

Fuzzy Mixed H_2/H_∞ Sampled-Data Control System

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Abstract: This paper is focus on fuzzy mixed H_2/H_∞ sampled-data controller design. A fuzzy sampled-data mixed H_2/H_∞ control scheme is presented for obtaining the system stability. Based on the proposed fuzzy mixed H_2/H_∞ controller, the suboptimal H_2 control performance can be implemented and H_∞ performance is also controlled in a limited condition. The main contribution is that a fuzzy sampled-data mixed H_2/H_∞ controller is proposed at the first time. Simulation results of two engineer systems show the feasibility of the mixed H_2/H_∞ sampled-data structure.

Keywords: Fuzzy system; sampled-data; mixed H_2/H_∞ performance.

1. INTRODUCTION

Recently, nonlinear control based on fuzzy/neural technique has witnessed rapidly growing attention. It is shown that this approach has been applied in engineering control (Chen et al., 2015; Li et al., 2015; Driss and Mansouri, 2016; Liu et al., 2016; Liu et al., 2016; Szymak, 2016; Liu and Tong, 2017). By employing Takagi–Sugeno fuzzy system, nonlinear system can be transformed into a simple linear system. This practice gives us a strong structure and helps us to analyse and synthesize the nonlinear system. So, there have many important results in (Lee et al., 2014; Dong and Yang, 2011; Tanaka and Sugeno, 1992).

With the development of computer technology, computers with digital signals are used to control the engineer systems. The computer can produce a discrete-time signal, the sampled-data signal with aid of a zero-order holder is used to control continuous-time systems. There are a lot of many important works to discuss the stability of sampled-data systems. In these references, stability analysis in (Lam and Leung, 2007; Lam and Seneviratne, 2009; Jiang, 2015), stabilization in (Koo et al., 2012; Yang et al., 2014), H_∞ control in (Yang and Cai, 2008; Yoneyama, 2010; Chen et al., 2011; Koo et al., 2012), H_2 GC control in (Yoneyama, 2011; Koo et al., 2013), and filtering in (Yoneyama, 2013; Huang et al., 2013; Ge et al., 2014) are investigated, respectively. Among these existing works, two main approaches have been used. The first one is that the discrete-time method is used to develop the sampled-data controllers, such as (Koo et al., 2013; Koo et al., 2012). The second one is the input delay method, which is employed to present sampled-data controllers, e.g., (Li et al., 2015; Yang et al., 2014; Yoneyama, 2011).

H_∞ performance focuses on a gain. An integral function is considered in H_2 performance. Only a single control performance is implemented in H_∞/H_2 control. H_∞ control and H_2 control of fuzzy sampled-data systems have been

discussed, respectively, where the dimension of the LMIs is enormous, which leads to the conservatism of the results. How to design a more simpler and easier to be implemented sampled-data controller with larger sampling interval still remains an open issue, especially for those engineering systems.

On the other hand, in the control system design, if H_∞/H_2 performance are meted at the same time, control system performance will be greatly improved. Thus, this approach is more promising in achieving a desired control performance because it ingests the advantages of H_2 performance and H_∞ performance. This problem is investigated in (Chen et al., 2000). However, in fuzzy sampled-data control, there is no report about H_2/H_∞ control problem, which motivates the work in this paper.

In our work, for nonlinear systems, we will present fuzzy mixed H_2/H_∞ sampled-data control scheme in Takagi–Sugeno fuzzy form. For performance conditions, we use mathematical tool to aid us in constructing a trustable and good fuzzy controller for nonlinear system.

Main contributions are summarized as follows:

- (i) Hybrid sampled-data approach is better than H_∞ control design and H_2 control design, and the control system performance will be greatly improved. The proposed fuzzy sampled-data control algorithm with a larger sampling interval is simple and easy to be implemented, which will provide more effective support in engineering condition.
- (ii) With a series of control base, fuzzy implemented sampled-data H_2/H_∞ controller for nonlinear systems is less conservative, where dimension of the LMIs is decreased.

In the following, the rest sections in this paper will be listed. Section 2 is problem formulation, next section provided principle results, another section shows simulative consequences, and final conclusion is given in last section.

2. PROBLEM FORMULATION

Give a fuzzy dynamic system, which shows:

If $\eta_1(t)$ is R_{i1} , $\dots$, $\eta_r(t)$ is R_{ir} , Then

$$\dot{x}(t) = A'_i x(t) + B'_i u(t) + \omega'(t), i = 1, \dots, L, \quad (1)$$

where $x(t)$, $u(t)$ and $\omega'(t)$ denote system state, control input and external disturbance, accordingly. The matrices A'_i and B'_i are constant.

And overall state equation is of the following form:

$$\dot{x}(t) = \sum_{i=1}^L \sigma_i(\eta(t)) [A'_i x(t) + B'_i u(t) + \omega'(t)], \quad (2)$$

where

$$\sum_{i=1}^L \sigma_i(\eta(t)) = 1.$$

We have the controller of fuzzy system:

If $\eta_1(t_k)$ is R_{j1} , $\dots$, $\eta_r(t_k)$ is R_{jr} , Then

$$u(t) = K_j x(t_k), j = 1, \dots, L \quad (3)$$

in which sampling interval $h_k = t_{k+1} - t_k \leq h$, h is a constant.

According to controller (3) rule, defuzzified shows

$$u(t) = \sum_{j=1}^L \sigma_j(\eta(t_k)) K_j x(t_k). \quad (4)$$

The sampled-data controller (4) is converted to a delayed input signal with aid of the input delay method. Substituting (4) into (2), we obtained

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t)) \sigma_j(\eta(t_k)) [A'_i x(t) \\ & + B'_i K_j x(t-s(t)) + \omega'(t)] \end{aligned} \quad (5)$$

in which $s(t) = t - t_k$, $0 \leq s(t) \leq h_k \leq h$.

Let's consider H_∞ control performance

$$\int_0^\infty x^T(t) Q_1 x(t) dt \leq \rho^2 \int_0^\infty \omega'^T(t) \omega'(t) dt, \quad (6)$$

where $\rho > 0$, $Q_1 > 0$.

Similarly

$$J = \int_0^\infty (x^T(t) Q_2 x(t) + u^T(t) R u(t)) dt \quad (7)$$

must be minimized free of disturbance $\omega'(t)$, where $Q_2 > 0$ and $R > 0$.

Control objective: For fuzzy system (2), to insure a condition that structure (6-7) of system (5) is obtained under system stability, a sampled-data controller (4) is designed.

Lemma 1 (Gu et al., 2003). There has

$$\left(\int_{r_1}^{r_2} \varpi(s) ds \right)^T M \left(\int_{r_1}^{r_2} \varpi(s) ds \right) \leq (r_2 - r_1) \int_{r_1}^{r_2} \varpi^T(s) M \varpi(s) ds. \quad (8)$$

Remark 1: In fuzzy sampled-data control techniques, there are two methodologies, such as the discrete-time designs and the input delay designs. The discrete-time method is used to develop the sampled-data controllers, such as (Koo et al., 2013; Koo et al., 2012). The input delay method is also employed to present sampled-data controllers, e.g., (Li et al., 2015; Yang et al., 2014; Yoneyama, 2011). In this paper, input time-delay technique will be used.

3. FUZZY MIXED H_2/H_∞ SAMPLED-DATA CONTROL

Here, we develop a fuzzy sampled-data controller for (4) to achieve mixed H_2/H_∞ control performance in (7) and (6) simultaneously. The design procedure is discussed step by step as the following. Firstly, section 3.1 shows the approach. Second, section 3.2. shows further approach. Finally, fuzzy we obtained controller in section 3.2.

3.1 Fuzzy Sampled-data H_∞ Control

In this part, fuzzy sampled-data H_∞ control is adopted and system stability with H_∞ performance (6) will be achieved.

Theorem 1. For a given matrix $Q_1 > 0$, given scalars $\rho > 0$, $h > 0$, $\delta > 0$, H_∞ control performance (6) is guaranteed under the bounded stability of fuzzy system (5) if matrices $\bar{P} > 0$, $\bar{T}_1 > 0$, $\bar{T}_2 > 0$ meeting the LMI conditions (9)

$$Y_{ij} = \begin{bmatrix} Y_{ij11} & Y_{ij12} & Y_{ij13} & Y_{ij14} & 0 \\ * & Y_{ij22} & 0 & 0 & 0 \\ * & * & Y_{ij33} & Y_{ij34} & 0 \\ * & * & * & Y_{ij44} & 0 \\ * & * & * & * & Y_{ij55} \end{bmatrix} < 0, i, j = 1, 2, \dots, L \quad (9)$$

where

$$Y_{ij11} = A'_i \bar{P} + \bar{P} A_i'^T + \bar{T}_1 - \bar{T}_2 + \frac{1}{\rho^2} I, Y_{ij12} = \bar{P},$$

$$Y_{ij13} = B'_i \bar{K}_j + \bar{T}_2, \Xi_{ij14} = \delta \bar{P} A_i'^T, Y_{ij33} = -\bar{T}_2$$

$$Y_{ij22} = -\frac{1}{1+\delta} Q_1^{-1}, Y_{ij34} = \delta \bar{K}_j B_i'^T,$$

$$Y_{ij44} = -2\delta \bar{P} + h^2 \bar{T}_2 + \frac{\delta}{\rho^2} I, Y_{ij55} = -\bar{T}_1.$$

And, we know $K_j = \bar{K}_j \bar{P}^{-1}$.

Proof. Use this Lyapunov candidate:

$$V(t) = V_a(x) + V_b(x_t) + V_c(x_t), \quad (10)$$

where

$$V_a(x) = x^T(t)Px(t), \quad V_b(x_t) = \int_{t-h}^t x^T(r)T_1x(r)dr,$$

$$V_c(x_t) = h \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(r)T_2\dot{x}(r)drd\theta$$

and $P > 0$, $T_1 > 0$, $T_2 > 0$.

Through the computation:

$$\begin{aligned} \dot{V}_a(x) &= \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t))\sigma_j(\eta(t_k)) [x^T(t)PA'_i x(t) \\ &\quad + x^T(t)PB'_i K_j x(t-s(t)) + x^T(t)P\omega'(t) \\ &\quad + x^T(t)A_i'^T Px(t) + x^T(t-s(t))K_j^T B_i'^T Px(t) \\ &\quad + \omega'^T(t)Px(t)] \\ &\leq \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t))\sigma_j(\eta(t_k)) [x^T(t)PA'_i x(t) \\ &\quad + x^T(t)PB'_i K_j x(t-s(t)) + x^T(t)A_i'^T Px(t) \\ &\quad + x^T(t)K_j^T (t-s(t))B_i'^T Px(t) \\ &\quad + \frac{1}{\rho^2} x^T(t)PPx(t) + \rho^2 \omega'^T(t)\omega'(t)]. \end{aligned} \quad (11)$$

$$\dot{V}_b(x_t) = x^T(t)T_1x(t) - x^T(t-h)T_1x(t-h). \quad (12)$$

By adopting Lemma 1, we have

$$\begin{aligned} -h \int_{t-h}^t \dot{x}^T(r)T_2\dot{x}(r)dr \\ \leq -\left(\int_{t-h}^t \dot{x}(r)dr\right)^T T_2 \left(\int_{t-h}^t \dot{x}(r)dr\right). \end{aligned} \quad (13)$$

By using (13)

$$\begin{aligned} \dot{V}_c(x_t) &= h^2 \dot{x}^T(t)T_2\dot{x}(t) - h \int_{t-h}^t \dot{x}^T(s)T_2\dot{x}(s)ds \\ &\leq h^2 \dot{x}^T(t)T_2\dot{x}(t) - x^T(t)T_2x(t) \\ &\quad + x^T(t-s(t))T_2x(t) + x^T(t)T_2x(t-s(t)) \\ &\quad - x^T(t-s(t))T_2x(t-s(t)). \end{aligned} \quad (14)$$

From (5), for a given $\mu > 0$,

$$\begin{aligned} 0 &= -2\delta \dot{x}^T(t)P\dot{x}(t) \\ &\quad + \delta \dot{x}^T(t)P \left\{ \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t))\sigma_j(\eta(t_k)) [A_i'x(t) + \omega'(t)] \right\} \\ &\quad + \delta \left\{ \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t))\sigma_j(\eta(t_k)) [A_i'x(t) \right. \end{aligned}$$

$$\begin{aligned} &\quad \left. + B_i'K_jx(t-s(t)) + \omega'(t)\right\}^T P\dot{x}(t) \\ &\leq -2\mu \dot{x}^T(t)P\dot{x}(t) \\ &\quad + \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t))\sigma_j(\eta(t_k)) [\delta \dot{x}^T(t)PA_i'x(t) \\ &\quad + \delta \dot{x}^T(t)PB_i'K_jx(t-s(t)) + \delta \dot{x}^T(t)A_i'^T P\dot{x}(t) \\ &\quad + \delta \dot{x}^T(t-s(t))K_j^T B_i'^T P\dot{x}(t) \\ &\quad + \delta \left(\frac{1}{\rho^2} \dot{x}(t)PP\dot{x}(t) + \rho^2 \omega'^T(t)\omega'(t) \right)]. \end{aligned} \quad (15)$$

According to (11-12) and (15), then

$$\dot{V}(t) \leq \sum_{i=1}^L \sum_{j=1}^L \sigma_i(\eta(t))\sigma_j(\eta(t_k)) [x'^T(t)S'_{ij}x'(t) + \rho'^2 \omega'(t)^T \omega'(t)] \quad (16)$$

where

$$x'(t) = [x^T(t), x^T(t-s(t)), \dot{x}^T(t), x^T(t-h)]^T, \quad \rho' = \sqrt{1+\mu}\rho,$$

$$S'_{ij} = \begin{bmatrix} S'_{ij11} & S'_{ij12} & S'_{ij13} & 0 \\ * & S'_{ij22} & S'_{ij23} & 0 \\ * & * & S'_{ij33} & 0 \\ * & * & * & S'_{ij44} \end{bmatrix} \quad (17)$$

with

$$\begin{aligned} S'_{ij11} &= A_i'^T P + PA_i' + T_1 - T_2 + \frac{1}{\rho^2} PP, \quad S'_{ij12} = PB_i'K_j + T_2, \\ S'_{ij13} &= \delta A_i'^T P, \quad S'_{ij22} = -T_2, \quad S'_{ij23} = \delta K_j^T B_i'^T P, \\ S'_{ij33} &= -2\delta P + h^2 T_2 + \frac{\delta}{\rho^2} PP, \quad S'_{ij44} = -T_1. \end{aligned} \quad (18)$$

If $S_{ij} = S'_{ij} + Q' < 0$ with $Q' = \text{diag}[(1+\delta)Q_1 \quad 0 \quad 0 \quad 0]$, then

$$S_{ij} = \begin{bmatrix} S'_{ij11} + (1+\delta)Q_1 & S'_{ij12} & S'_{ij13} & 0 \\ * & S'_{ij22} & S'_{ij23} & 0 \\ * & * & S'_{ij33} & 0 \\ * & * & * & S'_{ij44} \end{bmatrix} < 0. \quad (19)$$

(19) is equivalent to (20)

$$\hat{S}_{ij} = \begin{bmatrix} Y_{ij11} + (1+\delta)Q_1 & Y_{ij13} & Y_{ij14} & 0 \\ * & Y_{ij33} & Y_{ij34} & 0 \\ * & * & Y_{ij44} & 0 \\ * & * & * & Y_{ij55} \end{bmatrix} < 0. \quad (20)$$

According to Schur complement, (9) is equal to (20). If (9) is satisfied, then $\hat{S}_{ij} < 0$. And, (20) is equivalent to (19). Thus,

$S_{ij} = S'_{ij} + Q' < 0$ holds. That is to say, $S'_{ij} < -Q'$.

Substituting $S'_{ij} < -Q'$ into (17) yields

$$\dot{V}(t) \leq -x'^T(t)Q'x'(t) + \rho'^2 \omega'^T(t)\omega'(t). \quad (21)$$

Due to $Q' = \text{diag}[(1+\delta)Q_1 \quad 0 \quad 0 \quad 0]$, $\rho' = \sqrt{1+\delta}\rho$, (21) is rewritten as

$$\dot{V}(t) \leq -(1+\delta)x^T(t)Q_1x(t) + (1+\delta)\rho^2 \omega'^T(t)\omega'(t). \quad (22)$$

When $\|x(t)\| > \frac{\rho}{\lambda_{\min}(Q_1)} \|\omega'(t)\|$, $\dot{V}(t) < 0$. Thus, the fuzzy system (5) is uniformly ultimately bounded.

Integrating both sides of (22) with zero initial condition from 0 to t and as $t \rightarrow \infty$, then

$$\int_0^\infty x^T(t)Q_1x(t)dt \leq \rho^2 \int_0^\infty \omega'^T(t)\omega'(t)dt. \quad (23)$$

Remark 2: The H_∞ performance in (7) was supposed to remove the influence of $\omega(t)$ in the fuzzy system (2), but it can't meet a acquired control limit. Through an appropriate weighting matrices, it's easy to use H_2 optimal control approach to satisfy an acquired control performance. Then, we will discuss H_2 sampled-data control scheme.

3.2 Fuzzy H_2 Sampled-Data Control

In this subsection, fuzzy sampled-data H_2 control scheme is presented to diminish relating influence of (7) in great extent.

Theorem 2. Given $Q_2 > 0$, $R > 0$, $\mu > 0$, H_2 control performance (7) is guaranteed under asymptotic stability (5) without $\omega'(t)$ if matrices $\bar{P} > 0$, $\bar{T}_1 > 0$, $\bar{T}_2 > 0$ meeting corresponding LMIs (24)

$$\mathfrak{N}_{ij} = \begin{bmatrix} \mathfrak{N}_{ij11} & \mathfrak{N}_{ij12} & \mathfrak{N}_{ij13} & 0 & \mathfrak{N}_{ij15} & 0 \\ * & \mathfrak{N}_{ij22} & 0 & 0 & 0 & 0 \\ * & * & \mathfrak{N}_{ij33} & \mathfrak{N}_{ij34} & \mathfrak{N}_{ij35} & 0 \\ * & * & * & \mathfrak{N}_{ij44} & 0 & 0 \\ * & * & * & * & \mathfrak{N}_{ij55} & 0 \\ * & * & * & * & * & \mathfrak{N}_{ij66} \end{bmatrix} < 0, i, j = 1, 2, \dots, L \quad (24)$$

where

$$\mathfrak{N}_{ij11} = A_i' \bar{P} + \bar{P} A_i'^T + \bar{T}_1 - \bar{T}_2, \mathfrak{N}_{ij12} = \bar{P},$$

$$\mathfrak{N}_{ij13} = B_i' \bar{K}_j + \bar{T}_2, \mathfrak{N}_{ij15} = \delta \bar{P} A_i'^T, \mathfrak{N}_{ij22} = -Q_2^{-1},$$

$$\mathfrak{N}_{ij33} = -\bar{T}_2, \mathfrak{N}_{ij34} = \bar{K}_j^T, \mathfrak{N}_{ij35} = \delta \bar{K}_j^T B_i'^T,$$

$$\mathfrak{N}_{ij44} = -R^{-1}, \mathfrak{N}_{ij55} = -2\delta \bar{P} + h^2 \bar{T}_2, \mathfrak{N}_{ij66} = -\bar{T}_1.$$

And, we have

$$K_j = \bar{K}_j \bar{P}^{-1}.$$

Proof. Use Lyapunov candidate:

$$V(t) = V_a(x) + V_b(x_t) + V_c(x_t), \quad (25)$$

where

$$V_a(x) = x^T(t)Px(t), V_b(x_t) = \int_{t-h}^t x^T(r)T_1x(r)dr,$$

$$V_c(x_t) = h \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(r)T_2\dot{x}(r)drd\theta$$

and $P > 0$, $T_1 > 0$, $T_2 > 0$.

Making derivative of V for t in absence of $\omega'(t)$, we can obtain Theorem 2.

3.3 Fuzzy Mixed H_2/H_∞ Sampled-Data Control

Based on the analysis in sections 3.1-3.2, the fuzzy sampled-data mixed H_2/H_∞ control is proposed as follows.

Theorem3. Given matrices $Q_1 > 0$, $Q_2 > 0$, $R > 0$, and $\rho > 0$, $h > 0$, $\delta > 0$. The following, we have a sampled-data controller (4) such that mixed H_2/H_∞ performance in (6-7) is implemented simultaneously for (5). Control gains in controller are described as $K_j = \bar{K}_j \bar{P}^{-1}$.

Proof: According to sections 3.1-3.2, the conclusion in Theorem 3 could be achieved.

The fuzzy mixed H_2/H_∞ control design based on sampled-data is formulated as:

$$\min_{\bar{P}} x^T(0)\bar{P}^{-1}x(0)$$

$$\text{s.t. } \bar{P} > 0, \bar{T}_1 > 0, \bar{T}_2 > 0, (9) \text{ and } (24). \quad (26)$$

The above optimization problem is converted to a suboptimal control problem:

$$\min_{\bar{P}} \text{Trace}(U)$$

$$\text{s.t. } (26) \text{ and } \begin{bmatrix} U & I \\ * & \bar{P} \end{bmatrix} > 0. \quad (27)$$

Design Procedure: Fuzzy sampled-data mixed H_2/H_∞ control is listed:

Step 1: Suppose $\rho > 0$, $h > 0$ and $\delta > 0$. Select weighting matrices $Q_1 > 0$, $Q_2 > 0$, $R > 0$.

Step 2: Compute LMIs (9, 24) to obtain $\bar{K}_j$ ($j = 1, 2, \dots, L$).

Step 3: Decrease ρ and increase h until there is no solution of $\bar{K}_j$.

Step 4: Confirm the mixed H_2/H_∞ sampled-data control performance and stability.

To illustrate how the theoretical results from section 3 are applied in section 4 as follows:

Firstly, apply physics theorem to obtain the controlled nonlinear model.

Secondly, employ fuzzy system to describe nonlinear model with help of linearization technique.

Thirdly, use Theorem 3.3 to seek for feedback gains of fuzzy sampled-data controller under the optimal problem (27).

Finally, construct experiment platform of fuzzy closed-loop system to achieve state responses.

Remark 3: (16) is introduced in the proof of Theorem 3. Thus, now the results are better than existing ones.

Remark 4: By using various methods of (Yoneyama,2010), (Yoneyama,2011) and Theorems 1-2, Table 1 provides the dimensions of the LMIs, where n and m are the dimension of the state and the input, respectively. We find the mixed H_2/H_∞ is better than existing H_∞ and H_2 control method.

Table 1. The dimensions of the LMIs.

Method	(Yoneyama, 2010)	Theorem1	(Yoneyama, 2011)	Theorem2
$n=2, m=1$	20	10	19	11
$n=3, m=1$	30	15	28	16

4. ILLUSTRATIVE EXAMPLES

In this section, our fuzzy mixed H_2/H_∞ sampled-data control structure is applied to stabilize truck-trailer system and stirred tank reactor system to verify the presented method.

Example 1: Use truck-trailer system

$$\begin{aligned} \dot{x}_1(t) &= -\frac{vt}{Lt_0} x_1(t) + \frac{vt}{Lt_0} u(t) + \omega(t) \\ \dot{x}_2(t) &= \frac{vt}{Lt_0} x_1(t) \\ \dot{x}_3(t) &= \frac{vt}{Lt_0} \sin(x_2(t) + (vt/2L)x_1(t)) \end{aligned} \quad (28)$$

where $x_1(t)$, $x_2(t)$ mean angles, $x_3(t)$ means vertical position, $\omega(t)$ is the external disturbance, which is uncertain and bounded.

This system is linearized as two models, where

$$A'_1 = \begin{bmatrix} -\frac{vt}{Lt_0} & 0 & 0 \\ \frac{vt}{Lt_0} & 0 & 0 \\ \frac{v^2 t^2}{2Lt_0} & \frac{vt}{t_0} & 0 \end{bmatrix}, A'_2 = \begin{bmatrix} -\frac{vt}{Lt_0} & 0 & 0 \\ \frac{vt}{Lt_0} & 0 & 0 \\ \frac{dv^2 t^2}{2Lt_0} & \frac{dvt}{t_0} & 0 \end{bmatrix},$$

$$B'_1 = B'_2 = \begin{bmatrix} \frac{vt}{Lt_0} \\ 0 \\ 0 \end{bmatrix}, \omega'(t) = \begin{bmatrix} \omega(t) \\ 0 \\ 0 \end{bmatrix}, d = 10t_0 / \pi$$

and some parameters in (Tanaka and Sano;1994) .

Choose the membership functions $\sigma_1(t)$ and $\sigma_2(t)$, their definitions are same to that of in (Tanaka and Sano;1994).

This fuzzy sampled-data controller

$$u(t) = \sum_{i=1}^2 \sigma_i(t_k) K_i x(t_k) .$$

1)The minimum allowable disturbance attenuation ρ is 0.0008 under $h = 0.31$. Then we give $h = 0.31$, $\rho = 0.0008$ and $\delta = 0.5$, based on Theorem 3, we have

$$\begin{aligned} K_1 &= [1.4373 \quad -0.5029 \quad 0.0148], \\ K_2 &= [1.4373 \quad -0.5029 \quad 0.0148]. \end{aligned}$$

The minimum allowable disturbance attenuation is 0.0008, which is ineffective in the LMI conditions of H_∞ control (Yoneyama,2010). This implies that our approach is better than existing H_∞ control designs.

State responses x_1 , x_2 , x_3 and the control input u can be seen in Figures 1- 4.

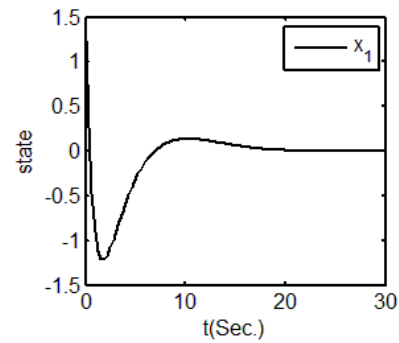


Fig. 1. The state x_1 .

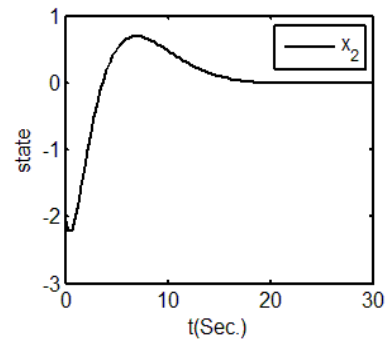
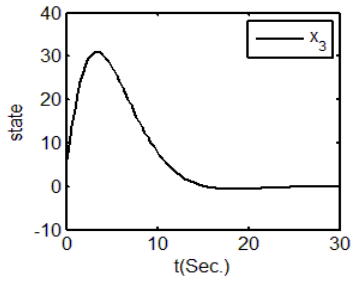
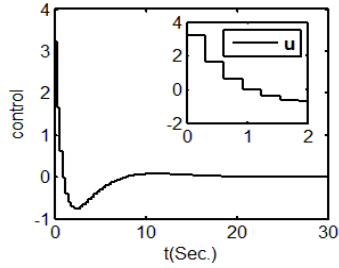


Fig. 2. The state x_2 .

Fig. 3. The state x_3 .Fig. 4. The sampled-data control u .

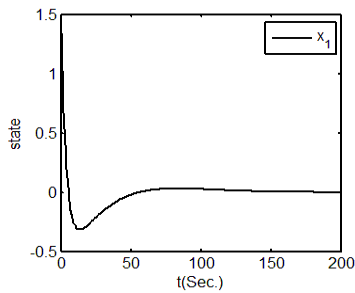
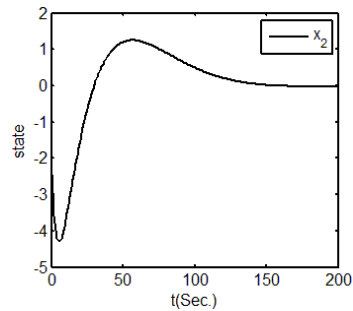
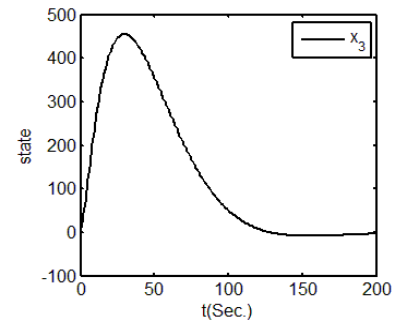
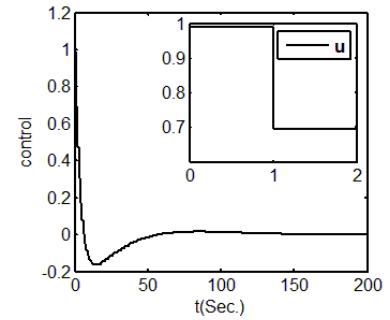
2) When $\rho=0.218$, Theorem 3 gives the maximum sampling interval $h=1.001$. The other parameters $\rho=0.218$, $\delta=5$ and fuzzy state feedback gains

$$K_1 = [0.6448 \quad -0.0102 \quad 3.7523 \times 10^{-5}],$$

$$K_2 = [0.6448 \quad -0.0102 \quad 3.7523 \times 10^{-5}].$$

The maximum sampling interval is 1.001, which is infeasible in the LMI conditions of H_2 control (Yoneyama, 2011). This implies our approach is better than existing H_2 control method.

Simulation results are in Figures 5- 8.

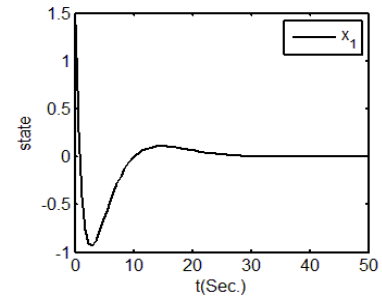
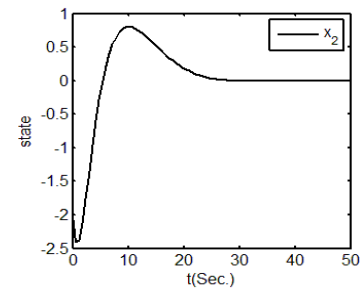
Fig. 5. The state x_1 .Fig. 6. The state x_2 .Fig. 7. The state x_3 .Fig. 8. The sampled-data control u .

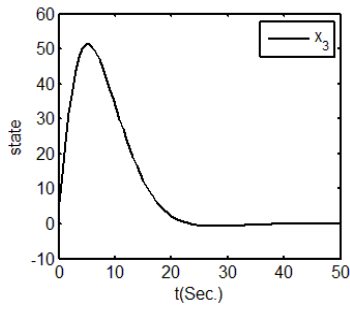
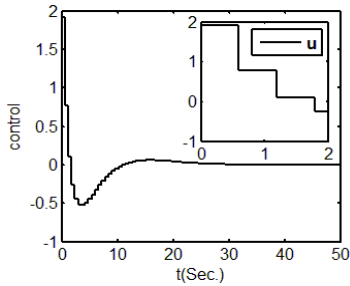
3) According to design procedure, the optimal design parameters are $h=0.6$, $\rho=0.0058$, $\delta=0.9$. Theorem 3 gives the control gains

$$K_1 = [1.0129 \quad -0.1898 \quad 0.0036],$$

$$K_2 = [1.0129 \quad -0.1898 \quad 0.0036].$$

Figures 9- 12 give the simulation results.

Fig. 9. The state x_1 .Fig. 10. The state x_2 .


 Fig. 11. The state x_3 .

 Fig. 12. The sampled-data control u .

Set $Q_1 = \text{diag}\{1 \ 10.1\} \times 10^{-8}$, $R = 1 \times 10^{-5}$,

$Q_2 = \text{diag}\{1 \ 10.1\} \times 10^{-8}$.

Example 2: Use CSTR system

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + D_\sigma(1-x_1(t))e^{\frac{x_2(t)}{1+x_2(t)/\gamma_0}}, \\ \dot{x}_2(t) &= -(1+\beta)x_2(t) + HD_\sigma(1-x_1(t))e^{\frac{x_2(t)}{1+x_2(t)/\gamma_0}} \\ &\quad + \beta u(t) + \beta w(t), \end{aligned} \quad (29)$$

where $\gamma_0 = 20$, $H = 8$, $D_\sigma = 0.072$, $v = 0.8$, $\beta = 0.3$.

CSTR is linearized as three models, where

$$A'_1 = \begin{bmatrix} -1.1774 & 0.0757 \\ -1.4189 & -0.6942 \end{bmatrix}, B'_1 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},$$

$$A'_2 = \begin{bmatrix} -1.8008 & 0.3958 \\ -6.4066 & 1.8768 \end{bmatrix}, B'_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},$$

$$A'_3 = \begin{bmatrix} -4.2779 & 0.3167 \\ -26.2228 & 1.1887 \end{bmatrix}, B'_3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},$$

$$\omega'(t) = \begin{bmatrix} 0 \\ \beta w(t) \end{bmatrix}, w(t) = 0.5e^{-0.1t} \sin(0.1t).$$

Choose the membership functions $\sigma_1(t)$, $\sigma_2(t)$, $\sigma_3(t)$, their definitions are same to that of in (Cao and Frank, 2000). This controller was designed for the CSTR system

$$u(t) = \sum_{i=1}^3 \sigma_i(t_k) K_i x(t_k).$$

For $\rho = 0.55$, Theorem 3 gives the maximum allowable sampling interval $h = 0.23$. The matrix parameters are given by $Q_1 = \text{diag}\{1 \ 1\} \times 10^{-6}$, $Q_2 = \text{diag}\{1 \ 1\} \times 10^{-5}$, $R = 1 \times 10^{-8}$.

With $h = 0.23$, $\rho = 0.55$ and $\delta = 0.2$, control gains are

$$K_1 = [22.3490 \ -9.3804], K_2 = [22.3490 \ -9.3804],$$

$$K_3 = [22.3490 \ -9.3804].$$

Under initial conditions $x_1(0)=0.5$, $x_2(0)=-1$, state responses x_1 , x_2 and the control input u are given in figures 13- 14.

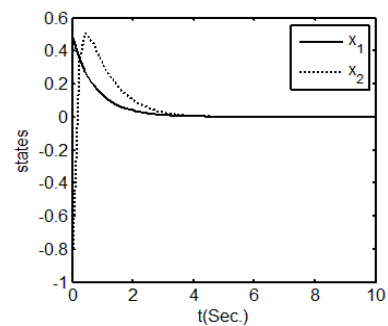
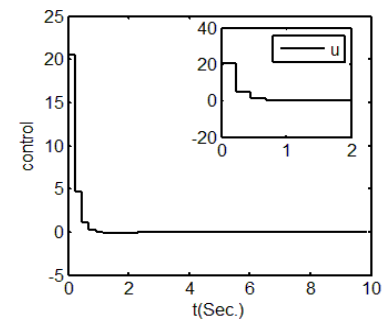

 Fig. 13. The curves of x_1, x_2 .

 Fig. 14. The sampled-data control u .

Figure 13 shows the stability of the continuous stirred tank reactor system by the fuzzy sampled-data mixed H_2/H_∞ controller.

Figure 14 shows the sampled-data behavior of the control signal.

By using the methods of (Yoneyama,2010)(H_∞ control), (Yoneyama,2011)(H_2 control) and Theorem 3, Table 2 provides the maximum sampling interval.

Table 2. The maximum sampling interval.

Method	(Yoneyama,2010)	(Yoneyama,2011)	Theorem 3
$h_{\max}$	0.186	0.223	0.230

According to the results, we find that the proposed method can harvest a larger sampling interval, which is less conservative than the approaches in (Yoneyama, 2010) and (Yoneyama, 2011). This implies that a better performance is achieved in our paper.

5. CONCLUSIONS

In conclusion, based on our scheme, not only the suboptimal H_2 control performance can be achieved, but also some influences were diminished to a limited condition. Simulation results also demonstrate the proposed fuzzy control scheme. Furthermore, we can expand this idea to time-delay systems, uncertain systems, etc.

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