Geometrical characterization of robust predictive control strategies

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Abstract: This paper reviews three methodologies: Model Predictive Control (MPC), Robust MPC based on LMIs and Desensitized MPC. These are applied for uncertainty LTI models and the necessary tools are provided for their maximal robust invariant set comparison. In the case of a classic MPC design one can consider an explicit controller for the nominal dynamic model (the close loop system becoming PWA polytopic) and compute its robust invariant set. If this set is not finetly determined, a contractive algorithm can be used to obtain an outer approximation. A certain degree of robustness can be added to such a construction by adjusting the cost function. Such a robustified control law can be compared in terms of maximal invariant set with the MPC schemes based on LMIs on one hand and with the desensitized version of MPC on the other hand.

Keywords: Model Predictive Control, Invariant set, Constrained linear system, Robust control.

1. INTRODUCTION

Model Predictive Control (MPC) is a constrained optimization based strategy of control in which a cost function is optimized at each sampling time and on finite receding horizon to obtain an optimal control to be applied to the system. This strategy of on-line optimization of the plant functioning don't guarantee a priori the stability of the closed system. Indeed if the system is unstable or nonminimal phase we can find optimal control sequences corresponding to an *inappropriate* cost functions which make the system to diverge. Also if the model of a system used to predict the behavior is not perfect (which is always true in practice) or if the system is subject to disturbance, finding an optimum to a cost function constructed on the nominal prediction is not sufficient to provide a robust control for the system.

In the same time, the optimization fundament imposes the feasibility as a crucial demand as long as it represents the main ingredient for the stability of the entire closed loop (Mayne et al., 2000). In this context, the positive invariance is an important ingredient assuring that the feasibility at the initial time will imply the feasibility at all future instants. Subsequently, the use of terminal constraints and the positive invariance arguments represent the current methodologies for obtaining stability guarantees.

The advances on the explicit solution for multiparametric programming made the analysis of the MPC laws easier (Bemporad et al., 2002). Even if the terminal conditions are not imposed at the design stage, one can obtain the region of the state space where the closed loop system will perform adequately for any receding horizon control by post-processing. Different other receding horizon control schemes need to construct the maximal positively invariant set or approximate it accordingly (Gondhalekar and Imura, 2007).

Knowing that for constrained linear systems, the predictive control problems lead to piecewise affine control laws, the problem of invariant set construction is equivalent to the analysis of the stability (invariance) of piecewise affine systems. In this framework, the difficulties are originated by the fact that iterative algorithms do not offer guarantees for the decidability. This is due to the fact that the sets are not finitely parameterized (see related discussions in Gilbert and Tan (1991), Rakovic et al. (2004), Vidal et al. (2000), Alamo et al. (2005)).

In the present paper we start from these basic facts and identify three predictive control schemes able to handle uncertainties in the nominal LTI prediction model. The first techniques is based on classical MPC design tuned to assure a robust behavior in a nonempty neighborhood of the origin. Secondly the well understood predictive control design based on LMI is recalled for its polytopic uncertainty handling capabilities. Finally, the desensitized MPC scheme will be presented as an alternative for the case when the structure of the model parameters' variation is known. Using the formulation of desensitized MPC explained by (Lana and Rotea, 2007), we want to look at the influence of this formulation on the robust invariant set (RIS), computed from an explicit solution without terminal constraints. A comparison between the RIS computed from an explicit solution without terminal constraints (Benlaoukli and Olaru, 2008), and the RIS considering the dynamic of the sensitivity trajectory as shown in (Lana and Rotea, 2007). The comparison of robust invariant sets will provide a geometric characterization of the classical MPC, robust MPC and the predictive control with sensitivity considered in the cost function.

A numerical example with uncertain parameters will be used to illustrate the concepts. In the case of classical MPC the RIS will be constructed using the equivalent PWA model affected by uncertainty, in the case of MPC based on LMI one can use the level set of the associated Lyapunov function while in the desensitized case one can use contractive algorithm in (Benlaoukli and Olaru, 2008).

In the following, section 2 recalls some basic facts about predictive control design, the explicit formulation of the control laws and LMI-based robust control laws. In section 3, the robust MPC, desensitized MPC and LMI-based predictive control will be presented. Sections 4 describes the principle of expansive and contractive algorithms for the construction of invariant sets for PWA systems, section 5 presents numerical example illustrating a comparison between the robust invariant sets computed with these several procedure. Finally the conclusions are drawn in section 6.

1.1 Preliminary notations

It is considered in the following that the polyhedral objects are bounded and can be expressed in a dual constraints-generators representation. Each region is defined as an intersection of a finite number of half space $P_i = \{x | H_i^T x \leqslant b_i\}$. Several operations on polyhedra will be currently used during the further polyhedral computations: the union and intersection of two polyhedra are defined respectively as $P_1 \cap P_2 = \{x | x \in P_1 \ and \ x \in P_2\}$ and $P_1 \cup P_2 = \{x | x \in P_1 \ or \ x \in P_2\}$. Polyhedra difference $P_1 \backslash P_2 = \{x | x \in P_1 \ and \ x \notin P_2\}$. Several software packages offer the capabilities to handle these operations in a trusty and efficient manner.

2. MPC PRINCIPLES AND LMI BASE CONTROL DESIGN

2.1 Classical MPC. Explicit formulation

Let a discrete time LTI system defined by :

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ Cx_k + Du_k \leq E \end{cases} \tag{1}$$

In Model Predictive Control (MPC), a constrained optimal control problem over a finite receding horizon must be solved at each sample time. Generally the cost function to be minimized has the form:

$$J_k = \sum_{i=1}^{N} x_{k+i}^T Q x_{k+i} + \sum_{i=0}^{N-1} u_{k+i}^T R u_{k+i} + x_N^T P x_N \tag{2}$$

such that N is the prediction horizon, $Q = Q^T$ and $R = R^T$ are the weighting factors on states error and control. P is the solution of Riccati equation:

$$P_{LQ} = Q + A^T P_{LQ} A - A^T P_{LQ} B [R + B^T P_{LQ} B]^{-1} B^T P_{LQ} A \tag{3}$$

and the corresponding stabilizing optimal controller is:

$$K_{LQ} = [R + B^T P_{LQ} B]^{-1} B^T P_{LQ} A. \tag{4}$$

Practically, by considering the constraints on the input signals and state trajectories, MPC apply the first component of the optimal control sequence:

$$\begin{array}{l} \min\limits_{u_k, u_{k+1}, \ldots, u_{k+N-1}} \sum\limits_{i=1}^{N} \|x_{k+i}\|_Q^2 + \sum\limits_{i=0}^{N-1} \|u_{k+i}\|_R^2 \\ \text{subject to: } \begin{cases} x_{k+i+1} = Ax_{k+i} + Bu_{k+i} \\ Cx_{k+i} + Du_{k+i} \leq E \end{cases} \end{array} \tag{5}$$

The methodology has real-time limitations for fast dynamical system and the construction of explicit solution (Bemporad et al., 2002) may overcome the on-line computational burden. This is equivalent to the resolution of multi-parametric quadratic programs (mp-QP) (Tøndel et al., 2003) where the parameters are the components of the state vector and the future references.

The explicit controller obtained is defined over a set of polytopic regions $P_{set} = \bigcup_{i=1}^{N} P_i$ where the intersections $P_i \cap P_j$ are not full dimensional $\forall i \neq j$. The controller is defined for each region P_i as:

$$K_i = F_i x + G_i \tag{6}$$

and the system (1) becomes autonomous one:

$$\begin{aligned} x_{k+1} &= (A + BF_i) x_k + BG_i \\ &= f_{PWA}^i(x_k) \end{aligned} \tag{7}$$

2.2 LMI-based robust control law

Consider an uncertainty system defined by its nominal dynamic (1) and d extreme realization (14). It is considered that the control u_k is bounded in norm as:

$$\|u_{k+i}\|_2 \leq u_{max}$$

and the output is bounded in norm as:

$$\|y_{k+i}\|_2 \leq y_{max}$$

The construction of a robust control law that stabilize (14) by considering its worst case (extreme dynamic) at each simpling time can be obtained if the following cost function finds a bounded optimal value:

$$\min_{u_k, u_{k+1}, \ldots, u_{k+H_u-1}} \max_{A_\Delta, B_\Delta} J_\infty(k) \tag{8}$$

where

$$J_\infty(k) = \sum_{i=0}^{\infty} (x_{k+i}^T Q x_{k+i} + u_{k+i}^T R u_{k+i}) \tag{9}$$

It is shown in (Kothare et al., 1996) that $\max\limits_{A_\Delta, B_\Delta} J_\infty(k)$ may be upper-bounded by:

$$V(x) = x^T P_{LMI} x \tag{10}$$

by using LMI formulation (Kothare et al., 1996) to obtain robust terminal cost P_{LMI}. The optimization problem to be solved is shown in (11). If this minimization problem is feasible, then the robust controller is:

$$K_{LMI} = Y S^{-1} \tag{12}$$

and the associated Lyapunov function is constructed as a quadratic function using:

$$P_{LMI} = \gamma S^{-1} \tag{13}$$

$$
\min_{\gamma,S,Y} \gamma
$$
$$
\text{s.t:}\begin{cases}
\begin{bmatrix} S & SA_j^T + Y^T B_j^T & SQ^{0.5} & Y^T R^{0.5} \\ A_j S + B_j Y & S & 0 & 0 \\ Q^{0.5} S & 0 & \gamma I & 0 \\ R^{0.5} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \\
\text{for } j = 1 \dots d. \\
\begin{bmatrix} u_{max}^2 I & Y \\ Y^T & S \end{bmatrix} \geq 0 \\
\begin{bmatrix} S & (A_j S + B_j Y)^T C^T \\ C(A_j S + B_j Y) & y_{max}^2 I \end{bmatrix} \geq 0 \\
\text{for } j = 1 \dots d.
\end{cases} \tag{11}
$$

3. ROBUST MPC

3.1 Robust MPC

In the case when the system is affected by uncertainties, one can retain the nominal prediction model as in (1) and consider a model affected by polytopic uncertainty:

$$
x_{k+1} = A_\Delta x_k + B_\Delta u_k \tag{14}
$$

described by d extreme realizations such that $\exists 0 \leq \alpha_i \leq 1$, $i = 1, \dots, d$ and:

$$
\begin{cases}
(A_\Delta, B_\Delta) = \sum_{i=1}^{d} \alpha_i (A_i, B_i) \\
\sum_{i=1}^{d} \alpha_i = 1.
\end{cases} \tag{15}
$$

It is supposed that the nominal model is inside this polytopic description (A_Δ, B_Δ).

Recalling the classical MPC design (5) based on the nominal prediction model, in the presence of uncertainty (14) the robustness become critical. The use of a the terminal cost matrix P_{LMI} obtained from a LMI-based robust control law (13):

$$
J_k = \sum_{i=1}^{N-1} x_{k+i}^T Q x_{k+i} + \sum_{i=0}^{N-1} u_{k+i}^T R u_{k+i} + x_N^T P_{LMI} x_N \tag{16}
$$

can provide an explicit controller that guarantee at least a stabilizing region (that contain origin) for which the constraints are satisfied. The invariance of this explicit controller over the entire partition of the state space $P_{set} = \bigcup_{i=1}^{N} P_i$ is not guaranteed, by consequence one needs to compute the invariant controller by using algorithms presented in the section 4.

3.2 Desensitized MPC

The desensitized MPC (DMPC) formulation (Lana and Rotea, 2007), considers the trajectory sensitivity as a complementary factor in the cost function (2). Such a construction is claimed to improve the robustness of the MPC design.

Let a dynamical system defined by:

$$
x_{k+1} = A_\mu x_k + B_\mu u_k \tag{17}
$$

such that $\mu = [\mu_1, \dots, \mu_m] \in \mathbb{R}^m$ is the vector of parameters. We define the variation of a given Matrix M in relation to μ by:

$$
\nabla_\mu M := \left[\frac{\partial M^T}{\partial \mu_1} \cdots \frac{\partial M^T}{\partial \mu_m} \right]^T \tag{18}
$$

and we consider μ_0 as the nominal vector of parameters. In (Lana and Rotea, 2007), the trajectory sensitivity p at μ_0 is defined as:

$$
p_k := \nabla_\mu x_k |_{\mu_0} \tag{19}
$$

and

$$
p_{k+1} = A_\mu x_k + A_b p_k + B_\mu u_k + B_b u_{\mu k} \tag{20}
$$

where:

$$
\begin{cases}
A_\mu = \nabla_\mu A_\mu |_{\mu_0}, \\
B_\mu = \nabla_\mu B_\mu |_{\mu_0}, \\
A_b = I_m \otimes A_{\mu_0}, \\
B_b = I_m \otimes B_{\mu_0}, \\
u_\mu = \nabla_\mu u_k |_{\mu_0}.
\end{cases}
$$

By considering the sensitivity trajectory the cost function to be minimized is defined as:

$$
\min_{u_k, u_{k+1}, \dots, u_{k+H_u-1}} \sum_{i=1}^{H_p} \|x_{k+i}\|_Q^2 + \|p_{k+i}\|_Q^2 + \sum_{i=0}^{H_u-1} \|\Delta u_{k+i}\|_R^2 \tag{21}
$$
$$
\text{s. t.:} \begin{cases}
x_{k+i+1} = A|_{\mu_0} x_{k+i} + B|_{\mu_0} u_{k+i} \\
p_{k+i+1} = A_\mu x_{k+i} + A_b p_{k+i} + B_\mu u_{k+i} \\
C x_{k+i} + D u_{k+i} \leq E
\end{cases}
$$

3.3 LMI-based predictive control

In the LMI-based control law an LMI-based optimization problem (11) is solved at each sampling time to compute the robust control action. The minimization problem is defined in (22), and in which constraints on control and output are considered. If this problem has solution, the controller at sampling time k is:

$$
u_{k+i} = K_k x_{k+i} \tag{23}
$$

and

$$
K_k = Y Q^{-1} \tag{24}
$$

In the next section the geometrical characterization of the invariant set for these control design techniques will be detailed.

4. INVARIANT SET CONSTRUCTION

4.1 Invariant set for PWA systems

The stability guarantee of MPC scheme can be obtained by the use of terminal constraints which assure the positive invariance of the feasible domain with respect to the closed loop dynamics. In the absence of terminal constraints which reinforce the invariance properties, the explicit

$$\min_{\gamma,S,Y} \gamma$$
$$\text{s.t:} \begin{cases} \begin{bmatrix} 1 & x_k^T \\ x_k & Q \end{bmatrix} \geq 0 \\ \begin{bmatrix} S & SA_j^T + Y^T B_j^T & SQ^{0.5} & Y^T R^{0.5} \\ A_j S + B_j Y & S & 0 & 0 \\ Q^{0.5} S & 0 & \gamma I & 0 \\ R^{0.5} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \\ \quad \text{for } j = 1 \dots d. \\ \begin{bmatrix} u_{max}^2 I & Y \\ Y^T & S \end{bmatrix} \geq 0. \begin{bmatrix} S & (A_j S + B_j Y)^T C^T \\ C(A_j S + B_j Y) & y_{max}^2 I \end{bmatrix} \geq 0 \\ \quad \text{for } j = 1 \dots d. \end{cases} \tag{22}$$

controller is defined over a region of the state space without guaranteeing the proper functioning of the close loop system. This has to be verified a posteriori by identifying the invariant set.

Contractive invariant set computation The principle of the contractive construction of an outer approximation for the maximal robust invariant set are described in (Benlaoukli and Olaru, 2008). A brief overview is given to illustrate the principle.

Let a piecewise affine autonomous dynamic system defined by (7). The following algorithm computes an upper bound $\Phi \supseteq \Phi^{MPI}$ where Φ^{MPI} is the maximal positively invariant set (MPI) of the piecewise affine system (7).

Algorithm 1. Contractive set construction.
Input: PWA system defined over polyhedron regions P_{set}.
Output: The (approximate) maximal positive invariant set Φ.

1: **while** ("precision condition" not true) $\wedge$ (max. no. of iterations)
2: $\quad N = cardinal(P_{set})$
3: $\quad i = 1$
4: $\quad$ **while** $(i \leq N)$
5: $\quad\quad po = \emptyset$
6: $\quad\quad$ **for** $j = 1, N$
7: $\quad\quad\quad po = po \bigcup preImag(Image(P_i) \cap P_j)$
8: $\quad\quad$ **end**
9: $\quad\quad m = cardinal(po)$
10: $\quad\quad P_{set} = \{P_1, ..., P_{i-1}, po, P_{i+1}, ..., P_N\}$
11: $\quad\quad i = i + m$
12: $\quad\quad N = cardinal(P_{set})$
13: $\quad$ **end**
14: **end**
15: $\Phi = P_{set}$

The algorithm eliminates at each iteration the subset of each P_i, $i = 1, ..., N$ that leaves P_{set} in one step with the forward dynamic (7). If P_{set} do not change at the updated (step 10), then the construction stops and the result is the exact invariant set, otherwise an expansive algorithm is used together with the contractive one to born the exact maximal positively invariant set.

Expansive invariant set computation The expansive algorithm proceeds in reverse way comparing to algorithm 1. A lower bound for the invariant set $\Psi \subseteq \Psi^{MPI} = \Phi^{MPI} \subseteq \Phi$ will be computed starting from an initial positive invariant set (for example the maximal output admissible region - (Gilbert and Tan, 1991) - for the region containing the origin). Alternatively, by choosing P_1 the region in P_{set} that contain the origin and by applying algorithm 1 one gets the initial invariant set Ψ_1. The result is stored in Ψ. subsequently, the idea is to compute all regions of P_{set} that transit in one step to Ψ. Iteratively Ψ is updated to contain all the sets reaching the invariant set in one step. It can be shown that Ψ expands and thus a lower bound for the maximal positively invariant set is obtained.

Algorithm 2. Expansive set construction.
Input: PWA system defined over polyhedron regions P_{set}.
Output: The (approximate) Invariant set $\Psi \subseteq \Psi^{MPI}$.

1: Let $P_1 \in P_{set}$ be the polytope that contains origin.
2: Use Algorithm 1 to compute the invariant set of P_1. Let the result be Ψ_1
3: $SSet = \Psi_1$
4: **while**("precision condition" not true) $\wedge$ (max. no. of iterations)
5: $\quad N = cardinal(P_{set})$
6: $\quad Interm = \emptyset$
7: $\quad L = cardinal(SSet)$
8: $\quad$ **for** $i = 1, N$
9: $\quad\quad po = \emptyset$
10: $\quad\quad$ **for** $j = 1, L$
11: $\quad\quad\quad po = po \bigcup preImag(Image(P_i) \cap SSet_j)$
12: $\quad\quad$ **end**
13: $\quad\quad Interm = Interm \bigcup po$
14: $\quad$ **end**
15: $\quad \Psi = \Psi \bigcup Interm$
16: $\quad SSet = Interm$
17: **end**

In same way when the exact invariant set cannot be obtained in finite time, then algorithms 1-2 provide an inner approximation of the maximal invariant set.

Remark 1. : If $\Psi_1 = \emptyset$ in (step 2), then the expansive algorithm can not compute any invariant set as long as the initialization step cannot be performed.

4.2 Robust invariant set computation for PWA uncertain systems

In section 4.1, we have given two algorithms to compute the maximal invariant set for PWA systems, and the same idea is developed here for the calculation of robust invariant set for an uncertain PWA system which can embed a large class of nonlinear systems.

The previous algorithms can be adapted to the case of polytopic PWA systems but precautions have to be taken in order to assure that an non-empty invariant set will be obtained. If the control law associated with the region containing the origin is robustly stabilizing the uncertain system, then the nonemptiness of the invariant set is guaranteed. This conditions are fulfilled by the use of the modified cost function (16).

Contractive and Expansive algorithms for robust invariant set computation follow for same idea as in Alg.1-Alg.2 by the use of $Image$ and $preImage$ operations with respect to all d extreme dynamics as long as the positive invariance has to be verified with respect to the set-valued mapping (15).

4.3 Ellipsoidal invariant sets

In the case of LMI-based predictive control in (22), (23) and (24), it is shown that for $u_{k+i} = K_k x_{k+i}$, $i \geq 0$ then if:

$$x_k^T Q^{-1} x_k \leq 1 \tag{25}$$

or, equivalently

$$x_k^T P_{LMI} x_k \leq \gamma \text{ with } P_{LMI} = \gamma Q^{-1} \tag{26}$$

Then

$$\max_{A_\Delta, B_\Delta} x_{k+i}^T Q^{-1} x_{k+i} < 1, \ i \geq 1 \tag{27}$$

and

$$\max_{A_\Delta, B_\Delta} x_{k+i}^T P_{LMI} x_{k+i} < \gamma, \ i \geq 1 \tag{28}$$

This level sets represent invariant ellipsoidal sets for the dynamics described by $[A_\Delta, B_\Delta]$. Considering the set of constraints to be satisfied by (1), the maximal robust positively invariant set is described by the union of ellipsoids (28) given by feasible pairs $(P_{LMI}(x), \gamma(x))$.

5. NUMERICAL EXAMPLE

Let the uncertain parameter system defined by:

$$\begin{cases} x_{k+1} = \begin{bmatrix} 1 & 1 \\ 1+\mu & 0.5 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k \\ y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k \end{cases} \tag{29}$$

such that the uncertain parameter $\mu \in [-0.1, +2.8]$, and its nominal value $\mu_0 = 0$. A simple optimal control problem with prediction horizon $N = 2$, $Q = I_2$ and $R = 1$ is considered. The constraints to be satisfied are:

$$\begin{cases} [-0.5, -0.5]^T \leqslant y_k \leqslant [0.5, 0.5]^T \\ -0.2 \leqslant u_k \leqslant 0.2 \end{cases} \tag{30}$$

and the extreme dynamics are defined by:

$$\begin{cases} x_{k+1}^1 = \begin{bmatrix} 1 & 1 \\ 0.9 & 0.5 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k \\ x_{k+1}^2 = \begin{bmatrix} 1 & 1 \\ 3.8 & 0.5 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k \end{cases} \tag{31}$$

In the following we consider these three predictive control strategies MPC, RMPC and DMPC.

5.1 MPC strategy

In this first case, we consider the cost function in (2). $P_{LQ} = \begin{bmatrix} 1.8243 & 0.6275 \\ 0.6275 & 1.6135 \end{bmatrix}$ is the solution of Riccati equation (3) that provides the terminal cost corresponding to the stabilizing controller $K_{LQ} = -[0.8243 \ 0.6275]$ for the nominal dynamic. Using multiparametric toolbox (Kvasnica et al., 2004) an explicit controller is obtained with 5 regions. From this explicit control law the contractive algorithm is used to compute the robust invariant set by considering its extreme dynamics.

Figure 1 shows the explicit controller which has an empty invariant set because the stabilizing controller K_{LQ} in region that contain origin can't stabilize the second extreme dynamic.

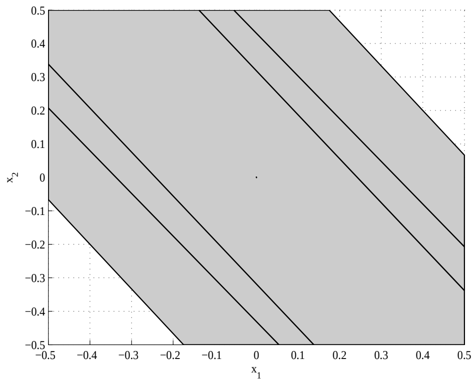


Fig. 1. Explicit controller with an empty invariant set.

5.2 RMPC strategy

Now by using LMI formulation as mentioned in (11) to compute the robust controller that stabilize the system with its all dynamics. The terminal cost $x_N^T P x_N$ for (2) is obtained by following these steps:

- compute $P_{LMI} = \begin{bmatrix} 29.0687 & -2.2891 \\ -2.2891 & 3.1386 \end{bmatrix}$ and its corresponding stabilizing controller $K_{LMI} = -[1.0280 \ 0.9419]$ that stabilizes all system's dynamics without considering any constraints,
- compute by inverse optimality (Larin, 2003) the new weight factors $Q_{inv} = \begin{bmatrix} 0.8900 & -0.6257 \\ -0.6257 & 0.4992 \end{bmatrix} 10^{-4}$ and $R_{inv} = 1.8362 \ 10^{-7}$,
- using Riccati equation (3), we compute the new $P_{LQ} = \begin{bmatrix} 0.8918 & -0.6240 \\ -0.6240 & 0.6658 \end{bmatrix} 10^{-4}$ corresponding to K_{LMI} and its new Q_{inv} and R_{inv},
- compute the explicit controller using the cost function (2).

Figure 2 shows the explicit controller and its corresponding robust invariant set.

The ellipsoidal set (28) corresponding to K_{LMI} computed without considering any constraints is shown in Fig. 3 as E_1, and the ellipsoidal set E_2 that satisfies all constraints is computed by contracting E_1 by factor $\gamma/10$. These ellipsoidal sets are shown in Fig. 3.

The superposition of the ellipsoidal and robust invariant set is shown in Fig. 4.

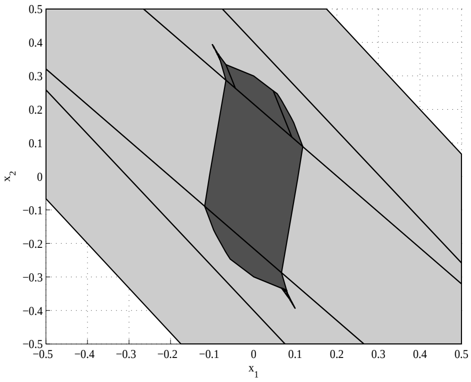


Fig. 2. Explicit controller (light grey) and the invariant controller (dark grey).

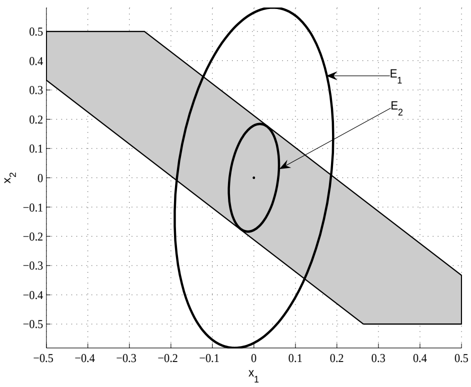


Fig. 3. The ellipsoidal invariant set satisfying all constraints E_2.

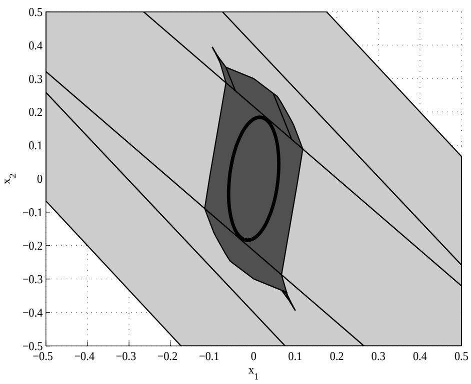


Fig. 4. Superposition of the invariant set and the ellipsoidal invariant set.

5.3 DMPC strategy

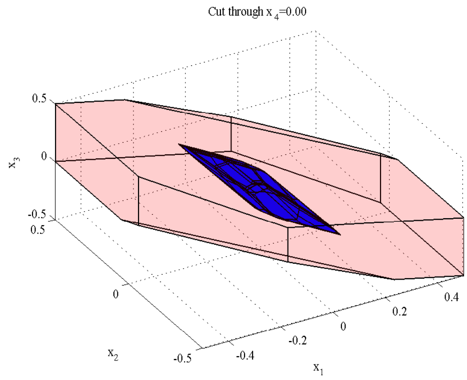
In the DMPC case, the dynamic of sensibility with respect to μ is considered as it is shown in (21). All coefficients of (20) are:

$$A_\mu = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},$$

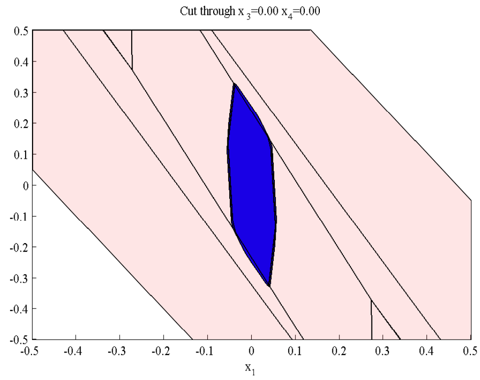
$$A_b = \begin{bmatrix} 1 & 1 \\ 1 & 0.5 \end{bmatrix},$$

$$B_\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

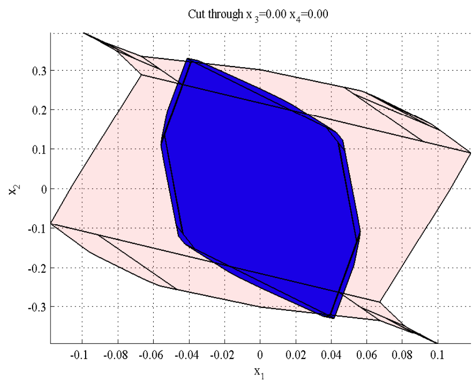
Now by following the same steps as in MPC strategy to compute the explicit controller and its invariant set, we obtain solutions shown in Fig. (5).



(a)



(b)



(c)

Fig. 5. (a) Convex hull of the explicit controller (13 regions in light red color) and its invariant set (133 regions in blue color). (b) Section throw dimensions [3 4] is done by considering $p_k = 0$ for all k; in comparison with the explicit controller and the corresponding invariant set. (c) Comparison between the invariant set for RMPC (light red color) and DMPC (blue color).

6. CONCLUSION

In this paper we have reviewed some existing methodologies to compute a robust control laws for uncertain systems. Using LMI modifications we can obtain a controller with invariant sets larger than classical constructions which don't include a robust terminal cost and in the same time larger than the ellipsoidal invariant set. We observe also that the desensitized MPC is augmenting the order of system, it renders the computation of the invariant set complex but by penalizing the sensitivity of the trajectory, a robust explicit controller can be conceived.

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