Improved State Estimation for Stochastic Nonlinear Chemical Reactor using Particle Filter based on Unscented Transformation

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Abstract: The state estimation problem in a stochastic nonlinear and non-Gaussian system is solved by the particle filter. Particle filters require an importance proposal distribution from which samples are drawn which would be equivalent to drawing samples from the posterior distribution. The choice of a suitable importance distribution to represent true posterior density is a crucial step in the design of particle filter. The unscented Kalman filter (UKF) using unscented transformation technique provides better state estimates and also has the capability of generating heavier tailed distributions than the widely used extended Kalman filter (EKF). Hence, this paper utilizes the UKF based on statistical linearization method in the particle filter for generating an importance proposal to achieve improved state estimation than the extended Kalman particle filter (EKPF). The effectiveness of the particle filter based on unscented transformation over EKPF is illustrated by conducting simulation studies on the stochastic nonlinear continuous stirred tank reactor (CSTR) and the divergence problem in the EKPF is also discussed.

Keywords: State estimation, importance proposal, particle filter, unscented transformation, statistical linearization, nonlinear continuous stirred tank reactor.

1. INTRODUCTION

Filtering is the problem of estimating the states (hidden variable) of a system from the available observations. The knowledge of such states can be used for monitoring, fault diagnosis and control in the field of process engineering. The state estimation of time varying system is an important prerequisite for safe and economical process operations (Ho and Lee, 1964). The Kalman filter is the optimal filter used for solving the state estimation problem in a linear system. The Kalman filter has limited applications in the engineering field because most of the systems exhibit nonlinear dynamics. The extended Kalman filter (EKF) which is an extension of the Kalman filter is used to provide solution to the nonlinear state estimation problem (Gelb, 1974; Anderson and Moore, 1979). The EKF requires the analytical computation of Jacobians at each time step but such computations become tedious for high dimensional systems. An alternative Kalman based estimator, the unscented Kalman filter (UKF) uses a deterministic sampling technique called unscented transformation (UT) in order to generate more accurate estimates of the covariance of the state than the well known EKF. The UKF does not involve the calculation of Jacobians. Hence, this filter is derivative free and it overcomes the limitations of the EKF (Julier and Uhlmann, 1997).

For nonlinear systems, the process and the measurement noise are normally assumed to be Gaussian but after passing them through the nonlinear system equations, evolve in to a non-Gaussian distribution (Shenoy et al., 2011). Both the nonlinear filters discussed above do not address non-Gaussian distributions. The particle filter (PF) based on the sequential Monte Carlo method addresses the distributions that are non-Gaussian (Gordon et al., 1993; Liu and Chen, 1998; Doucet et al., 2000). Earlier, it was used mostly for the tracking applications but currently, the researchers started applying the particle filter in the field of process engineering (Chen et al., 2005; Jayaprasanth and Jovitha, 2014). The complete representation of posterior distribution of the states can be easily computed using the PF. It requires the design of proposal distribution that can approximate the true posterior distribution reasonably well. The critical issue in the design of PF is the selection of the proposal distribution. The transition prior is chosen as proposal distribution in the basic sequential importance resampling particle filter (SIR-PF). The resampling is carried out to eliminate particles with lower weights and to retain only the particles with higher weights. Sometimes, the SIR-PF may become inefficient because the importance distribution does not take the current observation in to account (Pitt and Shephard, 1999).

The shortcomings in the SIR-PF motivated the researchers to design better importance proposal distribution. The importance distribution can be well approximated by incorporating the most recent measurement using either EKF or UKF as proposal (Daum, 2005; Ristic et al., 2004). The EKF usually relies on a first order Taylor series approximation, but such approximations can sometimes lead to poor representations of highly nonlinear functions, thereby resulting in divergence of the filter. Hence, the EKF as proposal distribution for the PF (i.e. EKPF) may not always provide good estimates due to its above limitations. If the EKF proposal is replaced by the UKF proposal, the resulting PF can perform much better because the UKF is more accurate and also allows one to control the rate at which the tails of the proposal distribution go to zero (Van et al., 2000). The objective of this paper is to investigate on the state estimation performance of PF based on unscented transformation over EKPF by applying to a stochastic nonlinear chemical reactor and then to compute and compare the performance index of filters using root mean squared error (RMSE). As both these PFs have in common its proposal distribution dependent on the current measurement, it can be an interesting problem to analyze and comment on the response of these filters when used for such an industrial system.

This paper is organized as follows: Section 2 provides the UKF algorithm and its advantages over EKF. The particle filtering algorithm based on unscented transformation is presented in Section 3. The state estimation performance of the local linearization particle filters for CSTR is presented in Section 4. Finally, conclusions are drawn in Section 5.

2. DERIVATIVE FREE KALMAN FILTER

The EKF algorithm uses analytic linearization approach for estimating the states of the nonlinear system. Hence, the EKF can be viewed as providing first-order approximations to the optimal terms. These approximations, however, can underestimate the estimates of the state covariance. An alternative nonlinear filter is the UKF where the above drawback has been overcome using the concept of sample statistics. The UKF addresses the problem faced by the EKF by using a deterministic sampling technique known as the unscented transformation (UT). The UT is a method involving the calculation of statistics of a random variable by selecting a minimal set of sample points (sigma points) around the mean. These sigma points are then propagated through the true nonlinear system and capture the posterior mean and covariance accurately to the third order Taylor series approximations for any nonlinearity. As this technique removes the requirement to analytically calculate Jacobians, the UKF is also called as derivative free Kalman filter (Wan and Van, 2000; Julier and Uhlmann, 2004).

Consider a nonlinear system represented by the following nonlinear state space equation:

$$x_k = f(x_{k-1}, u_{k-1}, v_{k-1}) \tag{1}$$

$$y_k = g(x_k, n_k) \tag{2}$$

where x_k is the system state vector, u_k is the system input, v_k is the process noise, y_k is the measured state variable, n_k is the measurement noise and the parameter k represents the time step. The nonlinear functions $f(.)$ and $g(.)$ represent the process model and measurement model respectively. The process noise can be either due to random fluctuations in the input variables or inaccuracies in the process model. The measurement noise can be due to inaccuracies in the sensor.

The UKF algorithm using the scaled UT for the time step k is as follows:

Time update: The estimated state $\hat{x}_{k-1}$ and covariance P_{k-1} at the previous time step are augmented with the mean and covariance Q of the process noise v_k respectively.

$$x_{k-1}^a = [\hat{x}_{k-1}^T \quad E[v_k^T]]^T \tag{3}$$

$$P_{k-1}^a = \begin{bmatrix} P_{k-1} & 0 \\ 0 & Q \end{bmatrix} \tag{4}$$

A set of $2L+1$ sigma points is derived from the augmented state and covariance. The sigma matrix which is the augmented sigma vector is obtained as follows:

$$\mathcal{X}_{k-1} = \left[x_{k-1}^a \quad x_{k-1}^a \pm \left(\sqrt{(L+\lambda)P_{k-1}^a} \right) \right] \tag{5}$$

where L is the dimension of the augmented state and $\lambda = \alpha^2(L+\kappa) - L$ is a scaling parameter. α controls the size of the sigma point distribution which is usually set to a small positive value and κ is a secondary scaling parameter which guarantees the positive semi definiteness of the covariance matrix.

The sigma points are propagated through the nonlinear system model to calculate the prior estimates.

$$\mathcal{X}_{k|k-1} = f(\mathcal{X}_{k-1}) \tag{6}$$

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2L} W_{(m)}^i \mathcal{X}_{i,k|k-1} \tag{7}$$

$$P_{k|k-1} = \sum_{i=0}^{2L} W_{(c)}^i [\mathcal{X}_{i,k|k-1} - \hat{x}_{k|k-1}][\mathcal{X}_{i,k|k-1} - \hat{x}_{k|k-1}]^T \tag{8}$$

where the weights for the predicted state and covariance are as follows:

$$W_{(m)}^0 = \frac{\lambda}{L+\lambda} \tag{9}$$

$$W_{(c)}^0 = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta) \tag{10}$$

$$W_{(m)}^i = W_{(c)}^i = \frac{1}{2(L+\lambda)} \quad i = 1,..,2L \tag{11}$$

The weighting term β is used to incorporate knowledge of the higher order moments of the distribution.

Measurement update: The predicted state $\hat{x}_{k|k-1}$ and covariance $P_{k|k-1}$ are augmented with the mean and covariance R of the measurement noise n_k respectively.

$$x^a_{k|k-1} = [\hat{x}^T_{k|k-1} \quad E[n^T_k]\,]^T \tag{12}$$

$$P^a_{k|k-1} = \begin{bmatrix} P_{k|k-1} & 0 \\ 0 & R \end{bmatrix} \tag{13}$$

The augmented sigma vector in this update step is obtained as follows:

$$\mathcal{X}_{k|k-1} = \left[x^a_{k|k-1} \quad x^a_{k|k-1} \pm \left(\sqrt{(L+\lambda)P^a_{k|k-1}} \right) \right] \tag{14}$$

The sigma points are propagated through the measurement model to calculate the predicted measurement and its covariance.

$$\mathcal{Y}_k = g(\mathcal{X}_{k|k-1}) \tag{15}$$

$$\hat{y}_k = \sum_{i=0}^{2L} W^i_{(m)} \mathcal{Y}_{i,k} \tag{16}$$

$$P_{y_k y_k} = \sum_{i=0}^{2L} W^i_{(c)}\, [\mathcal{Y}_{i,k} - \hat{y}_k][\mathcal{Y}_{i,k} - \hat{y}_k]^T \tag{17}$$

The state-measurement cross-covariance matrix,

$$P_{x_k y_k} = \sum_{i=0}^{2L} W^i_{(c)}\, [\mathcal{X}_{i,k|k-1} - \hat{x}_{k|k-1}][\mathcal{Y}_{i,k} - \hat{y}_k]^T \tag{18}$$

The posterior estimates of the state and covariance are calculated by using the Kalman gain,

$$K_k = P_{x_k y_k} P^{-1}_{y_k y_k} \tag{19}$$

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k(y_k - \hat{y}_k) \tag{20}$$

$$P_k = P_{k|k-1} - K_k P_{y_k y_k} K^T_k \tag{21}$$

The UKF algorithm can provide improved solution to the state estimation problem in the nonlinear domain when the distributions are Gaussian.

3. PARTICLE FILTERING

Particle filtering is based on recursive Bayesian filtering with Monte Carlo simulations. The PF provides better estimates than the EKF and UKF when non-Gaussian posterior are obtained for a nonlinear system (Doucet et al., 2001). It provides information on the entire posterior distribution of the state and not just the expectation of the state estimate. PFs rely on importance sampling and, as a result, more particles $\{x^i_k, i = 1,..., N_p\}$ are generated from the proposal distribution $\pi(x_k \mid y_{1:k})$ to approximate the posterior distribution $p(x_k \mid y_{1:k})$ of the states. As the number of particles N_p tends to infinity, the posterior density function can be approximated arbitrarily well by the point-mass estimate as follows:

$$p(x_k \mid y_{1:k}) \approx \sum_{i=1}^{N_p} w^i_k \delta(x_k - x^i_k) \tag{22}$$

where $\delta(.)$ denotes the Dirac delta function and the weights are normalized such that $\sum_i w^i_k = 1$. The normalized weights w^i_k are chosen using the principle of importance sampling as (Ristic et al., 2004)

$$w^i_k \alpha \frac{p(x^i_k \mid y_{1:k})}{\pi(x^i_k \mid y_{1:k})} \tag{23}$$

The choice of the importance proposal distribution $\pi(x^i_k \mid y_{1:k})$ is the key design issue in the PF. The SIR-PF assumes the transition prior $p(x_k \mid x_{k-1})$ which is independent of the current measurement as the proposal distribution. As a result, the state estimates are obtained from the SIR-PF without any knowledge of the measurement data. Hence, the performance of this filter may diverge because of the weaker assumptions (Arulampalam et al., 2002).

If the proposal distribution in the PF is designed by incorporating the most recent observation y_k through a bank of nonlinear filters such as EKF or UKF, then the corresponding filter is referred as local linearization particle filter. When the EKF is used as the importance proposal for PF, such a combination is called EKPF (De Frietas et al., 2000). The EKF is based on analytic local linearization method because the Jacobian matrices have to be worked out analytically at each time step which provides additional complexity. The EKPF also faces the above issue since it involves the EKF proposal.

3.1 Particle Filtering based on Unscented Transformation

When the UKF is used to generate the proposal for the PF, then the resulting estimator is referred as unscented particle filter (UPF). Such a filter can also be called as statistical local linearization particle filter because the UKF performs statistical local linearization. The UKF more accurately propagates the mean and covariance of the Gaussian approximation to the state distribution which leads to better estimation results (Jacob and Dhib, 2011; Ungarala, 2012). As the UKF algorithm is free from the calculation of Jacobians, the UPF also eliminates the need to perform any analytic differentiation. Since the UPF is free from analytic computations, it tends to be the finest solution provider to nonlinear estimation problem in the various fields of engineering (Ning and Fang, 2008; Prakash et al., 2011; Shen, 2014).

The basic framework for the UPF (and the EKPF) involves the estimation of the states of a stochastic nonlinear dynamic system given by process model (1) and measurement model (2). In the UPF, a separate UKF is used to generate a Gaussian proposal distribution as shown in (24) and it is used by each particle (index *i*) obtained from the particle filtering algorithm.

$$\pi(x_k^i \mid x_{k-1}^i, y_k) = \mathbb{N}(x_k^i; \hat{x}_k^i, P_k^i) \tag{24}$$

where $\hat{x}_k^i$ and P_k^i are the estimates of the mean and covariance of a particle respectively. The symbol $\mathbb{N}$ represents the Gaussian distribution. In summary, the particle filter based on unscented transformation for the time step k is as follows:

a) For $i = 1 : N_p$
- Run UKF
  $[\hat{x}_k^i, P_k^i] = \text{UKF}[x_{k-1}^i, P_{k-1}^i, y_k]$
- Draw a sample from the proposal distribution, i.e. $x_k^i \sim \mathbb{N}(x_k^i; \hat{x}_k^i, P_k^i)$
- Calculate importance weight
  $$w_k^i = \frac{p(y_k \mid x_k^i)\, p(x_k^i \mid x_{k-1}^i)}{\pi(x_k^i \mid x_{k-1}^i, y_k)}$$

End

b) Normalize the importance weights.

c) Resample to get an updated particle set $\{x_k^j, i^j\}_{j=1}^{N_p}$, where j refers to the index of the particle after resampling. The updated relationship in this framework is represented as parent$(j) = i$.

d) For $j = 1 : N_p$
- Assign Covariance: $P_k^j = P_k^{i^j}$

End

The output of the algorithm is the mean of the particle set that can be computed as follows:

$$\hat{x}_k = \frac{1}{N_p} \sum_{j=1}^{N_p} x_k^j \tag{25}$$

It must be noted that the statistical local linearization particle filter propagates the particles towards the high likelihood region to ensure improved estimation performance.

4. RESULTS AND DISCUSSIONS

In this section, the performance of the particle filtering algorithm based on unscented transformation is demonstrated by application to a stochastic non-adiabatic chemical reactor. The results of the EKPF are also presented for comparison.

4.1 Stochastic Nonlinear Chemical Reactor

A common chemical reactor used in the process industry is the continuous stirred tank reactor (CSTR). The reactor as shown in Fig. 1 has input-output operating curves that are nonlinear and it is cooled by a single coolant stream. It is normally operated at steady state and an exothermic and irreversible reaction takes place. It is used primarily for liquid phase reactions (Prakash and Senthil, 2008).

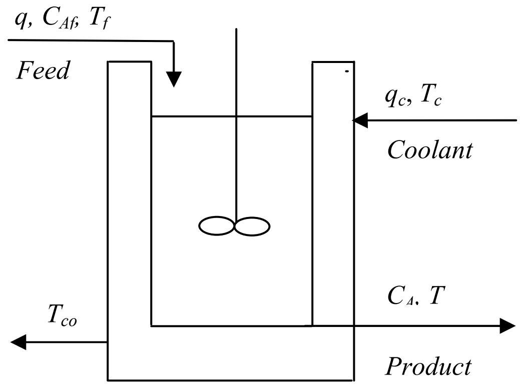


Fig. 1. Continuous stirred tank reactor with cooling jacket.

The nonlinear CSTR process is modeled by the following governing equations:

$$\frac{dC_A}{dt} = \frac{q}{V}\left(C_{Af} - C_A\right) - K_0 C_A e^{\left(\frac{-E}{RT}\right)} \tag{26}$$

$$\frac{dT}{dt} = \frac{q}{V}\left(T_f - T\right) - \frac{\left(-\Delta H\right) K_0 C_A}{\rho C_p} e^{\left(\frac{-E}{RT}\right)} + \frac{\rho_c C_{pc}}{\rho C_p V} q_c \left\{1 - e^{\left(\frac{-\text{hA}}{q_c \rho C_p}\right)}\right\}\left(T_c - T\right) \tag{27}$$

The steady state operating data for the process considered in this work is given in Table 1.

Table 1. Nominal operating data.

Process Variable	Steady State Operating Condition
Product concentration (C_A)	0.0885 mol/L
Reactor temperature (T)	441.1475 K
Coolant flow rate (q_c)	100 L/min
Process flow rate (q)	100 L/min
Feed concentration (C_{Af})	1 mol/L
Feed temperature (T_f)	350 K

Inlet coolant temperature (T_c)	350 K
CSTR volume (V)	100 L
Heat transfer term (hA)	7 x 10^5 cal/(min K)
Reaction rate constant (K_0)	7.2 x 10^{10} min^{-1}
Activation energy term (E/R)	1 x 10^4 K
Heat of reaction $(-\Delta H)$	-2 x 10^5 cal/mol
Liquid density (ρ, ρ_c)	1000 g/L
Specific heats (C_p, C_{pc})	1 cal/(g K)

The estimation problem requires the knowledge of the initial estimates and the noisy observations in order to recursively generate the estimates of the state variables. The true state is computed by solving the nonlinear differential equations using ode solver function in MATLAB. The state vector $x = \begin{bmatrix} C_A & T \end{bmatrix}^T$ comprises of the product concentration and reactor temperature. The measurement vector y for this process is of the form:

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} C_A \\ T \end{bmatrix} \qquad (28)$$

where C_A and T are the unmeasured state and the measured state respectively. The steady state operating condition of the states is chosen as the initial states for this problem. We have assumed that the random errors are present in the state variables (C_A and T) as well as in the measurement (T). Hence, the CSTR process considered in this work is regarded as stochastic system. However, the variance should not be increased more than one order of magnitude as it results in larger oscillations in the estimates of the reactor. The covariance matrices of process noise and measurement noise are assumed as

$$Q = \begin{bmatrix} (0.00088)^2 & 0 \\ 0 & (0.441)^2 \end{bmatrix} \text{ and } R = \begin{bmatrix} (0.441)^2 \end{bmatrix}$$

4.2 State Estimation Performance of Local Linearization Particle Filters

The state estimation scheme using local linearization particle filters for CSTR as shown in Fig. 2 has been developed with the sampling time of 0.083 min. It should be noted that using local linearization particle filter significantly reduces the number of particles required for generating improved estimates in comparison to the SIR-PF. Hence, the simulation studies are conducted on a nonlinear continuous reactor to analyze the performance of EKPF and UPF using the number of particles, $N_p = 30$.

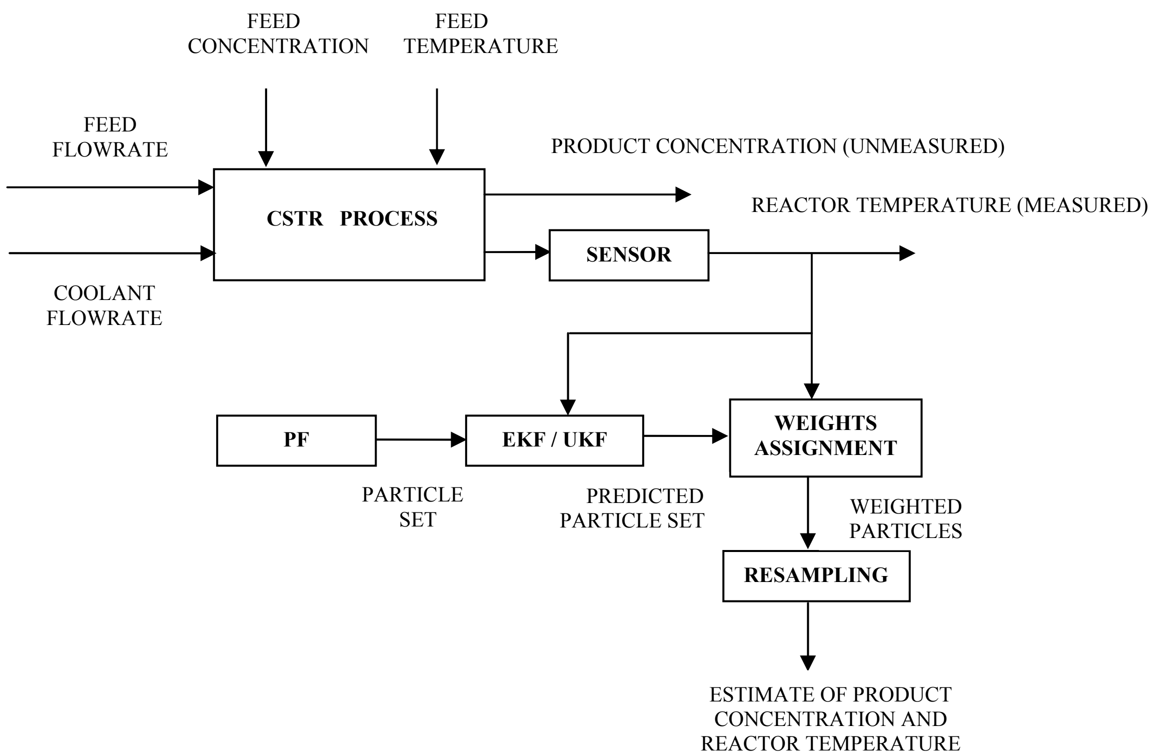


Fig. 2. Schematic layout of state estimation scheme using local linearization particle filters.

An evaluation of the estimation performance of the both the local linearization particle filters are carried out under the same conditions, so as to have an effective performance comparison. The filter tuning parameters are set as $\alpha = 0.01, \kappa = 3$ and $\beta = 5$ for the UKF algorithm considered in this work. Analysis of the filter performance is carried out by providing a step change in the q_c from 100 L/min to 106 L/min at 50^{th} instant, in order to provide nonlinearity in the CSTR operating region. It is noticed that in Fig. 3, the unmeasured product concentration estimated by the UPF tracks closely the true state trajectory as compared to the estimate of the EKPF. Fig. 4 depicts that the UPF estimates the measured reactor temperature much closer to the true reactor temperature than the EKPF. Figs. 5 and 6 show the state estimation error of local linearization particle filtering algorithms (EKPF and UPF) for the product concentration and reactor temperature respectively. Hence, it is observed that the UPF relatively achieves better tracking performance than the EKPF.

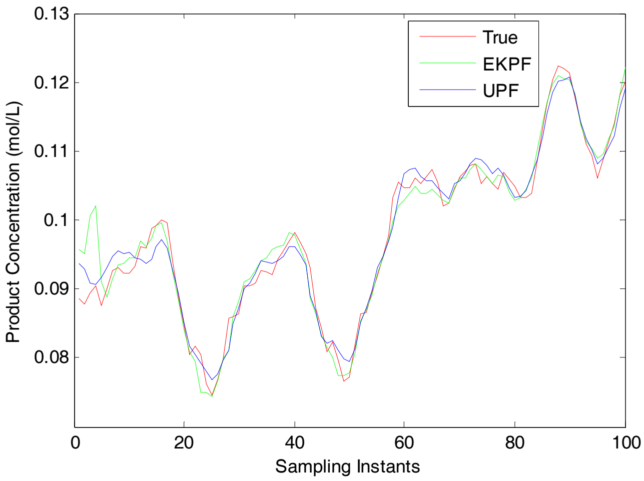


Fig. 3. Evolution of true and estimated state of product concentration in CSTR using EKPF and UPF.

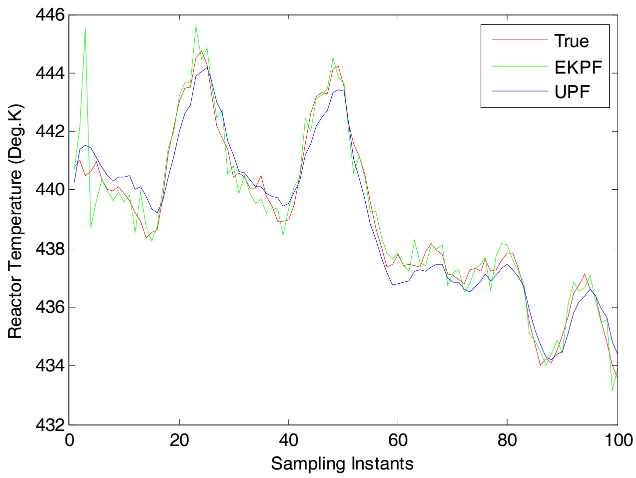


Fig. 4. Evolution of true and estimated state of reactor temperature in CSTR using EKPF and UPF.

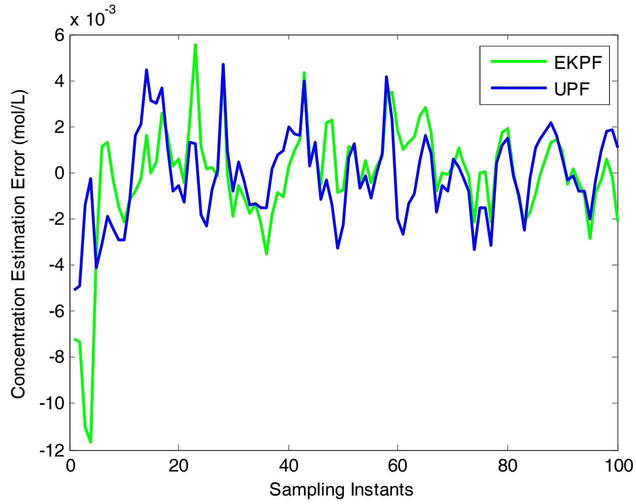


Fig. 5. Estimation error of EKPF and UPF for product concentration in CSTR.

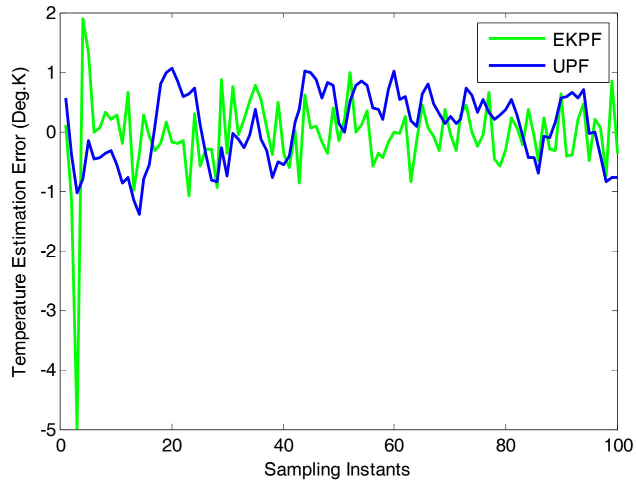


Fig. 6. Estimation error of EKPF and UPF for reactor temperature in CSTR.

The estimation performance of these PFs is compared by calculating the root mean squared error (RMSE). The RMSE over a Monte Carlo run is computed as follows:

$$\text{RMSE}=\left(\frac{1}{t}\sum_{k=1}^{t}\left(x_k-\hat{x}_k\right)^2\right)^{1/2} \tag{29}$$

where x_k and $\hat{x}_k$ are true and estimated state at the instant k respectively and t indicates the total number of time steps.

Table 2. Comparison of EKPF and UPF average RMSE values.

State Variable	Property	EKPF	UPF
C_A	Unmeasured	0.0025	0.0019
T	Measured	0.7037	0.5828

The average RMSE values for the PFs for 100 Monte Carlo runs are accounted in Table 2 which signifies that the PF based on statistical local linearization (UPF) provides better estimates than the PF based on analytic local linearization (EKPF). Fig. 7 depicts a bar chart for comparative performance analysis of the particle filters.

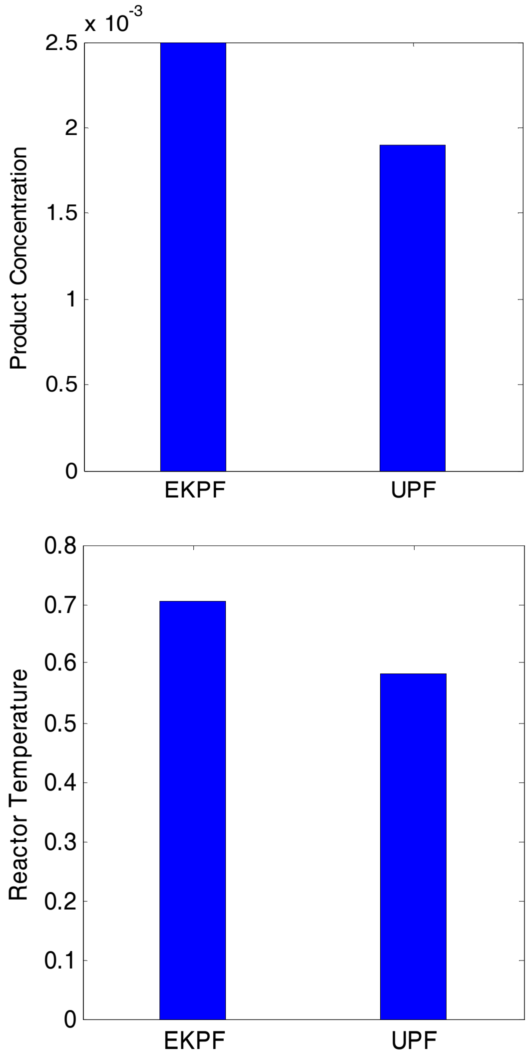


Fig. 7. Estimation performance analysis of the particle filters (EKPF and UPF).

As discussed earlier, the estimation of states from the EKF is only through analytic linearization technique and when such filter is used by each particle in the PF; the resulting EKPF may tend to diverge suddenly at any instant and fail to converge back to the true state. Figs. 8 to 11 reveal that due to the analytic approximations in the EKF algorithm, the EKPF diverges to a larger degree and results in large state estimation error but the UPF continues to estimate the system states satisfactorily.

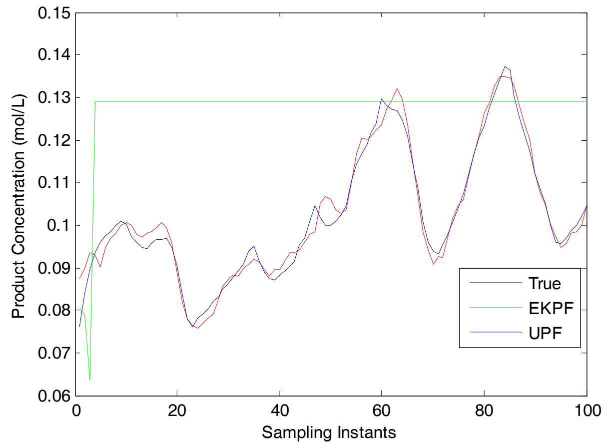


Fig. 8. Estimation performance of diverging EKPF and UPF for product concentration in CSTR.

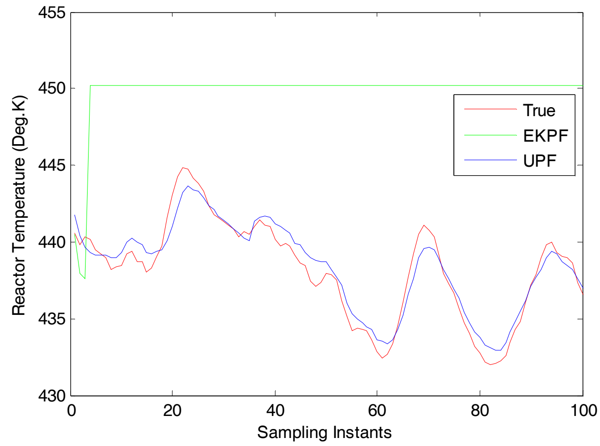


Fig. 9. Estimation performance of diverging EKPF and UPF for reactor temperature in CSTR.

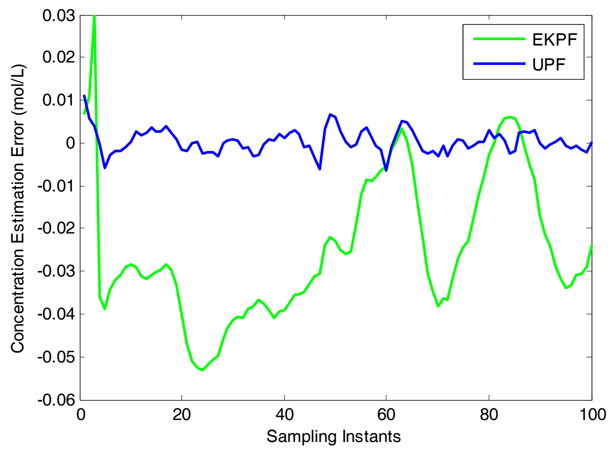


Fig. 10. Estimation error of EKPF under divergence and UPF for product concentration in CSTR.

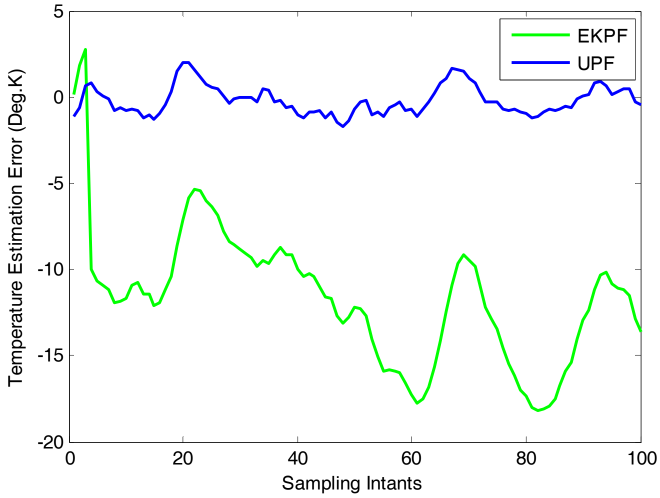
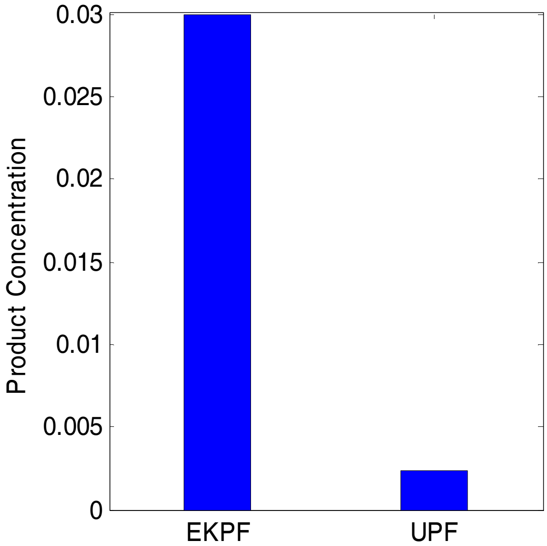


Fig. 11. Estimation error of EKPF under divergence and UPF for reactor temperature in CSTR.

The RMSE values of the estimates are provided in Table 3 which clearly indicates that the UPF completely outperforms the EKPF.

Table 3. Comparison of diverging EKPF and UPF RMSE values.

State Variable	Property	EKPF (under divergence)	UPF
C_A	Unmeasured	0.0299	0.0023
T	Measured	12.2759	0.6590



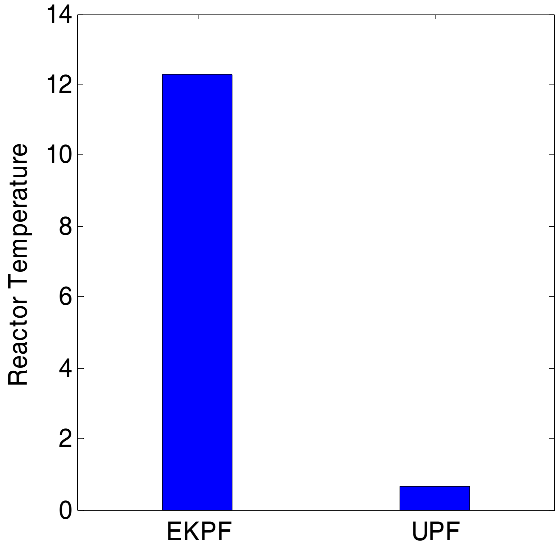


Fig. 12. Estimation performance analysis of the particle filters (diverging EKPF and UPF).

A bar chart shown in Fig. 12 illustrates the comparative performance analysis of the diverging EKPF and UPF. This signifies that the UPF can be chosen as the better state estimator than the EKPF for highly nonlinear state estimation problem in the advanced process control applications.

5. CONCLUSIONS

The local linearization particle filters are used in this work to provide solution to the nonlinear state estimation problem. Estimation of states is carried out for a stochastic nonlinear CSTR system by using EKPF and UPF, both having its proposal distribution dependent on the most recent measurement. It is observed from the simulation studies that the EKF also moves the prior towards the likelihood function, thus possibly creating a better proposal distribution for a PF, but it is done at the cost of introducing inaccuracies due to analytic linearization. The UPF addresses the shortcomings of the EKPF by generating proposal distribution with larger higher order moments using the UKF algorithm and obtaining the states that are closer to the true state of the system. Hence, from the obtained results, it is clear that the UPF performs relatively better nonlinear state estimation than the EKPF.

REFERENCES

Anderson, B.D. and Moore, J.B. (1979). *Optimal filtering.* Prentice-Hall, New Jersey, USA.

Arulampalam, M., Maskell, S., Gordon, N.J. and Clapp, T. (2002). A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. *IEEE Trans. Signal Process.*, 50(2), 174-188.

Chen, T., Morris, J. and Martin, E. (2005). Particle filtering for state and parameter estimation in batch processes. *J. Process Contr.*, 15(6), 665-673.

Daum, F. (2005). Nonlinear filters: Beyond the Kalman filter. *IEEE Aerosp. Electron. Syst. Mag.*, 20(8), 57-69.

De Frietas, N., Niranjan, M., Gee, A.H. and Doucet, A. (2000). Sequential Monte Carlo methods to train neural network models. *Neural Comput.,* 12(4), 955-993.

Doucet, A., Godsill, S. and Andrieu, C. (2000). On sequential Monte Carlo sampling methods for Bayesian filtering. *Stat. Comput.,* 10(3), 197-208.

Doucet, A., de Frietas, N. and Gordon, N.J. (2001). *Sequential Monte Carlo methods in practice*. Springer-Verlag, New York, USA.

Gelb, A. (1974). *Applied optimal estimation.* M.I.T. Press, Cambridge MA, USA.

Gordon, N.J., Salmond, D.J. and Smith, A.F.M. (1993). Novel approach to nonlinear/non-Gaussian Bayesian state estimation. *IEE Proc.-F.,* 140(2), 107-113.

Ho, Y.C. and Lee, R.C.K. (1964). A Bayesian approach to problems in stochastic estimation and control. *IEEE Trans. Autom. Control*., 9, 333-339.

Jacob, N.C. and Dhib, R. (2011). Unscented Kalman filter based nonlinear model predictive control of a LDPE autoclave reactor. *J. Process Contr.,* 21(9), 1332-1344.

Jayaprasanth, D. and Jovitha, J. (2014). Analytic local linearization particle filter for Bayesian state estimation in nonlinear continuous process. *WSEAS Trans. Syst.,* 13, 154-163.

Julier, S.J. and Uhlmann, J.K. (1997). A new extension of the Kalman filter to nonlinear systems. in *Proc. SPIE,* 3068, 182-193.

Julier, S.J. and Uhlmann, J.K. (2004). Unscented filtering and nonlinear estimation. in *Proc. IEEE*, 92(3), 401-422.

Liu, J.S. and Chen, R. (1998). Sequential Monte Carlo methods for dynamical systems*. J. Am. Stat. Assoc.,* 93, 1032-1044.

Ning, X. and Fang, J. (2008). Spacecraft autonomous navigation using unscented particle filter –based celestial/doppler information fusion. *Meas. Sci. Technol.,* 19(9), 1-8.

Pitt, M. and Shephard, N. (1999). Filtering via simulation: Auxiliary particle filters. *J. Am. Stat. Assoc.,* 94(446), 590-599.

Prakash, J. and Senthil, R. (2008). Design of observer based nonlinear model predictive controller for a continuous stirred tank reactor. *J. Process Contr.,* 18(5), 504-514.

Prakash, J., Patwardhan, S.C. and Shah, S.L. (2011). On the choice of importance distributions for unconstrained and constrained state estimation using particle filter. *J. Process Contr.,* 21(1), 3-16.

Ristic, B., Arulampalam, S. and Gordon, N.J. (2004). *Beyond the Kalman filter: Particle filters for tracking applications*. Artech House, Boston, USA.

Shen, Y. (2014). Hybrid unscented particle filter based state-of-charge determination for lead-acid batteries. *Energy,* 74(1), 795-803.

Shenoy, A.V., Prakash, J., McAuley, K.B., Prasad, V. and Shah, S.L. (2011). Practical issues in the application of the particle filter for estimation of chemical processes. in *Proc. 18th IFAC World Congress,* 2773-2778.

Ungarala, S. (2012). On the iterated forms of Kalman filters using statistical linearization. *J. Process Contr.,* 22(5), 935-943.

Van der Merwe, R., Doucet, A., de Freitas, N. and Wan, E. (2000). The unscented particle filter. *Tech. Rep.* CUED/F-INFENG/TR 380, Cambridge University Engineering Department, UK.

Wan, E. A. and Van der Merwe, R. (2000). The unscented Kalman filter for nonlinear estimation. in *Proc. IEEE Symposium on Adaptive system for signal processing*, *Communication, and Control*, 153-158.