Comparison of tracking performance and robustness of the simplified model of multirotor aerial robot with CDM and PID (with anti-windup compensation) controllers

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Abstract: The main aim of this article is a comparison of two tuning methods, CDM (Coefficient Diagram Method) and PID (proportional-integral-derivative) based on the pole placement with five different anti-windup compensators. For three most common used in multirotor aerial robots propulsion unit examples, that present different physical and dynamical properties (i.e. thrusts), authors proposed simplified mathematical models (first order with time-delay). First type of models was obtained by the estimation based on MATLAB System Identification Toolbox, the second one was estimated by graphical method and empirically fitting the date to the model. The thrust characteristics were recorded on the test stand and used for the model parametrization. Tuning methods (CDM algorithm and pole placement) are briefly described in further part of this paper in the context of tracking performance and tolerance for parameters uncertainty. The comparison was based on two integral quality indices types: IAE and ISE.

Keywords: Coefficient Diagram Method, anti-windup compensator, pole placement, PID control, motor-rotor simplified model, thrust controller, robust control.

1. INTRODUCTION

In last years the increased interest in multirotor aerial robots for different purposes can be observed – specification in (Austin, 2010). Specificity of those tasks impose to formulate new and more complex models. Developmental research (Nawrat and Kuś, 2013; Lozano et al., 2014; Valavanis et al., 2007) are focused on modern sensors, electronics, autonomous, as well as low-level layers of control responsible for stabilizing the robot and to harness its dynamics. There are many papers describing how to ensure apropriate thrust and torque (Magsino et al., 2013; Bouabdallah et al., 2004), but all of them are focusing mainly on coupling of light and mechanicaly durable construction with selected propulsion configuration. The classical constructions of multirotor aerial robots dominates. Those are based on cross-frame to which the propoulsion systems are connected – most common four or eight single drive units, it the last years often eight (Peng, et al., 2015) or six (Chen et al., 2014) in coaxial system configuration. The main advantage of coaxial systems is the better thrust to robot dimensions ratio. Drive unit selection is the other area of research (Bondyra et al., 2016) and it is not taken into account in this paper – however authors selected three different sets of motor-rotor systems (different thrust, power consumtion, etc.) to show the performance of controllers presented in further section of the article. The perspective of limited energy sources of the robot (large and heavy energy cells in relations to entire construction) oblige to provide the apropriate „energy management" to increase the application area of the robot. To assure this solution, it is neccesary to implement effective control algorithms. In not expensive, commercial multirotors (used for entertainment and photography), control systems are based on manual control of the robot orientation (Euler angles or a Tait-Bryan angles) in local coordinate system and setting of thrust value. The stability loop is implemented for each of angles (Pitch, Roll, Yaw) and thrust (most frequently with PID controllers). The computation algorithm (colloquialy called „mixer") give an ability to perform certain maneuvers by changing the rotation speed of each propulsion unit. In more complex solutions (Valavanis et al., 2007) the cascade control systems are used. The inner loop of the cascade system (robot orientation) is based on three orientation controllers towards x,y,z axis and the outer loop (robot positioning) based on three controllers that are responsible for robot positioning in regard to the observer.

Despite the accurate method of control, the main issue is offen forgotten: propulsion unit construction. That is the key to success which is the assurance of appropriate rotational speed in relation to expected thrust and propeller torque (Magsino et al., 2013). It seems that simulating the propulsion units (most common the brushless DC motors) is not a challenge, but coupled with propellers that are spinning with high speed (few to over a dozen thousands rotations per minute) and the influence of numbers of aerodynamical effect, is a challenge. This is the reason why the dynamics of the propulsion units is approximated by simple linear models and with multidimensional nonlinears models (Szafrański, 2015). Authors are focusing on necessary aerodynamical effects that the aerial robot construction is submitted (such as the propulsion units which cause this effect) and can be expressed in the dynamical model of the aerial robot throughout implementing additional measurements (rotational speed, current, etc. on single unit). It is also possible to propose additional control systems (rotational speed, thrust, torque control system, etc.) from the Fig.1 for simple propulsion models. In this situation the certainty of declared parameters, dispite any other disturbances, are achieved (mostly the step-like changes of the wind blast, etc.)

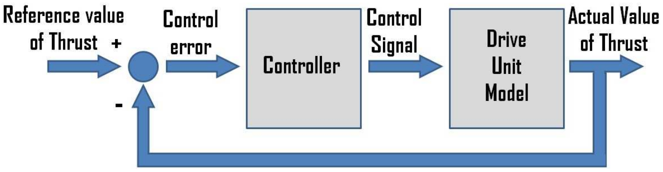


Fig. 1. Block diagram of proposed closed-loop control system of thrust (for particular drive unit)

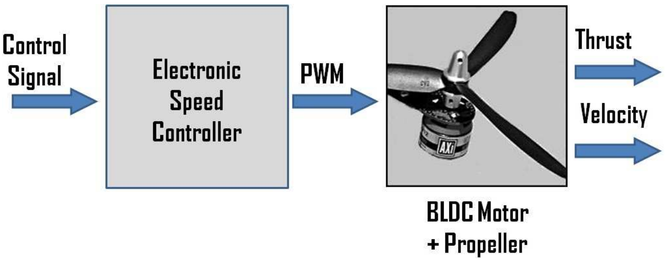


Fig. 2. Block diagram of propulsion unit (ESC, BLDC motor and propeller) with signals

In this paper authors present the effective control of desired thrust that is produced in system precisly called: motor-rotor (brushless DC motor embeded directly on the shaft) fed by proper voltage input modulation from ESC system (Electronic Speed Controller) – Fig.2 (Giernacki, et al., 2016). In many papers speed control loop is commonly used (Magsino, et al., 2013) because of the measurement simplicity directly on the the robot in real time. It is worth noting that speed and thrust measurement is not a subject of this paper, but its practical apsect, methodology and considerations can be successfully generalized and implemented on mentioned speed control systems (if one will use different characteristics of the motor-rotor system in the place of presented equivalents of thrust). Prospective results of simulations (presented briefly in (Giernacki, et al. 2016)) provided a closer look at this topic and to performed more comprehensive researches.

Article is organized as follow – in section II the analysed models are presented, as well as the methodology how they were created and the gathering of necessary data and parameters estimation. Section III is divided into two subsections that are describing synthesis of the thrust control system, that is CDM (Coefficient Diagram Method), which is robust on parameter uncertainty and the PID controller based on pole placement tuning method (without and with anti-windup compensation). The windup compensation problem, robustness on parameter uncertainty and control quality is discussed in section IV based on comparison of those two tuning methods. In Section V the conclusions and further research are described.

2. PROPULSION UNIT MODELS

2.1 Data gathering for the models

On the specially prepared test stand (more comprehensive description (Bondyra et al., 2016)) for different quadrotor and coaxial quadrotor generation such as *Falcon* and *Dropter* realized at *Institute of Control and Information Engineering* in *Poznan University of Technology* many simulations were performed for gathering important data. Those works are the inspiration for creating models of propulsion unit systems used in aerial robots (from the simple linear models to complex multidimensional nonlinear models that include electric and aerodynamical aspects) for which it is possible to design different types of controllers and tuning methods. For this purpose three different types of propulsion unit were applied on the test stand (Fig.3) from T-Motor company:

- energy efficient U8-16 PRO BLDC motor with 26" propeller – nominal rotation speed of about 2500 rpm, up to 2 kg of thrust and 300 W of maximum power consumption,

- maximizing thrust MN4014 BLCD motor with 16"x5.4" propeller – up to 3 kg of thrust, 900 W of power consumption,

- MN3110-15 BLCD motor with small propeller size 10"x3.3" – nominal rotation speed of 11544 rpm, up to 1.03 kg of thrust and 481 W of maximum power consumption.

After calibration of the test stand for each propulsion unit ten samples of thrust force were recorded on dedicated software: *DYNO Terminal*. The PWM step-like changes of the voltage were the input of the system in range from 0% to 98% and the system response was recorded as the thrust force characteristics. The average response from ten samples is shown in Fig.4. Authors recorded as well one sample of changing the PWM and response signals in time (Fig.5). This data were used for walidation of the models.

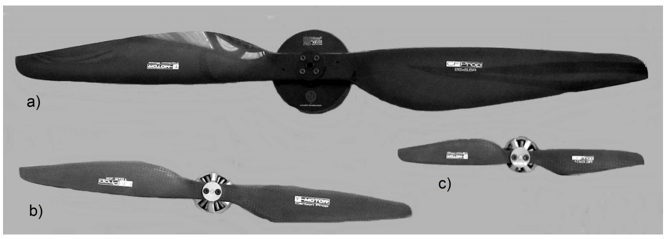


Fig. 3. Propulsion units: a) U8 Pro + 26" propeller, b) MN4014 + 16" propeller, c) MN3110 + 10" propeller

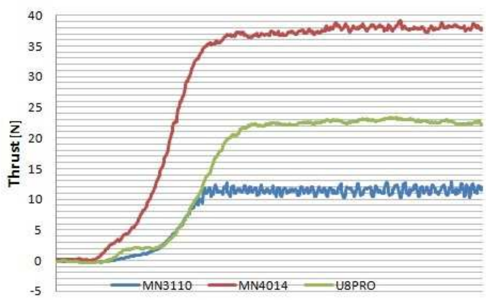


Fig. 4. Step responses of drive units with MN3110, MN4014 and U8PRO BLDC motors (as a function of samples)

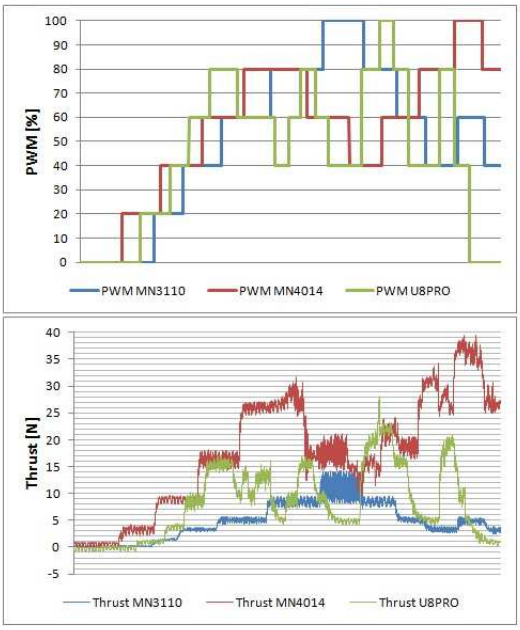


Fig. 5. Thrust responses of the MN3110, MN4014 and U8PRO BLDC motors models for PWM input function

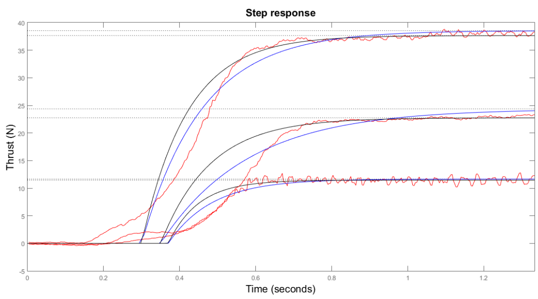


Fig. 6. Step responses (from the top: MN4014, U8PRO, MN3110) of real motor-rotor units (red), simplified models obtained via graphical method (black) and by the estimation of the models created with MATLAB's System Identification Toolbox (blue).

Table 1. Parameter estimation and proposed models of motor-rotor (models used in the paper are marked with grey color)

$G_p(s)$	MN3110	MN4014	U8PRO
IDENT	11.65/(0.115s+1) *exp(-0.37s)	38.55/(0.1585s+1) *exp(-0.3s)	24.37/(0.2301s+1) *exp(-0.35s)
Graphical	11.46/(0.08s+1) *exp(-0.37s)	37.65/(0.12s+1) *exp(-0.3s)	22.74/(0.13s+1) *exp(-0.35s)
Padé	(-11.46s+61.94)/ ($0.08s^2$+1.432s+5.405)	(-37.65s+251)/ ($0.12s^2$+1.8s+6.667)	(-22.74s+130)/ ($0.13s^2$+1.743s+5.714)
Approximation $e^{-sT_o}=j_0/(h_1s+h_0)$	11.46/ ($0.0064s^2$+0.16s+1)	37.65/ ($0.0144s^2$+0.24s+1)	22.74/ ($0.0169s^2$+0.26s+1)

2.2 Parameter estimation of the models

The range of useful thrust forces used during the fly of the aerial multirotor robots does not exiceed 70% of the maximum value (above this value the thrust force is occupied by large increase of power consumption). The analysis of the recorded data from Fig.4 with estimation of the parameters (with the validation on data set from Fig.5) gives a possibility to create a linear models of propulsion units – to be exact: a first order model with time-delay (Table 1). In this case two different estimation methods were used. First one was based on toolbox from MATLAB 2015a software – System Identification Toolbox. In the second one, it was based on graphical analysis of the step responses from which the specific dynamical parameters were collected. In each considered case, the graphical method was more precise in approximation of the real plant, than the estimaton of the model with usage of the MATLAB System Identification Toolbox (Fig.6).

To simplify the computations, the expotential form of time-delay was approximated by first order linear model. The Pade approximation creates problem of non-minimum phaze system (zero in the numerator of the closed-loop transfer function) and it was assumed that the time-delay approximation takes the form: $e^{-sT_o}=j_0/(h_1s+h_0)$ (assuming that $T_0<0.5$), where: $j_0=1$, $h_0=1$. Results are presented in Table 1.

3. SYNTHESIS OF CONTROL SYSTEMS

In the case of control system synthesis, the type of notation and model structure, determine the type of controller, control law nad tuning method – see (Raptis and Valavanis, 2011). In the reference to control systems of multirotor aerial robots, their multilayer nad cascade characterisctics, besides the simplified linear mathematical model, it is neccessary to use fast and universal controllers. Those requirements can be achieved with PID controllers – still commonly used (Flores et al., 2012; Li & Li, 2011; Bouabdallah, 2004). However it main advantage – universality of usage, gives a good control performance. In most of situations the optimization criteria and meaningful aspects that are omitted cause difficulties in control quality and it is described in further part of the paper. The most important aspect is an influence of the integral part of controller in the case when the control signal reaches the saturation limit. The tuning of PID type controller with different methods (Ziegler-Nichols or pole placement used in this paper) does not cosider this aspect – that why it is necessary to use windup compensators (Sadalla and Horla, 2015a). Another important aspect form the control quality perspective is the robustness of the controller for parameters model uncerainty. Even if most complex methods will be used, the obtained model will be always more or less precise approximation of its real equivalent. This aspect is analysed in this paper. To achive reference of results, the PID type controller (with antiwindup compensation) tuned with pole placement method where compared to results achieved with coefficient diagram method, which is also cosidered as a polynomial control class (tuning is based on placing the poles of the closed-loop characteristic equation (Manabe, 1998). Necessary informations with references to important papers are presented in next *Section*.

3.1 Coefficient Diagram Method

The CDM algorithm is based on an idea of the use of relationship between obtained closed-loop system time characteristics and the placement of its characteristic polynomial poles on the complex plane *s* (Manabe, 1998). It is worth to mention, that this control design method objective is to easily obtain a good controller with minimum user effort (Coelho et al. 2014). From the user point-of-view, the main feature of CDM it is simplicity. If only the system model is known (transfer function), the control designer only needs to define two things: the value of time constant equivalent and the controller order for the expected system disturbance shape. Then the controller transfer function is automatically obtained via an algebraic method (analogically to pole placement) (Coelho et al. 2014). The simplicity of this procedure, stability assurance in the first iteration of algorithm, possibilty of usage of a special tool (coefficient diagram – more in (Manabe and Kim, 2000)) to analyse the dynamics, stability and robustness of the system, proves that CDM method is useful and practical applicable method of the PID type controller tuning. Especially if it is considered to use in aerial systems that are known of high dynamics (Budiyono et al., 2009). Full description of this method, necessary mathematical formulas and tuning algorithm can be found in (Manabe, 2005; Coelho et al. 2014; Bir and Tibkin, 2009).

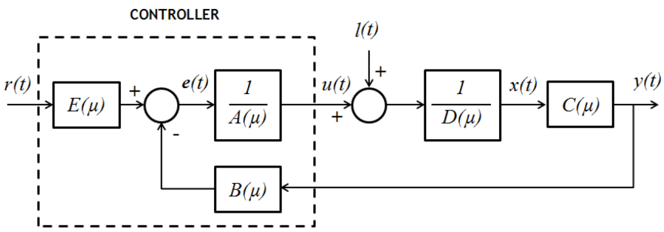


Fig. 7. Block diagram of CDM closed-loop system.

The synthesis of controller for the system from Fig.7 starts from the notation of plant model expressed in the time domain (regarding to (Coelho et al. 2014)) – by the use of μ operator defined in equation (1):

$$\frac{d^i}{dt^i} = \mu^i \; . \quad (1)$$

A general linear differential equation with constant coefficient a_i and b_j, for $i=0,\ldots,n$ and $j=0,\ldots,m$, with the following generic structure:

$$a_0 y(t) + \sum_{i=1}^{n} a_i \frac{d^i y(t)}{dt^i} = b_0 u(t) + \sum_{j=1}^{m} b_j \frac{d^j u(t)}{dt^j} \quad (2)$$

can assume an alternative formulation using the μ operator as shown is (3):

$$y(t)\left(a_0 + \sum_{i=1}^{n} a_i \cdot \mu^i \right) = u(t)\left(b_0 + \sum_{j=1}^{m} b_j \cdot \mu^j \right). \quad (3)$$

This representation resembles a polynomial in μ with a_i and b_j as coefficients (of $A(\mu)$ and $B(\mu)$ polynomials) and n and m as their orders. The system differential equation may be written in a more compact way, as:

$$A(\mu) y(t) = B(\mu) u(t) \quad (4)$$

where the dot operation represent the product between each polynomial term and the related signal.

In CDM controller design (Coelho et al. 2014), the pre-filter $E(\mu)$ is a zero order polynomial and his only coefficient is computed in order to achieve closed-loop zero steady state error. Model of plant dynamics consist of two polynomials: $C(\mu)$ and $D(\mu)$. For the sake of simplicity, let the order of both polynomials $A(\mu)$ and $B(\mu)$ be equal to m and the order of both polynomials $C(\mu)$ and $D(\mu)$ be equal to n (even if some higher order coefficient of $B(\mu)$ must be set to zero). After that, the $m+n$ order characteristic polynomial $P(\mu)$ of closed-loop control system may be written as (5):

$$P(\mu) = A(\mu) \cdot D(\mu) + C(\mu) \cdot B(\mu). \quad (5)$$

Characteristic polynomial from equation (5) may be rewritten as the one from equation (6):

$$P(\mu) = \sum_{i=0}^{n+m} p_i \cdot \mu^i . \quad (6)$$

Let's define two additional figures: the stability index, denoted by γ_i for $i=1,\ldots,(n+m)-1$ and the predominant time constant τ. Both are described in further detail in (Manabe, 1998) and presented hereafter in (7) and (8):

$$\gamma_i = \frac{p_i^2}{p_{i-1} p_{i+1}}, \quad (7)$$

$$\tau = \frac{p_1}{p_0}. \quad (8)$$

Each of the characteristic polynomial coefficient p_i in (6) can be written as a function of both stability indexes and predominant time constant. Hence the (normalized) characteristic polynomial can be expressed alternatively as (9):

$$\frac{P(\mu)}{p_0} = \sum_{i=2}^{n+m} \left\{ (\tau\mu)^i \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_j^{i-j}} \right) \right\} + \tau\mu + 1. \quad (9)$$

For Manabe's polynomial (one of the best, ready-to-use characteristic polynomial structures), the coefficients are chosen in order to have the following stability index values (Manabe, 1998):

$$\gamma_i = \begin{cases} 2.5 & if \quad i = 1 \\ 2 & if \quad i = 2,\ldots, n+m \end{cases}. \quad (10)$$

In classic version of a pole placement method it is not specified how to obtain the desired characteristic polynomial in particular system (Coelho et al. 2014) – there is only mentioned that it should provide stable closed-loop system. The CDM method presents a simple way to obtain such a polynomial (by just defining the desired equivalent time constant) that will provide good performance (stability, robustness and quality of control). Then having the desired characteristic polynomial, the next step is just algebraic and requires solving an system of equations with the formulation described by (5).

Giving p_0, τ and γ_i beforehand, the problem resumes to the pole-placement problem (Koksal and Hamamci, 2004). However in the CDM method the structure of the Sylvester matrix differs from the pole placement one. This is due to the knowledge of some $A(\mu)$ coefficients due to a priori assumptions about the type of system disturbances (consider Table 2).

Table 2. Controller order selection as a function of signal disturbance type

Disturbance type	$A(\mu)$ degree	$B(\mu)$ degree	$P(\mu)$ degree	Condition
None	n-1	n-1	2n-1	-
Step	n	n	2n	$a_0=0$
Impulse	n-1	n-1	2n-1	-
Ramp	n+1	n+1	2n+1	$a_0=0$ $a_1=0$
k order	n+k-1	n+k-1	2n+k-1	$a_0=0$ … a_k-1=0

Assuming the Sylvester matrix **A** has the structure represented in (12) and that the unknown polynomial coefficients, considering zero the lower *k* coefficients of $A(\mu)$, are arranged in a vector ***x*** as expressed in (13) then the CDM controller solution is obtained by solving equation (11):

$$x = \mathrm{A}^{-1} \cdot p \quad (11)$$

where:

$$\mathrm{A} = \begin{bmatrix} d_m & c_m & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ d_{m-n+k} & c_{m-n+k} & & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ d_0 & c_0 & \ddots & 0 & 0 \\ 0 & 0 & & d_m & c_m \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & d_0 & c_0 \end{bmatrix} \quad (12)$$

and

$$x = \begin{bmatrix} a_n & b_n & \cdots & a_k & b_k & b_{k-1} & \cdots & b_0 \end{bmatrix}^T, \quad (13)$$

$$p = \begin{bmatrix} p_{n+m} & \cdots & p_1 & p_0 \end{bmatrix}^T. \quad (14)$$

3.2 PID pole placement control with anti-windup compensation (AWC systems)

The PID pole placement method is based on comparison of the closed-loop characteristic equation with its desired form. The model of such a system with anti-windup compensation (AWC) is presented in Fig. 4 (Giernacki, et al., 2016).

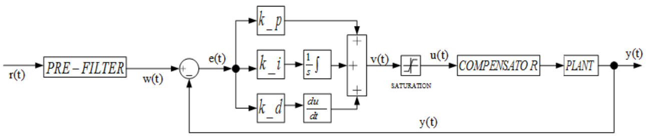


Fig. 8. Matlab-Simulink diagram of tracking system with time-delay

The model of the plant $G_p(s)$ is described by transfer function (15):

$$G_p(s) = \frac{b_0}{a_1 s + a_0} e^{-sT}. \quad (15)$$

The gain of the model is b_0 and T_0 is a time-delay. All coefficients are assumed known. The PID controller takes the form (16):

$$G_{PID}(s) = \frac{k_d s^2 + k_p s + k_i}{s} \quad (16)$$

and its parameters should be tuned off-line with respect to open-loop transfer function (17) as below:

$$G_o(s) = \frac{\mathrm{k_d} s^2 + k_p s + k_i}{s} \cdot \frac{b_0}{a_1 s + a_0} e^{-sT}. \quad (17)$$

In order to streamline the derivation of the closed loop transfer function, a first-order approximation of time-delay (18) is assumed:

$$e^{-sT_0} \approx \frac{j_0}{h_1 s + h_0}. \quad (18)$$

Now, the closed-loop characteristic polynomial is compared to its desired form (left part of equation 6 with arbitrary chosen coefficients: *a*, m_3, m_2, m_1 and m_0), forming Diophantine equation with approximation of the delay, and is given as (19):

$$am_3 s^3 + am_2 s^2 + am_1 s + am_0 = a_1 h_1 s^3 + (b_0 k_d j_0 + a_1 h_0 + a_0 h_1) s^2 + (a_0 h_0 + b_0 k_p j_0) s + b_0 k_i j_0 \quad (19)$$

For the pole placement problem (Aström, 2002) after computations, the solution of this Diophantine equation is (20):

$$k_i = \frac{am_0}{b_0 j_0}, \quad k_p = \frac{am_1 - a_0 h_0}{b_0 j_0}, \quad k_d = \frac{am_3 - a_1 h_1 - a_0 h_1}{b_0 j_0}. \quad (20)$$

The cut-off level of the computed control signal *v(t)* (Fig. 8) set at $\pm\alpha_{min}$ allows asymptotic tracking in the closed-loop system and is used to give relative measure of the hardness of the constraints. After comparison with the desired characteristic equation the requirements for the compensator parameters are: $j_0=1$, $h_1=am_3 a_1$, $h_0=1$. According to (20) the primary meaning of time-delay approximation model is preserved, but due to solving the Diophantine equation, there's a possibility to place the poles of the closed loop system in chosen locations (Giernacki, et al., 2016).

In order to cancel its zero-dynamics, a pre-filter is added in series with the closed-loop system and is modeled by the transfer function (21):

$$G_F(s) = \frac{k_{\text{filter}}}{k_d b_0 s^2 + k_p b_0 s + k_i b_0}. \quad (21)$$

where k_{filter} is a pre-filter gain. The main purpose of this filter is to decrease the speed of transients and reduce the probability of consecutive re-saturations of control signal.

Five different anti-windup compensator are listed to enable the authors analyse the impact of AWC on the control quality. A short description of each is presented below:

-Tracking system with limitation of the Integrator (AWC1) - the main disadvantage of this type of compensator is that the integral part of the controller can be stopped even if the signal *v(t)* is not saturated,

-Tracking system with limitation of Integrator with error signal *e(t)* (AWC2) - this method is based on difference between reference signal *r(t)* and the output signal *y(t)* which is fed to the integrator. If the signal *v(t)* and *u(t)* resaturates then the integration should be stopped,

-Tracking system with control signal conditioning (AWC3)

The third compensation method is described by equation

$$e_i(t) = e(t) k_i + \Delta u(t) K_f T_N,$$

where *e(t)* is the difference between reference signal *r(t)* and *y(t)* and $\Delta u(t)$ is the difference between unsaturated *v(t)* and saturated control signal *u(t)* multiplied by known coefficients K_f and T_N.

-Tracking system with dead-zone of control signal (AWC4) - in the system with AWC4 large initial values of unsaturated control signal *v(t)* are caused by proportional and derivative part of the controller due to step changes of the reference signal *r(t)*. The integral part is not able to compensate this changes fast enough and this problem is eliminated by adding another saturation block to *P* and *D* elements,

-Tracking system with external reset of Integrator (AWC5) - the last system is based on reset the integral part of controller whenever the control signal *v(t)* exceeds the given limit. The range of dead-zone to which the control signal is fed is equal to the double cut-off limit in constrained control signal *u(t)*.

More comprehensive information and schemes of the anti-windup compensator can be found in (Sadalla and Horla, 2015a, Sadalla and Horla, 2015b).

4. SIMULATION TESTS

The complex analysis of tracking and robustness quality in systems with PID and CDM controllers, was performed. The main purpose was not the direct comparison between both methods, but the general analysis and further utilization of advantages of both methods.

4.1 Tracking performance

The first order intertia models with time delay presented in Table 1 (approximation is based on equation (18)), were implemented in MATLAB 2015a / Simulink (standard parameters configuration) with tuning methods mentioned in *Section 3*. To evaluate the performance, integral of absolute error (IAE) and integral of squared error (ISE) indices have been used.

In the first analysis the tracking performance of square signal (thrust force) for CDM controller was performed for simulation time of 10 seconds (Fig.8) in the presence of step type disturbances that had an impact on the model. Another aspect that was analysed is the level of the amplitudes of control signal. It can be noted from the recorded data that similar raising times were achieved for each of three motor-rotor units (the expected values of time constant τ were achieved each time exactly as they were declared in CDM tuning procedure). However, different effectiveness of disturbances damping, was observed. For the most dynamical system (MN4014) it can be noted that there are high overshoots that the controller manage it in a proper way.

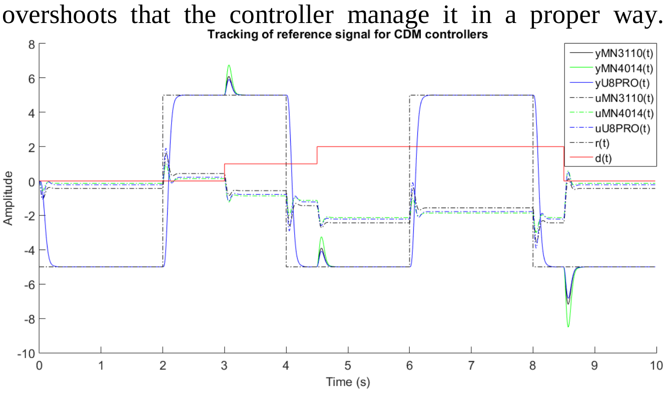
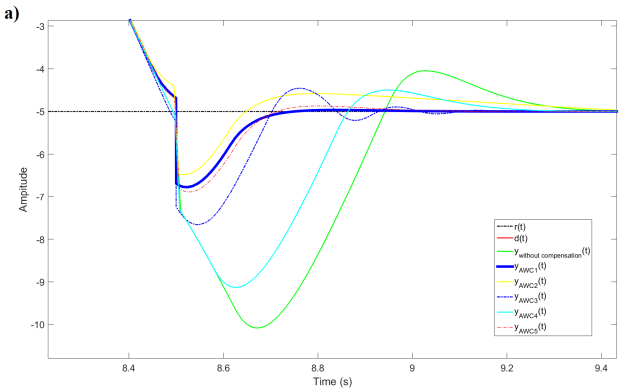
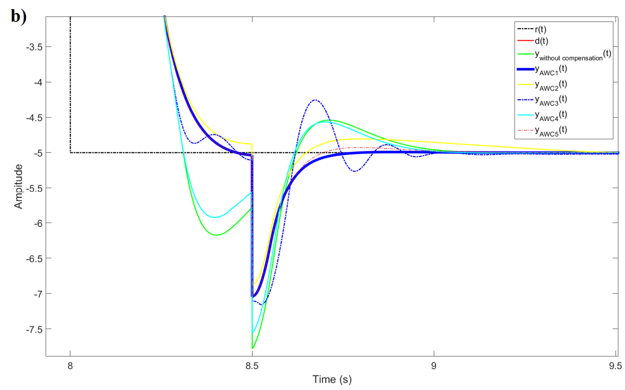


Fig. 9. Reference signal *r(t)*, control signal *u(t)*, output signal *y(t)* and disturbance signal *d(t)* for the control systems with CDM controllers

In the second analysis (analogously to previous one) six systems with PID controller based on pole placement tuning method and with: AWC0 (model without windup compensation), AWC1-AWC5 (model with windup compensation metods that were described in *Section 3*). The saturation limit of the control signal for AWC1, AWC3, AWC4 and AWC5 was set to ±4 and for AWC system it was ±1. For AWC3 system there are two additional parameters: K_f = 1 and T_N = 0.1. Additional description of these parameters can be found in (Hyppe, 2006) and it is omitted here for undisturbed presentation of work results.

Current research are the extension of previous presented in (Giernacki, et al., 2016), where it was investigated that using proper windup compensation method with pole placement tuning method of PID controller leads to decrease of disturbance damping, however the consequences are that it lowers the dynamics of the closed-loop system (increase of steady-state time response). Results presented in Fig.10 show the subtle differences between each type of AWC according to the system without compensation AWC0. In Table 3, the integral quality indices, are presente. In each of three motor-rotor units the most effective was the AWC1 system which has fastest output response and good disturbances damping.





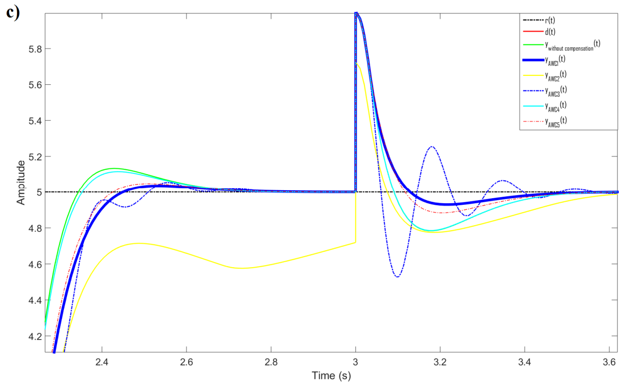


Fig. 10. Zoom of time courses for control systems with PID controllers and a) MN3110, b) MN4014, c) U8PRO propulsion unit plant models

Table 3. Integral quality indices IAE i ISE for motor-rotor models (AWC0-AWC5)

	AWC 0	AWC1	AWC 2	AWC3	AWC 4	AWC 5
	MN3110					
IAE	18.096	11.746	11.944	12.236	15.667	11.920
ISE	87.337	57.177	55.149	61.073	75.595	58.016
	MN4014					
IAE	13.844	12.650	12.559	12.846	13.635	12.749
ISE	91.639	76.159	71.219	78.859	89.167	77.170
	U8PRO					
IAE	16.003	15.515	15.423	15.638	15.959	15.585
ISE	111.57	104.247	97.688	105.327	110.86	105.13

4.2 Robustness analysis for parameters uncertainty

In authors area of interests was looking for the answer to the question: how will the controllers (CDM and pole placement with AWC1) manage with parameter uncertainty of the models. Thinking about the placing the poles at specific point is just a theorize this aspect because the model is being created is always simplified model with worst or better approximation. This approximation has an impact on tracking performance of a real plant if the controller is tunned for its simplified model.

First test was focused on recording the integral quality indices IAE and ISE (Fig.11) for changing a_1 and T_0 values of motor-rotor units models with CDM controller (analogous to model from subsection 4.1). Parameters T_0 i a_1 were chaning in specified range +/- of nominal value of each unit parameters.

The controller tunned with coefficient diagram method is robust to the changes of the a_1 parameter (time constant of the first order interia plant), however it effectiveness decreases with increasing of the time-delay parameter T_0.

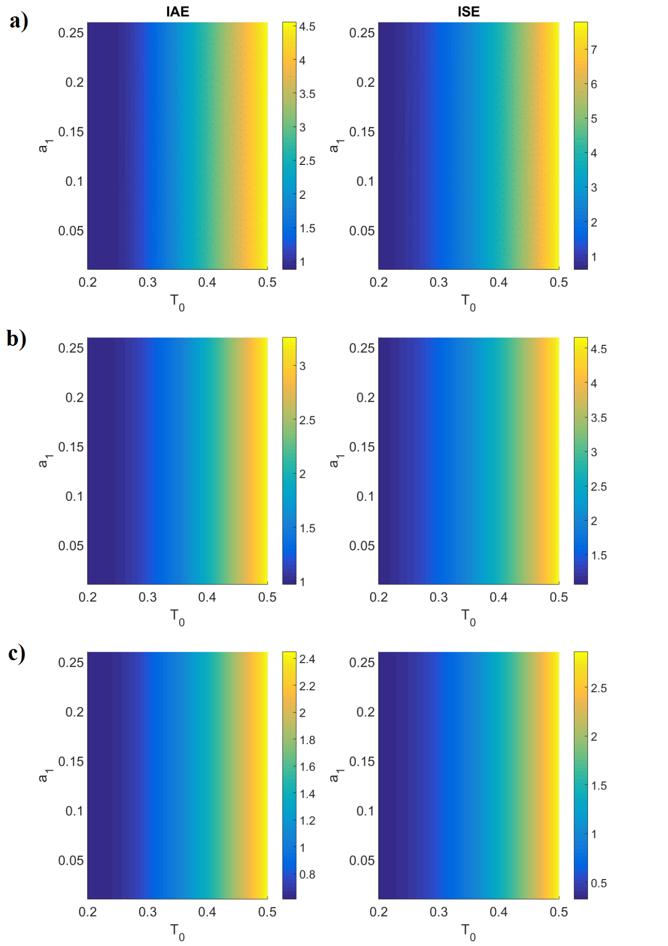


Fig. 11. Integral quality indices IAE and ISE for the change of T_0 i a_1 parameters of the motor-rotor model: a) MN3110, b) MN4014, c) U8PRO with CDM controller

The second test was performed for systems with AWC1 model and PID controller. To verify its parametrical robustness results have been compared to system without anti-windup compensation AWC0 and two integral quality indices were used, defined as (22)-(23):

$$\Delta J_1 = IAE_{AWC1}(T_{0i}, a_{1j}) - IAE_{AWC0}(T_{0i}, a_{1j}), \quad (22)$$

$$\Delta J_2 = ISE_{AWC1}(T_{0i}, a_{1j}) - ISE_{AWC0}(T_{0i}, a_{1j}), \quad (23)$$

where i and j are respectively values of T_0 and a_1 in range of theirs uncertainty (see Fig.12). Positive values of ΔJ_1 and ΔJ_2 correspond to degree of robustness improvement when AWC1 scheme is used. Recorded values shows similar results as it was before for CDM controller, high robustness for a_1 parameter unceraintly, however the robustness of system with AWC1 is increasing in comparison to system with AWC0 while the time-delay parameter T_0 in increasing. For smaller time-delays the differences are small – especially for U8PRO unit. From the recorded values of ΔJ_1 and ΔJ_2 it can be noted that for smaller dimensions of propellers its more important to used the anti-windup compensators – for bigger dimensions (bigger inertia) there is no meaningful improvement in the robustness.

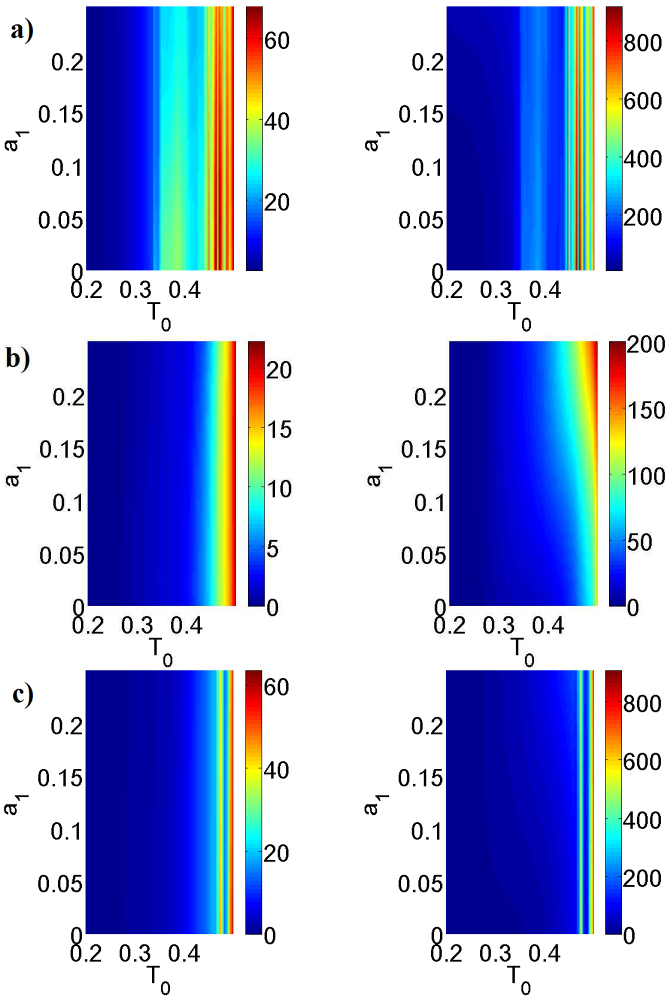


Fig. 12. Integral quality indices ΔJ_1 i ΔJ_2 for the change of T_0 i a_1 parameters of the motor-rotor model: a) MN3110, b) MN4014, c) U8PRO with PID controller (system with AWC1 related to AWC0)

Third test (analogoulsy to first one) verifies the robustness of the CDM controller on parameter b_0 (model gain) and time-delay T_0 uncertainty. Simillar results were achieved as earlier, the CDM controller is robust for b_0 parameter uncertainty (the controller tuned for nominal model tolerates gain uncertainty adjusting the output signal from the controller) – see Fig. 13.

Last test was performed for PID controller with anti-windup compenstator AWC1. To verify its parametrical robustness results have been compared to the system without anti-windup compensation AWC0 and two integral quality indices were used: ΔJ_3 and ΔJ_4, defined as (24)-(25):

$$\Delta J_3 = IAE_{AWC1}(b_{0i}, T_{0j}) - IAE_{AWC0}(b_{0i}, T_{0j}), \quad (24)$$

$$\Delta J_4 = ISE_{AWC1}(b_{0i}, T_{0j}) - ISE_{AWC0}(b_{0i}, T_{0j}), \quad (25)$$

where i and j are respectively values of b_0 and T_0 in range of theirs uncertainty (see Fig.14). From the achieved results it can be observed based on ΔJ_4 quality index that system with AWC1 definitely increases the effectiveness of robustness for time-delay parameter uncertainty in comparison to system without compensation AWC0. Analogously to CDM controller, the model gain uncertainty are effectively compensated by controller gains.

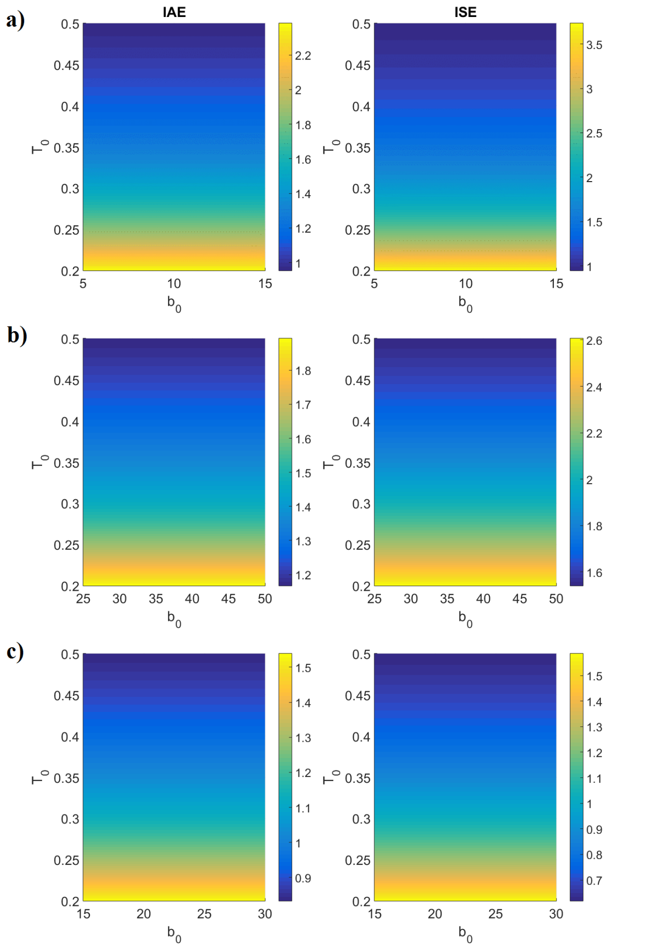


Fig. 13. Integral quality indices IAE i ISE for the change of b_0 i T_0 parameters of the motor-rotor model: a) MN3110, b) MN4014, c) U8PRO with CDM controller

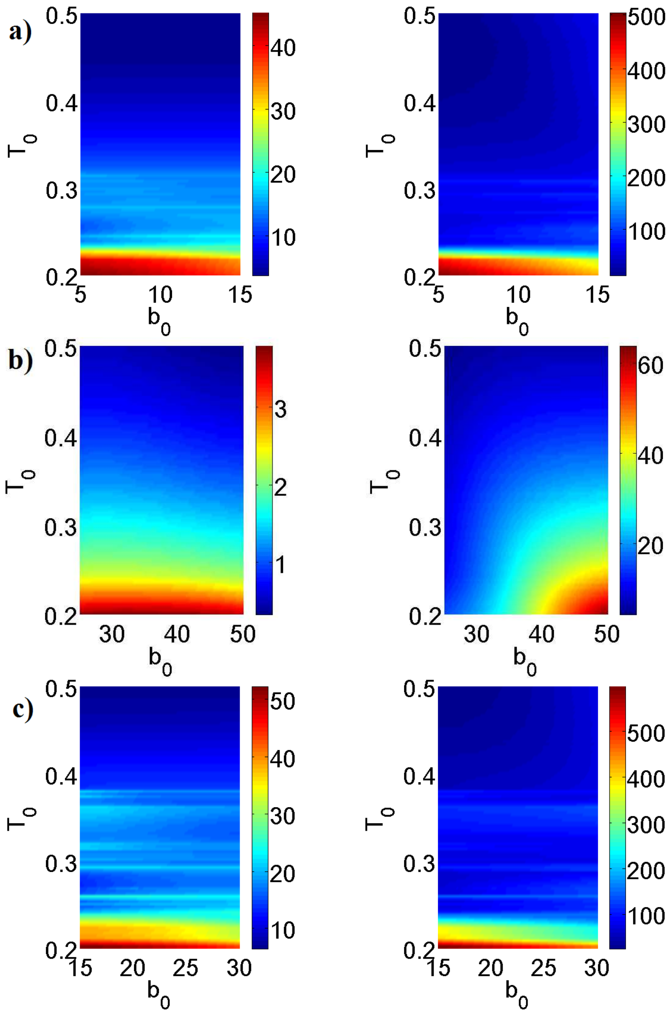


Fig. 14. Integral quality indices ΔJ_3 i ΔJ_4 for the change of b_0 i T_0 parameters of the motor-rotor model: a) MN3110, b) MN4014, c) U8PRO with PID controller (system with AWC1 related to AWC0)

5. CONCLUSIONS

The comparison of two contoller tuning methods based on pole placement were presented in this article. Both of them gives an ability to solve the Diophantine equation if the structure of model and controller is known. The main difference is the method of how the poles of the closed-loop transfer function are being calculated. It has an impact on damping the disturbances, output response characteristics and the parametric robustness. Those aspects were analysed and the CDM tuning method confirms that it can be used for controlling the thrust force in motor-rotor units, that has different dynamical characteristics and are applied in multirotor aerial robots.The biggest advantage of this metod is that it has fast output response after the change of the reference signal, but the disturbance damping (such as wind gusts) aspect needs to be imporved and this new aspect of further research.

On the other hand pole placement for PID controllers is an effective alternative – however it is neccessary to choose to correct type of AWC and it parameters. It helps to overcome one of the most known issues of the PID controllers and that

is the control signal protraction when it gets saturated. The effectiveness of control with AWC1 (that was carried) characterizes with good dynamics (lower that CDM), most effective disturbances damping and high parametric robustness. It is important in aspect of applying this method in aerial robots. In authors opinion the dynamics of steady-state time response after the change of reference signal can be improved because in this article the control was not optimized in accordance to predefinied aim function (e.g. minimization of the control errors). It is worth to consider the optimization of the controller gains or to implement the non-integer controller (such as fractional PID type controller). This gives possibility to achieve two additional tuning parameters (orders of integral and derivative part of the controller). Further research will be focused on this aspect.

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