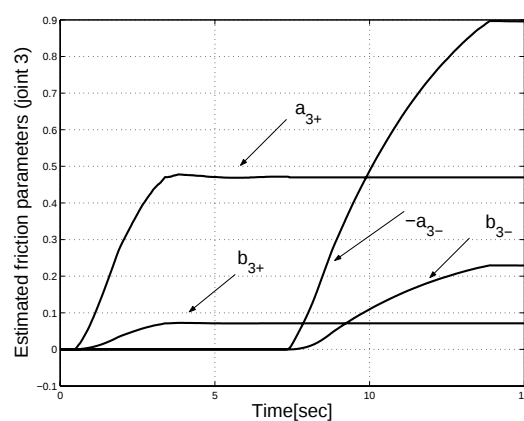


Fig. 6 Estimated Friction Parameters (joint 3)