Robust H_∞ Control for Sampled-data Dynamic Positioning Ships

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Abstract: This study is concerned with the robust H_∞ sampled-data control problem for dynamic positioning (DP) ships. Using the input delay approach, the ship dynamic positioning (DP) system is converted to a time-varying delay system. Sufficient conditions were then established to make the system exponentially stable with prescribed H_∞ performance using Lyapunov stability theorems. Then, the H_∞ sampled-data controller is obtained by means of linear matrix inequalities to guarantee that the DP ships can maintain the desired position, heading and velocities. Simulation result is shown that the proposed method and the designed controller for DP ships are effective in the existence of varying environment disturbance.

Keywords: dynamic positioning ships; H_∞ Control; sampled-data control; Lyapunov-Krasovskii functional; time delays

1. INTRODUCTION

The dynamic positioning (DP) is a system which is controlled by computer, can keep a ship's desired position and heading or sail an exact track by using of active thrusters and propellers(see T. I. Fossen, 2002), has been employed in various vessel types, such as offshore support vessels, Semi-submersible crane vessels, cruise ships, oceanographic research vessels and so on. Compared with the conventional anchor moored position-keeping methods such as jack-up barge and anchoring, the DP mode has its own advantages: High positioning accuracy and flexibility; Easy to be changed position; No anchor handling tugs needed; Not dependent on water depth and so on. With the expansion of ocean to the deeper water, the DP ships technology has attracted considerable attention and become an important research topic.

In the beginning, Linearization Proportional integral derivative (PID) controller was first used and still used today in DP system (see Fossen, 1994). Subsequently, optimal control theory and Kalman filtering technology were proposed (see Balchen et al. , 1976; A. J. Sørensen et al., 1996 and Xie, 2014). Recently, more advanced control algorithms have been proposed for DP ships. The estimators and nonlinear controllers were applied to deal with the vessel dynamics' intrinsic nonlinear characteristics (see Strand et al., 1999; Zhao et al., 2010; Wang et al., 2014). Vectorial backstepping techonology, which is used to guarantee the exponentially stable of DP ships system is presented in (Fossen et al., 1998) and (Snijders, 2005). In (Loria et al., 2000), separation principle combined with PD-type control law is used to prove the globally asymptotically stable of DP ships. In (Fossen et al.(1999)); (Lin et al., 2013); (Wang, 2012) and (Lindegaard, 2003), a passive nonlinear observer was designed to estimate the vessel velocity using Lyapunov methods.

In the last decades, sampled-data system has been an important topic, because modern control systems widely used the digital computers to control continuous-time systems. For example, in a DP ship (see Fig. 1), a variety of sensors, such as DGPS for obtaining higher accuracy and reliability position of ships; gyrocompasses for determining heading of ships; VRUs or MRUs or VRSs, for determining the ship's roll pitch and heave; wind sensors for anticipating wind gusts etc. are used by digital computers to determine the ship's motion state. The sensors produce continuous-time signals which are transformed into discrete-time control signals by being sampled and quantized with a microcontroller, and they will be transformed into continuous-time signals again by the zero-order holder. Until now, Considerable research methods have been proposed for analysing these sampled-data systems (see Fridman, 2006; Hu et al., 2006; Gao et al., 2010; Rubagotti et al., 2011; Kim, 2012; Wu, 2013; Chen et al., 2014; Lall et al., 2001 ; Shi, 1998; Wu, 2014; Abedi, 2015; Chen, 2015; Moarref, 2016; Liaquat, 2016; Zhang, 2016). Input delay approach (see Fridman et al., 2004, 2005; Wang, 2016; Liu, 2015; Krishnasamy, 2015; Chen, 2015; Wu, 2014; Suplin et al., 2007), is one of the main methods proposed by (Fridman et al., 2004). In the approach, the sampling period can be converted to a bounded time-varying delay. Moreover, the sampling distances are not required to be constant, which is its most significant advantage. Besides, the approach can be applied to the system with non-uniform uncertain sampling (see Gao et al., 2010), which is difficult for traditional lifting techniques to deal with.

It should be pointed out that, the control methods existing in literatures for DP ships are mainly focused on the design of continuous-time controller for continuous-time model. However, the DP ships system, commonly utilizes embedded computers for producing discrete-time control signal. So, the results obtained from the existing methods can't be applied directly for DP ships control systems. Hence, in view of the advantage of the input delay approach mentioned above, how to design sampled-data controllers directly for DP ships using the input delay approach is an important and meaningful question. To the best of our knowledge, literature cannot be found to consider the issue, which motivated the paper to handle the sampled-data control problem for DP ships, and it has significance both in practice and theory.

In this brief, the issue about H_∞ control for DP ships based on sampled-data is discussed. Using the input delay approach, the DP control system is converted to a system with time-varying delay. In terms of LMI approach, adequate conditions are established to make the system exponentially stable with prescribed H_∞ performance using Lyapunov stability theorems. Then, the H_∞ sampled-data controller is obtained to guarantee that the DP ships can maintain the desired position, heading and velocities under the external disturbances like waves, wind, ocean currents. Finally, a simulation of DP ships is provided to demonstrate that the proposed method is effective.

2. PROBLEM FORMULATION

The mathematical model of DP ships is described in [1] as follows:

$$\begin{aligned} M\dot{v} + Dv &= u + w \\ \dot{\eta} &= J(\psi)v \end{aligned} \tag{1}$$

Where

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix}$$

$$D = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & -Y_{\dot{r}} \\ 0 & -Y_{\dot{r}} & -N_{\dot{r}} \end{bmatrix}$$

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where $\eta = [x \quad y \quad \psi]^T$ represents the ship's position x, position y and heading ψ related to the earth-fixed frame. $\upsilon = [p \quad v \quad r]^T$ represents the ship's velocity of the body-fixed frame, where p represents surge velocity, v represents the sway velocity and r represents the yaw velocity (the body-fixed coordinate system is depicted in Fig.2). $J(\psi)$ represents the transformation matrix of the two frames mentioned above ; u represents the vector of control forces and moment; w represents a disturbance vector including waves, wind, ocean currents; M is the inertia matrix; D is a linear damping matrix.

As the heading ψ is very small, and it can be obtained that

$$J(\psi) \cong I_{3\times 3}. \tag{2}$$

Under the condition (2), the linearized low frequency motion of the model is got as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

Where

$$x(t) = [\eta \quad \upsilon]^T = [x \quad y \quad \psi \quad p \quad v \quad r]^T,$$

$$A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$

$$C = [I \quad 0]$$

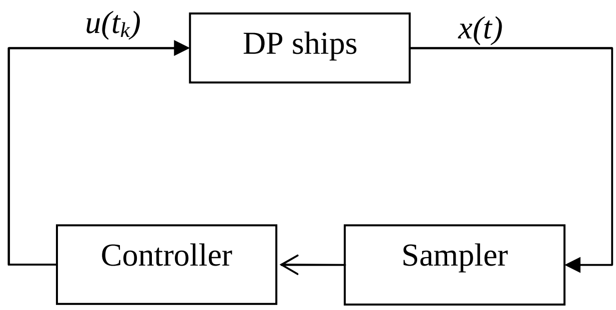

Fig. 1. DP ships control system

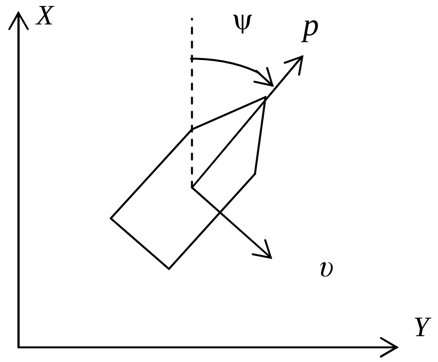

Fig. 2. Coordinate systems of DP ships

In this paper, the state variables of DP ships is assumed to be measured at the sampling instant $0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots$, that is, in the interval $t_k \le t \le t_{k+1}$, only $x(t_k)$ is available. The sampling period follow the assumption that it is bounded by a constant d, that is,

$$t_{k+1} - t_k \le d, \forall k \ge 0, d > 0, \tag{4}$$

Then, a state-feedback control law of the system (1) is considered in the form,

$$u(t)=u(t_k)=Kx(t_k),\quad t_k\le t<t_{k+1}, \tag{5}$$

Where t_k represents the sampling instant , $u(t_k)$ represents the discrete-time control signal , K is a state feedback gain matrix. By substituting (5) into (1), then

$$\begin{cases}\dot{x}(t)=Ax(t)+BKx(t_k)+Ew(t), & t>0\\ y(t)=Cx(t)\end{cases} \tag{6}$$

Remark 1: Note that Eq. (6) involves both discrete and continuous signals, which is more different and practical than the existing continuous-time control method for DP ships system. Besides, because the parameter uncertainties exist in the sampled-data control system, it is difficult for the traditional lifting technique to deal with the problem.

The paper's purpose is to find the state feedback gain K to satisfy the following requirements with the parameter uncertainties:

1) The closed-loop system (6) with $w(t)$=0 is exponential stable, if there exist scalars α>0 and β>0 such that

$$\| x(t)\|\le \beta\sqrt{e^{-\alpha t}}\ \| x_0\|_c, \qquad \forall t\ge 0$$

where $\| x_0\|_c = \sup_{-d\le\theta\le 0}\{\| x(\theta)\|,\| \dot{x}(\theta)\|\}$

2) To reject the varying environment disturbances like waves, ocean currents and wind, the closed-loop system is assumed to satisfy that $\| y(t)\|_2\le\gamma\| w(t)\|_2$ for all nonzero $w(t)\in L_2[0,\infty)$ under zero condition, where $\gamma>0$.

Then, using the input delay approach, the state-feedback controller $u(t)$ is rewritten as

$$\begin{aligned}&u(t)=u(t_k)=u(t-(t-t_k))=u(t-\tau(t)),\\ &t_k\le t<t_{k+1},\quad \tau(t)=t-t_k\end{aligned} \tag{7}$$

Where the time-varying delay $\tau(t)$ is piecewise-linear satisfying

$$\begin{aligned}&0\le\tau(t)\le d,\\ &\dot{\tau}(t)=1,\quad t\ne t_k\end{aligned} \tag{8}$$

Thus, the sampled-data system in Eq. (6) is converted to a system with time-varying delay as follow:

$$\begin{cases}\dot{x}(t)=Ax(t)+BKx(t-\tau(t))+Ew(t), & t>0\\ y(t)=Cx(t)\\ x(t)=\phi(t), & t\in[-d,0]\end{cases} \tag{9}$$

3. MAIN RESULTS

The robust H_∞ control problem for sampled-data DP ships is studied in this section. By using Lyapunov-Krasovskii functional method, sufficient condition is obtained to guarantee that the system is exponentially stable.

Theorem 1: Given scale d> 0, α> 0,γ>0, the closed-loop system (9) is exponentially stable with H_∞ performance γ, if there exist symmetric positive-definite matrices Z , P , Q , H_i, R_i, i=1,2,3,4 such that

$$\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}A^T & \sqrt{d}H_1 & \sqrt{d}R_1\\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}K^TB^T & \sqrt{d}H_2 & \sqrt{d}R_2\\ * & * & \Xi_{33} & -N_4^T & 0 & \sqrt{d}H_3 & \sqrt{d}R_3\\ * & * & * & -\gamma^2 I & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}E^T & \sqrt{d}H_4 & \sqrt{d}R_4\\ * & * & * & * & -Z^{-1} & 0 & 0\\ * & * & * & * & * & -Z & 0\\ * & * & * & * & * & * & -Z\end{bmatrix}<0, \tag{10}$$

Where

$$\Xi_{11}=\alpha P+PA+A^TP+e^{\alpha d}Q+H_1+H_1^T+C^TC$$
$$\Xi_{12}=PBK-H_1+H_2^T+R_1$$
$$\Xi_{13}=H_3^T-R_1$$
$$\Xi_{14}=PE+H_4^T$$
$$\Xi_{22}=-H_2-H_2^T+R_2+R_2^T$$
$$\Xi_{23}=-H_3^T-R_2+R_3^T$$
$$\Xi_{24}=-H_4^T+R_4^T$$
$$\Xi_{33}=-Q-R_3+R_3^T$$

Proof. First, a Lyapunov-Krasovskii functional for the system Eq.(9) is chosen as follows

$$V(t)=\sum_{i=1}^{3}V_i(t),\quad t\in[t_k,t_{k+1}) \tag{11}$$

$$V_1(t)=e^{\alpha t}x(t)^TPx(t)$$

$$V_2(t)=\int_{t-d}^{t}e^{\alpha(s+d)}x(s)^TQx(s)ds$$

$$V_3(t)=\int_{-d}^{0}\int_{t+\theta}^{t}e^{\alpha(s-\theta)}\dot{x}(s)^TZ\dot{x}(s)dsd\theta$$

Calculating the derivative of $V(t)$, it can be obtained that:

$$\dot{V}_1(t)=\alpha e^{\alpha t}x(t)^TPx(t)+2e^{\alpha t}x(t)^TP\dot{x}(t)$$
$$\dot{V}_2(t)=e^{\alpha(t+d)}x(t)^TQx(t)-e^{\alpha t}x(t-d)^TQx(t-d)$$

$$\dot{V}_3(t)=e^{\alpha t}\dot{x}(t)^T Z\dot{x}(t)\frac{e^{\alpha d}-1}{\alpha}-e^{\alpha t}\int_{t-d}^{t}\dot{x}(s)^T Z\dot{x}(s)ds \tag{12}$$

Using the Leibniz-Newton formula, for any matrices of proper dimension R_i, H_i, i=1,2,3 , the equations can be obtained as follow:

$$\begin{aligned}&2e^{\alpha t}[x^T(t)H_1+x^T(t-\tau(t))H_2+x^T(t-d)H_3]\\&\times[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds]=0\end{aligned} \tag{13}$$

$$\begin{aligned}&2e^{\alpha t}[x^T(t)R_1+x^T(t-\tau(t))R_2+x^T(t-d)R_3]\\&\times[x(t-\tau(t))-x(t-d)-\int_{t-d}^{t-\tau(t)}\dot{x}(s)ds]=0\end{aligned} \tag{14}$$

Similarly, it can be obtained that

$$\begin{aligned}e^{\alpha t}\int_{t-d}^{t}\dot{x}(s)Sx(s)ds=&e^{\alpha t}\int_{t-\tau(t)}^{t}\dot{x}(s)Sx(s)ds\\&+e^{\alpha t}\int_{t-d}^{t-\tau(t)}\dot{x}(s)Sx(s)ds\end{aligned} \tag{15}$$

Substituting (13)-(15) into (12) that

$$\begin{aligned}\dot{V}(t)=&\alpha e^{\alpha t}x(t)^T Px(t)+2e^{\alpha t}x(t)^T P\dot{x}(t)+e^{\alpha(t+d)}x(t)^T Qx(t)\\&-e^{\alpha t}x(t-d)^T Qx(t-d)+e^{\alpha t}\dot{x}(t)^T Z\dot{x}(t)\frac{e^{\alpha d}-1}{\alpha}\\&-e^{\alpha t}\int_{t-d}^{t}\dot{x}(s)^T Z\dot{x}(s)ds\\&\le\alpha e^{\alpha t}x(t)^T Px(t)+2e^{\alpha t}x(t)^T P\dot{x}(t)+e^{\alpha(t+d)}x(t)^T Qx(t)\\&-e^{\alpha t}x(t-d)^T Qx(t-d)+e^{\alpha t}\dot{x}(t)^T Z\dot{x}(t)\frac{e^{\alpha d}-1}{\alpha}\\&-e^{\alpha t}\int_{t-d}^{t}\dot{x}(s)^T Z\dot{x}(s)ds\\&+2[x^T(t)H_1+x^T(t-\tau(t))H_2+x^T(t-d)H_3]\\&\times[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds]\\&+2[x^T(t)R_1+x^T(t-\tau(t))R_2+x^T(t-d)R_3]\\&\times[x(t-\tau(t))-x(t-d)-\int_{t-d}^{t-\tau(t)}\dot{x}(s)ds]\\&\le e^{\alpha t}\zeta^T(t)(\Theta+\frac{e^{\alpha d}-1}{\alpha}[A\quad BK\quad 0]^T\cdot Z[A\quad BK\quad 0]\\&+dHZ^{-1}H^T+dRZ^{-1}R^T)\zeta(t)\\&-e^{\alpha t}\int_{t-\tau(t)}^{t}[\zeta^T(t)H+\dot{x}^T(s)Z]Z^{-1}[H^T\zeta(t)+Z\dot{x}(s)]ds\\&-e^{\alpha t}\int_{t-d}^{t-\tau(t)}[\zeta^T(t)R+\dot{x}^T(s)Z]Z^{-1}[R^T\zeta(t)+Z\dot{x}(s)]ds\end{aligned} \tag{16}$$

Where

$$\zeta(t)=[x^T(t)\quad x^T(t-\tau(t))\quad x^T(t-d)]^T \tag{17}$$

$$H=\begin{bmatrix}H_1\\H_2\\H_3\end{bmatrix},R=\begin{bmatrix}R_1\\R_2\\R_3\end{bmatrix},$$

$$\Theta=\begin{bmatrix}\Xi_{11}-C^TC & \Xi_{12} & H_3{}^T-R_1\\ * & \Xi_{22} & -H_3{}^T-R_2+R_3{}^T\\ * & * & -Q-R_3-R_3{}^T\end{bmatrix}$$

With

$$\begin{aligned}\Xi_{11}&=\alpha PA+PA+A^TP+e^{\alpha d}Q+H_1+H_1^T+C^TC\\\Xi_{12}&=PBK-H_1+H_2^T+R_1\\\Xi_{22}&=-H_2-H_2^T+R_2+R_2^T\end{aligned} \tag{18}$$

Since Z>0, then the last two parts in (16) are all less than 0. Therefore, if

$$\Theta+[A\quad BK\quad 0]^T Z[A\quad BK\quad 0]+dHZ^{-1}H^T+dRZ^{-1}R^T<0$$

which is equivalent to (10) by Schur complements, hence, $\dot{V}(x_t)<0$ can be obtained.

Thus, it follows that, for $t\in[t_k,t_{k+1})$

$$V(t)\le V(t_k)\le V(t_{k-1})\le\ldots\le V(0) \tag{19}$$

It can be calculated that

$$\begin{aligned}V(0)=&x(0)^T Px(0)+\int_{-d}^{0}e^{\alpha(s+d)}x(s)^T Qx(s)ds\\&+\int_{-d}^{0}\int_{\theta}^{0}e^{\alpha(s-\theta)}\dot{x}(s)^T Z\dot{x}(s)dsd\theta\\&\le x(0)^T Px(0)+\int_{-d}^{0}x(s)^T Qx(s)ds\\&+d\int_{-d}^{0}\int_{\theta}^{0}\dot{x}(s)^T Z\dot{x}(s)dsd\theta\\&\le\lambda_{\max}(P)\|x(0)\|^2+d\lambda_{\max}(Q)\sup_{-d\le\theta\le0}\|x(\theta)\|^2\\&+d^2\lambda_{\max}(Z)\sup_{-d\le\theta\le0}\|\dot{x}(\theta)\|^2\\&\le\eta\left(\sup_{-d\le\theta\le0}\{\|x(\theta)\|,\|\dot{x}(\theta)\|\}\right)^2\end{aligned} \tag{20}$$

Where

$$\eta=\lambda_{\max}(P)+d\lambda_{\max}(Q)+d^2\lambda_{\max}(Z)$$

On the other hand, there is

$$V(0) \geq e^{\alpha t} x(t)^T Px(t) \geq e^{\alpha t} \lambda_{\min}(P) \| x(t) \|^2 \qquad (21)$$

Using the (20) and (21), it can be got that

$$\begin{aligned} \| x(t) \|^2 &\leq e^{-\alpha t} \frac{1}{\lambda_{\min}(P)} V(0) \\ &\leq e^{-\alpha t} \frac{\xi}{\lambda_{\min}(P)} \left(\sup_{-d \leq \theta \leq 0} \{ \| x(\theta) \|, \| \dot{x}(\theta) \| \} \right)^2 \end{aligned} \qquad (22)$$

Then

$$\| x(t) \| \leq \sqrt{\frac{\xi}{\lambda_{\min}(P)} e^{-\alpha t}} \| x_0 \|_c \qquad (23)$$

Thus, according to Definition 1, the system (9) is exponential stability. This completes the proof.

Now, the H_∞ performance will be considered for system (9). By choosing the same Lyapunov-Krasovskii functional given in (11) and following similar proof, it can be got that

$$\begin{aligned} & y^T(t)y(t) - \gamma^2 w^T(t)w(t) + e^{-\alpha t}\dot{V}(t) \leq \\ & \begin{bmatrix} \zeta(t) \\ w(t) \end{bmatrix}^T \{ \Theta + \frac{e^{\alpha d}-1}{\alpha} [A \quad BK \quad 0 \quad E]^T Z \cdot \\ & [A \quad BK \quad 0 \quad E] + dHZ^{-1}H + dRZ^{-1}R \} \begin{bmatrix} \zeta(t) \\ w(t) \end{bmatrix} \end{aligned} \qquad (24)$$

Where

$$H = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix}, R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix},$$

$$\Theta = \begin{bmatrix} \Xi_{11} & \Xi_{12} & H_3^T - R_1 & PB + H_4^T \\ * & \Xi_{22} & -H_3^T - R_2 + R_3^T & -H_4^T + R_4^T \\ * & * & -Q - R_3 - R_3^T & -R_4^T \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

By Schur complement , inequalities (10) guarantee

$$\begin{aligned} & \Theta + \frac{e^{\alpha d}-1}{\alpha} [A \quad BK \quad 0 \quad E]^T Z \cdot \\ & [A \quad BK \quad 0 \quad E] + dHZ^{-1}H + dRZ^{-1}R < 0 \end{aligned}$$

So, it can be obtained from Eq. (16) that

$$y^T(t)y(t) - \gamma^2 w^T(t)w(t) + e^{-\alpha t}\dot{V}(t) < 0 \qquad (25)$$

Therefore, from Eq.(25), $\| y(t) \|_2 \leq \gamma \| w(t) \|_2$ can be got for all nonzero $w(t) \in L_2[0, \infty)$, then the H_∞ performance is established.

Remark 2: Theorem 1 gives a sufficient condition to solve the exponential stability problem for system Eq. (9). Besides, it should be pointed out that using the free-weighting matrices can reduce conservativeness of proposed delay-dependent results (see (Fridman et al., 2004b, 2005c) and(Suplin et al., 2007)). In addition, it is worth noting that in this work the value of decay rate α is free and can be chosen for different situations, which is more excellent and significant than the result existing in sampled-data systems.

Now, the H_∞ sampled-data controller (7) will be designed to guarantee the exponential stable of system (9) based on Theorem 2.

Theorem 2: Given scales $d > 0$, $\alpha > 0$, $\gamma > 0$, the system (9) is exponentially stable under the assumption that the sampling period is bounded by a constant d, that is, $t_{k+1} - t_k \leq d, \forall k \geq 0$,if there exist symmetric positive-definite matrices Z, P ,Q , H_i, R_i, i=1,2,3,4, such that the LMIs hold as follow :

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{P}A^T & \sqrt{d}\bar{H}_1 & \sqrt{d}\bar{R}_1 & \bar{P}C^T \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{K}^T B^T & \sqrt{d}\bar{H}_2 & \sqrt{d}\bar{R}_2 & 0 \\ * & * & \Xi_{33} & -\bar{N}_4^T & 0 & \sqrt{d}\bar{H}_3 & \sqrt{d}\bar{R}_3 & 0 \\ * & * & * & -\gamma^2 I & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}E^T & \sqrt{d}\bar{H}_4 & \sqrt{d}\bar{R}_4 & 0 \\ * & * & * & * & \bar{Z} - 2\bar{P} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{Z} & 0 & 0 \\ * & * & * & * & * & * & -\bar{Z} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \qquad (26)$$

Where

$$\begin{aligned} \bar{\Xi}_{11} &= \alpha\bar{P} + \bar{P}A + A^T\bar{P} + e^{\alpha d}\bar{Q} + \bar{H}_1 + \bar{H}_1^T \\ \bar{\Xi}_{12} &= B\bar{K} - \bar{H}_1 + \bar{H}_2^T + \bar{R}_1 \\ \bar{\Xi}_{13} &= \bar{H}_3^T - \bar{R}_1 \\ \bar{\Xi}_{14} &= E + \bar{H}_4^T \\ \bar{\Xi}_{22} &= -\bar{H}_2 - \bar{H}_2^T + \bar{R}_2 + \bar{R}_2^T \\ \bar{\Xi}_{23} &= -\bar{H}_3^T - \bar{R}_2 + \bar{R}_3^T \\ \bar{\Xi}_{24} &= -\bar{H}_4^T + \bar{R}_4^T \end{aligned}$$

$$\Xi_{33} = -\bar{Q} - \bar{R}_3 + \bar{R}_3^T$$

Moreover, a suitable controller with H_∞ performance γ in forms of (7) is designed to satisfy the proposed conditions . And the control gain matrix K is given by

$$K = \bar{K}\bar{P}^{-1} \tag{27}$$

Proof: By noticing that $-\bar{P}\bar{Z}^{-1}\bar{P} \le \bar{Z} - 2\bar{P}$

Let $\pounds = diag\left\{P^{-T}, P^{-T}, P^{-T}, I, I, P^{-T}, P^{-T}\right\}$. Denoting

$$\bar{P} = P^{-1},\ \bar{K} = KP^{-1},\ \bar{Q} = P^{-T}QP^{-1},\ \bar{H}_1 = P^{-T}H_1P^{-1},$$
$$\bar{H}_4 = H_4P^{-1},\ \bar{Z} = P^{-T}ZP^{-1}$$

and pre-multiplying and post-multiplying Eq. (10) by $\pounds$ and $\pounds^T$ respectively, Eq. (26) and the following inequality can be obtained:

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{P}A^T & \sqrt{d}\bar{H}_1 & \sqrt{d}\bar{R}_1 \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{K}^TB^T & \sqrt{d}\bar{H}_2 & \sqrt{d}\bar{R}_2 \\ * & * & \Xi_{33} & -\bar{R}_4^T & 0 & \sqrt{d}\bar{H}_3 & \sqrt{d}\bar{R}_3 \\ * & * & * & -\gamma^2 I & \sqrt{\frac{e^{\alpha d}-1}{\alpha}}\bar{P}E^T & \sqrt{d}\bar{H}_4 & \sqrt{d}\bar{R}_4 \\ * & * & * & * & -\bar{P}Z^{-1}\bar{P} & 0 & 0 \\ * & * & * & * & * & -\bar{Z} & 0 \\ * & * & * & * & * & * & -\bar{Z} \end{bmatrix} < 0 \tag{28}$$

Where

$$\tilde{\Xi}_{11} = \alpha\bar{P} + \bar{P}A + A^T\bar{P} + e^{\alpha d}\bar{Q} + \bar{H}_1 + \bar{H}_1^T + \bar{P}C^TC\bar{P}$$

By Schur complement, inequality (28) is equivalent to inequality (26). The proof is completed.

Remark 3: According to the Theorem 2, sufficient conditions are provided to deal with the robust H_∞ control problem for DP ships based on sampled-data, and the proper sampled-data controller is proposed. It should be pointed that the conditions are formulated by means of LMI, which is easy to be determined by standard numerical software. The tool provided by LMI technique is effective for designing the sampled-data controller of DP ships. Moreover, the proposed methods of the work can be extended to other ships easily.

4. NUMERICAL EXAMPLES

For validating the effectiveness of the proposed H_∞ sampled-data controller for DP system, a numerical simulation is carried out for a DP ship with environment disturbance in the section. The DP ship's main parameters are referenced to the Ship Handling Simulator designed by the Institute of Navigation Jimei University. The length and beam of vessel are respectively 170m and 25.8m, tonnage is 2.4×10^7 kg, draft is 9.5m. The M and D in model (1) are given by:

$$M = \begin{bmatrix} 2.6415\times10^7 & 0 & 0 \\ 0 & 3.345\times10^7 & 1.492\times10^7 \\ 0 & 1.492\times10^7 & 6.52\times10^9 \end{bmatrix}$$

$$D = \begin{bmatrix} 2.22\times10^4 & 0 & 0 \\ 0 & 2.22\times10^5 & -1.774\times10^6 \\ 0.2 & -1.774\times10^6 & 7.151\times10^8 \end{bmatrix}$$

Using the parameters given above and formula (3), the system matrices for DP model can be obtained. The extern disturbances like waves, wind, ocean currents are considered in the simulation. The direction and significant height of wave are respectively 150° and 5.5 m. The direction and speed of current are respectively 100° and 0.25 m/s. The direction and speed of wind are respectively 225° and 10 m/s. The ship's initial position and heading is η_i = [0m 0m 2°], and body-fixed velocities v_i = [5m/s 5m/s 0.2°/s]. The final desired state is η_f = [10 10 0°], v_f = [0m/s 0m/s 0°/s].Assuming the sampling interval d=1.0s, the minimum value achieved by Theorem 2 to guarantee H_∞ performance γ_{min}=1.630. Then, the value of feedback gain matrix is obtained that

$$K = \begin{bmatrix} -0.0332 & 0 & 0 & -2.1871 & 0 & 0 \\ 0 & 0.8548 & -0.178 & 0 & 1.5367 & -1.308 \\ 0 & -0.178 & 0.652 & 0 & 0.0303 & -0.2838 \end{bmatrix}$$

The simulation results are shown in Fig. 3-8. Fig. 3 and Fig. 4 show the ship's x direction and y direction, Fig. 5 shows the ship's heading ψ, which are respectively response of the proposed controller. This indicates that the proposed sampled-data controller can stabilize the ship and keep the ship at the desired target position and heading. Similarly, Fig. 6, 7 and 8 show velocity response [u, v, r] of the controller, which indicates that the sampled-data controller can also stabilize the ship's velocities. The above results illustrate that our proposed sampled-data controller for the DP ships is effective and robust.

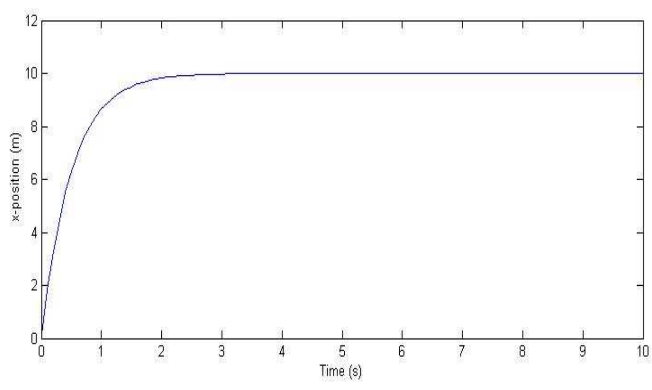

Fig. 3. Position of the x direction of the ship

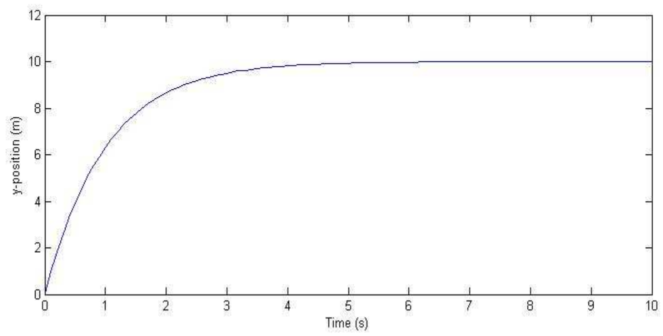

Fig. 4. Position of the y direction of the ship

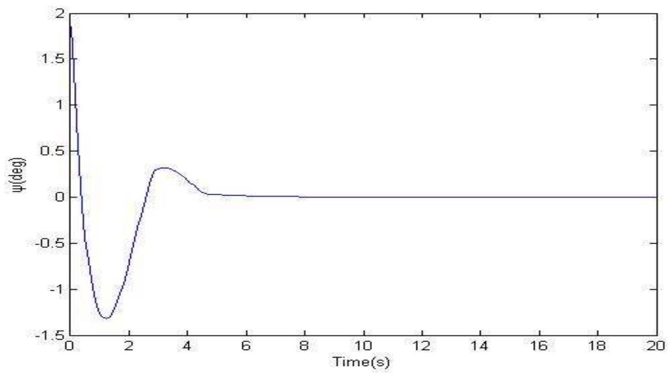

Fig. 5. Heading ψ of the ship

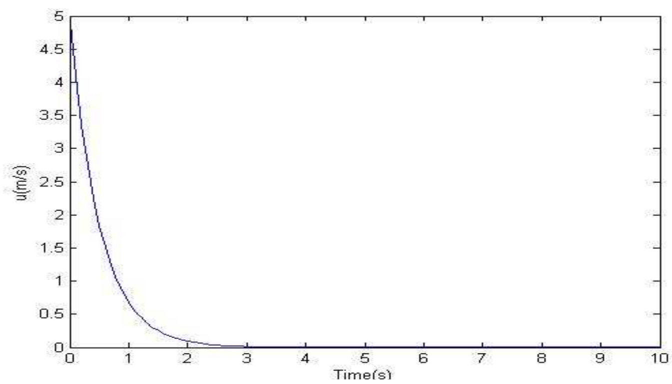

Fig. 6. Surge velocity u of the ship

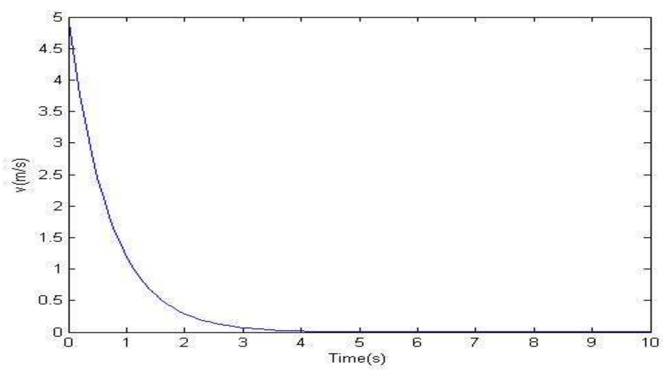

Fig. 7. Sway velocity v of the ship

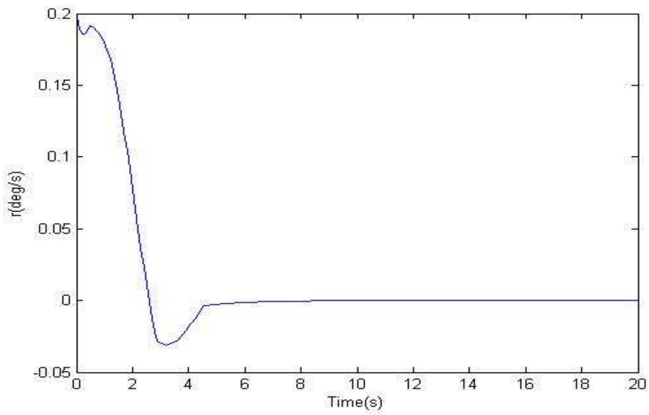

Fig. 8. Yaw rate r of the ship

5. CONCLUSIONS

In the paper, the robust H_∞ control technique for sampled-data DP ships is proposed. Using the input delay approach, the ship DP system is converted to a system with time-varying delay. By using the Lyapunov approach, sufficient condition is derived to guarantee that the DP ships system is exponentially stable. It has been proven from the a DP ship simulation that the designed controller can make the position, heading and velocities of DP ship achieve the desired target value under the extern environment disturbance. The proposed methods can be extended to other ships.

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